Random Experiment → An exp. whose outcomes are unpredictable.

A single performance of random exp is trial

- → Rolling a die
- → Tossing coin
- → choose one from a deck.
- → Present or Absent
- → Appearing for exam

Sample space: (of a rand exp)

Set of all outcomes of a random experiment

$S = \{H, T\}$

Event :

A collection of certain outcomes from the sample space. An event is a subset of sample space.

Eg:

Tossing 2 dice $S = \{(1,1)\ (1,2)\ \dots\ (1,6)$
$(2,1)\ (2,2)\ \dots\ (2,6)$
$\vdots$
$(6,1)\ (6,2)\ \dots\ (6,6)\}$

A = event of getting the sum equal to 6

$A = \{(1,5)\ (2,4)\ (3,3)\ (4,2)\ (5,1)\}$

Any statement of conditions that defines this subset is an event

An event is a statement [proposition] whose value (i.e) truth value is determined

Sure event: [If the sample space is a subset]

An event whose occurence is inevitable.

Any random exp. sample space is a sure event

Rolling dice, Let A => event of even   A = {2, 4, 6}
B => event of odd   B = {1, 3, 5}
C => Event of no ≥ 2 C = {2, 3, 4, 5, 6}

NULL event : No event
⇓
Impossible event / Empty event.
=> Null event is an impossible event.

* An event containing single outcome => elementary / Atomic event

Mutually exclusive events (me)

Two events A & B the mutually exclusive (me)

if $A \cap B = \phi$.

$\bigcap_{i=1}^{n} A_i = \phi$

Eg: Rolling dice : A & B => so no intersection.
=> If A occurs than B should not occur
one occur then other should not occur,

From the above example of rolling dice
=> A & B are mutually exclusive
=> A & C are not
=> B & C are not

A list of event $A_1, \ldots A_n$ is me, if $A_i \cap A_j = \phi$
$A_1 = \{1, 2\}$
$A_2 = \{3, 4\}$
$A_3 = \{5, 6\}$

No sample point is included in more than one event in the list.

Collective exhaustive events.

A list of events $A_1 \dots A_n$ is collectively exhaustive if $A_1 \cup A_2 \cup \dots \cup A_n = S$

$$\bigcup_{i=1}^{n} A_i = S$$

A collection of mutually exclusive, non empty, collectively exhaustive events forms a partition of a sample space

PROBABILITY:

It is a measure of chance or livelihood of an event to happen.

3 approaches for probability.

→ classical Approach
→ Relative frequency approach
→ Axiomatic approach

Classical Approach:

If an event E happens in 'h' no. of favourable ways out of a total no. of 'n' possible ways, all of which are equally likely, then the probability of E is given by,

$$P(E) = \frac{h}{n}$$

Limitations:

Equally likely is not realistic

Unbias in real world is not possible all the timing.

Not approachable when n is ∞

Theoretical Approach

It is actually an apriori approach : without actually performing expt, you can find probability

Relative frequency approach: [Empirical Approach]

(past data)

If an expt is performed 'n' times (n is large) and an event 'E' happens 'h' no. of times, then the probability of E is

$$\boxed{P(E) = \lim_{n \to \infty} \frac{h}{n}}$$

$h \to$ relative frequency

Limitation:

-> Repetition of expt many times may not be feasible (costly) / destructive expts)

-> Approximate value for probability.

-> 'large' is vague

-> The main subject of probability theory is to develop tools to find probabilities for different events.

AXIDAMATIC APPROACH

Let 'S' be sample space of a random expt. To each event $A \subseteq S$, we assign a real number $P(A)$

$P[\ ]$ is called Probability function if the following axioms are satisfied.

$A1$: $P(A) \geq 0$ for any $A \leq S$

$A2$: $P(S) = 1$

$A3$: For any list of mutually exclusive events $A_1, A_2, \ldots$

$$P\left[\bigcup_i A_i\right] = P(A_1) + P(A_2) + \cdots\cdots$$

A3: For m.e events $P[\cup A_i] = \sum_i P(A_i)$

Domain of the function P : Event space $\rightarrow R$

$P(A)$ is ~~the~~ called the probability of A

Result :

Result 1 : $P(\varphi) = 0$

Proof

→ This is true for all the ~~s~~tatements like something abl union with empty set means the union is that same set

$\boxed{S \cup \varphi = S}$ where S, φ are mutually exclusive

$$P(S \cup \varphi) = P(S)$$

By Axiom 3:

$$P(S) + P(\varphi) = P(S)$$

$$P(\varphi) = 0$$

Result 2 : $P(\bar{A}) = 1 - P(A)$

* For any set lets take A and its complement $\bar{A}$ are mutually exclusive $(A \cap \bar{A}) = \phi$

Proof

$A \cup \bar{A} = S$, where $A, \bar{A}$ are mutually exclusive

$P[A \cup \bar{A}] = P(S)$

By A_2. $P(S) = 1$

$P[A \cup \bar{A}] = 1$

A_3

$P(A) + P(\bar{A})$

~~P[A~~

$P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$

Result 3 : If $\boxed{A \subseteq B}$, then (i) $P(A) \leq P(B)$

(ii) $P(B - A) = P(B) - P(A)$

whenever A occurs B also occurs

occurrence of A ; then occurrence of B

occurrence of B , then not the occurrence of A

Eg: $A = \{3, 6\}$ $B = \{3, 4, 5, 6\}$

$A \subseteq B$

O/c is 3 => A is occured

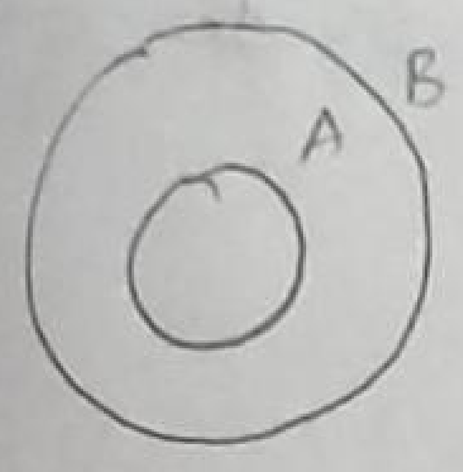
B is also occured

O/c is 4 => B is occured

A did not occured

Proof.

