Method 2.

$$M_x(t) = E(e^{tx})$$

$$= E\left(1 + \frac{tx}{1!} + \frac{t^2x^2}{2!} + \frac{t^3x^3}{3!} + \frac{t^r x^r}{r!} + \dots\right)$$

$$= 1 + \frac{t}{1!}E(x) + \frac{t^2}{2!}E(x^2) + \frac{t^3}{3!}E(x^3) + \dots \frac{t^r}{r!}(Ex^r)$$

the coefficient of $\frac{t^r}{r!}$ in the expansion of mgf is the r^{th} Moment.

$E(x^t) \Rightarrow r^{th}$ moment

$E(x-\mu)^r \Rightarrow r^{th}$ central moment

mgf $M_x(t) = E(e^{tx})$

Find the mgf of Binomial distribution with parameter n & Hence find the mean & variance.

Proof of Binomial distribution : $f(x) = {}^nC_x\, p^x q^{n-x}$ $x = 0 \dots n$

mgf $M_x(t) = E(e^{tx})$

$$= \sum e^{tx}\, nC_x\, p^x q^{n-x}, \quad x = 0 \dots n$$

$$= \sum nC_x\, (pe^t)^x q^{n-x}$$

WKT

$\sum nC_x\, p^x q^{n-x} = (p+q)^n$

$$\boxed{M_x(t) = (pe^t + q)^n}$$

$\cdot\, E(x) = M_x'(t)\Big|_{t=0} = n(pe^t + q)^{n-1} \cdot pe^t \Big|_{t=0}$

$$\boxed{M_x(t)\Big|_{t=0} = np}$$

$E(x^2) = M''_x(t)\Big|_{t=0} = \cancel{(np)'}\Big|_{t=0}$

$= np\left[(pe^t+q)^{n-1}e^t + e^t(n-1)(pe^t+q)^{n-2}pe^t\right]$

$= np\left[1+(n-1)p\right]$

$= np\left[1+np-p\right]$

$= np\left[q+np\right]$

$= npq + n^2p^2$

Variance $= E(x^2) - E(x)^2$

$= npq + n^2p^2 - n^2p^2$

$= npq$

Mean $= E(x)$

$= np$

Find the mgf of exponential distribution with parameter λ

$M_x(t) = E(e^{tx})$

$= \int_0^\infty e^{tx}\lambda e^{-\lambda x}dx$ $= \lambda\int e^{-(\lambda-t)x}dx$

$= \lambda\left[\frac{e^{-(\lambda-t)x}}{-(\lambda-t)}\right]_0^\infty$

$= \frac{\lambda}{-(\lambda-t)}\,[0-1]$

$= \frac{\lambda}{\lambda-t}$

$= \frac{\lambda}{\lambda-t}$

$E(x) = M_x'(t)\Big|_{t=0}$

$= \lambda\,\frac{-1}{(\lambda-t)^2}(-1)\Big|_{t=0}$

$= 1/\lambda$

$$E(x^2) = \lambda \frac{(-2)}{(\lambda - t)^3} (-1) \Big|_{t=0}$$

$$= \frac{2}{\lambda^3}$$

$$\text{Variance}(x) = E(x^2) - E(x)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$= \frac{1}{\lambda^2}$$

$$\boxed{\begin{array}{l} \text{Mean} = E(x) = 1/\lambda \\ \text{Variance} = E(x^2) - E(x)^2 = 1/\lambda^2 \end{array}}$$

$$M_x(t) = E(e^{tx})$$

$$= \sum e^{tx} \frac{e^{-\lambda} \lambda^x}{x!} \qquad x = 0, 1, 2$$

$$= e^{\lambda(e^t - 1)} \qquad = e^{-\lambda} \frac{\sum (e^t \lambda)^x}{x!}$$

$$= e^{-\lambda} e^{e^t \lambda} \Big|_0^2$$

$$= e^{\lambda(e^t - 1)}$$

$$E(x) = M_x'(t) \Big|_{t=0}$$

$$= e^{\lambda(e^t - 1)} \lambda e^t \Big|_{t=0}$$

$$= \lambda$$

Properties of mgf :

1) Two r.v x & y with same mgf have same probability distribution

$\Rightarrow$ mgf of a r.v is unique

If $M_x(t) = M_y(t)$

2) $M_x(0) = 1$

3) mgf of $cx = M_{cx}(t)$ In mgf of x, replace t by ct

$E(x) = \mu$

$E(cx) = c\,E(x)$

$\mu_x(t) = E(e^{tx})$

$\mu_{c+x}(t) = E[e^{(t(c+x))}]$

$= E[e^{ct} \cdot e^{xt}]$

$= e^{ct} \mu_x(t)$

$\mu_{c+x}(t) = e^{ct} \mu_x(t)$

Q) mgf of the sum of 2 independent rvs.

If x & y are independent $\Rightarrow \mu_{x+y}(t) = \mu_x(t) \cdot \mu_y(t)$

$\mu_{x+y}(t) = E(e^{t(x+y)})$

$= E[e^{tx} e^{ty}]$

$= E(e^{tx}) E(e^{ty})$ .... x & y are independent

$= \mu_x(t) \cdot \mu_y(t)$

Q) using mgf of S ND find mgf of ND with mean μ & SD σ mgf of SND $M_x(t) = e^{t^2/2}$ & $z = \frac{x-\mu}{\sigma}$

$x = \mu + z\sigma$

mgf of ND $M_x(t) = M_{\mu+\sigma x}(t)$

$= e^{\mu t} M_{\sigma z}(t)$ - using prob. $M_{x+c}(t) = e^{ct} M_x(t)$

$= e^{\mu t} M_x(\sigma t)$ using $M_{cx}(t) = M_x(ct)$

$= e^{\mu t} e^{(\sigma t)^2/2}$

$M_x(t) = e^{\mu t} + e^{\sigma^2 t^2/2}$.

