KyberSwap Elastic

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Abstract

1 Introduction

1.1 Motivation & Observation.

Kyber DMM (KyberSwap Classic) offers a tool for liquidity providers (LPs) to concentrate their liquidity into a specific price range thanks to the introducion of an amplification factor [1]. However, KyberSwap Classic (KS Classic) still has limitation in its offerings, such as lacking of dynamic price range support. Liquidity provider has only to choose between adding their capital into a specific price range defined by the pool creators, or create a new pool with their liquidity preference.

Uniswap v3, that was introduced shortly after KS Classic, offers such a feature in a nice and elegant design [2]. UniswapV3 allows user to specify the price range that they want to support their liquidity, and combines different liquidity positions (i.e. price ranges) from ther users into one single liquidity pool. This design allows flexibility in liquidity provisioning for liquidity providers, at the same time avoid fragmentation of liquidity pools.

Uniswap V3, however, did not address how to utilize the collected liquidity provider (LP) fees. Specifically, the fees are kept non-utilised inside the smart contract instead of being reinvested into the pool. The reason is that continuously compounding fees for different liquidity positions are complex and gas consuming.

In addition, UniswapV3 allows a single type of liquidity distribution within the selected price range. For example, if a LP provides liquidity for the price range of (p_1, p_2), in which $(p_1 \leq p_2)$, the liquidity will be spread out equally for the entire range.

1.2 Contributions

To address the challenges mentioned above, in this paper, firstly, KS Classic is reformulated to be more generalized. Secondly, based on this form, a better KS Classic, called KyberSwap Elastic (KS Elastic), is proposed to aggregate different KS Classic positions. Furthermore, a reinvestment curve to combine fees from KS Classic positions into an AMM pool is introduced to reinvest the collected fee from KS Elastic positions. And last but not least, the specific reinvestment curve for pair of similar assets is also presented.

2 Background

2.1 Generalize KS Classic

In order to propose a better AMM, first and foremost, KS Classic needs to be generalized. Few would deny that a good starting point is to compare it to the normal AMM. In the normal AMM, the relation between two reserves are depicted as follows, where x and y are reserves of two tokens X, Y respectively in a pool and k is a constant.

Normal AMM:

$$x \cdot y = k^2$$

The principal contribution of KS Classic is to bring an amplification factor α to concentrate liquidity into a specific price range:

KS Classic:

$$\alpha x_0 \cdot \alpha y_0 = \alpha^2 \cdot k^2 = L^2$$

In the above formula, $\alpha \cdot k$ or L is a constant and represents the liquidity of a position in KS Classic. Suppose that $\Delta x, \Delta y$ are the swapped volumes of each token, and x_0, y_0 are the initial balances. The real balances and virtual balances are represented as follows:

Real balances:

$$\begin{cases} x = x_0 + \Delta_x \\ y = y_0 + \Delta_y \end{cases}$$

Virtual balances:

$$\begin{cases} x' = a \cdot x_0 + \Delta_x \\ y' = a \cdot y_0 + \Delta_y \end{cases}$$

From the two formulas, the virtual balances can be written in terms of real balances:

$$\begin{cases} x' = x + (\alpha - 1) \cdot x_0 \\ y' = y + (\alpha - 1) \cdot y_0 \end{cases}$$

One point that should be noted is x_0, y_0, α are constant. Moreover, as the two virtual balances follow normal AMM reserves curve, we have:

$$x' \cdot y' = L^2 \Rightarrow (x + (\alpha - 1) \cdot x_0) \cdot (y + (\alpha - 1) \cdot y_0) = L^2$$

Meanwhile, the reference price is defined as $p_r = \frac{y_0}{x_0}$, as a result, x_0, y_0 can be written in term of the reference price p_r and the constant L:

$$p_r = \frac{y_0}{x_0}, x_0 \cdot y_0 = \frac{L^2}{\alpha^2}$$

$$\Rightarrow \frac{x_0}{L} = \frac{1}{\alpha \cdot \sqrt{p_r}}; \frac{y_0}{L} = \frac{\sqrt{p_r}}{\alpha}$$

Moreover, we can also deduce the maximum and minimum of virtual reserves:

$$x_{min} = 0 \Rightarrow \begin{cases} x'_{min} = (\alpha - 1) \cdot x_0 \\ y'_{max} = \frac{\alpha^2}{\alpha-1} \cdot y_0 \end{cases}$$

$$\Rightarrow y_{max} = \frac{\alpha^2}{\alpha-1} \cdot y_0 - (\alpha - 1) \cdot y_0 = (\frac{\alpha}{\alpha-1} - \frac{\alpha-1}{\alpha})L \cdot \sqrt{p_r}$$

$$\mathrm{y}_{min} = 0 \Rightarrow \begin{cases} y'_{min} = (\alpha - 1) \cdot y_0 \\ x'_{max} = \frac{\alpha^2}{\alpha-1} \cdot x_0 \end{cases}$$

$$\Rightarrow x_{max} = \frac{\alpha^2}{\alpha-1} \cdot x_0 - (\alpha - 1) \cdot x_0 = (\frac{\alpha}{\alpha-1} - \frac{\alpha-1}{\alpha}) \cdot \frac{L}{\sqrt{p_r}}$$

The next step is to determine the supported price range in the virtual pool:

$$p_{max} = \frac{y'_{max}}{x'_{min}} = \frac{\alpha^2}{(\alpha - 1)^2} \cdot p_r$$

$$p_{min} = \frac{(\alpha - 1)^2}{\alpha^2} \cdot p_r$$

$$\Rightarrow p_r = \sqrt{p_{max} \cdot p_{min}}$$

From equation mentioned above, the relation between real balances can be rewritten in another form:

$$(x + \frac{(\alpha - 1)L}{\alpha\sqrt{p_r}})(y + \frac{\alpha - 1}{\alpha} \cdot L \cdot \sqrt{p_r}) = L^2 \Rightarrow (x + \frac{L}{\sqrt{p_{max}}})(y + \text{Ł} \cdot \sqrt{p_{min}}) = L^2$$

Figure 1 shows us how the reserves curves are established in KS Classic. At first, the virtual reserves curve (shown in violet) is like other normal AMM,