(ii)



Since we need to proceed with mutually exclusive event

$B = A \cup (B - A)$ where A, $B-A$ are mutually exclusive

$P(B) = P[A \cup (B-A)]$

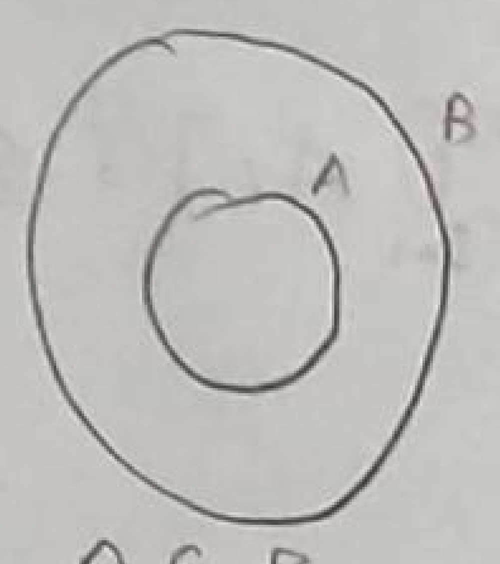
$P(B) = P(A) + P(B-A)$ By A3

$P(B-A) = P(B) - P(A)$

Body of big event, small event, big event, small event

(i) $P(A) \le P(B)$ using A1 for $B-A$

Result 4: $0 \le P(A) \le 1$



$A \subseteq B$

$P(A) \geq 0$ by A $\longrightarrow$ ①

we know $A \subseteq S$

So, B will be the sample space $A \subseteq S$

then by result 3.

$P(A) \le P(S) = 1$

$P(A) \le 1 \longrightarrow$ ②

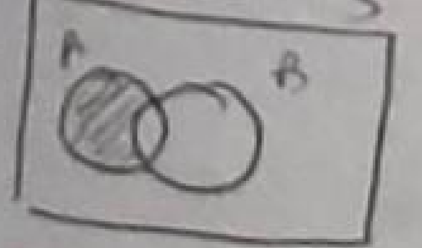
①, ② $\rightarrow 0 \le P(A) \le 1$

Result 5: Addition rule

For any 2 events A, B

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

For mutually exclusive events the intersection is 0



Proof

$A \cup B = (A - B) \cup B$ where $A-B$, B are mutually exclusive.

We need to find

$A - B = A - (A \cap B)$

$\because A \cap B \subseteq A$

$P(A-B) = P[A - (A \cap B)]$

$= P(A) - P(A \cap B)$

$P(A \cup B) = P[(A-B) \cup B]$

using Result 3(ii) $= P(A-B) + P(B)$

$= P(A) - P(A \cap B) + P(B)$

$= P(A) + P(B) - P(A \cap B)$

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Another method

$\#(A \wedge B) = \#(A) + \#(B) - \#(A \cap B)$

\# is total number of elements in the set

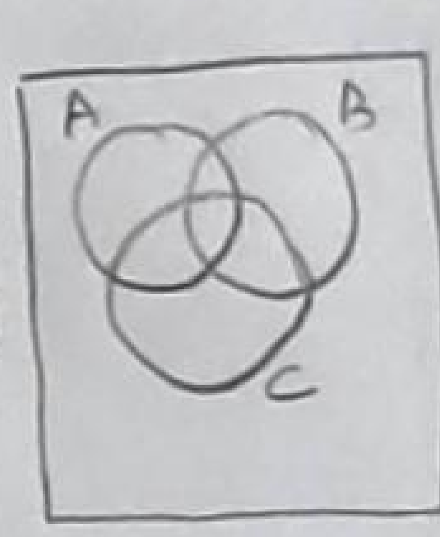
- by

$\#(S) \Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

<u>Note</u>

For 3 events A, B, C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$



In general for n events $A_1, \ldots A_n$ $\quad A_1, A_2, A_3$

$$\boxed{P\left[\bigcup_{i=1}^{n} A_i\right] = \sum P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - + \cdots \pm \cdot (-1)^{n+1} P\left[\bigcap_{i=1}^{n} A_i\right]}$$

Conditional probability:

Let S be the sample space

Let A, B be 2 event

Assume that WKT the O/C is in B

Given this info, wt is prob that the O/C is in A?

This prob is denoted by $P(A|B)$ [Prob of A given B]

<u>Example</u>

toss 2 dice

$S = \{(1,1) \cdots (1,6)$

$\vdots$

$(6,1) \cdots (6,6)\}$

without this the prob will be $\frac{5}{36}$

$\boxed{\text{Suppose the } \cancel{\text{sum}} \text{ o/c of first die is } 3}$

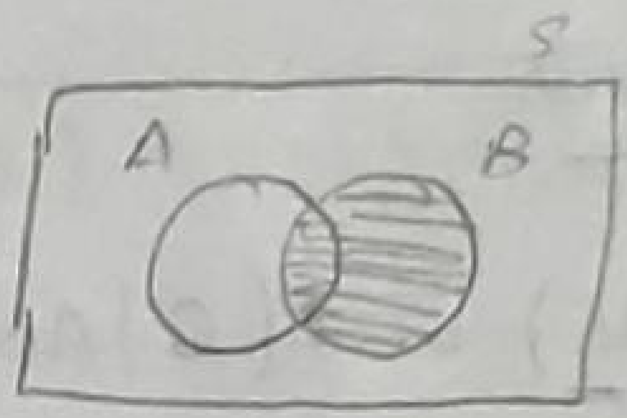
given this info, wt is the prob that the sum of the 2 die is 8

If the first die is 3 $\Rightarrow$ condition

$B = \{ (3,1)\ (3,2)\ (3,3)\ (3,4)\ \boxed{(3,5)}\ (3,6) \}$

$= 1/6.$

$P(A) = \dfrac{\#(A)}{\#(S)}$



$P(A/B) = \dfrac{\# \cancel{P}(A \cap B)}{\#(B)}$

$\div$ by $\#S$ in Nr & Dr

$= \dfrac{P(A \cap B)}{P(B)}$

Result 6:

Let S be the sample space.

