Camlin Page
Date / /

JOINT PROBABILITY DISTRIBUTION:

So far single rv.
Now 2 or more rvs defined on the same sample space. It is called joint probability distribution.

Consider X: height of student
Y: weight of same student

$f_{xy}(x, y)$ joint pdf

$P_{xy}(x, y) = P[X = x, Y = y]$ joint pmf

$F_{xy}(x, y) = P[X \leq x, Y \leq y]$ joint cdf

The individual prob distributions of rvs X and Y are called marginal distributions denoted by $f_x(x)$ and $f_Y(y)$

MARGINAL pmf:

Let $P_{xy}(x, y)$ be the joint pmf of X and Y then marginal dist of x $P_x(x) = \sum_{y \in Y} P_{xy}(x, y)$

marginal dist of y $P_y(y) = \sum_{x \in X} P_{xy}(x, y)$

eg: 2 dice are rolled. Let X denotes max of 2 throws. Let Y denotes no. of times an even no. appears. Find the joint pmf of X & Y. Also find marginal dist of X & Y.

X \ Y	0	1	2		
1	1/36	0	0	1/36	$P_x(1)$
2	0	2/36	1/36	3/36	$P_x(2)$
3	3/36	2/36	0	5/36	$P_x(3)$
4	0	4/36	3/36	7/36	$P_x(4)$
5	5/36	4/36	0	9/36	$P_x(5)$
6	0	6/36	5/36	11/36	$P_x(6)$
	$P_Y(0) = 9/36$	$P_Y(1) = 18/36$	$P_Y(2) = 9/$	1	

Marginal dist of X:

X	1	2	3	4	5	6
$P_X(x)$	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/36$

Marginal dist of Y:

Y	0	1	2
$P_Y(Y)$	$9/36$	$18/36$	$9/36$

Properties of joint pmf/pdf:

* pmf	pdf
1. $P_{xy}(x,y) \geq 0$	1. $f_{xy}(x,y) \geq 0$
2. $\sum\sum_{x \in X, y \in Y} P_{xy}(x,y) = 1$	2. $\int\int f_{xy}(x,y)\,dy\,dx = 1$

* If $f_{xy}(x,y)$ is the joint pdf of x and y then marginal dist of x $f_x(x) = \int f(x,y)\,dy$

marginal dist of Y $f_Y(Y) = \int f(x,y)\,dx$

Note: If X and Y are independant rv

$f(x,y) = f_x(x)\, f_Y(Y)$

Conditional distribution of Y given X = x

For 2 events A, B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

For 2 rvs X and Y with joint pmf/pdf $f(x,y)$ the conditional distribution of Y given X = x is given by

(Y's)

$$f_{Y|X=x}(Y|x) = \frac{f(x,Y)}{f_X(x)}, \quad f_x(x) \neq 0$$

$$f_{x|Y=y}\overset{(x|y)}{\cancel{(y|x)}} = \frac{f(x,Y)}{f(u)}, \quad f_Y(Y) \neq 0$$

Conditional expectation and variance

The conditional expectation of Y given X = x is given by

$$E(Y|X=x) = \begin{cases} \sum_{y \in Y} y\, f_{Y|x}(y|x), & \text{if } x \,\&\, Y \text{ are discret} \\ \int y\, f_{Y|x}(y|x)\,dy, & \text{if } x \,\&\, Y \text{ are contn.} \end{cases}$$

Illy $E(Y^2|X=x) = \begin{cases} \sum_{y \in Y} y^2 f_{Y|x}(y|x) \\ \int y^2 f_{Y|x}(y|x)\,dy \end{cases}$

Then variance $(Y|X=x) = E(Y^2|X=x) - (E(Y|X=x))^2$

Result: For 2 rv X and Y with joint pmf/pdf $f(x,y)$ $\boxed{E(E(X|Y)) = E(x)}$

* $E(X|Y)$ values change for diff values of y
* hence we multipl by $f_Y(y)$

Proof:

$$E(E(X|Y)) = \int_{-\infty}^{\infty} E(X|Y)\, f_Y(y)\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f_{x|y}(x|y)\,dx\, f_Y(y)\,dy$$