$E(x) = M_x'(t)_{t=0} = e^{\mu t + \sigma^2 t^2/2}$

$= \mu \leftarrow (\mu + \sigma^2 t)\big|_{t=0}$

Q) Let x be a r.v with mgf $M_x(t) = e^{2t^2}$ with mgf of $N(\mu, \sigma)$

$\mu = 0$ & $\frac{\sigma^2}{2} \Rightarrow \sigma^2 = 4$

$\sigma = 2$

$x \sim N(\mu=0, \sigma=2) \Rightarrow z = \frac{x-\mu}{\sigma} = \frac{x-0}{2}$

$P[0 < x < 1] = P[0 < z < 1/2] = \phi(0.5) - \phi(0)$

$= 0.6915 - 0.5 = 0.1915$

Q) Let x - rv with pmf $p(x) = 2(1/3)^x$, $x = 1, 2, \ldots$ Find mgf of x & $E(x)$

$M_x(t) = E[e^{tx}]$

$= \Sigma e^{tx} 2(1/3)^x$, $x = 1, 2 \ldots$

$= 2 \Sigma (e^t/3)^x$

$= 2\left\{ \frac{e^t}{3} + \left(\frac{e^t}{3}\right)^2 + \cdots \right\}$

$= 2\left\{ \frac{1}{1-e^t/3} - 1 \right\}$

$= 2\left(\frac{3}{3-e^t} - 1 \right)$

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Sum of independent random variables

If $x_1, \dots x_n$ are independent rvs with mgfs $M_{x_1}(t) \dots M_{x_n}(t)$, then the mgf of $X = x_1 + \dots + x_n$ is

$$M_X(t) = M_{x_1}(t) + \dots M_{x_n}(t)$$

1) Sum of 2 independent ~~random~~ normal random variable is a normal random variable

<u>Proof</u>

let $x_1 \sim N(\mu_1, \sigma_1)$

$x_2 \sim N(\mu_2, \sigma_2)$

Normal of mgf

$$M_x(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$$

let $x = x_1 + x_2$

$$M_x(t) = M_{x_1}(t) \cdot M_{x_2}(t)$$

$$= e^{\mu_1 t + \frac{\sigma_1^2 t}{2}} \cdot e^{\mu_2 t + \frac{\sigma_2^2 t}{2}}$$

$$= \boxed{e^{(\mu_1 + \mu_2)t + \frac{(\sigma_1^2 + \sigma_2)^2 t}{2}}}$$

This is in the form of Normal of mgf

x is normal with mean $\mu_1 + \mu_2$

& variance $\sigma_1^2 + \sigma_2^2$

2) If $x_1 \dots x_n$ are independent binomial random variables with parameters $(n_1, p) \dots\dots (n_k, p)$

then $x_1 + \dots x_n$ is binomial with parameters $(n_1 + \dots n_k, p)$

(Becos p is a common parameter for all x's)

Proof:

mgf of $x = x_1 + \dots + x_n$

mgf of Binomial $(q + pe^t)^n$

$$M_x(t) = (q + pe^t)^{n_1} \times \dots \times (q + pe^t)^{n_k}$$

$$= (q + pe^t)^{n_1 + \dots n_k}$$

3) If $x_1 \dots x_k$ are independent poisson random variables with parameters $\lambda_1 \dots \lambda_k$ then $x_1 + \dots x_k$ is a poission random variable with mean $\lambda_1 + \dots + \lambda_k$

Proof:

mgf of poisson $e^{\lambda(e^t - 1)}$

$$M_x(t) = e^{\lambda_1 (e^t - 1)} \times \dots e^{\lambda_k (e^t - 1)}$$

$$= e^{\underbrace{(\lambda_1 + \dots \lambda_k)}_{\lambda \text{ value}} (e^t - 1)}$$

ERLONG / GAMMA distribution.

ERLONG destribution

exponential distribution describes the time b/w successive events

Generalization of expo distribution

While expo distribution describes the time b/w successive events, the k erlong random variable describes the time b/w any and the kth occurence of the event.

exponential x : Time for the next arrival

(Time b/w one arrival & next arrival)

Erlong y : Time for the K^{th} arrival

eg:

Time for 4^{th} arrival.

y : 4 Erlong.

<u>Pdf</u>

$$\boxed{f(x) = \frac{\lambda e^{-\lambda x} (\lambda x)^{K-1}}{\Gamma(K)} \quad , x \geq 0}$$

parameters λ & K

when $K = 1$ -> erlong distribution reduces to exponential distribution with parameter λ

Be os,

y = 1 Erlong = exponential distribution

<u>Mean & variance</u>.

Mean $E(x) = \frac{K}{\lambda}$

$var(x) = K/\lambda^2$

Erlong is for K^{th} arrival.

expo is for 1^{st} arrival.

For expo distribution

$E(x) = 1/\lambda$

$var(x) = 1/\lambda^2$

The generalized form of Erlang dis: is called gamma distribution

For any real number r, a continuous r.v, X follows gamma ~~differentiation~~ distribution is its pdf is given by

pdf

$$\boxed{f(x) = \frac{\lambda e^{-\lambda x} (\lambda x)^{r-1}}{\Gamma(r)}, \quad x \geq 0}$$

$X \sim \text{Gamma}(r, \lambda)$

Mean $E(X) = r/\lambda$

variance $= r/\lambda^2$

sum of ~~all~~ exponential r.v is Erlang ~~r~~v

mgf of Exponential r.v $= \dfrac{\lambda}{\lambda - t}$

mgf of r - Erlong $= \left(\dfrac{\lambda}{\lambda - t}\right)^r$

To find mgf of r-erlong distribution.

pdf of r- Erlong

$$f(x) = \frac{\lambda e^{-\lambda x} \cdot (\lambda x)^{r-1}}{\Gamma(r)}, \quad x \geq 0$$

mgf ~~of~~ $M_x(t) = E(e^{tx})$

$$= \int_0^\infty e^{tx} \frac{\cdot \lambda e^{-\lambda x} (\lambda x)^{r-1}}{\Gamma(r)} dx.$$

$$= \frac{\lambda^r}{\Gamma(r)} \int_0^\infty e^{-(\lambda-t)x} x^{r-1} dx$$

WKT

$$\int_0^\infty e^{-ax} x^{n-1} dx = \frac{\Gamma(n)}{a^n}$$

$$= \frac{\lambda^r}{\Gamma(r)} \cdot \frac{\Gamma(r)}{(\lambda-t)^r}$$

$$= \left(\frac{\lambda}{\lambda-t}\right)^r$$

Sum of r exponential r.v with parameter λ is r- Erlong r.v ~~is the~~ with parameter r and λ

$$X = x_1 + \dots + x_r$$

Proof:

$$M_x(t) = M_{x_1}(t) + \cdots M_{x_r}(t)$$

$$= \left(\frac{\lambda}{\lambda-t}\right) \times \cdots \times \left(\frac{\lambda}{\lambda-t}\right)$$

$$= \left(\frac{\lambda}{\lambda-t}\right)^r .$$

Mgf of r Erlong $= \left(\frac{\lambda}{\lambda-t}\right)^r$

Joint probability distribution

When ~~an~~ 2 or more random variable defined on the ~~joint~~ same sample space.

consider,

$x \to$ height of a student

$y \to$ weight of the same student.

$f_{xy}(x, y) \to$ joint pdf

WKT

$F(x) = P[X \leq x]$

$P_{xy}(x, y) = P[X = x, Y = y] \to$ joint pmf

$F_{xy}(x, y) = P[X \leq x, Y \leq y] \to$ joint cdf.

The individual random variable.