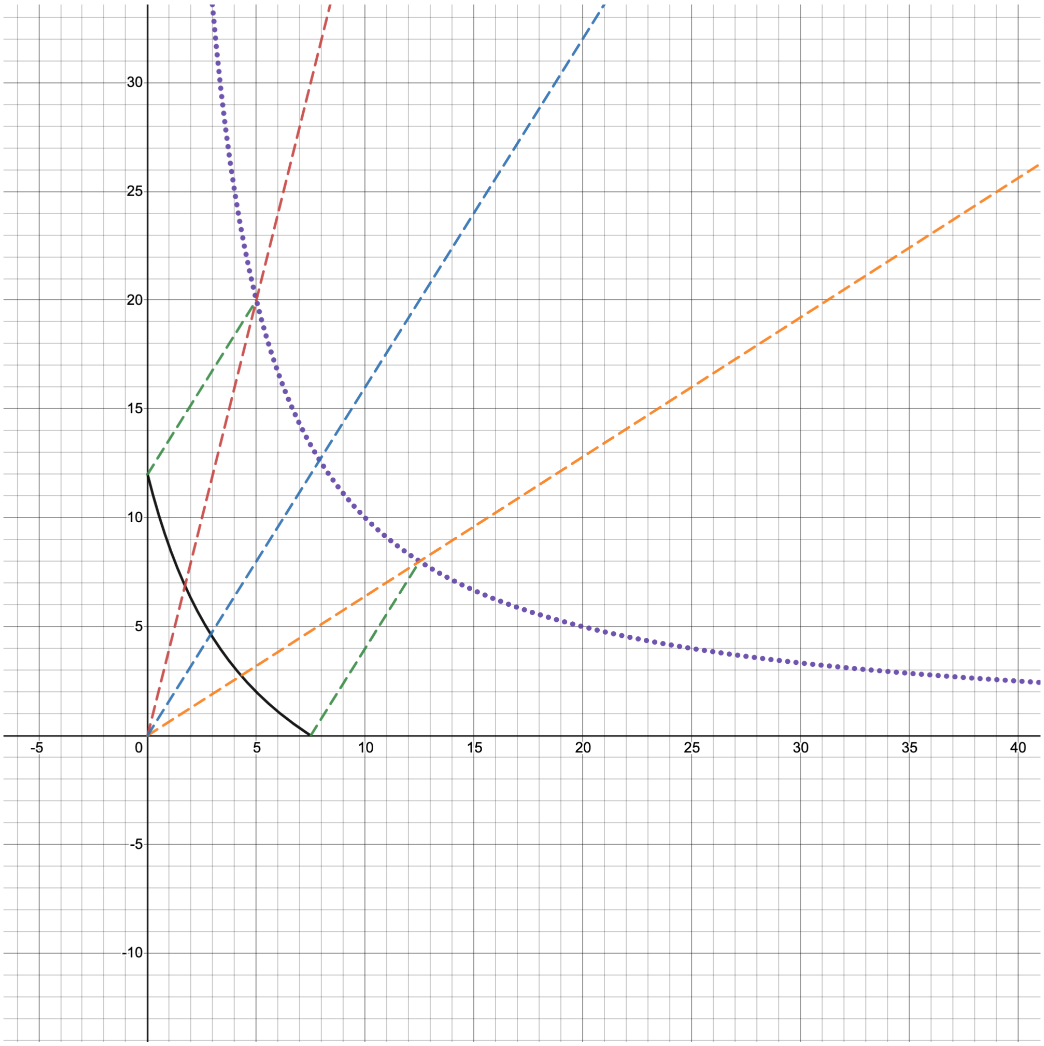

Figure 1: Example of KS Classic curve with
$L = 10; Pr = 1.6, \alpha = 2.72, p_{max} = 4, p_{min} = 0.64$

a constant product. Meanwhile, the black curve shows us the real reserve balances. The red, blue, and orange lines represent the linear functions $y = p \cdot x$, where p is the slope of the line. In other words, it represents the price of y by x. As a result, the interception points between these linear functions and the virtual reserves curve give us the state of reserve of each token at each given price. The two linear functions with two slopes p_{min}, p_{max} intercept the reserves curve at two points: the limit range of the position.

Moreover, we can also observe that the real reserves curve is a translation of the virtual curve and is limited by the maximum reserve of X and Y.

Finally, a position in KS Classic can be generalized in two different ways:

$$(L, p_r, \alpha) \Rightarrow \begin{cases} L : Liquidity \\ p_r : Reference \quad price \\ \alpha : Amplification \quad factor \end{cases}$$

Or

$$(L, p_{max}, p_{min}) \Rightarrow \begin{cases} L : Liquidity \\ p_{max} : Max \quad price \quad supported \\ p_{min} : Min \quad price \quad supported \end{cases}$$

We should note that one form can be switched to another easily as the maximum and minimum price supported can be calculated using the reference price and amplification factor.

Move to another aspect, let P_c is the current price, the real balances can be rewritten in term of L, p_c, p_{max}, p_{min}:

$$\Rightarrow x = L(\frac{1}{\sqrt{p_c} - \frac{\alpha - 1}{\alpha \cdot \sqrt{p_r}}}) = L(\frac{1}{\sqrt{p_c}} - \frac{1}{\sqrt{p_{max}}})$$

$$\Rightarrow y = L(\sqrt{p_c} - \frac{\alpha - 1}{\alpha} \cdot \sqrt{p_r}) = L(\sqrt{p_c} - \sqrt{p_{min}})$$

Moveoever, suppose that Δx is the vol of token X that need to move the curve from p_c to p'_c and the vol of Token Y that taken from the pool is ΔY. The two Δs can be deduced in term of L, p_c, p'_c:

$$\Rightarrow x + \Delta x = L(\frac{1}{\sqrt{p'_c}} - \frac{\alpha - 1}{\alpha \sqrt{p_r}}) \Rightarrow \Delta x = L(\frac{1}{\sqrt{p'_c}} - \frac{1}{\sqrt{p_c}})$$

$$\Rightarrow \Delta y = L(\sqrt{p'_c} - \sqrt{p_c})$$

To sum up, in this section, the generalized form of a position in KS Classic has been formalized; from now on, this form will be used to propose novel contributions.

3 KyberSwap Elastic

3.1 Aggregated KS Classic

The first contribution proposed in this light paper is how to aggregate different positions into one reserves curve. A good starting point is to aggregate two different positions into one; then, multiples positions could be aggregated one by one using the same methodology. In general, two positions in KS Classic can be represented as follows:

$$(L_1, p_{r_1}, \alpha_1); (L_2, p_{r_2}, \alpha_2)$$

$$\begin{aligned}
\mathrm{p}_{max_1} &= (\tfrac{(\alpha_1)}{(\alpha_1)-1})^2 p_{r_1} \\
\mathrm{p}_{max_2} &= (\tfrac{(\alpha_2)}{(\alpha_2)-1})^2 p_{r_2} \\
\mathrm{p}_{min_1} &= (\tfrac{(\alpha_1-1)}{(\alpha_1)})^2 p_{r_1} \\
\mathrm{p}_{min_2} &= (\tfrac{(\alpha_2-1)}{(\alpha_2)})^2 p_{r_2}
\end{aligned}$$

So, from the two price ranges, we have 3 cases:

