

Roll No:

(To be filled in by the candidate)

PSG COLLEGE OF TECHNOLOGY, COIMBATORE - 641 004**SEMESTER EXAMINATIONS, MODEL QP2****MSc - SOFTWARE SYSTEMS Semester: 3****20XW31 PROBABILITY AND STATISTICS****Duration : 3 Hours****Maximum Marks : 100****INSTRUCTIONS:**

1. Answer **ALL** questions. Each question carries 20 Marks.
2. Subdivision (a) carries 3 marks each, subdivision (b) carries 7 marks each and subdivision (c) carries 10 marks each.
3. Statistical table brought by the candidates shall be permitted.

1. a) Suppose A and B are events such that $P[A|B] = 0.2$ and $P[A|\bar{B}] = 0.3$ and $P[\bar{B}] = 0.2$. What is $P[A]$?
- b) i) If the events A and B are independent and the events B and C are independent, is it true that the events A and C are also independent? Why or why not? [3]
 ii) Alice and Bob go target shooting together. Both shoot at a target at the same time. Suppose Alice hits the target with probability 0.7, whereas Bob, independently, hits the target with probability 0.4. Given that exactly one shot hit the target, what is the probability that it was Bob's shot? Given that the target is hit, what is the probability that Bob hit it? [4]
- c) A company producing electric relays has three manufacturing plants producing 50%, 30%, and 20%, respectively, of its product. Suppose the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.
 (1) If a relay is selected at random from the output of the company, what is the probability that it is defective?
 (2) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?
2. a) From the interval (0, 1), five points are selected at random and independently. What is the probability that the first decimal point of exactly two of them is 3?
- b) i) Guests arrive at a hotel, in accordance with a Poisson process, at a rate of five per hour. Suppose that for the last 10 minutes no guest has arrived. What is the probability that the next one will arrive in less than 2 minutes. [3]
 ii) Consider the following game that involves tossing a fair die. If the outcome of a toss is an even number, you win \$2. If the outcome is 1 or 3, you lose \$1. If the outcome is 5, you lose \$3. What is the expected gain/loss? [4]
- c) (1) The heights of students in a college are normally distributed with mean 68 inches and standard deviation 3 inches. Find the probability that the height of a randomly selected student will be greater than 72 inches, between 65 and 71 inches. [5]
 (2) If the service life, in hours, of a semiconductor follows Weibull distribution with parameters $\alpha = 0.025$ and $\beta = 0.5$. What is the probability that the service life is more than 4000 hours? Find the mean service life of such semiconductors? [5]
3. a) Suppose that, for a discrete random variable X, $E(X) = 2$ and $E[X(X - 4)] = 5$. Find the variance of $-4X + 12$.
- b) i) Average occupancy of packets in the buffer of a switch in the Internet is known to be 40%. Buffer occupancy is a random variable X. Without knowing the PDF of X, what can be said about the probability that buffer occupancy equals or exceeds 60%? [3]
 ii) Find the moment generating function of geometric distribution. Hence find its mean. [4]

- c) Let X and Y be continuous random variables having joint density function given by

$$f(x, y) = \begin{cases} c(x^2 + y^2), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}. \text{ Determine the following:}$$

- (1) Value of c;
- (2) Marginal density functions of X and Y; Are X and Y independent?
- (3) $P[1/4 < X < 3/4]$ and $P[Y < 1/2]$.

4. a) Distinguish between point estimation and interval estimation.

- b) i) What do you mean level of significance in hypothesis testing? [3]

- ii) A random sample of 200 ball bearings made by a certain machine during one week showed a mean diameter of 0.824 inch and a standard deviation of 0.042 inch. Find 95% confidence limits for the mean diameter of all the ball bearings. [4]

- c) (1) The mean life (in hours) of a sample of 100 fluorescent light bulbs produced by a company is found to be 1570 with a standard deviation of 120. If μ is the mean lifetime of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ against the alternative hypothesis $\mu < 1600$, using a level of significance of 0.05. [5]

- (2) In order to determine the possible effect of a chemical treatment on the rate of germination of cotton seeds a pot culture experiment was conducted. The results are given as in Table.1. Use chi-square test at 5% LOS and test the hypothesis that the chemical treatment improves the germination rate of cotton seeds. [5]

	Germinated	Not germinated
Chemically treated	118	22
Untreated	120	40

Table.1

5. a) Prove: If the correlation coefficient between X and Y is -1, then $y = ax + b$ where a, b are constants and $a < 0$.

- b) i) The regression equations of two variables X and Y are $x + 2y = 5$ and $2x + 3y = 8$. Which one of these is the regression line of Y on X? What is the coefficient of correlation between X and Y? [3]

- ii) Suppose that random variables X_1, \dots, X_n are independent and from a Poisson distribution with mean λ . Assume that the prior distribution of λ is exponential with mean 1. Find the posterior distribution of λ when $\bar{x} = 3$ with $n = 10$. [4]

- c) i) A sample consisting of the values, 9.6, 13.5, 5.4, 7.9, 4.8, 5.1, -1.2, and 6.8 comes from a population with the density function $f(x, \theta) = \frac{1}{\theta \sqrt{2\pi}} e^{-\frac{x^2}{2\theta^2}}, -\infty < x < \infty$. Obtain the maximum likelihood estimator of the parameter, θ .

[OR]

- ii) The data in Table.3 give the resting pulse rates Y, in beats per minute, and the years of schooling X of 10 individuals. Obtain the regression line of Y on X. Also find the coefficient of correlation between X and Y.

X	12	16	13	18	19	12	18	19	12	14
Y	73	67	74	63	73	84	60	62	76	71

Table.3

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