The individual prob distr of rv x & y are called marginal distributions denoted by $f_x(x), f_y(y)$

Marginal distributions of x and y

Let $P_{xy}(x, y)$ be the joint prob of x and y.

then marginal distribution of x

$$P_x(x) = \sum_{y \in Y} P_{xy}(x, y)$$

$$P_y(y) = \sum_{x \in X} P_{xy}(x, y)$$

Eg:

2 dice are rolled. Let x denotes max of the 2 throws, y denotes the no. of times an even no appears. Find the joint pmf of x and y. Also find the marginal distribution of x & y.

$y = \{0, 1, 2\}$

$x = \{1, 2, 3, 4, 5, 6\}$

(x, y)

	x \ y	0	1	2	Pdf	
(1,1)	1	1/36	0	0	1/36	$P_x(1)$
(1,2) (2,1) (2,2)	2	0	2/36	1/36	3/36	$P_x(2)$
(3,1) (3,3) (1,3) (3,2) (2,3)	3	3/36	2/36	0	5/36	$P_x(3)$
(4,1) (1,4) (4,3) (3,4) (4,4) (4,2) (2,4)	4	0	4/36	3/36	7/36	$P_x(4)$
	5	5/36	4/36	0	9/36	$P_x(5)$
11	6	0	6/36	5/36	11/36	$P_x(6)$
		9/36	18/36	9/36	$\boxed{1}$	
		$P_y(0)$	$P_y(1)$	$P_y(2)$		

(3,6) (5,6) (6,1) (6,3) (6,5) (1,6) (6,6) (2,6) (6,2) (4,6) (6,4)

$x \Rightarrow$ max of 2 throws

$y \Rightarrow$ no. of times an even no. appears

This is ~~table~~ Pmf of x, y.

pdf here is the marginal probability distribution of x

$P_y(0)$, $P_y(1)$, $P_y(2)$ is the marginal distribution of y

Marginal distribution of x

x	1	2	3	4	5	6
$P_x(x)$	$1/36$	$3/36$	$5/36$	$7/36$	$9/36$	$11/3$

Marginal distribution of y

y	0	1	2
$P_y(y)$	$9/36$	$18/36$	$9/36$

Properties of joit pmf / pdf.

Pmf

→ $P_{xy}(x,y) \geq 0$

→ $\sum_{x \in X} \sum_{y \in Y} p_{xy}(x,y) = 1$

Pdf

→ $f_{xy}(x,y) \geq 0$

→ $\int \int f_{xy}(x,y)\, dy dx = 1$

If $f_{xy}(x,y)$ is the joint pdf of x & y then

marginal distribution of x : $f_x(x) = \int f(x,y)\, dy$

marginal distribution of y : $f_y(y) = \int f(x,y)\, dx$

Note

If x & y are independent rv

$f(x,y) = f_x(x)\, f_y(y)$

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Conditional probability ·

Conditional distribution of Y given X = x

For 2 events A, B

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad , \; P(B) \neq 0$$

For 2 random variable X & Y with joint pmf / pdf

f(x, y), the conditional distribution of Y given X = x

is given by

$$\boxed{f_{Y|X=x}(y|x) = \frac{f(x,y)}{f_x(x)} \quad , \; f_x(x) \neq 0}$$

function of y

Conditional expectation & variance

the conditional expectation of Y given X = x is

given by

// If X & Y are discrete

$$E[Y|X=x] = \sum_{y\in Y} y \cdot f_{Y|x}(y|x)$$

value sum of the random variable and conditional probability

// If X & Y are continuous

$$E[Y|X=x] = \int y \, f_{y|x}(y|x) \, dy$$

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$$E[y^2 \mid X = x] = \sum y^2 f_{y|x}(y^2 \mid x)$$

Variance

$$Var[Y \mid X = x] = E[Y^2 \mid X = x] - [E[Y \mid X = x]]^2$$

Result

For 2 r.v X & Y with joint pmf / pdf $F(x,y)$

$$\boxed{E[E(X|Y)] = E(X)}$$

proof :

LHS

in terms of y

$$E[E(X|Y)] = \int_{-\infty}^{\infty} E(X|Y) \, f_Y(y) \, dy$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \, \boxed{f_{x|y}(x|y) \, f_y(y)} \, dy$$

WKT

$$f_{x|y}(x|y) = \frac{f(x,y)}{f_y(y)}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x \cdot f(x,y) \, dx \, dy$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \, dy \; x \, dx$$