$\because f_{x|y}(x|y) = \frac{f(x,y)}{f_Y(y)}$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f(x,y)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\; x\,dx$$

$$= \int_{-\infty}^{\infty} f_x(x)\, x\,dx = E(x)$$

eg: The joint pdf of rv's x and y is

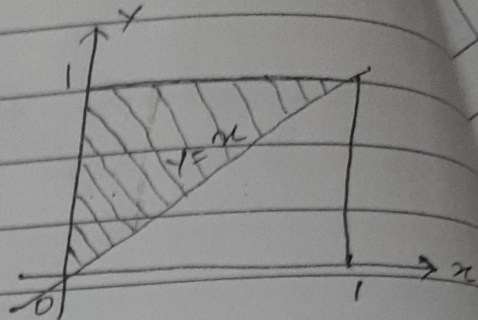
$$f(x,y) = \begin{cases} kxy^2 & 0 < x < y < 1 \\ 0 & o/w \end{cases}$$

Find i. value of K ii. marginal dist of x & y
iii. $E(x)$ & $E(Y)$ iv. X and Y are independant

$E(xy) = E(x)E(Y)$, covar = 0, $f(x,y) = f_x(x) f_y(y)$

Date

Soln: For joint pdf: $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$



$1 = \int_0^1 \int_x^1 f(x,y)\,dy\,dx$

$1 = \int_0^1 \int_x^1 k(xy^2)\,dy\,dx$

$\int_0^1 \frac{kxy^3}{3}\Big|_x^1 dx = 1$

* all points in region obey condition

if $x=1$ $y=0$ $x \le y$ its false

$\int_0^1 kx\left(\frac{1}{3} - \frac{x^3}{3}\right) dx = 1$

$\int_0^1 \frac{kx}{3}\,dx - \int_0^1 \frac{kx^4}{3}\,dx = 1$

$\frac{kx^2}{6}\Big|_0^1 - \frac{k}{3\times 5} x^5\Big|_0^1 = 1$

$\frac{k}{6} - \frac{k}{15} = 1 \qquad \frac{9k}{15\times 6} = 1 \qquad \boxed{k=10}$

ii. $f_x(x) = \int f(x,y)\,dy$

$= \int_x^1 kxy^2\,dy = \left[kx\frac{y^3}{3}\right]_x^1$

$= \frac{kx}{3} - \frac{kx^4}{3} = \frac{kx(1-x^3)}{3} = \frac{10x(1-x^3)}{3} \qquad 0 \le x \le 1$

$f_y(y) = \int_0^y f(x,y)\,dx = \int_0^y kxy^2\,dx = \frac{ky^2x^2}{2}\Big|_0^y$

$f_y(y) = \frac{ky^4}{2} = 5y^4 \qquad 0 \le x \le 1$

iii. $E(x) = \int_{-\infty}^{\infty} x f_x(x)\,dx = \int_{-\infty}^{\infty} x \frac{10x(1-x^3)}{3}\,dx$

$= \frac{10}{3}\int_{-\infty}^{\infty} x^2 - x^5\,dx$

$= 10/3 \left[x^3/3 - x^6/6\right]_0^1$

$= 10/3\,(1/3 - 1/6) = 10/3\left(\frac{1}{6}\right) = 10/18$

$E(x) = 5/9$

Camlin Page
Date / /

$$E(Y) = \int_{-\infty}^{\infty} y\, f_Y(y)\, dy = \int_0^1 5y^4 y\, dy$$

$$E(Y) = 5\int_0^1 y^5 dy = \frac{5y^6}{6}\Big|_0^1 = \frac{5}{6}$$

iv. X, Y are independant, $f(x,y) = f_X(x) f_Y(y)$

$$f_X(x) f_Y(y) = \frac{10}{3} x(1-x^3) 5y^4 \neq Kxy^2 \; (f(x,y))$$

$\therefore$ X and Y are not independant

(or)

$$\boxed{E(x,y) = \iint xy\, f(x,y)\, dy\, dx}$$

$$= \int_0^1 \int_x^1 xy\, 10xy^2\, dy\, dx = 10\int_0^1\int_x^1 x^2 y^3\, dy\, dx$$

$$= 10\int_0^1 x^2 \frac{y^4}{4}\Big|_x^1 = \frac{10}{4}\int_0^1 x^2 - x^5\, dx = \frac{10}{4}\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{10}{4}\left[\frac{1}{3} - \frac{1}{7}\right] = \frac{10}{4}\left[\frac{4}{21}\right] = \frac{10}{21}$$