A, B events with $P(B) > 0$

then, (i) $P(A/B) \geq 0$

(ii) $P(S/B) = 1$

(iii) For m.e events $A_1, A_2 \ldots\ldots$

$P\left[\bigcup_i A_i / B\right] = \sum_i P(A_i / B)$

$P[(A_1 \cup A_2 \cup \ldots)/B] = P(A_1/B) + P(A_2/B) + \ldots$

Multiplication rule :

$P(A|B) = \frac{P(A\cap B)}{P(B)}$

$P(A\cap B) = P(A|B) \cdot P(B)$, $P(B) \neq 0$

$P(B|A) = \frac{P(A\cap B)}{P(A)}$

or

$P(A\cap B) = P(B/A)\, P(A)$, $P(A) \neq 0$

$P(AA) = P(A\cap A)$

For 3 events

$P(A\cap B\cap C) = P(C|AB)\; P(AB)$

$= P(C|AB)\; P(A|B) \cdot P(B)$

$P(AA)\; P(C|AB)$

$P(B)\, P(A|A)\, P(C|AB)$

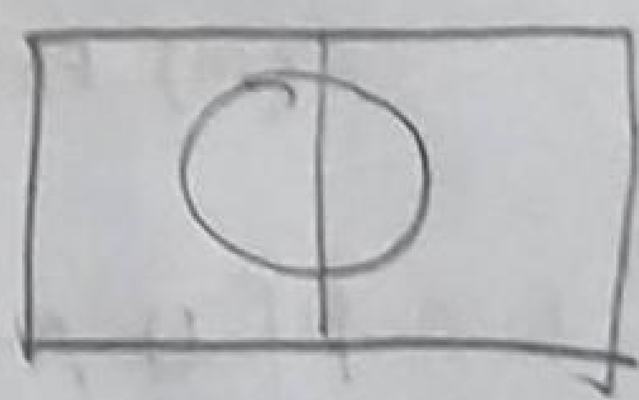
For n events

$P(A_1 A_2 \ldots A_n) = P(A_1) * P(A_2|A_1) * P(A_3|A_1A_2) * \ldots * \cancel{P(A_n)}$
$P(A_n|A_1 A_2 \ldots A_{n-1})$

Total probability and bayer's rule

Some times it may not 1 possible to find P(A) directly . it is possible to find

$P(A|B)$ & $P(A|\bar{B})$



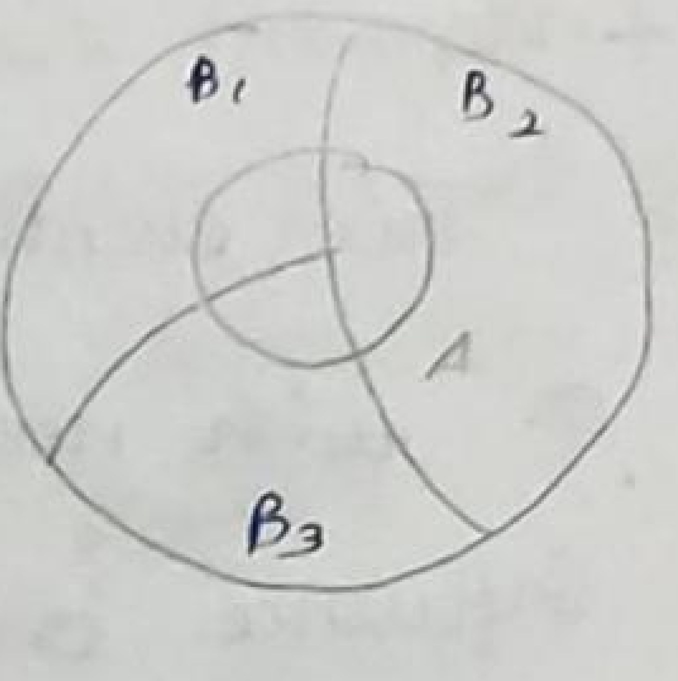
$A = (A\cap B) \cup (A\cap \bar{B})$ M.e

$P(A) = P(A\cap B) + P(A\cap \bar{B})$

$\Rightarrow P(A) = P(A|B)\,P(B) + P(A|\bar{B})\; P(\bar{B})$

This is called Total prob A .

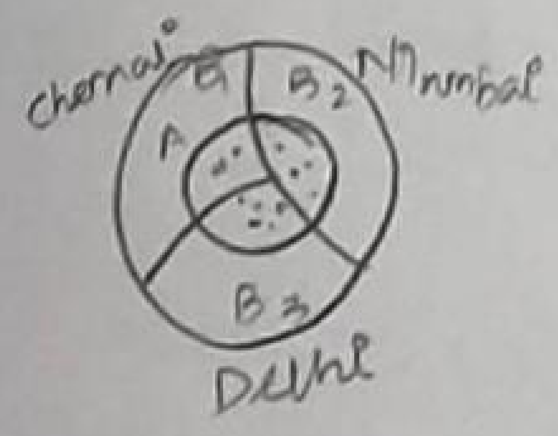
Total probablity



let $B_1 \ldots B_n$ be a list of m.e and c.e events in a sample space S with $P(B_i) \neq 0 \;\forall i$

the for any event A,

$$\boxed{P(A) = P(A|B_1)P(B_1) + \cdots + P(A|B_n)P(B_n)}$$



1) What is the prob that a randomly selected phone is defective? $p(A)$

2) Defective phone from mumbai plant

Bayes's rule

$B_1, \ldots B_n$ is a . of me & c.e events

$A \to$ any event

thus $P(B_k|A) = ?$

$$P(B_k|A) = \frac{P(A \cap B_k)}{P(A)}$$

$$= \frac{P(B_k)\,P(A|B_k)}{P(B_1)\,P(A|B_1) + \cdots + P(B_n)\,P(A|B_n)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Independent events :

Two events A & B are independent of the occurence or with non-occurence of one event has no influence on the occurence or non-occurence of the other event

$$P(A|B) = P(A)$$

$$P(A \cap B) = P(B)\, P(A)$$

$$\boxed{P(A \cap B) = P(A)\, P(B)}$$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(A)\, P(B|A)$

$P(A \cap B) = P(B)\, P(A|B)$

<u>Note</u>

For event,

$$\left.\begin{aligned} P(AB) &= P(A)\, P(B) \\ P(BC) &= P(B)\, P(C) \\ P(AC) &= P(A)\, P(C) \end{aligned}\right\}$$ ABC are pair wise independent

If $P(ABC) = P(A)\, P(B)\, P(C)$ then A, B, C are ind

Mutually exclusive v/s Independent

A, B are m.e $\Rightarrow$ $A \cap B = \varphi \Rightarrow P(A \cap B) = 0$

A, B are independent $\rightarrow$ $P(A \cap B) = P(A)\, P(B)$

mutually exclusive events cannot happen simultaneously.