$$= \int_{-\infty}^{\infty} f_x(x) \; x \, dx$$

$$= E(X)$$

the joint pdf of r.v's x and y is

$$f(x,y) = \begin{cases} kxy^2 & 0 \le x \le y \le 1 \\ 0 & o/w \end{cases}$$

Find i) the value of k

ii) Marginal distribution of x and y

iii) $E(x)$ and $E(y)$

iv) if x & y are independent

$$f(x,y) = f_x(x)\, f_y(y)$$

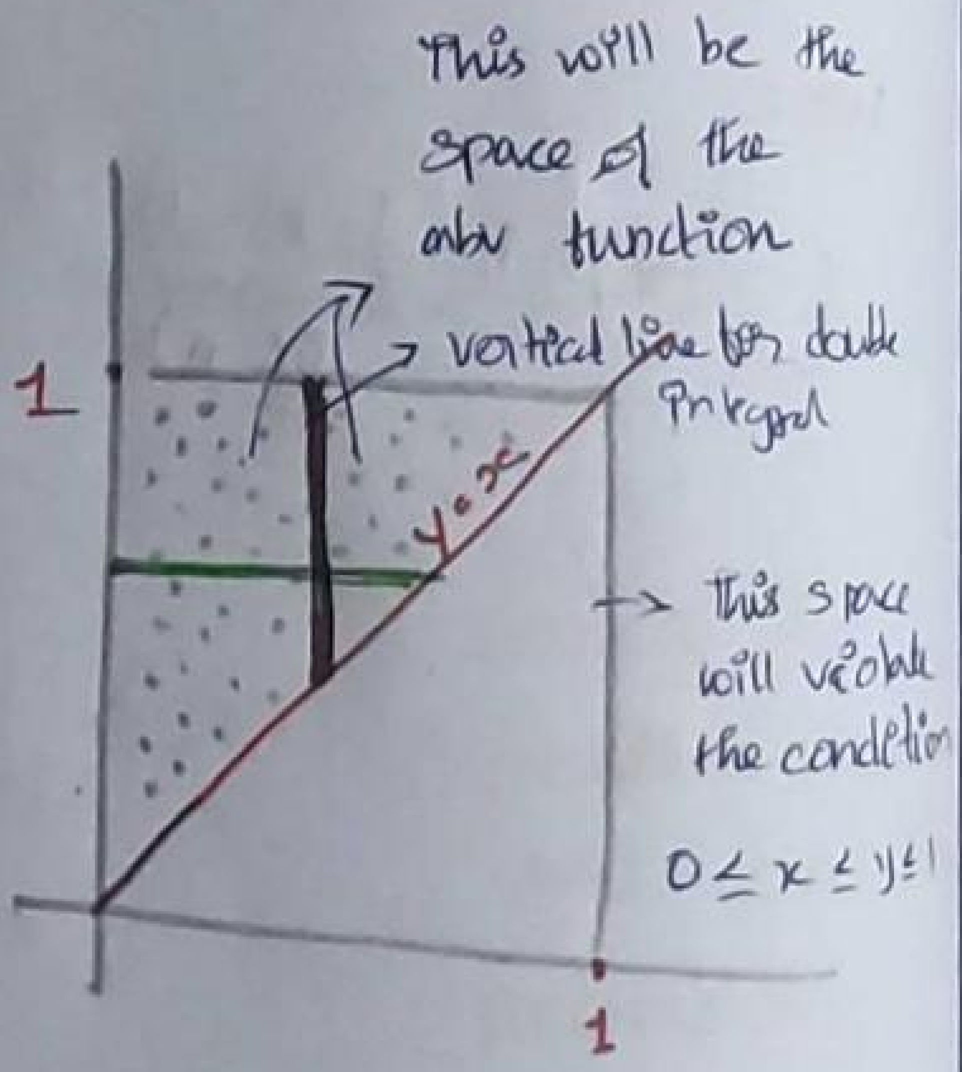
$$E(xy) = E(x)\, E(y)$$

$$var(x+y) = var(x) + var(y) + 2 \cdot$$

i) For joint pdf

with respect to y

$$\boxed{\iint f(x,y)\, dy\, dx = 1}$$



$$\int_0^1 \int_x^1 kxy^2\, dy\, dx = 1$$

$$\int_0^1 kx \left[\frac{y^3}{3}\right]_x^1 dx = 1$$

$$\frac{k}{3} \int_0^1 (x - x^4)\, dx = 1$$

$$\frac{k}{3} \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = 1$$

$$\frac{k}{3} \left[\left(\frac{1}{2} - \frac{1}{5}\right) - 0\right] = 1$$

$$\boxed{k = 10}$$

$$\boxed{\iint f(x,y)\,dx\,dy = 1}$$ with respectto ...

$$\int_0^1 \int_0^y kxy^2\,dx\,dy = 1$$

$$\int_0^1 k y^2 \left[\frac{x^2}{2}\right]_0^y dy = 1$$

$$\frac{k}{2}\int_0^1 y^4\,dy = 1$$

$$\frac{k}{2}\left[\frac{y^5}{5}\right]_0^1 = 1$$

$$\frac{k}{2}\left[\frac{1}{5}\right] = 1$$

$$\boxed{k = 10}$$

Marginal distribution of x with respect to y

$$f_x(x) = \int_x^1 f(x,y)\,dy$$

Marginal distribution of $\cancel{y}$ with respect to x

$$f_Y(y) = \int_0^y f(x,y)\,dx$$

$$f_x(x) = \int f(x,y)\,dy$$

$$= \int_x^1 10xy^2\,dy$$

$$= 10x\left[\frac{y^3}{3}\right]_x^1$$

$$= \frac{10}{3}x(1-x^3)$$

$$f_x(x) = \frac{10}{3}x(1-x^3) \quad 0 \le x \le 1$$

$$f_y(y) = \int f(x,y)\,dx$$

$$= \int_0^y 10 x y^2\, dx$$

$$= 5y^4$$

iii]

$$E(X) = x f_x(x)\,dx$$

$$= \int_0^1 x\left(\frac{10}{3} x (1-x^3)\right) dx$$

$$= \frac{10}{3} \int_0^1 (x^2 - x^5)\,dx$$

$$= \frac{10}{3}\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_0^1$$

$$= \frac{10}{3}\left[\frac{1}{3} - \frac{1}{6} - 0\right]$$

$$= \frac{10}{3} \times \frac{1}{6} = \frac{5}{9}$$

$$E(Y) = \int Y f_Y\, y\, dy$$

$$= \int_0^1 y\, 5y^4\, dy$$

$$= 5/6.$$

iv] If x & y are independent

WKT $f(x,y) = f_x(x)\, f_y(y)$

But here

$$f_x(x)\, f_y(y) = \frac{10}{3} x(1-x^3)\, 5y^4$$

$$\neq f(x,y)$$

So they are not dependent

$$E(x \cdot y) = \iint xy\, f(x,y)\, dy\, dx$$

$$= \int_0^1 \int_x^1 xy\, 10x\, y^3\, dy\, dx.$$

$$= 10 \int_0^1 x^2 \left[\frac{y^4}{4}\right]_x^1 dx$$

$$= 10 \int_0^1 x^2 \left[\frac{1}{4} - \frac{x^4}{4}\right] dx$$

$$= \frac{10}{4} \int_0^1 x^2(1-x^4)\, dx$$

$$= \frac{5}{2}\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{5}{\cancel{2}} \times \frac{\cancel{4}^{\,2}}{21}$$

$$= \frac{10}{21}$$

Then $E(X) \cdot E(Y) = \frac{5}{9} \cdot \frac{5}{6}$

$$= \frac{25}{54} \neq E(xy)$$

Not independent

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$$f(x,y) = \begin{cases} xy^2 + \underline{x^2} & 0 \le x \le 2,\ 0 \le y \le 1 \\ 0 & O/\omega \end{cases}$$