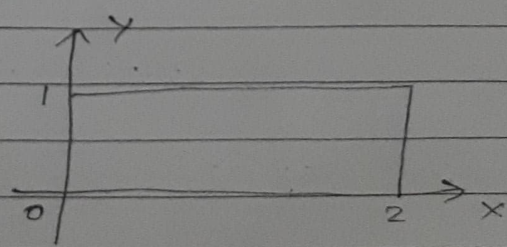
$$E(x,Y) \neq E(x)E(Y) \qquad 10/21 \neq 25/54$$

eg: The joint pdf of rv's X and Y is

$$f(x,y) = \begin{cases} xy^2 + \frac{x^2}{8} & 0 \le x \le 2,\ 0 \le y \le 1 \\ 0 & \text{o/w} \end{cases}$$

Find i. $P(X>1)$ ii. $P(Y<1/2)$ iii. $P(X>1, Y<1/2)$
iv. $P(X<Y)$ v. $P(Y<1/2 \mid X>1)$ vi. $P(X+Y \le 1)$

Soln:



$$f_X(x) = \int_0^1 f(x,y)\, dy$$

$$= \int_0^1 xy^2 + \frac{x^2}{8}\, dy$$

$$= \frac{xy^3}{3} + \frac{x^2}{8} y\Big|_0^1 = \frac{x}{3} + \frac{x^2}{8}, \quad 0 \le x \le 2$$

$f_Y(y) = \int_0^2 f(x,y)\,dx$

$= \int_0^2 xy^2 + \frac{x^2}{8}\,dx = \frac{x^2y^2}{2} + \frac{x^3}{3\times 8}\Big|_0^2$

$= \frac{y^2}{2}(4) + \frac{8}{3\times 8} = 2y^2 + \frac{8}{3\times 8}$

$f_Y(y) = 2y^2 + 1/3 \quad 0 \le y \le 1$

i. $P(X>1) = \int_1^2 f_X(x)\,dx = \int_1^2 \frac{x}{3} + \frac{x^2}{8}\,dx$

$= \frac{x^2}{3\times 2}\Big|_1^2 + \frac{x^3}{3\times 8}\Big|_1^2 = \frac{1}{2} + \frac{7}{24} = \frac{12+7}{24} =$

$P(X>1) = 19/24$

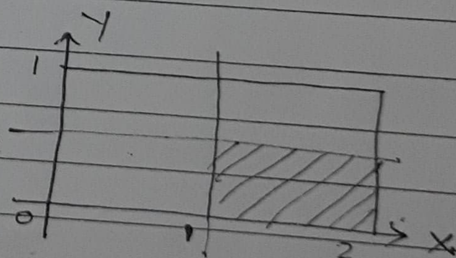
ii. $P(Y<1/2) = \int_0^{1/2} f_Y(y)\,dy = \int_0^{1/2} 2y^2 + 1/3\,dy$

$= 2y^3/3 + 1/3\,y\Big|_0^{1/2} = \frac{2}{3}\left(\frac{1}{8}\right) + \frac{1}{3}\left(\frac{1}{2}\right)$

$= \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$

iii. $P(X>1, Y<1/2) = \iint f(x,y)\,dy\,dx$

$= \int_1^2\int_0^{1/2} xy^2 + \frac{x^2}{8}\,dy\,dx$

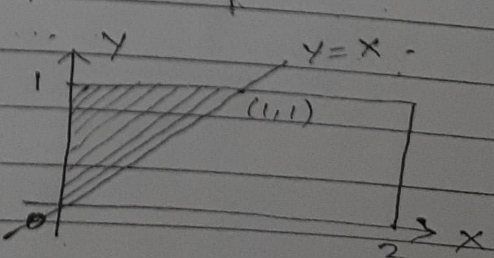


$= \int_1^2 \frac{xy^3}{3}\Big|_0^{1/2} + \frac{x^2}{8}y\Big|_0^{1/2}\,dx$

$= \int_1^2 x/24 + x^2/16\,dx = \int_1^2\left[\frac{x^2}{48} + \frac{x^3}{48}\,dx\right]$