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An elevator with 2 passengers stops at the 2^{nd}, 3^{rd}, 4^{th} floor. If it is equally likely that a passenger get off at any of the three floors, wt is the probability that the passengers get off at different floors?

$a_i\, b_j$ - Passenger 1 gets at i^{th} floor
Passenger 2 gets at j^{th} floor

If 2 dice are thrown, what is the probability that the sum is

(i) 8 (ii) neither 7 nor 11

$$S = \{ (1,1) \cdots (1,6)$$
$$\vdots \qquad \vdots$$
$$(6,1) \cdots (6,6) \}$$

(i) sum is 8 $= \frac{5}{36}$

(ii) P (sum is neither 7 nor 11)

Where A → sum is 7
B → sum is 11

$= P(\bar{A} \cancel{and} \cap \bar{B})$

$= P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B)\}$

$= 1 - \frac{6}{36} - \frac{2}{36} = 28/36$

3) A card is drawn from a well-shuffled pack of 52 cards. Find the prob that it is either a spade or an ace.

$\# (S) = 52$

$P(\text{spade or ace}) = P(A) + P(B) - P(AB)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

4) The odds against A solving a prob are 4 to 3

The odds in favour of B solving the same prob are 7 to 5

What is the prob that the problem is solved if they both independently?

$P(\bar{A}) = \frac{4}{7} \Rightarrow P(A) = \frac{3}{7}$

$P(B) = \frac{7}{12}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$

(or)

$P(A \cup B) = 1 - P(\overline{A \cup B})$

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A})P(\bar{B})$

Result

If A & B are independent, then

(i) $\bar{A}$ and $\bar{B}$ are also independent

(ii) A & $\bar{B}$ are independent

(iii) $\bar{A}$ & B are independent

Proof

(i). $A, B \Rightarrow$ Independent

$P(A \cap B) = P(A) P(B)$

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$$
$$= 1 - P(A \cup B)$$
$$= 1 - \{P(A) + P(B) - P(A \cap B)\}$$
$$= 1 - P(A) - P(B) + P(A) P(B)$$
$$= [1 - P(A)] - P(B) . \{1 - P(A)\}$$
$$= [1 - P(A)] \cdot [1 - P(B)]$$
$$= P(\bar{A}) P(\bar{B})$$

Hence proved.

(ii) $A, B \Rightarrow$ independent.

$P(A \cap B) = P(A) P(B)$

$$P(A \cap \bar{B}) = P[A - (A \cap B)]$$
$$= P(A) - P(A \cap B)$$
$$= P(A) - P(A) P(B)$$
$$= P(A) [1 - P(B)]$$
$$= P(A) P(\bar{B})$$

5) Three horses A, B, C are in a race

A is twice as likely to win as B

B is twice . C

What are their respective chance of wining.

Since they are mutually exclusive.

becos, only one horse can win a race

$P(A) = 2P(B)$

$P(B) = 2P(C)$

$P(A \cup B \cup C) = 1$. where A, B, C are m.e

$P(A) + P(B) + P(C) = 1$

$4P(C) + 2P(C) + P(C) = 1$

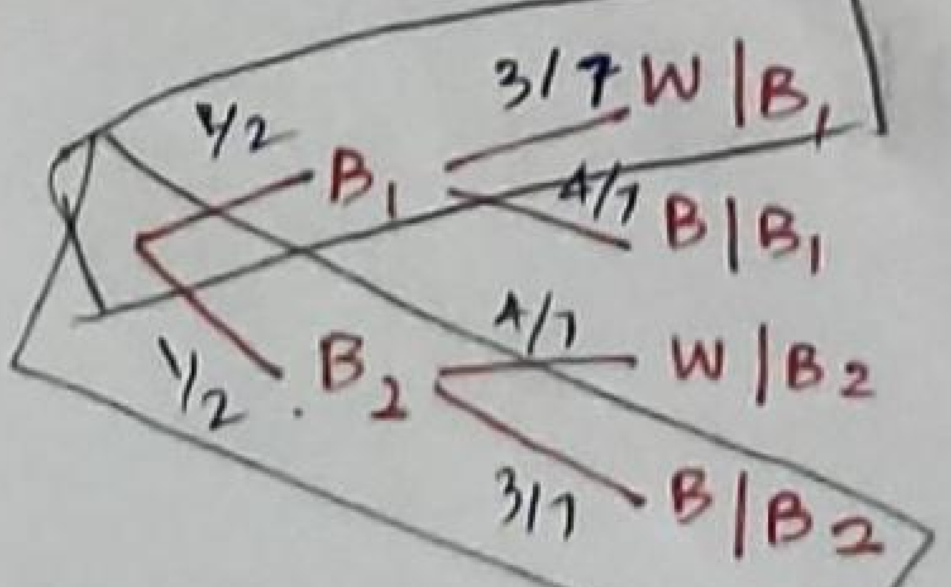
$P(C) = 1/7$

$P(B) = 2/7$ $P(A) = 4/7$.

6) 2 boxes contain 3W, 4B and 4W, 3B balls if a box is chosen and a ball is drawn from it. What is the prob that it is a white ball?