<u>Find</u>

(i) $P[X > 1]$

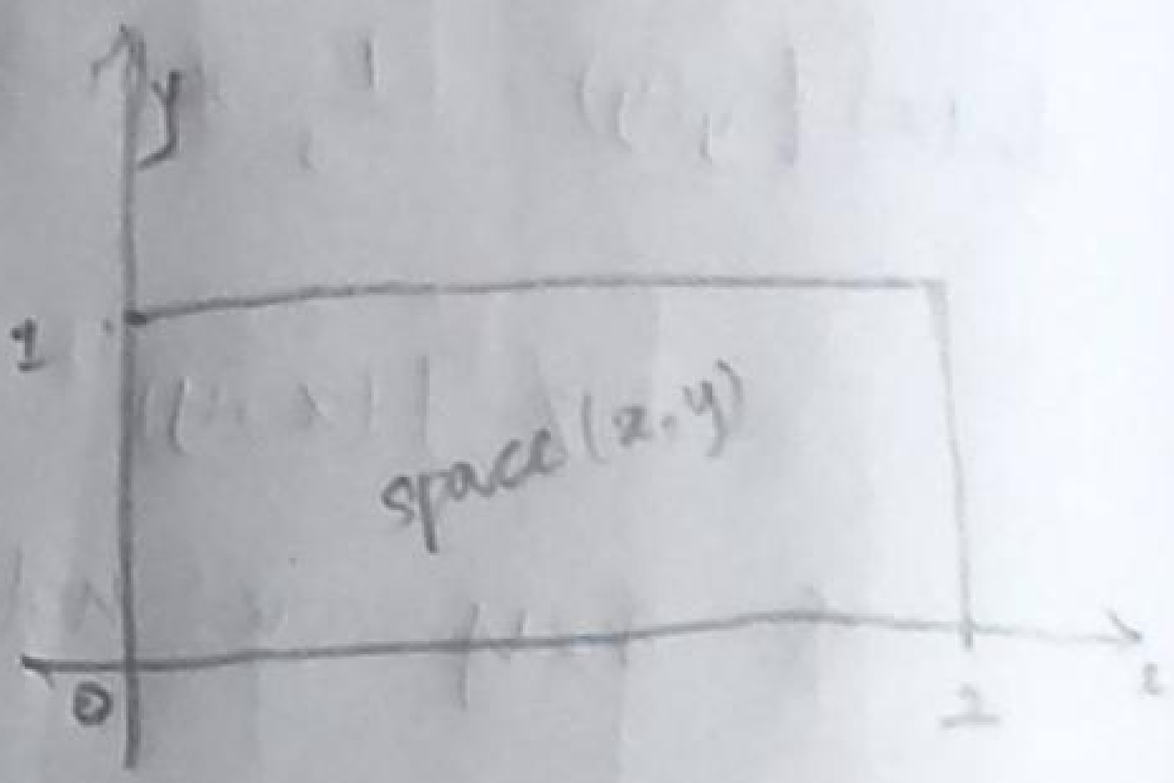
(ii) $P[Y < 1/2]$

(iii) $P[X > 1, Y < 1/2]$

(iv) $P[X < Y]$

(v) $P[Y < 1/2 \mid X > 1]$

(vi) $P[X + Y \le 1]$



$$f_X(x) = \int f(x,y)\,dy$$

$$= \int_0^1 \left(xy^2 + \frac{x^2}{8}\right) dy$$

$$= \left[\frac{xy^3}{3} + \frac{x^2 y}{8}\right]_0^1$$

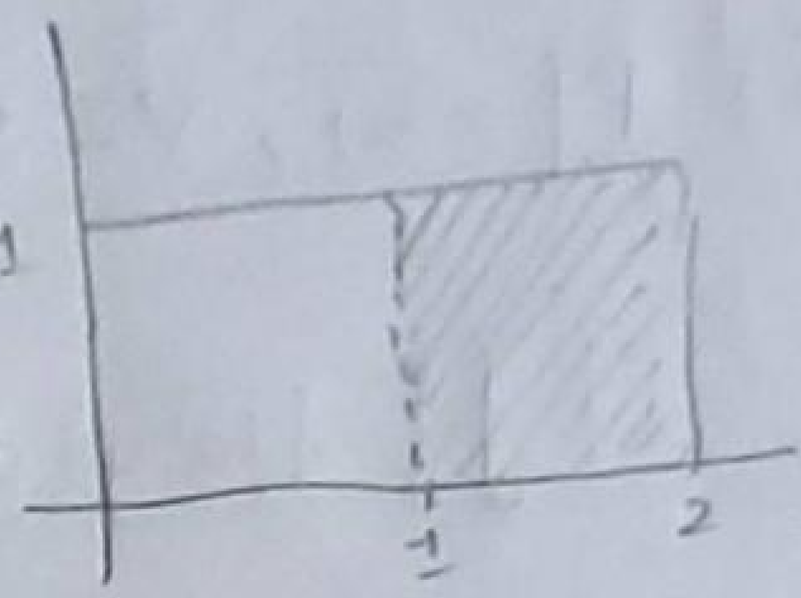
$$= \frac{x}{3} + \frac{x^2}{8},\ 0 \le x \le 2.$$

$$= \frac{8x + 3x^2}{24}$$

$$f_Y(y) = \int f(x,y)\,dx$$

$$= \int 2y^2 + \frac{1}{3}.$$

(i) $P[x>1]$



$$= \int_1^2 f_x(x)\,dx$$

$$= \int_1^2 \left(\frac{x^2}{3} + \frac{x^2}{8}\right) dx$$

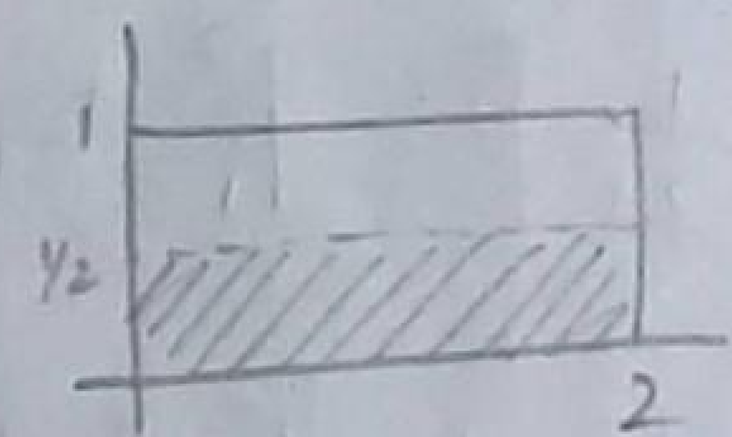
$$= \frac{1}{3}\left[\frac{x^2}{2}\right]_1^2 + \frac{1}{8}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{6}(4-1) + \frac{1}{24}[8-1]$$

$$= \frac{1}{\cancel{6}_2}(\cancel{3}^1) + \frac{1}{24}(7)$$

$$= \frac{12+7}{24} = \frac{19}{24}$$

(ii) $P[y < 1/2]$



$$= \int_0^{1/2} f_y(y)\,dy$$

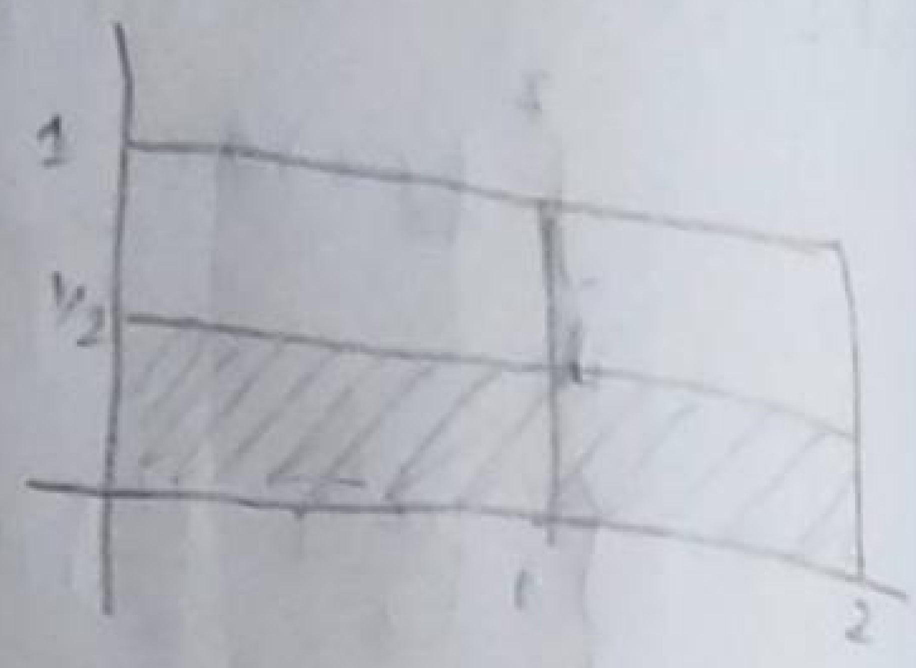
$$= \int_0^{1/2} \left(2y^2 + \frac{1}{3}\right) dy$$