Suppose that $p_{max1} \geq p_{max2}$

3.1.1 Case 1: $p_{max_1} > p_{min_1} \geq p_{max_2} > p_{min_2}$

In the first case, the two price ranges are distinct. From these two price ranges, the maximum reserves of X and Y in two positions, respectively, are represented as follows:

$$\begin{aligned}
\mathrm{x}_{max_2} &= (\tfrac{\alpha_2}{\alpha_2-1} - \tfrac{\alpha_2-1}{\alpha_2})\tfrac{L_2}{\sqrt{p_{r_2}}} \\
\mathrm{x}_{max_1} &= (\tfrac{\alpha_1}{\alpha_1-1} - \tfrac{\alpha_1-1}{\alpha_1})\tfrac{L_1}{\sqrt{p_{r_1}}} \\
\mathrm{y}_{max_2} &= (\tfrac{\alpha_2}{\alpha_2-1} - \tfrac{\alpha_2-1}{\alpha_2})L_2\sqrt{p_{r_2}} \\
\mathrm{y}_{max_1} &= (\tfrac{\alpha_1}{\alpha_1-1} - \tfrac{\alpha_1-1}{\alpha_1})L_1\sqrt{p_{r_1}}
\end{aligned}$$

The two price ranges do not have any overlap range; hence, the combined reserves curve naturally includes two parts. The first part is from 0 to x_{max_1} where the liquidity of position 1 is poured. The second part is from x_{max_1} to $x_{max_1} + x_{max_2}$, where we pour the liquidity of the position 2. So the relation between reserves in two ranges are depicted as follows:

If x $\in [0, x_{max_1}) \Rightarrow (x + \frac{(\alpha_1 - 1)L_1}{\alpha_1\sqrt{p_{r_1}}})(y + \frac{(\alpha_1-1)L_1\sqrt{p_{r_1}}}{\alpha_1} - y_{max_2}) = L_1^2$

If x $\in [x_{max_1}, x_{max_1} + x_{max_2}) \Rightarrow (x + \frac{(\alpha_2-1)L_2}{\alpha_2\sqrt{p_{r_2}}} - x_{max_1})(y + \frac{(\alpha_2-1)}{\alpha_2} L_2\sqrt{p_{r_2}}) = L_2^2$

Figure 2 illustrates the reserves curve, which combines two reserves curves that continue and its derivative function also continues (allows the price range continues).

The second and third cases are similar and based on a common idea: in the overlap price range, the liquidity equal is the sum of liquidity in two positions.

3.1.2 Case 2: $p_{max_1} \geq p_{max_2} > p_{min_1} \geq p_{min_2}$

We have 3 aggregated positions:

$$\begin{cases} (L_1, \frac{\alpha_1\alpha_2}{(\alpha_1-1)(\alpha_2-1)}\sqrt{p_{r_1}p_{r_2}}, \frac{1}{1-\sqrt{\frac{\alpha_2(\alpha_1-1)}{\alpha_1(\alpha_2-1)}\sqrt{\frac{p_{r_2}}{p_{r_1}}}}}) \\ (L_1 + L_2, \frac{\alpha_2(\alpha_1-1)}{(\alpha_1)(\alpha_2-1)}\sqrt{p_{r_1}p_{r_2}}, \frac{1}{1-\sqrt{\frac{(\alpha_2-1)(\alpha_1-1)}{\alpha_1\alpha_2}\sqrt{\frac{p_{r_1}}{p_{r_2}}}}}) \\ (L_2, \frac{(\alpha_1-1)(\alpha_2-1)}{\alpha_1\alpha_2}\sqrt{p_{r_1}p_{r_2}}, \frac{1}{1-\sqrt{\frac{\alpha_1(\alpha_2-1)}{(\alpha_1-1)\alpha_2}\sqrt{\frac{p_{r_2}}{p_{r_1}}}}}) \end{cases}$$

$$or \begin{cases} (L_1, p_{max_1}, p_{max_2}) \\ (L_1 + L2, p_{max_2}, p_{min_1}) \\ (L_2, p_{min_1}, p_{min_2}) \end{cases}$$

