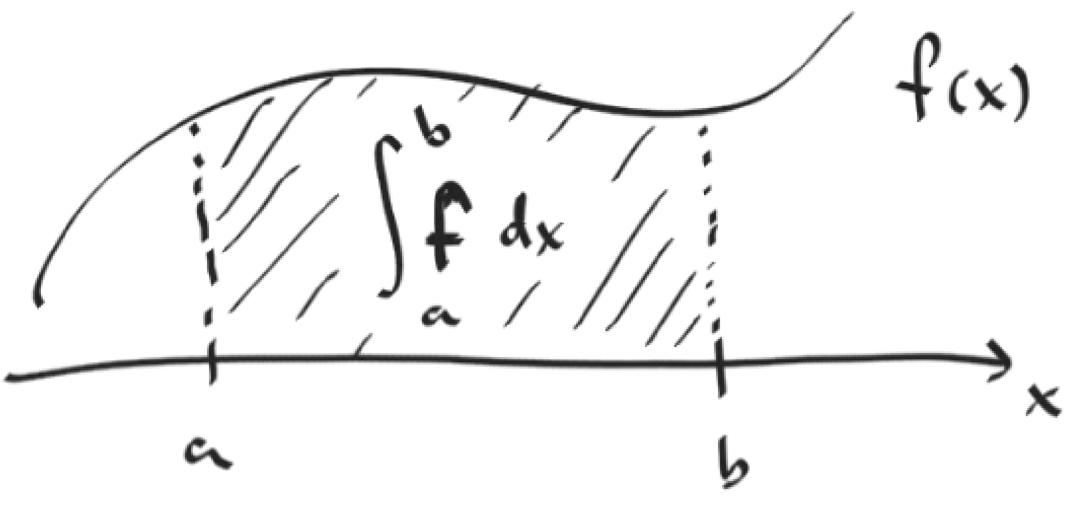
Simpson

0. Numerical Integration

1. O.D.E ''

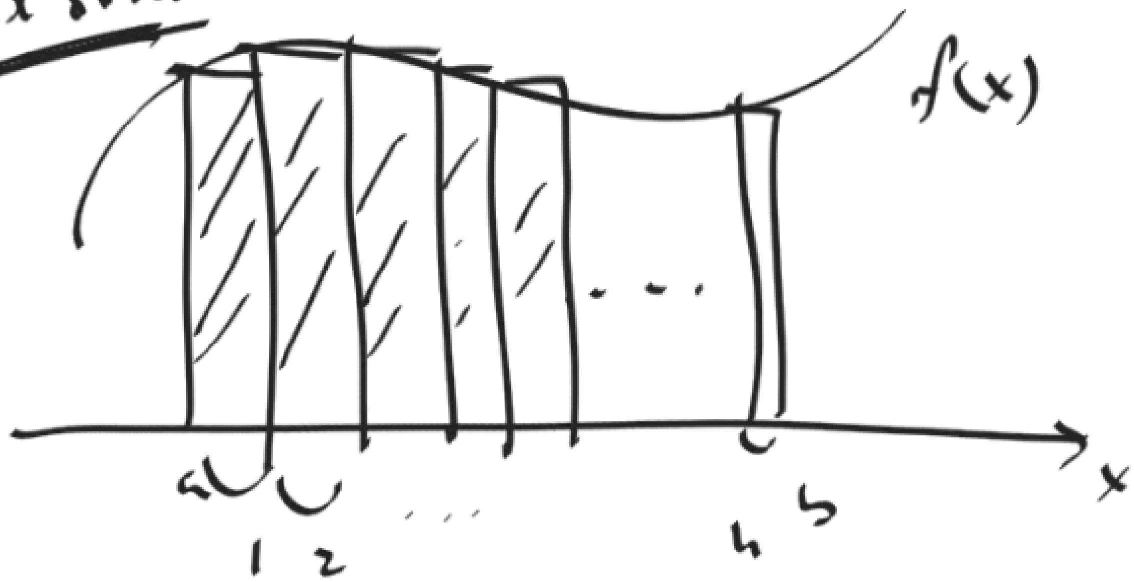