$$= \left[\frac{2y^3}{3} + \frac{y}{3}\right]_0^{1/2}$$

$$= \frac{1}{12} + \frac{1}{6} - 0$$

$$= \frac{3}{12} = 1/4.$$

(iii) $P[x>1, Y<1/2]$



$$= \iint f(x,y)\, dy\, dx$$

$$= \int_1^2 \int_0^{1/2} \left(xy^2 + \frac{x^2}{8}\right) dy\, dx$$

$$= \int_1^2 x\left[\frac{y^3}{3}\right]_0^{1/2} + \left[\frac{x^2 y}{8}\right]_0^{1/2} dx$$

$$= \int^2 \frac{x}{3}\left[\frac{1}{8}\right] + \frac{x^2}{8}\left[\frac{1}{2}\right] dx.$$

$$= \int_1^2 \left[\frac{x}{24} + \frac{x^2}{16}\right] dx$$

$$= \frac{1}{24}\left[\frac{x^2}{2}\right]_1^2 + \frac{1}{16}\left[\frac{x^3}{3}\right]_1^2$$

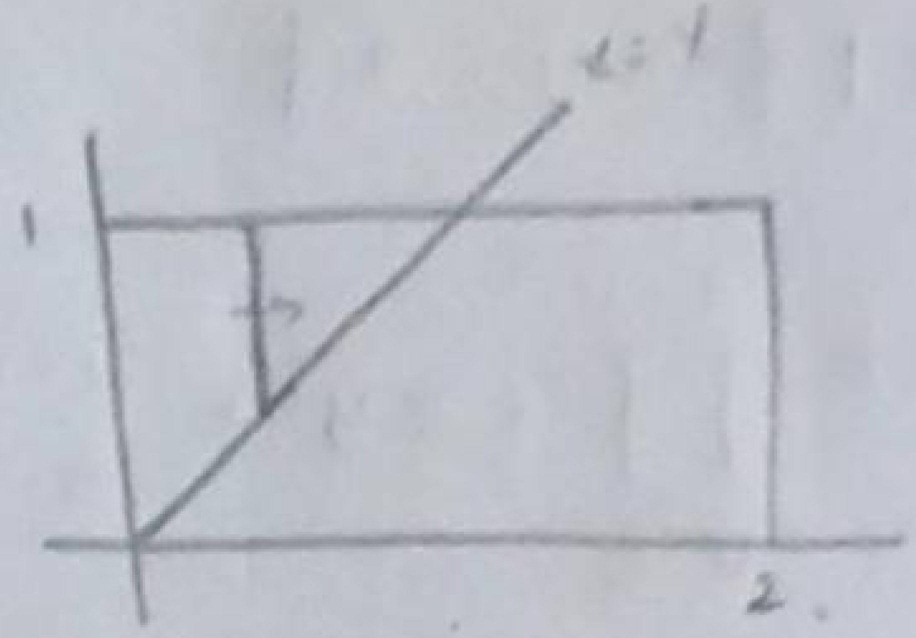
$$= \frac{1}{24 \cdot 2}[4]-[1] + \frac{1}{16\times 3}[8-1]$$

$$= \frac{1}{24 \cdot 2}(3) + \frac{1}{16\times 3}(7)$$

$$= \frac{3}{48} + \frac{7}{48}$$

$$= \frac{10}{48} = \frac{5}{24}$$

iv) $P[X < Y]$



$$= \iint f(x,y)\, dy\, dx$$

$$= \int_0^1 \int_x^1 \left(xy^2 + \frac{x^2}{8}\right) dy\, dx$$

$$= \int_0^1 \left[\frac{xy^3}{3} + \frac{x^2 y}{8}\right]_x^1 dx$$

$$= \int_0^1 \left[\frac{x}{3} + \frac{x^2}{8}\right] - \left[\frac{x^4}{3} + \frac{x^3}{8}\right] dx$$

$$= \int_0^1 \left[\frac{x}{3} + \frac{x^2}{8} - \frac{x^4}{3} - \frac{x^3}{8}\right] dx$$

$$= \int_0^1 \left[\frac{8x + 3x^2 - 8x^4 - 3x^3}{24}\right] dx$$

$$= \left[\frac{x^2}{6} + \frac{x^3}{24} - \frac{x^5}{15} - \frac{x^4}{32}\right]_0^1$$

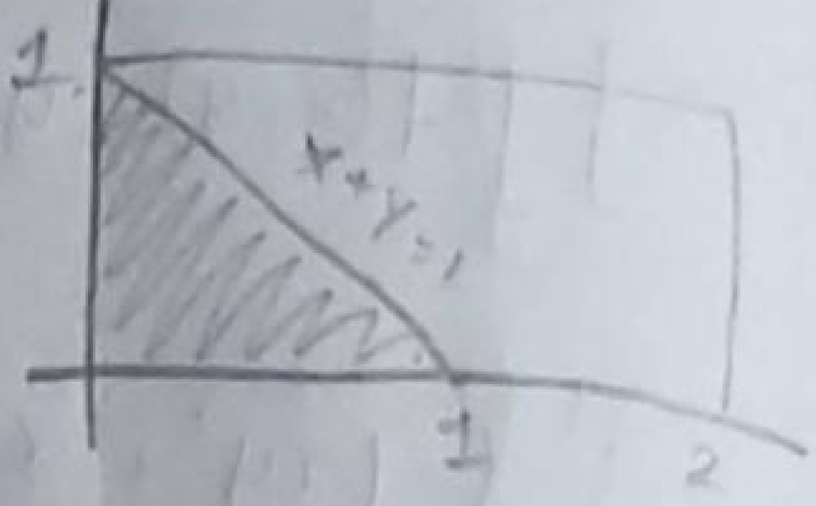
$$= \left[\frac{1}{6} + \frac{1}{24} - \frac{1}{15} - \frac{1}{32} - 0\right] = \frac{80 + 20 - 32 - 15}{480} = \frac{53}{480}$$