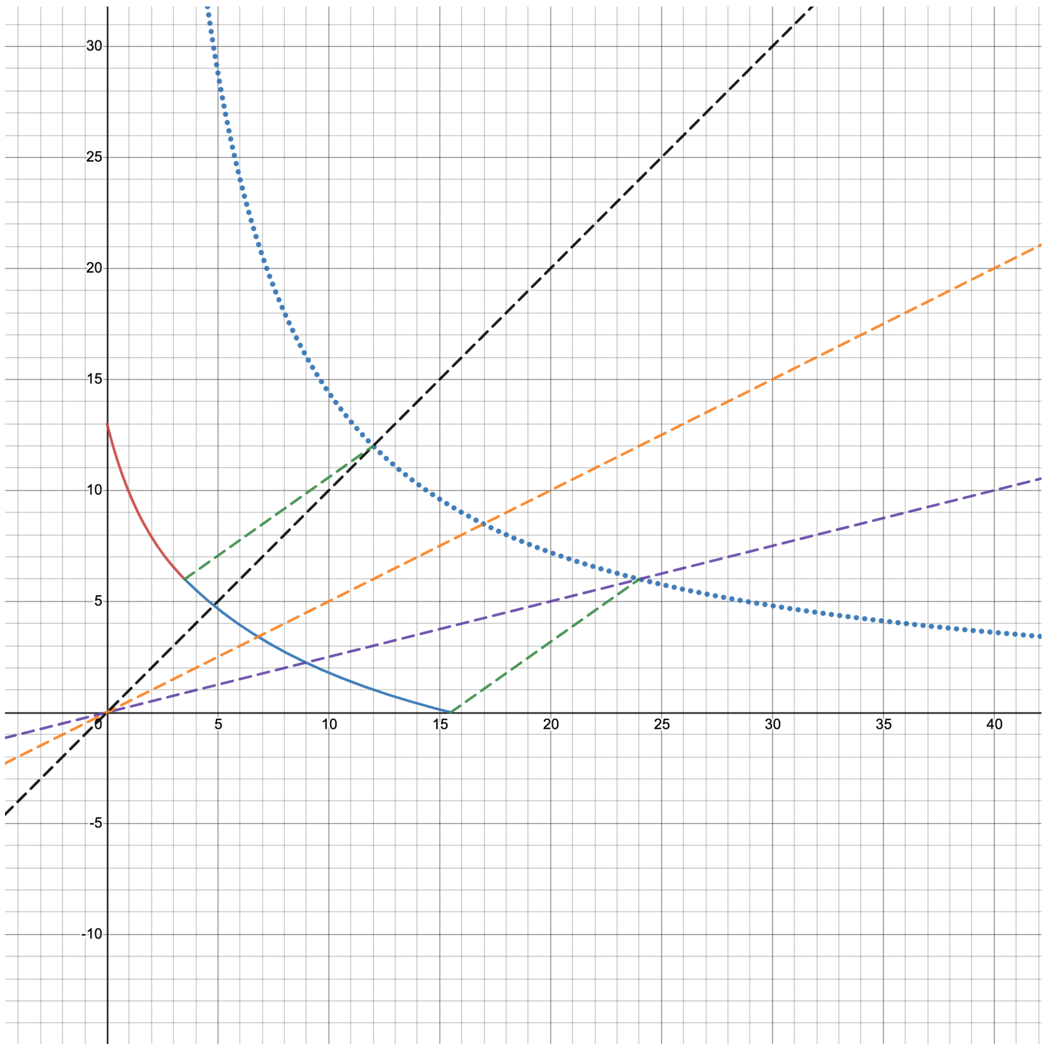

Figure 2: Aggregation two positions:
$(12, 1/2, \alpha = \sqrt{(}2)/(\sqrt{(}2) - 1) = 3.4)$ and $(7, 2, \alpha = \sqrt{(}2)/(\sqrt{(}2) - 1) = 3.4)$

Figure 3 illustrates how the reserves curve in case 2 looks like, combining two positions that overlap into one single reserves curve. The middle range (shown in blue) is the part that includes the liquidity of both positions.

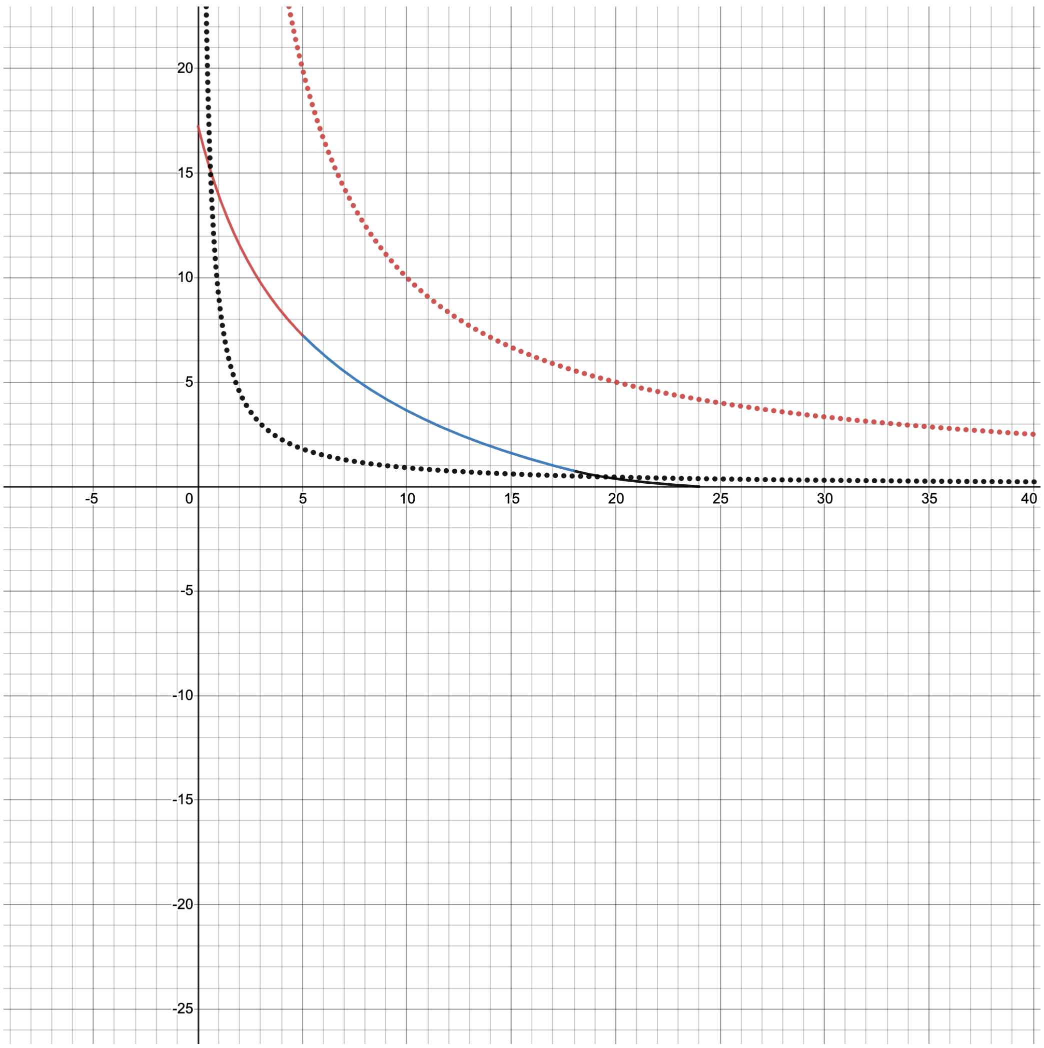

Figure 3: Aggregation two positions:
$(3, 1/4, 2)$ and $(10, 1, 2)$

3.1.3 Case 3: $p_{max_1} \geq p_{max_2} > p_{min_2} \geq p_{min_1}$

We have 3 aggregated positions:

$$\begin{cases} (L_1, \frac{\alpha_1\alpha_2}{(\alpha_1-1)(\alpha_2-1)}\sqrt{p_{r_1}p_{r_2}}, \frac{1}{1-\sqrt{\frac{\alpha_2(\alpha_1-1)}{\alpha_1(\alpha_2-1)}\sqrt{\frac{p_{r_2}}{p_{r_1}}}}}) \\ (L_1+L_2, p_{r_2}, \alpha_2) \\ (L_1, \frac{(\alpha_1-1)(\alpha_2-1)}{\alpha_1\alpha_2}\sqrt{p_{r_1}p_{r_2}}, \frac{1}{1-\sqrt{\frac{\alpha_1(\alpha_2-1)}{(\alpha_1-1)\alpha_2}\sqrt{\frac{p_{r_1}}{p_{r_2}}}}}) \end{cases}$$

$$or \begin{cases} (L_1, p_{max_1}, p_{max_2}) \\ (L_1 + L2, p_{max_2}, p_{min_2}) \\ (L_1, p_{min_2}, p_{min_1}) \end{cases}$$

From now on, using the same methodology, we can aggregate multiple positions. Figure 4 shows an example of aggregation of multiple positions.

3.2 Reinvestment curve

The previous section proposed a method to aggregate different KS Classic positions. However, one issue that arises is how to compound the KS Elastic fee. KS Elastic fee cannot be poured directly into the pool due to the NFT nature of KS Elastic positions. Liquidity in KS Elastic can be active or inactive according to the position and the current price, making the earned fees different. To address this issue, we propose that the fee received by KS Elastic will be poured directly into an AMM pool called the reinvestment curve, and new reinvestment curve liquidity tokens are minted for the KS Elastic position.