$= \frac{1}{48}\left[3+7\right] = 10/48 = 5/24$

iv. $P(X<Y) = \int_0^1\int_x^1 \left(xy^2 + \frac{x^2}{8}\right)dy\,dx$



$= \int_0^1 \frac{xy^3}{3} + \frac{x^2}{8}y\Big|_x^1\,dx$

Camlin Page
Date / /

$$= \int_0^1 \frac{x}{3}(1-x^3) + \frac{x^2}{8}(1-x)\,dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 \frac{1}{8}$$

$$= \frac{1}{3}\underset{-1/5}{\left[\tfrac{1}{2}\cancel{4}\right]} + \frac{1}{8}\left[\frac{1}{12}\right] = \frac{1}{10} + \frac{1}{96} = \frac{96+10}{960} = \frac{106}{960}$$

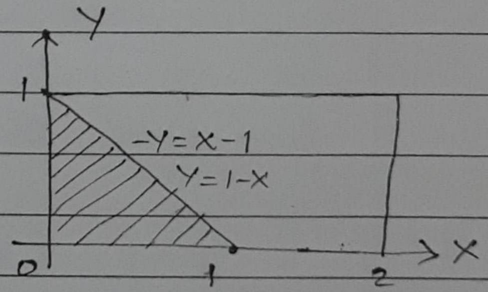
$P(X<Y) = \cancel{3/4}\ \frac{53}{480}$

v. $P(Y<1/2 \mid X>1) = \dfrac{P(X>1,\ Y<1/2)}{P(X>1)}$

$$= \frac{5/24}{19/24}$$

$P(Y<1/2 \mid X>1) = 5/19$

vi. $P(X+Y \le 1) = \int_0^1\int_0^{1-x} f(x,y)\,dy\,dx$



$$= \int_0^1\int_0^{1-x} xy^2 + \frac{x^2}{8}\,dy\,dx$$

$$= \int_0^1 \frac{xy^3}{3}\Big|_0^{1-x} + \frac{x^2}{8}y\Big|_0^{1-x}\,dx$$

$$= \int_0^1 \frac{x}{3}(1-x)^3 + \frac{x^2}{8}(1-x)\,dx$$

$$= \int_0^1 \frac{1}{3}(x-3x^2+3x^3-x^4) + \frac{1}{8}\int_0^1 x^2 - x^3\,dx$$

$$= \frac{1}{3}\left(\frac{x^2}{2} - x^3 + \frac{3x^4}{4} - \frac{x^5}{5}\right)\Big|_0^1 + \frac{1}{8}\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$$

$$= \frac{1}{3}\left(\frac{1}{2} - 1 + \frac{3}{4} - \frac{1}{5}\right) + \frac{1}{8}\left(\frac{1}{3} - \frac{1}{4}\right)$$

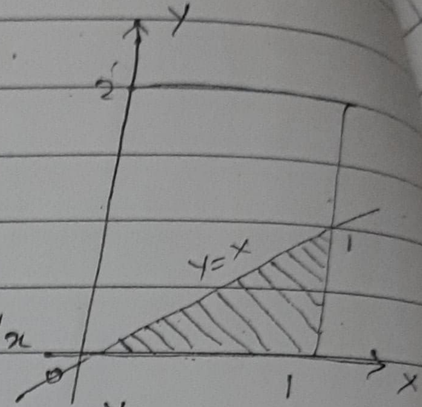
$$= \frac{1}{3}\left(\frac{1}{20}\right) + \frac{1}{8}\left(\frac{1}{12}\right) = \frac{1}{60} + \frac{1}{96} = \frac{8+5}{480} = \frac{13}{480}$$

eg: $f(x,y) = \frac{6}{7}(x^2 + xy/2)$ $0 < x < 1;\ 0 < y < 2$

Find i. $P(X > Y)$ ii. $P(Y > 1/2, X < 1/2)$

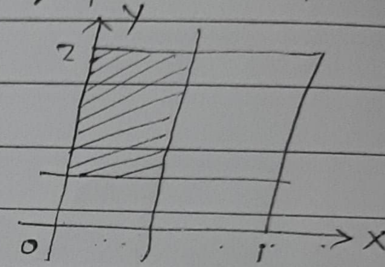
$$P(X>Y) = \int_0^1 \int_0^x \frac{6}{7}(x^2 + xy/2)\,dy\,dx$$

$= 15/56$



$$P(Y>1/2, X<1/2) = \int_0^{1/2} \int_{1/2}^{2} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right)dy\,dx$$

$= 69/448$



eg: Roll a fair die. Let the o/c be X then toss a fair coin X times and let Y denotes the no. of tails. Find the joint pmf of X & Y. Also find the marginal pmf of X and Y.