$\boxed{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \frac{4}{7}}$ => as per

$P(W) = P(B_1)\, P(W/B_1) + P(B_2)\, P(W/B_2)$



A box1 contains 5 red and 3 Blue balls

Box2 contains 4 red and 5 Blue balls

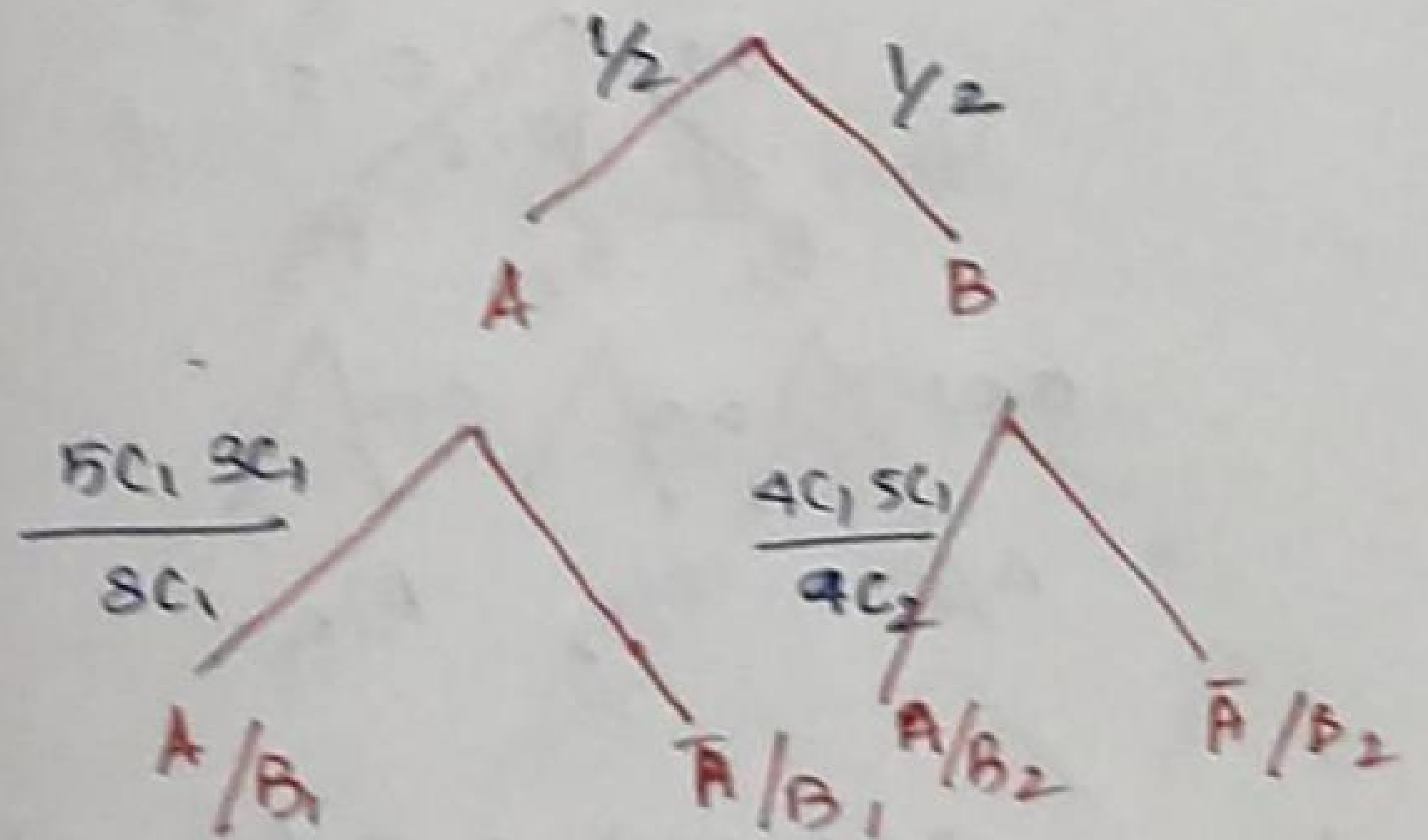
A box is chosen 2 balls are drawn from it

what is the prob that one is red & other is blue

$B_1 \Rightarrow$ Box 1

$B_2 \Rightarrow$ Box 2

$A \rightarrow$ selecting 1R and 1B ball



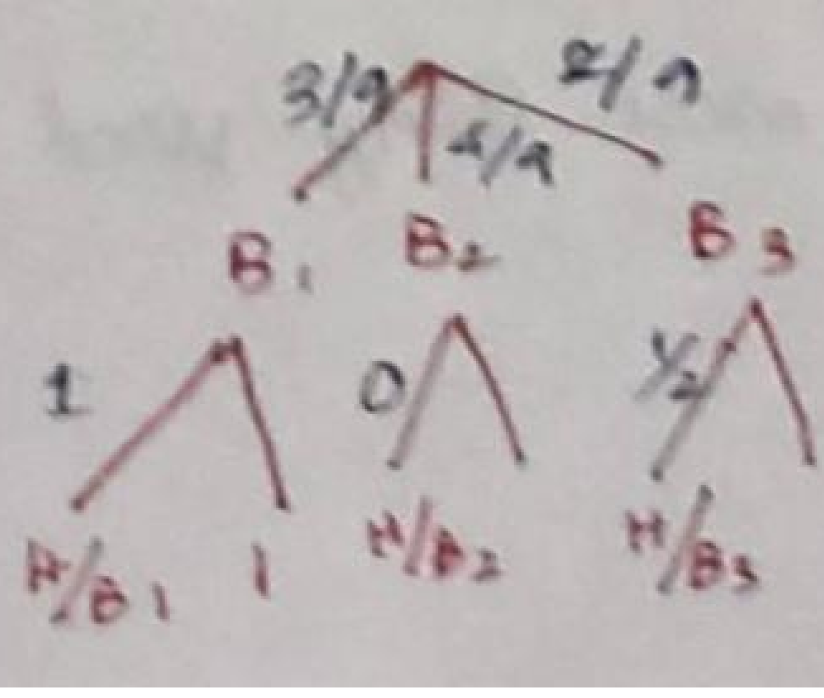
$$P(A) = \frac{1}{2} \times \frac{5C_1 \times 3C_1}{8C_2} + \frac{1}{2}\ \frac{4C_1\ 5C_1}{9C_2}$$

A box contains 3 coins with a head on each side, 4 coins with a tail on each side and 2 fair coins. If one of these coins is selected at random and tossed once. Wt is the prob that head is obtained.