3.2.1 Combination with a constant-product curve

The previous section addresses the first challenge: the aggregation between multiple positions into one single curve. The reinvestment curve is a constant-product curve; hence, to reinvest the fee we got from KS Elastic positions, we must be able to combine a KS Elastic curve and a constant-product

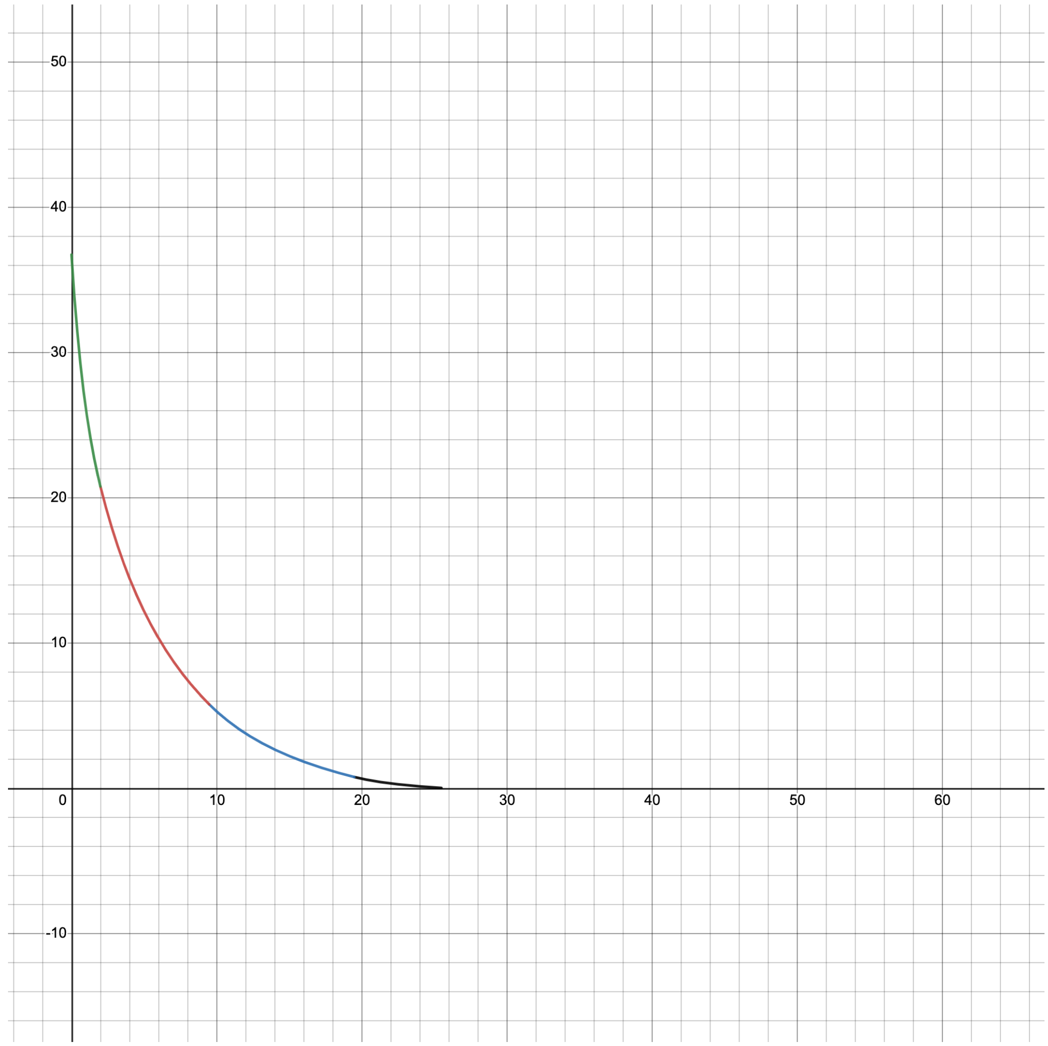

Figure 4: Aggregation four positions

one. However, until now, only KS Elastic curves are mentioned. Our goal is not only the aggregation of KS Elastic curves but more extensive. A combination between a KS Elastic curve and a constant-product curve would be a profitable feature for LP. The challenge that arises is that we need to maintain and utilize the liquidity of two curves simultaneously and guarantee the price in two curves is identical after each swap. Hence in this section, an aggregation with the constant-product curve is introduced.

Firstly, the constant-product curve is represented as follow:

$$x_f \cdot y_f = L_f^2$$

On the other hand, a KS Elastic curve is also represented as follow:

$$(L, p_r, \alpha);$$

$$\mathrm{x}_{p_{max}} = (\frac{\alpha}{\alpha_1 - 1} - \frac{\alpha - 1}{\alpha})\frac{L_p}{\sqrt{p_r}}$$

$$\mathrm{y}_{p_{max}} = (\frac{\alpha}{\alpha_1 - 1} - \frac{\alpha - 1}{\alpha})L_p\sqrt{p_r}$$

Moreover, when the current price p_c reaches one of the two limits, the reserve of token X in the constant-product curve is calculated in term of L_f and p_r:

at $\mathrm{p}_c = p_{p_{max}} : x_f(p_{max}) = \frac{L_f}{(\frac{\alpha}{\alpha - 1}\sqrt{p_r})} = \frac{L_f}{\sqrt{p_{max}}}$

at $\mathrm{p}_c = p_{p_{min}} : x_f(p_{min}) = \frac{L_f}{(\frac{\alpha - 1}{\alpha}\sqrt{p_r})} = \frac{L_f}{\sqrt{p_{min}}}$

Now, similar to the case 2 and 3 in the aggregation between KS Elastic curves, we have 3 aggregated ranges:

If $\mathrm{x} \in (0, x_f(p_{max})) \Rightarrow x(y - y_{p_{max}}) = L_f^2$

If $\mathrm{x} \in [x_f(p_{min}) + x_{p_{max}}, \infty) \Rightarrow (x - x_{p_{max}})y = L_f^2$

If $\mathrm{x} \in [x_f(p_{max}), x_f(p_{min}) + x_{p_{max}}] \Rightarrow (x + \frac{(\alpha-1)}{\alpha}\frac{L_p}{\sqrt{p_r}})(y + \frac{(\alpha-1)}{\alpha}L_p\sqrt{p_r}) = (L_f + L_p)^2$

$\iff (x + \frac{L_p}{\sqrt{p_{max}}})(y + L_p\sqrt{p_{min}}) = (L_f + L_p)^2$

Figure 6 shows us how an constant-product curve (shown in violet) and a KS Elastic curve (shown in red) are combined. In more detail, when the real balance is in the range $[x_f(p_{max}), x_f(p_{min}) + x_{p_{max}}]$, the real reserves curve is the translation of the KS Elastic virtual reserves curve (shown in red), and