Soln:

$X = \{1, 2, 3, 4, 5, 6\}$
$Y = \{0, 1, 2, 3, 4, 5, 6\}$

X \ Y	0	1	2	3	4	5	6
1	$\frac{1}{6} \times \frac{1}{2}$	$\frac{1}{6} \times \frac{1}{2}$	0	0	0	0	0
2	$1/6 \times 1/4$	$\frac{1}{6} \times \frac{1}{2}$	$\frac{1}{6} \times \frac{1}{4}$	0	0	0	0
3	$1/6 \times 1/8$	$1/6 \times 3/8$	$1/6 \times \frac{3}{8}$	$\frac{1}{6} \times \frac{1}{8}$	0	0	0
4	$\frac{1}{6} \times \frac{4C_0}{2^4}$	$\frac{1}{6} \times \frac{4C_1}{2^4}$	$\frac{1}{6} \times \frac{4C_2}{2^4}$	$\frac{1}{6} \times \frac{4C_3}{2^4}$	$\frac{1}{6} \times \frac{4C_4}{2^4}$	0	0
5	$1/6 \times \frac{5C_0}{2^5}$	$\frac{1}{6} \times \frac{5C_1}{2^5}$	$\frac{1}{6} \times \frac{5C_2}{2^5}$	$\frac{1}{6} \times \frac{5C_3}{2^5}$	$\frac{1}{6} \times \frac{5C_4}{2^5}$	$\frac{1}{6} \times \frac{5C_5}{2^5}$	0
6	$1/6 \times \frac{6C_0}{2^6}$	$\frac{1}{6} \times \frac{6C_1}{2^6}$	$\frac{1}{6}\ \frac{6C_2}{2^6}$	$\frac{1}{6}\ \frac{6C_3}{2^6}$	$\frac{1}{6}\ \frac{6C_4}{2^6}$	$\frac{1}{6}\ \frac{6C_5}{2^6}$	$\frac{1}{6}\ \frac{6C_6}{2^6}$

H, T
HH, HT, TH, TT
HHH, HTH, THH, TTH
HHT, HTT, THT, TTT

Note that $p(x,y) = \frac{1}{6}\ {}^{x}C_y\ (1/2)^x$ $y = 0, 1, 2, \ldots x$

Camlin Page
Date / /

Marginal dist of X:

$P_x(x) = 1/6 \qquad x = 1, 2, 3, 4, 5, 6$

Marginal dist of Y

$Y=y$	0	1	2	3	4	5	6
$P_y(y)$	$63/384$	$120/384$	$99/384$	$64/384$	$29/384$	$8/384$	$1/384$

eg: Suppose that 3 cards are drawn from a deck of 52 cards. If X and Y denote the no. of diamonds and spades respectively. Find the joint pmf of X and Y.