$B_1 \rightarrow$ selecting a 2-headed coin

$B_2 \rightarrow$ selecting a 2-tailed coin

$B_3 \rightarrow$ selecting a fair coin



$$P(H) = \frac{3}{9}(1) + \frac{4}{9}(0) + \frac{2}{9}\left(\frac{1}{2}\right)$$

$$= \frac{4}{9}$$

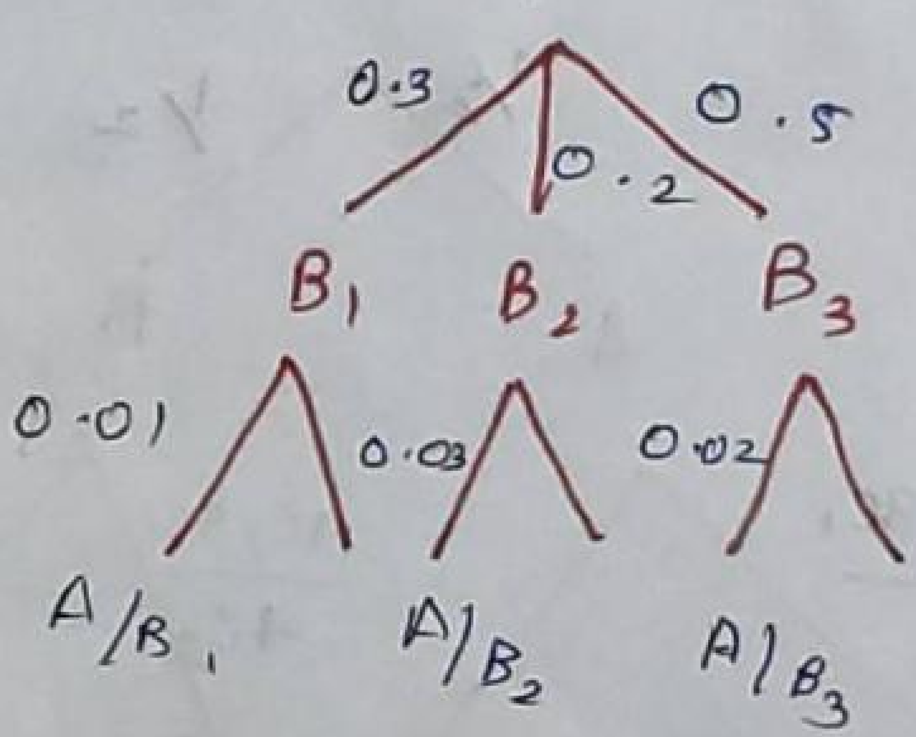
A company employs 3 analytical plans for the design & development of a product plans 1, 2 and 3 are used for 30%, 20% & 50% of the products respectively. The probabilities of defective products of these plans are 0.01, 0.03 & 0.02 (i) If a random product was observed, wt is the prob that it is defective

(ii) If it is found defective, which plan was most likely used & thus responsible?

$B_1 \Rightarrow$ using plan 1

$B_2 \Rightarrow$ using plan 2

$B_3 \Rightarrow$ using plan 3



(i) $P(A) = 0.3(0.01) + (0.2)(0.03) + 0.5(0.02) = 0.019$

(ii) $$P(B_1/A) = \frac{P(A \cap B_1)}{P(A)} = \frac{P(B_1)\,P(A/B_1)}{P(A)}$$

$$= \frac{0.3 \times 0.01}{0.019} = 0.157$$

$$P(B_2/A) = \frac{0.2 \times 0.03}{0.019} = 0.315$$

$$P(B_3/A) = \frac{0.5 \times 0.02}{0.019} = 0.526$$

Plan 3 was most likely used

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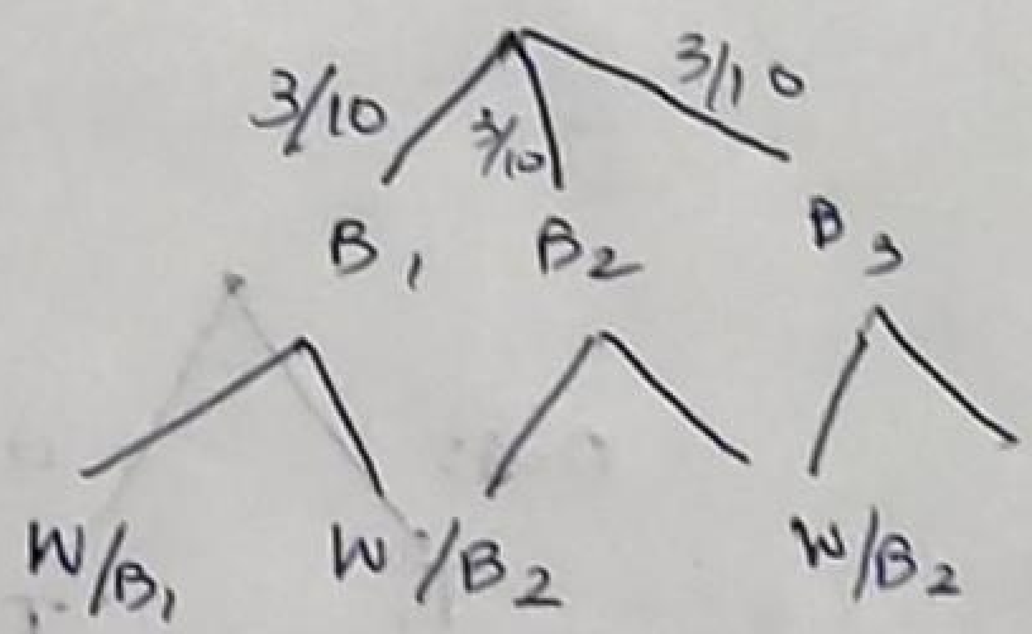
10) There are 10 boxes each of three contains 1W and 9B balls each of another 3 contains 9W & 1B balls and remaining 4 contains 5W & 5B balls. One ball selected at random and a ball is chosen at. is found to be white wt is prob that box contain 1W 9B are selected.