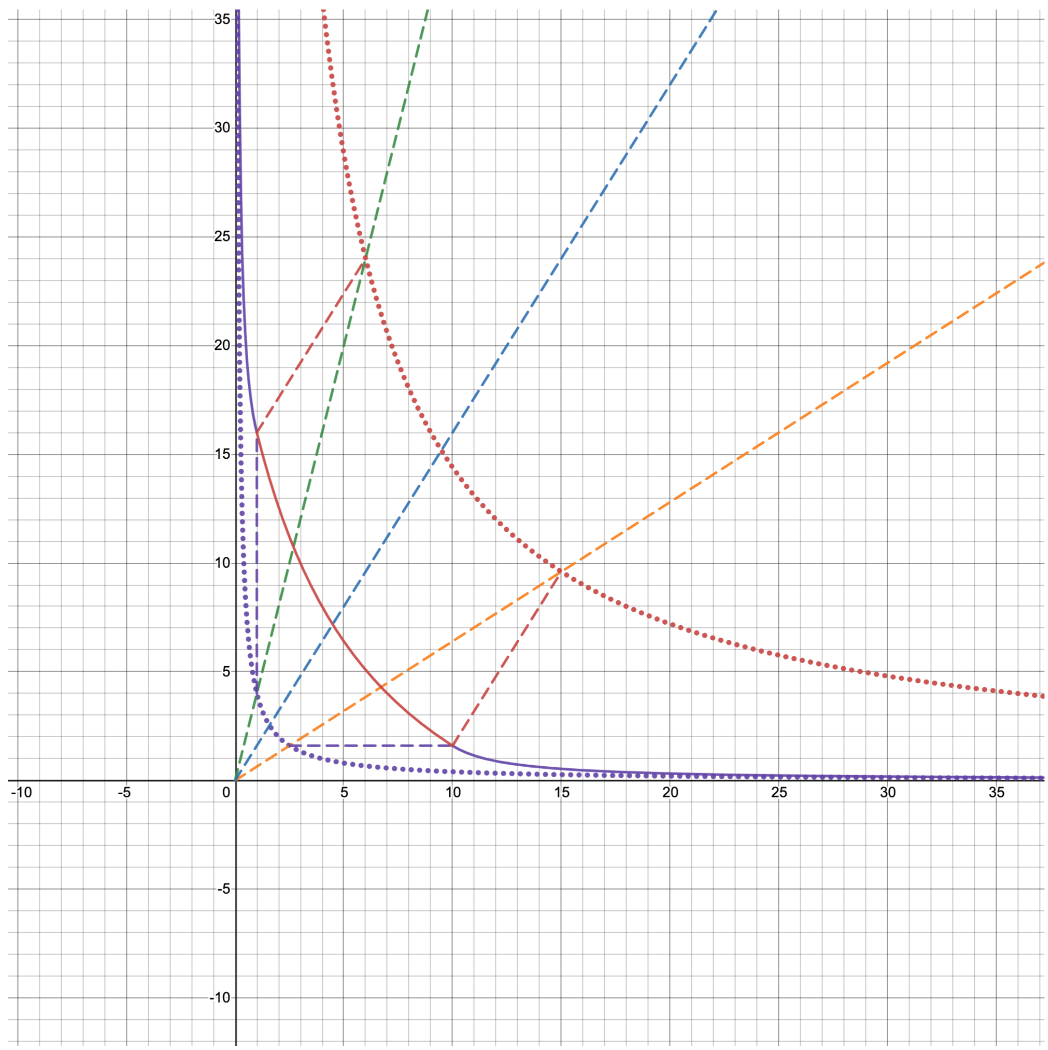

Figure 5: Aggregation two positions:
$L_p = 10; p_r = 1.6, \alpha = 2.72, p_{max} = 4, p_{min} = 0.64, L_f = 2$

its liquidity is the total of liquidity of both curves. Whenever the prices are out of the range, only the liquidity of the constant-product curve remains, and the reserves curve is the translation of the infinite reserves curve (shown in violet).

Besides, the reserves of token X at any time can be rewritten in term of

x_f, x_p :

$$\begin{cases} x(p_c) = x_f(p_c) + x_p(p_c) \\ y(p_c) = y_f(p_c) + y_p(p_c) \end{cases}$$

To sum up, thank to the same methodology, we can aggregate an constant-product curve and a KS Elastic one.

3.2.2 Concentrated reinvestment curve for pair of similar assets

One would note that reinvesting the entire collected fee into an infinite range for pair of similar assets is a waste. As market price for a couple of similar assets usually moves in a narrow range. Even if it moves out of the range, it quickly recovers back into the stable range. To optimize the benefit of LPs, we should concentrate the liquidity generated by the collected fee in a custom range. In fact, we can use other models as the concentrated reinvestment curve for pair of similar assets; the model of Curve is such an example. However, the integration with such an external curve like Curve is difficult, so it is better to propose a familiar curve for Kyber KS Elastic positions.

Hence we propose the reinvestment curve for pair of similar assets with the following set of parameters $L_r, A_1, A_2, p_r, \alpha$, where $A_2 >> A_1$. In more detail, we propose to add a KS Elastic position with a significant amplification factor into the constant-product curve:

$$\Rightarrow (A_1 L_r, p_r = 1, \alpha_1 = 1); (A_2 L_r, p_r = 1, \alpha_2 = \alpha) \tag{1}$$

As a result, the reinvestment curve is the aggregation of two curves, where the total reserves of X and Y are :

$$\begin{cases} x_r = x_1 + x_2 \\ y_r = y_1 + y_2 \end{cases} \tag{2}$$

Moreover the price minimum and maximum for $(A_2 L_r, p_r = 1, \alpha_2 = \alpha)$ are:

$$p_{r_{max}} = (\frac{\alpha}{\alpha - 1})^2$$

$$p_{r_{min}} = (\frac{\alpha - 1}{\alpha})^2$$

$$(\mathrm{A}_1 L_r, p_r = 1, \alpha_1 = 1) \iff x_1 y_1 = A_1^2 L_r^2$$
$$(\mathrm{A}_2 L_r, p_r = 1, \alpha_2 = \alpha) \iff (x_2 + \tfrac{A_2 L_r}{\sqrt{p_{r_{max}}}})(y_2 + A_{2Lr}\sqrt{p_{r_{min}}}) = A_2^2 L_r^2 \tag{3}$$

