No of Pages :3 **Course Code : 20XW31**

Roll No:
(To be filled in by the candidate)

PSGCOLLEGE OF TECHNOLOGY, COIMBATORE - 641 004

SEMESTER EXAMINATIONS, MODEL QP1

MSc - SOFTWARE SYSTEMS Semester: 3

20XW31 PROBABILITY AND STATISTICS

Duration : 3 Hours **Maximum Marks : 100**

INSTRUCTIONS:

1. Answer **ALL** questions. Each question carries 20 Marks.
2. Subdivision (a) carries 3 marks each, subdivision (b) carries 7 marks each and subdivision (c) carries 10 marks each.
3. Statistical table brought by the candidates shall be permitted.

1. a) If two events A and B are independent, does it follow that $\overline{A}$ and $\overline{B}$ are also independent? Justify.

   b) i) An urn A contains 5 white and 3 black balls and urn B contains 4 white and 4 black balls. An urn is selected and a ball is drawn from it. What is the probability that the ball is white? [3]

   ii) Suppose that each of two dice is biased so that when either die is rolled, the probability that the number k will appear is 0.1 for k = 1, 2, 5, or 6 and is 0.3 for k = 3 or 4. If these two dice are rolled independently, what is the probability that the sum of the two numbers that appear will be 7? [4]

   c) A machine part may be selected from any of the three manufacturers with probabilities 0.25, 0.5, and 0.25 respectively. The probabilities that it will function properly during a specified period of time are 0.2, 0.3, and 0.4 respectively for these manufacturers.

   (1) Find the probability that a randomly chosen machine part will function properly for the specified period of time.

   (2) Given the machine part functions properly for the specified period of time, what is the probability that it was from manufacturer 2?

2. a) A point is selected at random from the interval $(0,1)$. What is the probability that it is less than 1/3?

   b) i) The incidence of occupational disease in an industry is such that the workmen have a 20% of chance of suffering from it. What is the probability that out of six workmen, 4 or more will contact the disease? [3]

   ii) Suppose that the time to failure (in minutes) of a certain electronic components follows Weibull distribution with $\alpha = 1/5$ and $\beta = 1/3$. What is the probability that such a component will fail in less than 5 hours? How long can such a component be expected to last? [4]

   c) i) The number of customers arriving at a cafeteria has Poisson distribution with a mean of 0.3 per minute. Suppose that for the last 10 minutes no customer has arrived. What is the probability that

   (1) the next one will arrive in less than 2 minutes;
   (2) two or more customers arrive in a 5-minute span. [4]

   ii) The amount of soft drink in a bottle is a normal random variable. Suppose that 7% of the bottles with this soft drink have less than 155 ml and 10% of the bottles have more than 163 ml. What are the mean and standard deviation of the amount of soft drink in a randomly selected bottle? [6]

3. a) If X is a random variable having Poisson distribution with parameter λ, then what is the moment generating function of Y = X - 2 ?

b) i) For a random variable X, $E[(X-1)^2] = 10$ and $E[(X-2)^2] = 6$. Find $E[X]$. [3]

ii) Obtain the moment generating function of exponential distribution. Hence find its mean and variance. [4]

c) Suppose that the random variables X and Y have a joint density function given by $f(x,y) = \begin{cases} cx(1+3y^2), & 0 < x < 2,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$. Determine the following:

(1) The value of c;
(2) The marginal density functions of X and Y;
(3) The conditional density function of X given Y=y;
(4) $P\left[\frac{1}{4} < X < \frac{1}{2} \mid Y = \frac{1}{3}\right]$.

4. a) In hypothesis testing, suppose you reject the null hypothesis at 1% LOS. Can you reject it at 3% LOS? Why or why not?

b) i) In interval estimation, what do you mean by 'a level of confidence of 95%'? [3]

ii) A sample of 10 measurements of the diameter of a sphere gave a mean of 4.38 inches and a standard deviation of 0.06 inch. Find a 95% confidence limits for the actual diameter. [4]

c) i) The manufacturer of a patent medicine claimed that it was 90% effective in relieving an allergy for a period of 8 hours. In a sample of 200 people who had the allergy, the medicine provided relief for 160 people. Determine whether the manufacturer's claim is legitimate by using 0.05 as the level of significance. [5]

ii) The number of visits to the web site www.psgtech.edu using one of the two operating systems (Windows and Mac) and one of the two browsers (Firefox and Chrome) in a certain period was as in Table.1. Conduct a Chi-square test of the hypothesis that the choice of browser is independent of the choice of operating system using 5% LOS. [5]'

Table.1

	Mac	Windows
Firefox	538	1788
Chrome	290	1126

5. a) If the coefficient of correlation between the variables X and Y is 0.5, find the coefficient of correlation between the variables $2X-4$ and $3-2Y$.

b) i) The regression equations of two variables X and Y are $3x + 2y = 26$ and $6x + y = 31$.. Also find the means of X and Y. [3]

ii) Suppose 10 rats are used in a biomedical study where they are injected with cancer cells and then given a cancer drug that is designed to increase their survival rate. The survival times, in months, are 14, 17, 27, 18, 12, 8, 22, 13, 19, and 12. Assume that the exponential distribution applies. Give a maximum likelihood estimate of the mean survival time. [4]

c) i) Assume that the prior distribution for the proportion p of defectives produced by a machine is *p* is uniform (i.e., $\pi(p) = 1$, for $0 < p < 1$). Denote by X the number of defectives among a random sample of size 2. Find theposterior probability distribution of *p*, given that *x=1* is observed.

[OR]

ii) Scraps of iron were selected on the basis of their densities, X, and their iron contents, Y, were measured. The results were as shown in Table.4. Find the equation of regression line of y on x. Also find the coefficient of correlation between X and Y.

Table.4

X	2.8	2.9	3.0	3.1	3.2	3.2	3.2	3.3	3.4
Y	27	23	30	28	30	32	34	33	30

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