Today

Numerical Integration

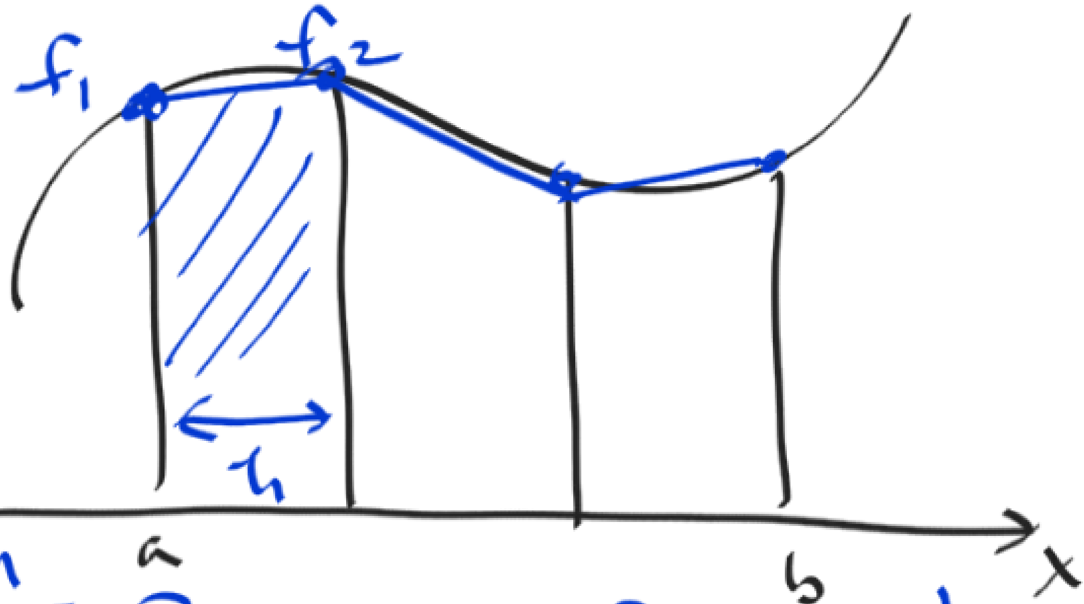


Numerical Integ.