Hence we can easily calculate the set of $x_{r_{p_{max}}}, y_{r_{p_{max}}}, x_{r_{p_{min}}}, y_{r_{p_{min}}}$

$$\begin{cases} x_{r_{p_{max}}} &= L_r \frac{A_1}{\sqrt{p_{r_{max}}}} \\ y_{r_{p_{min}}} &= L_r A_1 \sqrt{p_{r_{min}}} \\ y_{r_{p_{max}}} &= L_r (A_1 \sqrt{p_{r_{max}}} + A_2 \sqrt{p_{r_{max}}} - A_2 \sqrt{p_{r_{min}}}) \\ x_{r_{p_{min}}} &= L_r (\frac{A_1}{\sqrt{p_{r_{min}}}} + \frac{A_2}{\sqrt{p_{r_{min}}}} - \frac{A_2}{\sqrt{p_{r_{max}}}}) \end{cases} \tag{4}$$

From now on, we can clarify three different ranges in the reinvestment curve for pair of similar assets:

$$\begin{cases} 0 \leq x_r \leq x_{r_{max}} \Rightarrow x_r (y_r - y_{r_{max}}) = A_1^2 L_r^2 \\ x_{r_{max}} \leq x_r \leq x_{r_{min}} \Rightarrow (x_r + \frac{A_2 L_r}{\sqrt{p_{r_{max}}}})(y_r + A_2 L_r \sqrt{p_{r_{min}}}) = ((A_1 + A_2) L_r)^2 \\ x_{r_{min}} \leq x_r \Rightarrow (x_r - x_{r_{min}}) y_r = A_1^2 L_r^2 \end{cases} \tag{5}$$

As we can see on the following figure illustrates three different ranges of the reinvestment curve for pair of similar assets:

Figure 6: Reinvestment curve

Finally, the reinvestment curve is integrated into the KS Elastic curve, of which its liquidity is represented by L_p:

$$\begin{cases} 0 \leq x_r \leq x_{r_{max}} \Rightarrow (x_r + \frac{L_p}{\sqrt{p_{p_{max}}}})(y_r - y_{p_{p_{min}}} + \frac{L_p}{\sqrt{p_{p_{min}}}}) = (A_1 L_r + L_p)^2 \\ x_{r_{max}} \leq x_r \leq x_{r_{min}} \Rightarrow (x_r + \frac{L_p}{\sqrt{p_{p_{max}}}} + \frac{A_2 L_r}{\sqrt{p_{r_{max}}}})(y_r + L_p\sqrt{p_{min}} + A_2 L_r \sqrt{p_{r_{min}}}) \\ = ((A_1 + A_2)L_r + L_p)^2 \\ x_{r_{min}} \leq x_r \Rightarrow (x_r + \frac{L_p}{\sqrt{p_{p_{max}}}})(y_r - y_{p_{r_{min}}}) = (A_1 L_r + L_p)^2 \end{cases} \tag{6}$$

3.3 Calculate fee

3.3.1 Calculate fee for normal pairs

In such an aggregation pool we have KS Elastic positions which are represented by (L_p, p_r, α). On other hand AMM states which are represented by (x_f, y_f), from x_f, y_f we have $L_f = \sqrt{x_f \cdot y_f}; p_c = \frac{y_f}{x_f}$.

In the KS Elastic pool, we can get the price range from the amplification factor α and the reference price p_r.

$$p_{max} = (\frac{\alpha}{\alpha - 1})^2 \cdot p_r$$

$$\mathrm{p}_{min} = (\tfrac{\alpha-1}{\alpha})^2 \cdot p_r$$

Suppose that $p_{min} < p_c < p_{max}$ the reserves balances of X, Y can be written in term of L_p, p_c, p_{max}

$\Rightarrow x_p = \frac{L_p}{\sqrt{p_c}} - \frac{L_p}{\sqrt{p_{max}}}; y_p = L_p(\sqrt{p_c} - \sqrt{p_{min}})$

Suppose that a swap with $\Delta x > 0$ does not move p_c out of $[p_{min}, p_{max}]$. The fee is poured into the constant-product curve, so the new reserve and liquidity in AMM are depicted:

$$\begin{cases} x_{f_{new}} = x_f + \Delta x \cdot fee \\ L_{f_{new}} = \sqrt{(x_f + \Delta x \cdot fee) \cdot y_f} (3) \end{cases}$$

As a result, a new reserves curve that combines KS Elastic and reinvestment has the total liquidity of two curves and is represented:

$$(x_{f_{after}} + x_{p_{after}} + \frac{L_p}{\sqrt{p_{max}}})(y_{f_{after}} + y_{p_{after}} + \sqrt{p_{min}}) = (L_{f_{new}} + L_p)^2 \quad (7)$$

In more detail, the total reserve of constant-product curve can be rewritten:

$$\mathrm{x}_{f_{after}} + x_{p_{after}} = x_f + x_p + \Delta x$$

Moreover, the ratio of the reserve of the constant-product curve after the swap over the total virtual reserve is equal to the new liquidity of the constant-product curve over the total liquidity:

$$\frac{y_{f_{after}}}{y_{f_{after}} + y_{p_{after}} + L_p\sqrt{p_{min}}} = \frac{L_{f_{new}}}{L_{f_{new}} + Lp} \quad (8)$$

$$(1)\&(2) \Rightarrow y_{f_{after}} = \frac{L_{f_{new}}(L_{f_{new}} + L_p)}{\frac{L_p}{\sqrt{p_{max}}} + x_f + x_p + \Delta x}$$

Meanwhile reserves of KS Elastic curve are:

$$y_{p_{after}} = \frac{L_p(L_{f_{new}} + L_p)}{x_f + x_p + \Delta x + \frac{L_p}{\sqrt{p_{max}}}} - L_p\sqrt{p_{min}}$$

$$x_{f_{after}} = \frac{L_{f_{new}}(x_f + x_p + \Delta x + \frac{L_p}{\sqrt{p_{max}}})}{L_{f_{new}} + L_p}$$

$$x_{p_{after}} = \frac{L_p(x_f + x_p + \Delta x + \frac{L_p}{\sqrt{p_{max}}})}{L_{f_{new}} + L_p} - \frac{L_p}{\sqrt{p_{max}}}$$

The amount of token Y moved out of pool can be calculated thanks to the difference between the two states:

$$\Delta y = y_f + y_p - y_{f_{after}} - y_{p_{after}}$$

Hence, after the swap, the state of the pool is:

$$\begin{cases} x_{f_{after}}, y_{f_{after}} \\ L_p, p_r, \alpha \end{cases}$$

As a result, the total protocol fee is represented as a portion in reinvestment curve liquidity:

$$\Delta L = L_{f_{new}} - L_f$$

So the total protocol fee should be distributed proportionally to liquidity providers in both pools.