v) $P[y < 1/2 \mid x > 1]$

$$= \frac{P[x > 1,\ y < 1/2]}{P[x > 1]}$$

$$= \frac{\frac{5}{24}}{\frac{19}{24}} = \frac{5}{\cancel{24}} \times \frac{24}{19} = \frac{5}{19}$$

vi) $P[x+y \le 1]$



$$= \int_0^1 \int_0^{1-x} \left(xy^2 + \frac{x^2}{8} \right) dy\, dx$$

$$= \int_0^1 \left[\frac{xy^3}{3} + \frac{x^2 y}{8} \right]_0^{1-x} dx$$

$$= \int_0^1 \left[\frac{x(1-x)^3}{3} + \frac{x^2(1-x)}{8} \right] dx$$

$$= \int_0^1 \left[\frac{x(1-3x+3x^2-x^3)}{3} \right] dx + \int_0^1 \frac{x^2-x^3}{8}\, dx$$

$$= \frac{1}{3} \int_0^1 (x-3x^2+3x^3-x^4)\, dx + \frac{1}{8} \int_0^1 (x^2-x^3)\, dx$$

$$= \frac{1}{3} \left[\frac{x^2}{2} - \frac{3x^3}{3} + \frac{3x^4}{4} - \frac{x^5}{5} \right]_0^1 + \frac{1}{8} \left[\frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{1}{3} \left[\frac{1}{2} - 1 + \frac{3}{4} - \frac{1}{5} \right] + \frac{1}{8} \left[\frac{1}{3} - \frac{1}{4} \right]$$

$$= \frac{1}{3} \left[\frac{10-20+15-4}{20} \right] + \frac{1}{8} \left[\frac{1}{3} - \frac{1}{4} \right]$$

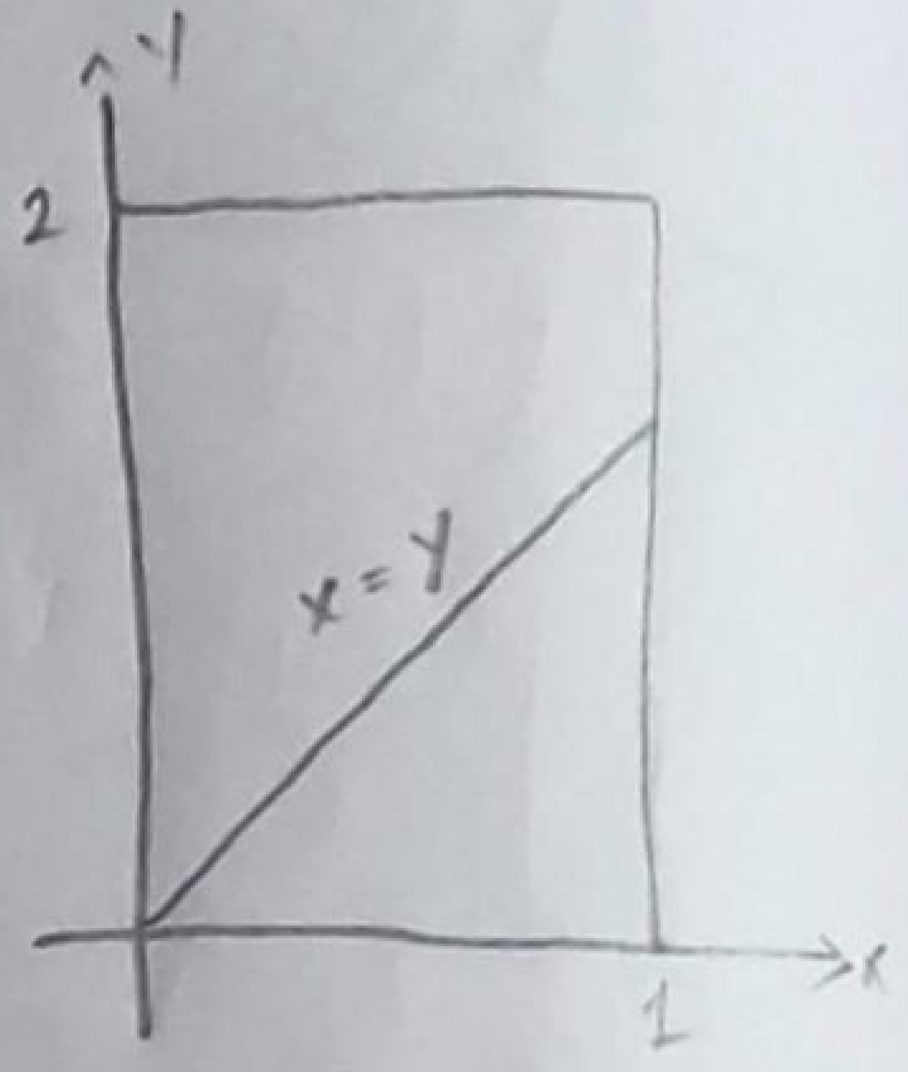
$$= \frac{1}{3} \left[\frac{1}{20} \right] + \frac{1}{8} \left[\frac{1}{12} \right] = \frac{1}{3} \cdot \frac{1}{20} + \frac{1}{96}$$

$$= \frac{96+60}{96 \times 60} = \frac{13}{480}.$$

$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right)$, $0 < x < 1$, $0 < y < 2$

Find (i) $P(X > Y)$

(ii) $P(Y > 1/2 , x < 1/2)$



$x = 0$ to 1

$y = 0$ to x

(i) $P[X>Y] = \int_0^1 \int_0^x \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\, dx$