B_1 : type 1 => 1W, 9B

B_2 : type 2 => 9W, 1B

B_3 : type 3 → 5W, 5B

W: white ball



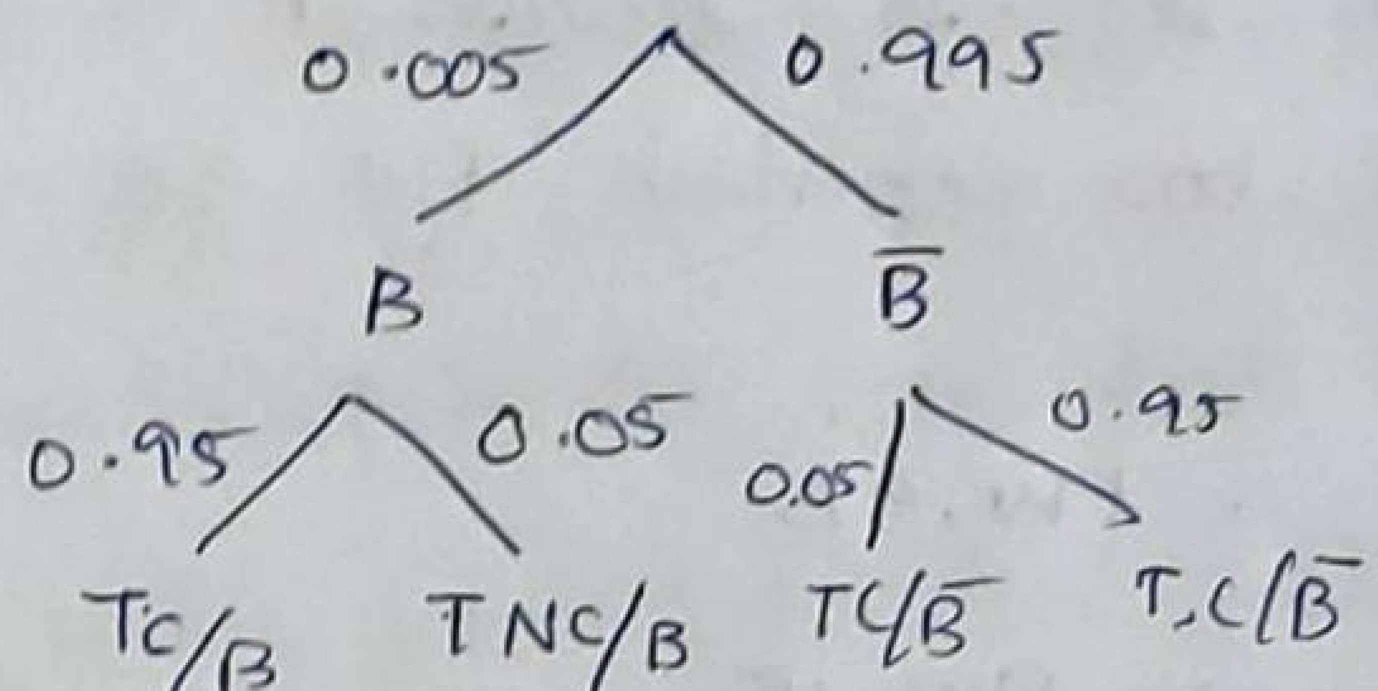
$$P[B_1|W] = \frac{P(B_1) \times P(W/B_1)}{P(W)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{10}}{\frac{3}{100} + \frac{27}{100} + \frac{20}{100}} = \frac{3}{50}$$

4 roads

1) B -> person has cancer

B̄ ⇒ No cancer.



$$P(B/T_c) = \frac{P(B)\,P(T_c/B)}{P(T_c)} = \frac{0.005 \times 0.95}{0.05 \times 0.95 \times 0.005 + 0.995}$$