In case of KS Elastic: $\frac{L_p(L_{f_{new}} - L_f)}{L_p + L_f}$

In case of reinvestment curve: $\frac{L_f(L_{f_{new}} - L_f)}{L_p + L_f}$

Suppose that S = total supply of reinvestment liquidity token, the reinvestment fee go directly to the pool and raise liquidity token value:

$$\frac{Value_{new_{LT}}}{Value_{old_{LT}}} = \frac{L_f + \frac{L_f(L_{f_{new}} - L_f)}{L_p + L_f}}{L_f} = \frac{L_p + L_{f_{new}}}{L_p + L_f}$$

The reinvestment curve mints additional liquidity tokens and attribute to the KS Elastic pool:

$$\frac{S_{mint}}{S} = \frac{L_p(L_{f_{new}} - L_f)(L_p + L_f)}{L_f(L_p + L_{f_{new}})(L_p + L_f)}$$

$$=\mathrm{L}_p \frac{(L_{f_{new}} - L_f)}{L_f(L_p + L_{f_{new}})}$$

$$\mathrm{S}_{new} = S_{mint} + S = \frac{L_{f_{new}}(L_p + L_f)}{L_f(L_p + L_{f_{new}})} S$$

3.3.2 Calculate fee for pair of similar assets

First and foremost we must calculate the reserve of token X in the reinvestment curve x_f

$$\begin{cases} x_{after} = x + \Delta X \\ x_{f_{new}} = x_f + \Delta X \cdot fee \end{cases}$$

When

$$p_c = p_{f_{max}} \Rightarrow \begin{cases} x_f = \frac{A_1 L_f}{\sqrt{p_{f_{max}}}} \\ y_f = A_1 L_f \sqrt{p_{f_{max}}} + A_2 L_f(\sqrt{p_{f_{max}}} - \sqrt{p_{f_{min}}}) \end{cases}$$

From now on, we can calculate the ratio of y_f and x_f easily:

$$f_{p_{f_{max}}} = \frac{y_f}{x_f} = p_{f_{max}} + \frac{A_2}{A_1}(1 - \frac{\sqrt{p_{f_{min}}}}{\sqrt{p_{f_{max}}}})$$

When

$$p_c = p_{f_{min}} \Rightarrow$$

$$\mathrm{f}_{p_{f_{min}}} = \frac{y_f}{x_f} = \frac{A_1\sqrt{p_{min}}}{\frac{A_1}{\sqrt{p_{min}}} + A_2(\frac{1}{\sqrt{p_{min}}} - \frac{1}{\sqrt{p_{max}}})}$$

From (5) and the ratio of y_f and x_f, we can also comfortably calculate the new liquidity L'_f:

$$p_{f_{max}} \leq p'_c \Rightarrow x_{f_{new}}(y_f - L'_f A_2(\sqrt{p_{f_{max}}} - \sqrt{p_{f_{min}}})) = (L'_f A_1)^2$$

$$p_{f_{min}} \leq p'_c \leq p_{f_{max}} \Rightarrow (x + \frac{L'_f A_2}{\sqrt{p_{f_{max}}}})(y + L'_f A_2 \sqrt{p_{f_{min}}}) = (L'_f(A_1 + A_2))^2$$

$$p'_c \leq p_{f_{min}} \Rightarrow ((x_{f_{new}} - L'_f A_f(\frac{1}{\sqrt{p_{min}}} - \frac{1}{\sqrt{p_{max}}})))y = (L'_f A_2)^2$$

However, the result of these functions are quadratic equations is costly in term of computation. To overcome this issue, we can approximate the new liquidity in the following section.

4 Practical for Implementation

4.1 Characterize a position

In KS Classic, we characterize a position using L, P, α; however, the way of tracking is complicated, especially when we need to aggregate different positions. As a result, in the KS Elastic, it could be simply using L, P_{min}, P_{max}.

4.2 Keep track $\sqrt{p}$

We have learned from Uniswap v3 that they keep tracks $\sqrt{p}$ instead of p. As the calculation of $\sqrt{(}p)$ is costly in terms of computation.

4.3 Approximate new liquidity

4.3.1 For pairs of general pairs

As we observed in (3), the formula to calculate $L_{f_{new}}$ is complicated for a smart contract and costly in terms of computation. As a result, we aim to propose a smart contract-friendly function that approximates the proposed function. The function to calculate an approximate of $L_{f_{new}}$, called $L_{f_{approx}}$:

$$L_{approx} = L_f + \frac{\Delta x fee \sqrt{p_c}}{2}$$

The difference between $L_{f_{new}} = \sqrt{(x_f + \Delta x \cdot fee) y_f}$ and $L_{f_{approx}}$ is:

$$\frac{L_{f_{new}} - L_{f_{approx}}}{L_f} = -\frac{\Delta_X^2 \cdot fee^2}{2(\sqrt{x_f^2 + \Delta x \cdot fee \cdot x_f} + x_f)^2}$$

$$\Rightarrow |\frac{L_{f_{new}} - L_{f_{approx}}}{L_f}| \leq \frac{1}{8}(\frac{\Delta x \cdot fee}{X_f})^2$$

Hence, if $\Delta x \cdot fee << x_f$ then $L_{f_{approx}} \approx L_{f_{new}}$.

4.3.2 For pairs of similar assets

$$p_{f_{max}} \leq p_c \Rightarrow \Delta L = \Delta x fee(\frac{1}{\frac{2A_1}{\sqrt{p_c}} + \frac{A_2(\sqrt{p_{max}} - \sqrt{p_{min}})}{p_c}})$$

$$The p_{f_{min}} \leq p_c \leq p_{f_{max}} \Rightarrow \Delta L = \Delta x fee(\frac{1}{\frac{2(A_1+A_2)}{\sqrt{p_c}} - A_2(\frac{\sqrt{p_{min}}}{\sqrt{p_c}} - \frac{1}{\sqrt{p_{max}}})})$$

$$p_c \leq p_{f_{min}} \Rightarrow \Delta L = \Delta x fee(\frac{1}{\frac{2A_1}{\sqrt{p_c}} + A_2 \frac{1}{\sqrt{p_{min}}} - \frac{1}{\sqrt{p_{max}}}})$$

4.4 Minting protocol

Instead of minting directly after each swap, we update only L_f of AMM liquidity. We also keep a snapshot $L_{f_{Last}}$ of the last time liquidity token is minted or burned. When a swap change the KS Elastic curve to another position or when users add/remove liquidity from the pool, the total fee is calculated as: $fee = L_{f_{new}} - L_{f_{last}}$ and new liquidity tokens are minted for KS Elastic position owners.

5 Conclusions

In this paper, a novel model for KS Elastic and DEFI, in general, is proposed to improve KS Classic and open a new methodology for the DEFI community in terms of manage aggregation positions and their fees. Motivated by the demand of LPs, the proposed model promises to address multiple issues and will be implemented in the next version of KS Elastic.

References

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