$X = \{0, 1, 2, 3\} \qquad Y = \{0, 1, 2, 3\}$

wrong

X \ Y	0	1	2	3
0	$\frac{4C_1\,13C_0}{52C_3} \times \frac{4C_1\,13C_0}{52C_3}$	$\frac{4C_1\,13C_0}{52C_3} \times \frac{4C_1\,13C_1}{52C_3}$	$\frac{4C_1\,13C_0}{52C_3} \times \frac{4C_1\,13C_2}{52C_3}$	$\frac{4C_1 \times 13C_0}{52C_3} \times \frac{4C_1\,13}{52C}$
1	$\frac{4C_1\,13C_1}{52C_3} \times \frac{4C_1\,13C_0}{52C_3}$	$\frac{4C_1\,13C_1}{52C_3} \times \frac{4C_1\,13C_1}{52C_3}$	$\frac{4C_1\,13C_1}{52C_3} \times \frac{4C_1\,13C_2}{52C_3}$	$\frac{4C_1\,13C_1}{52C_3} \times \frac{4C_1\,13C}{52C_3}$
2	$\frac{4C_1\,13C_2}{52C_3} \times \frac{4C_1\,13C_0}{52C_3}$	$\frac{4C_1\,13C_2}{52C_3} \times \frac{4C_1\,13C_1}{52C_3}$	$\frac{4C_1\,13C_2}{52C_3} \times \frac{4C_1\,13C_2}{52C_3}$	$\frac{4C_1\,13C_2}{52C_3}\ \frac{4C_1\,13C}{52C_3}$
3	$\frac{4C_1\,13C_3}{52C_3}\ \frac{4C_1\,13C_0}{52C_3}$	$\frac{4C_1\,13C_3}{52C_3} \times \frac{4C_1\,13C_1}{52C_3}$	$\frac{4C_1\,13C_3}{52C_3} \times \frac{4C_1\,13C_2}{52C_3}$	$\frac{4C_1\,13C_3}{52C_3} \times \frac{4C_1\,13C}{52C_3}$

$x \backslash Y$	0	1	2	3
0	$\frac{26C_3}{52C_3}$	$\frac{13C_1\, 26C_2}{52C_3}$	$\frac{13C_2\, 26C_1}{52C_3}$	$\frac{13C_3\, \cancel{26C_0}}{52C_3}$
1	$\frac{13C_1\, 26C_2}{52C_3}$	$\frac{13C_1\, 13C_1\, 26C_1}{52C_3}$	$\frac{13C_1\, 13C_2}{52C_3}$	$\frac{0}{52C_3}$
2	$\frac{13C_2\, 26C_1}{52C_3}$	$\frac{13C_2\, 13C_1}{52C_3}$	0	0
3	$\frac{13C_3}{52C_3}$	0	0	0

eg: RV's X and Y have joint pdf
$f(x,y) = kxy \quad 0 \le x \le y \le 1$
Find i. the value of k
ii. marginal distributions of X & Y
iii. if X and Y are independant
iv. $P[X \le 1/2, Y < 1/4]$
v. conditional distributions of X & Y

eg: RV's X and Y have joint pdf
$f(x,y) = k(x+y) \quad 0 \le x, y \le 1$
Find i. value of k
ii. marginal distributions of X & Y
iii. $P[X > Y]$, $P[Y < 1/2]$, $P[X < Y^2]$

JOINT MOMENTS, COVARIANCE, CORRELATION

Joint moments of 2 rvs X and Y gives the info about their joint behaviour.

$E[X^j Y^k]$ = jk^{th} joint moment of X and Y

If $j=0 \Rightarrow k^{th}$ moment of Y
If $k=0 \Rightarrow j^{th}$ moment of X

Camlin Page
Date / /

For $\boxed{j=k=1 \quad E(XY) \text{ is the correlation of } X \,\&\, Y}$

※ If $E(xy)=0$, we say X and Y are orthogonal

✓ $E[(X-\bar{X})^j (Y-\bar{Y})^k] \rightarrow jk^{th}$ joint central moment

$\left.\begin{array}{l}\bar{X} \,\&\, \mu_x \\ \bar{Y} \,\&\, \mu_y\end{array}\right\}$ are different

$\bar{X} \,\&\, \bar{Y} \rightarrow$ sample mean

$\mu_x \,\&\, \mu_y \rightarrow$ population mean

$j=0, k=2 \quad E(Y-\bar{Y})^2 \rightarrow$ variance (Y)

$j=2, k=0 \quad E(X-\bar{X})^2 \rightarrow$ variance (X)

$j=k=1 \quad E[(X-\bar{X})(Y-\bar{Y})] \rightarrow$ covariance of X & Y

$E(XY)-E(X)E(Y)$

Covariance of independent rv:

If X and Y are independant, then $E(XY)=E(X)E($

ie) covariance $(X,Y)=0$

※ Converse is not true

eg: Let X be a uniform rv over $(-1,1)$ and $Y=X$

$E(X) = \frac{a+b}{2} = \frac{-1+1}{2} = 0 \qquad pdf = \frac{1}{b-a} = \frac{1}{1-(-1)} = \frac{1}{2}$