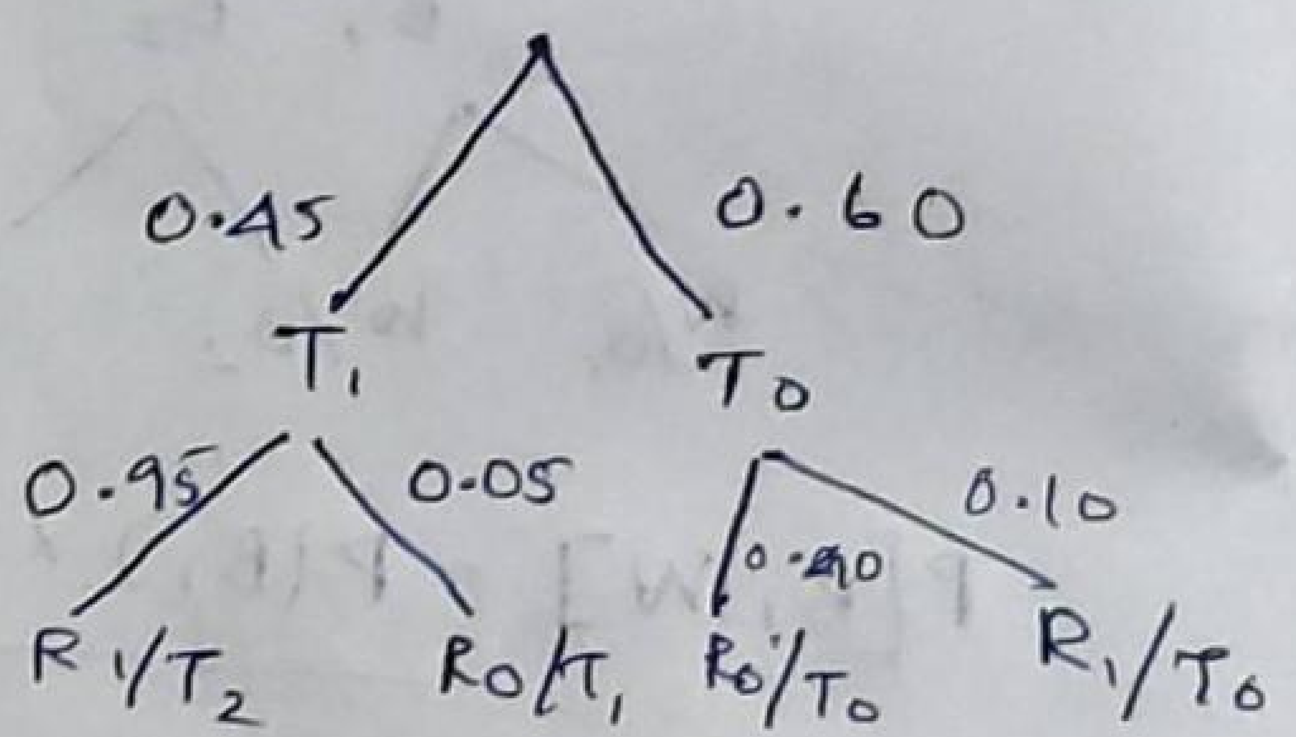
Worksheet

12) $P(T_1) = 0.40$

$P(T_0) = 0.60$

$P(R_0/T_0) = 0.90$

$P(R_1/T_1) = 0.95$



(i) P(1 being received)

$P(R_1) = 0.40 \times 0.95 + 0.60 \times 0.10$

$= 0.44$

(ii) $P(T_1/R_1)$

$$= \frac{P(T_1)\,P(R_1/T_1)}{P(R_1)} = \frac{0.40 \times 0.95}{0.44} = 0.863$$

(iii) $P(\text{Error}) = 0.40 \times 0.05 + 0.60 \times 0.10$

$= 0.08$

1) 4 → Green

8 → Red

8 → Yellow

Reduced sample space (contain only green & yellow)

$P(\text{getting Green}) = \frac{4}{12} = \frac{1}{3}$

2) A => head on first toss

B → head on second toss

$S = \{HH, HT, TH, TT\}$

C → both outcomes are same

$P(A) = \frac{2}{4} = \frac{1}{2}$

$P(B) = \frac{2}{4} = \frac{1}{2}$

$P(C) = 1/2$

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$\frac{1}{4} \neq \frac{1}{8}$

So, not endependent but pairwise independent.

3) $S = \{HH, TT, HT, TH\}$

$P(A) = \frac{3}{4}$, $P(B) = \frac{2}{4}$

$P(A \cap B) = P(A) \cdot P(B)$

$\neq \frac{3}{8}$

4) $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$, $P(A \cup B) = \frac{11}{12}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{11}{12} = \frac{1}{3} + \frac{3}{4} - P(A \cap B)$$

$$\frac{11}{12} = \frac{4+9}{12} - P(A \cap B)$$

$$\frac{11}{12} - \frac{13}{12} = -P(A \cap B)$$

$$\frac{1}{6} = \frac{2}{16} = P(A \cap B)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{3/4}$$

$$= \frac{1}{6} \times \frac{4}{3} = 2/9.$$

5) Alice $0.75 = P(A)$

Bob $0.80 = P(B)$

$$P(\bar{A} \cap B) + P(\bar{B} \cap A) \Rightarrow$$

$$= P(\bar{A}) \cdot P(B) + P(\bar{B}) \cdot P(A)$$

$$= 0.25 \times 0.80 + 0.2 \times 0.75$$

6) $P(A/B) = 0.2$, $P(A/\bar{B}) = 0.3$, $P(B) = 0.8$

$P(A) = ?$

$$\frac{P(A \cap B)}{P(B)} : 0.2 = \frac{P(A \cap B)}{0.8}$$

$$0.3 = \frac{P(A \cap \bar{B})}{0.2}$$

$$P(A) = 0.22$$

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7) $P(B) = 2 \times P(B)$

$S = \{HH$

$P(A \cup B) = P(A) + P(B)$

$= 2(P(A)) + P(A)$

$S = 3P(A)$

$P(A) = \frac{P(S)}{3} = \frac{1}{3} = P(A)$

$\frac{2}{3} = P(B)$

8) $0.6 + 0.7 - 0.2 = P(A \cup B) > 1$

So, not possible

9) $S = \{HHH, HHT, HTH, THH, \cancel{H}TTT, TTH, THT, HTT\}$

$P(\text{no tails}) = 1/8.$

11) $P(K) = \begin{cases} 0.1 & K = 1, 2, 5, 6 \\ 0.3 & K = 3, 4. \end{cases}$

O/C's for sum 7 $= (1, 6)\ (2, 5)\ (3, 4)\ (4, 3)$
$(5, 2)\ (6, 1)$

$P(\text{sum is } 7) = (0.1)(0.1) + (0.1)(0.1) + (0.3)(0.3)$
$+ (0.3)(0.3) + (0.1)(0.1) +$
$(0.1)(0.1)$

$= 4(0.1)^2 + 2(0.3)^2$

$= 4(0.01) + 2(0.09)$

$= 0.04 + 0.18$

$= 0.22$

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$= P(1 \text{ in die } 1, 6 \text{ in Die } 2)$

$= P(1 \text{ in die } 1) * P(6 \text{ in Die } 2)$

10) Alice wins if he throws 6

Bob wins if he throws 7

~~P(Alice wins)~~ = P[SU FS U FFS U FFSU ---]

$P(\text{Alice wins}) = P[SU\overset{\text{No 6}}{F}\overset{\text{No 7}}{F}\overset{\text{6 by Alice}}{S}UFFFFSU\cdots]$

$$= \frac{5}{36} + \frac{31}{36} \cdot \frac{30}{36} \cdot \frac{5}{36} + \frac{31}{36} \cdot \frac{30}{36} \cdot \frac{31}{36} \cdot \frac{30}{36} \cdot \frac{5}{36}$$

$$= \frac{5}{36}\left[1 + x + x^2 + \cdots\right]$$

Where $x = \frac{31}{36} \cdot \frac{30}{36}$.

Binomial formula $\Rightarrow 1 + x + x^2 + \cdots = \frac{1}{1-x}$

$$= \frac{5}{36}\left[\frac{1}{1 - \frac{31}{36} \cdot \frac{30}{36}}\right]$$