$$= \int_0^1 \frac{6}{7}\left[x^2 y + \frac{xy^2}{4} \right]_0^x . dx$$

$$= \int_0^1 \frac{6}{7}\left[x^3 + \frac{x^3}{4} \right] dx$$

$$= \frac{6}{7} \int_0^1 \frac{5x^3}{4}\, dx$$

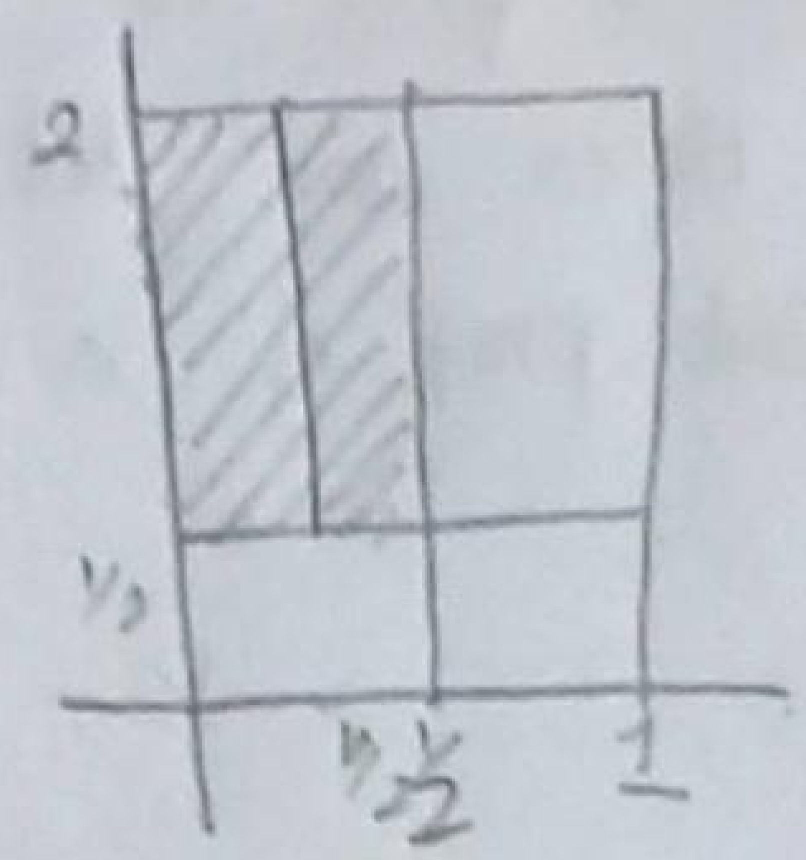
$$= \frac{6}{7} \times \frac{5}{4} \left[\frac{x^4}{4} \right]_0^1$$

$$= \frac{30}{28} \left[\frac{1}{4} \right]$$

$$= \frac{15}{56}$$

$P[X>Y] = 15/56$

i) $P[y > 1/2 , x < 1/2]$



$$= \int_0^{1/2}\int_{1/2}^{2} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dy\, dx$$

$$= \int_0^{1/2} \frac{6}{7} \left[x^2y + \frac{xy^2}{4} \right]_{1/2}^{2} dx$$

$$= \int_0^{1/2} \frac{6}{7} \left[2x^2 + \frac{4x}{4} \right] - \left[\frac{x^2}{2} + \frac{x}{16} \right] dx$$

$$= \frac{6}{7}\int_0^{1/2} \left[2x^2 + x - \frac{x^2}{2} - \frac{x}{16} \right] dx$$

$$= \frac{6}{7} \int_0^{1/2} \left[\frac{4x^2 - x^2}{2} + \frac{16x - x}{16} \right] dx$$

$$= \frac{6}{7} \int_0^{1/2} \left[\frac{3x^2}{2} + \frac{15x}{16} \right] dx$$

$$= \frac{6}{7} \left[\frac{\cancel{3}x^3}{\cancel{3}\cdot 2} + \frac{15x^2}{16\cdot 2} \right]_0^{1/2}$$

$$= \frac{6}{7} \left[\frac{1}{16} + \frac{15}{32\cdot 4} \right]$$

$$= \frac{6}{7} \left[\frac{8+15}{128} \right] = \frac{\cancel{6}^{\,3}}{7} \left[\frac{23}{\cancel{128}_{\,64}} \right] = 69/448$$

Roll a fair die let the o/c be x, they toss a fair coin x times and let y denotes the no. of tails. Find the joint pmf of x & y also find the marginal pmf of x, y

$X = \{1, 2, 3, 4, 5, 6\}$

$Y = \{0, 1, 2, 3, 4, 5, 6\}$

$x \backslash y$	0	1	2	3	4	5	6	$P_x(x)$
1	$\frac{1}{6}\cdot\frac{1}{2}$	$\frac{1}{6}\cdot\frac{1}{2}$	0	0	0	0	0	1/6
2	$\frac{1}{6}\cdot\frac{1}{4}$	$\frac{1}{6}\cdot\frac{2}{4}$	$\frac{1}{6}\cdot\frac{1}{4}$	0	0	0	0	1/6
3	$\frac{1}{6}\cdot\frac{1}{8}$	$\frac{1}{6}\cdot\frac{3}{8}$	$\frac{1}{6}\cdot\frac{3}{8}$	$\frac{1}{6}\cdot\frac{1}{8}$	0	0	0	1/6
4	$\frac{1}{6}\cdot\frac{1}{16}$	$\frac{1}{6}\cdot\frac{4}{16}$	$\frac{1}{6}\cdot\frac{6}{16}$	$\frac{1}{6}\cdot\frac{4}{16}$	$\frac{1}{6}\cdot\frac{1}{16}$	[illegible]	0	1/6
5	$\frac{1}{6}\cdot\frac{1}{32}$	$\frac{1}{6}\cdot\frac{5}{32}$	$\frac{1}{6}\cdot\frac{10}{32}$	$\frac{1}{6}\cdot\frac{10}{32}$	$\frac{1}{6}\cdot\frac{5}{32}$	$\frac{1}{6}\cdot\frac{1}{32}$	0	1/6
6	$\frac{1}{6}\cdot\frac{1}{64}$	$\frac{1}{6}\cdot\frac{6}{64}$	$\frac{1}{6}\,{}^6C_2\,\frac{1}{32}$	$\frac{1}{6}\,{}^6C_3\,\frac{1}{32}$	$\frac{1}{6}\,{}^6C_4\,\frac{1}{32}$	$\frac{1}{6}\cdot\frac{6}{64}$	$\frac{1}{6}\cdot\frac{1}{64}$	1/6
	63/384	$\frac{120}{384}$	$\frac{90}{384}$	$\frac{64}{384}$	$\frac{29}{384}$	$\frac{8}{384}$	$\frac{1}{384}$	

$$P(x, y) = \frac{1}{6} p^y q^{x-y}$$

Marginal pmf of x

x	1	2	3	4	5	6
$P_x(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Marginal pmf of y

Y	0	1	2	3	4	5	6
$P_y(y)$	$\frac{63}{384}$	$\frac{120}{384}$	$\frac{99}{384}$	$\frac{64}{384}$	$\frac{29}{384}$	$\frac{8}{384}$	$\frac{1}{384}$

suppose three cards are drawn from a deck of 52 cards. If x & y denote the no. of diamonds & spades respectively

$x = \{0, 1, 2, 3\}$

$y = \{0, 1, 2, 3\}$

$x \backslash y$	0	1	2	3
0	$\frac{26C_3}{52C_3}$	$\frac{13C_1\ 26C_2}{52C_3}$	$\frac{13C_2\ 26C_1}{52C_3}$	$\frac{13C_3}{52C_3}$
1	$\frac{13C_1\ 26C_3}{52C_3}$	$\frac{13C_1\ 13C_1\ 26C_1}{52C_3}$	$\frac{13C_2\ 13C_1}{52C_3}$	0
2	$\frac{13C_2\ 26C_1}{52C_3}$	$\frac{13C_2\ 13C_1}{52C_3}$	0	0
3	$\frac{13C_3}{52C_3}$	0	0	0