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# Homework 1.
## Probability.

① $p(\text{male}) = p(\text{female}) = 0.5$

$P(3M \text{ and } 3F) = C_6^3 \, p^3 q^{6-3} = C_6^3 \cdot 0.5^3 \cdot 0.5^3 = 20 \cdot 0.5^6 = 0.3125$

answer: 0.3125

② $P(0/4 \text{ ace in one hand}) = \dfrac{C_4^0 \, C_{48}^{13}}{C_{52}^{13}} \doteq 0.3038 = P_1$

$P(0/4 \text{ aces in three hand}) = P_1^3 = 0.028$

③ $p = 0.1$, $1-0.1$

$1-(0.9)^n \geqslant 0.9$ – prob at least one

$-(0.9)^n \geqslant -0.1 \Longleftrightarrow (0.9)^n \leqslant 0.1$

$n \lg 0.9 \leqslant \lg 0.1 \quad | : \lg 0.9 \ (<0)$

$n \geqslant \dfrac{\lg 0.1}{\lg 0.9} \Longleftrightarrow n \geqslant 21.85$

$n \in \mathbb{Z} \Rightarrow n \geqslant 22$

answer: 22

④ $1-(1-P(4/4))^n \geqslant 0.5$

$(1-P(4/4))^n \leqslant 0.5$

$n \cdot \lg(1-P(4/4)) \leqslant \lg 0.5$

$n \geqslant \dfrac{\lg 0.5}{\lg(1-P(4/4))}$, where $P(4/4) = 0.00264$

$n \geqslant 262.1$, but $n \in \mathbb{Z} \Rightarrow n \geqslant 263$

answer: 263.

⑤ $P(\text{hitting}) = 0.2$

$$\underbrace{\underbrace{1 - C_{10}^0 \cdot (0.8)^{10} \cdot 0.2^0}_{\text{at least one}} - C_{10}^1 \cdot 0.2^1 \, 0.8^9}_{\text{at least two}} \ominus$$

$\ominus \ 1 - 0.8^{10} - 0.2 \cdot 10 \cdot 0.8^9$

⑥ 5 problem assuming at least one

$$\frac{1-0.8^{10} - 2 \cdot 0.8^9}{1-0.8^{10}}$$

⑦ $P(\text{two red}) = \dfrac{C_{26}^2 \cdot C_{26}^{11}}{C_{52}^{13}}$

According to Bernoulli: $(p = 0.5)$

$P(\text{two red}) = C_{13}^2 \cdot 0.5^2 \cdot 0.5^{11}$

⑧ $P = \dfrac{C_{12}^2 (2^6 - 1)}{12^6}$

$C_{12}^2 (2^6$

⑨ (a) $p(ace) = \frac{1}{6}$

at least one: $1 - \left(\frac{5}{6}\right)^6 = 0.665!$

(b) equal one: $C_6^1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^5$

(c) exactly two: $C_6^2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^4$

Poisso distribution

$P_m \approx \frac{\lambda^m}{m!} e^{-\lambda}$, where $\lambda = np$

for big $n$ and small $p$

(a) $n = 6$, $p = \frac{1}{6} \Rightarrow \lambda = \frac{1}{6} \cdot 6 = 1$
$m = 1$

$P_1 = 1 - \frac{1^1}{1!} \cdot e^{-1} \approx 0.6321$ for at least one

(b) $P_1 = \frac{1^1}{1!} e^{-1} \approx 0.3679$

(c) $P_2 = \frac{1^2}{2!} e^{-1} \approx 0.1839$

⑩ $p(\text{left-handers}) = 0.01$, $n = 200$.

$\lambda = np = 0.01 \cdot 200 = 2$.

more 4: 1 - less 4.

$1 - \frac{\lambda^0}{0!} \exp(-\lambda) - \frac{\lambda^1}{1!} \exp(-\lambda) - \frac{\lambda^2}{2!} \exp(-\lambda) - \frac{\lambda^3}{3!} \exp(-\lambda)$

$= 0.142877$.

⑪ $N_{papers} = 500$ pages

$N_{misprints} = 500$ misprints

$p = \frac{N_{mis}}{N_{symbols}} = \frac{500}{500b}$, where $b$ - symbol per page

$\lambda = n \cdot p$, $m > 3$, $n = b$

$\lambda = b \cdot \frac{500}{500b} = 1$.

$1 - \frac{\lambda^0}{0!} \exp(-\lambda) - \frac{\lambda^1}{1!} \exp(-\lambda) - \frac{\lambda^2}{2!} \exp(-\lambda) =$

$= 0.803$.

⑫ $p(\text{color blindness}) = 0.01$.

$\lambda = n \cdot p = 0.01n$

$P_1 = \frac{(n \cdot 0.01)^1}{1!} \cdot \exp(0.01n)$

$n - \frac{(n \cdot 0.01)^1}{1!} \exp(0.01) \geq 0.95n$

(12 cont.) $n \exp(-0.01n) \le 0.05n$

$-0.01n \le \ln 0.05$

$n \geq \frac{\ln 0.05}{-0.01}$

$n \geq 300.$

(13) (a) $n = 100, p = 0.01, \lambda = n \cdot p = 0.01 \cdot 100 = 1.$

$P_0 = \frac{1^0}{0!} \exp(-1) = 0.36788$

(b) $1 - \frac{1^0}{0!} e^{-1} - \frac{1^1}{1!} e^{-1} = 1 - 2e^{-1} = 0.264$

(14) $p = 0.01; n, m = 0, \lambda = n \cdot p = 0.01n$

$n = 100x; \lambda = 1x = x$

$P_0 = \frac{\lambda^{0}}{0!} \exp(-\lambda) = e^{-x}$ $(\lambda^0 = 1)$

$e^{-x} \le 0.01$

$-x \ln e \le \ln 0.01$

$x \geq -\ln 0.01$

$n \geq 4.6, n \in \mathbb{Z} \Rightarrow n \geq 5$

(15) $P_0 = \frac{\lambda^0}{0!} \exp(-\lambda) \le \frac{1}{e}$

$e^{-\lambda} \le \frac{1}{3}$ $\left(\frac{1}{e} \approx \frac{1}{3}\right)$

$3^{-\lambda} \le \frac{1}{3}$

$\lambda \geq 1$

$np \geq 1$ $\quad p = \frac{1}{649740}$

$n \geq \frac{1}{p}$

$n \geq 649740.$

(16) $\lambda = n \cdot p, n, k$

at least one page will contain more $k$ misprints

$1 - \frac{\lambda^0}{0!} \exp(-\lambda) \ldots \frac{\lambda^k}{k!} \exp(-\lambda)$

$1 - \left(\sum_{m=0}^{k} \frac{\lambda^m}{m!} \exp(-\lambda)\right)^n$