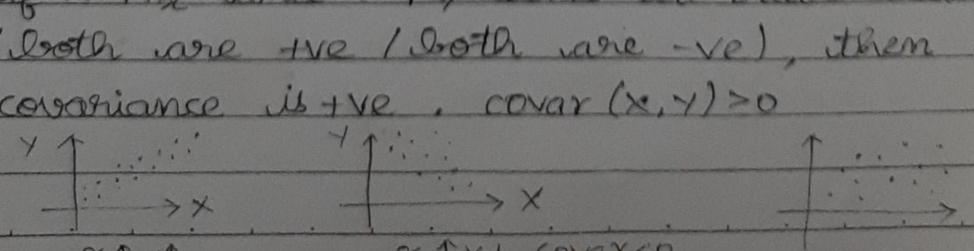
$E(X^3) = \int_{-1}^{1} x^3 \frac{1}{2}\, dx = 0 \qquad \because$ it is odd function

$cov(X,Y) = E(XY)-E(X)E(Y) = E(X^3)-E(X)E(Y)$

$= 0$

Covariance measures the deviations of the points (X,Y) from $E(X)=\mu_x$ & $E(Y)=\mu_y$

* If $X-\mu_x$ and $Y-\mu_y$ tend to have same sign (both are +ve / both are -ve), then covariance is +ve. covar $(x,y)>0$



covar >0 $\quad x\uparrow y\uparrow$ $\qquad x\uparrow y\downarrow$ covar <0 $\qquad$ covar $(x,y)=0$

Pranathi

* If $X-\mu_x$ and $Y-\mu_x$ tend to have opp. sign, then covariance is -ve. covar(X,Y)

H/W Question previous page

1\. i. $\iint f(x,y)\,dy\,dx = 1$

$$\int_0^1\int_x^1 kxy\,dy\,dx = 1$$

$$\int_0^1 kx\frac{y^2}{2}\Big|_x^1 dx = 1 \qquad \int_0^1 kx\,\frac{1-x^2}{2}\,dx = 1$$

$$\tfrac{1}{2}k\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 1$$

$$k/2\left[\tfrac{1}{2} - \tfrac{1}{4}\right] = 1 \qquad \frac{k}{2}\,\frac{1}{4} = 1$$

$$\boxed{k = 8}$$

ii. $f_x(x) = \int f(x,y)\,dy = \int_x^1 8xy\,dy$

$$= 8x\frac{y^2}{2}\Big|_x^1 = 8x\,\frac{1-x^2}{2} = 4x - 4x^3$$

$f_y(y) = \int f(x,y)\,dx = \int_0^y 8xy\,dx$

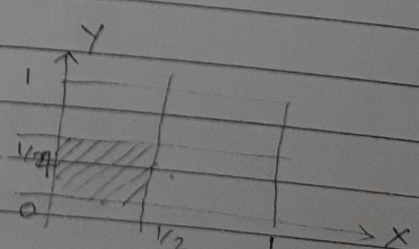
$$= \frac{8x^2}{2}y\Big|_0^y = 4y^3$$

iii. $f(x,y) \neq f_x(x)\,f_y(y)$

$8xy \neq (4x-4x^3)(4y^3)$

$\therefore$ X & Y are not independant

iv. $P[X < 1/2,\ Y < 1/4]$



$$= \int_0^{1/2}\int_0^{1/4} 8xy\,dy\,dx$$

$$= \int_0^{1/2} \frac{8xy^2}{2}\Big|_0^{1/4} dx$$

$$= \int_0^{1/2} 4x\left(\tfrac{1}{16}\right)dx = \frac{x^2}{8}\Big|_0^{1/2} = \frac{1}{8}\left(\frac{1}{4}\right) = \frac{1}{32}$$

Camlin Page

Date / /

v. $f_{Y|X=x}(Y|X) = \dfrac{f(x,Y)}{f_x(x)} = \dfrac{\cancel{8}xY}{\cancel{4}\cancel{x}(1-x^2)} = \dfrac{2Y}{1-x^2}$

$f_{X|Y=y}(X|Y) = \dfrac{f(x,Y)}{F_Y(Y)} = \dfrac{8xY}{4Y^3} = \dfrac{2x}{Y^2}$

2.5 i. $\int_0$