1st order

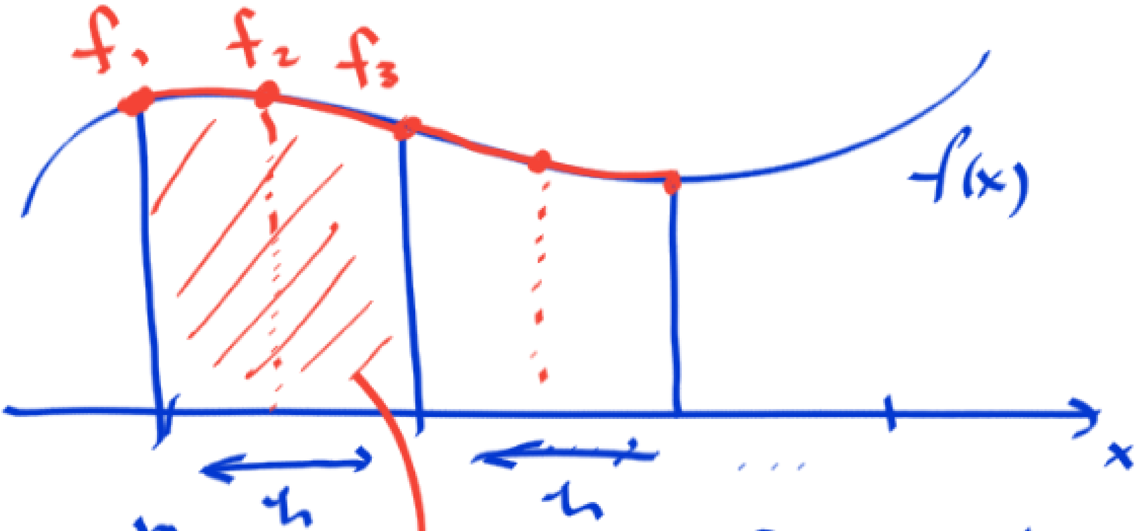


Trapezoidal Rule

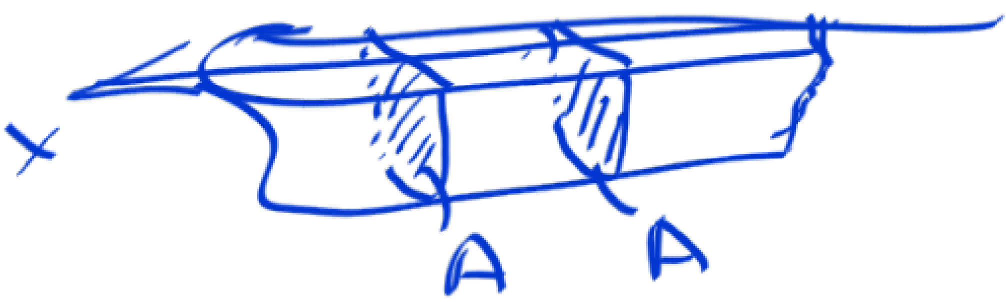


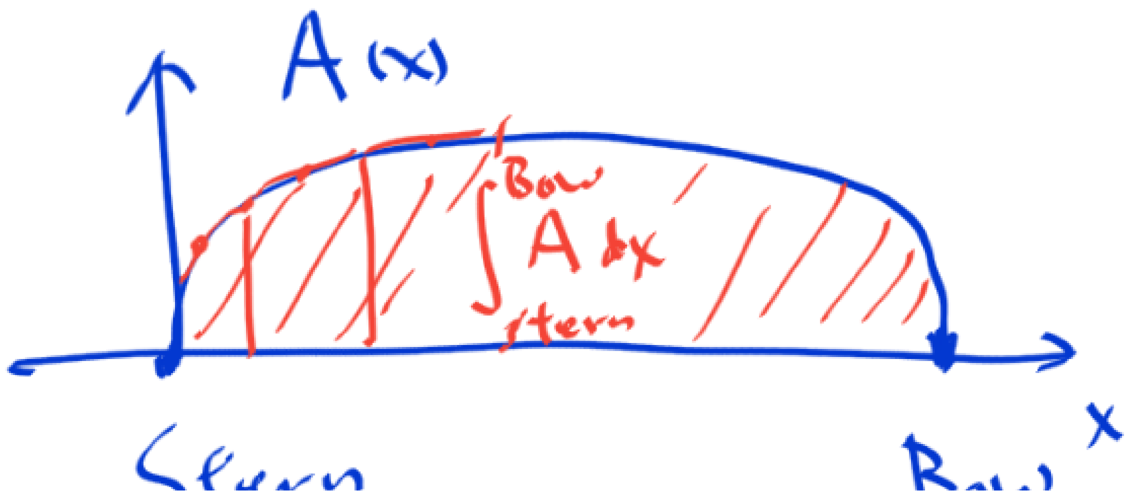
$$\sum_{i=1}^{n}\left[h\left(\frac{f_1+f_2}{2}\right)\right]_n \cong \int_a^b f\,dx$$

Simpson Rule



$$\sum_{i=1}^{n}\left[h\cdot\frac{(f_1+4f_2+f_3)}{6}\right]_i \cong \int_a^b f\,dx$$





Solution Ordinary Diff. Eq.

$\dot{y}(x) = f(x,y)$

$\dot{y}(t) = f(t,y)$

$\Rightarrow$ Initial Value Prob.

$$\dot{y}(t) = \boxed{\frac{dy(t)}{dt} = f(t,y)}\ ,\ t > t_0$$

I.V.P

$$y(t=t_0) = y_0 \leftarrow \text{Ini. Cond.}$$

$\uparrow$ given.

Find $\underline{y(t) = ?}$, $t > t_0$

e.g. $\left\{ \frac{dy}{dt} = 2y \ldots \right.$ $t > 0$

$$\boxed{\frac{dy}{dt} = 2y}$$

$$y(0) = y_0 \quad \leftarrow \text{I.C.}$$

$$y(t) = c\, e^{2t} \quad \Rightarrow \quad \boxed{y(t) = y_0 e^{2t}}$$

?

I.C.

$$\frac{dy}{dt} = 2y$$

$$\frac{y(t+\Delta t) - y(t)}{\Delta t} = 2y(t)$$

$t \leftarrow$ given.

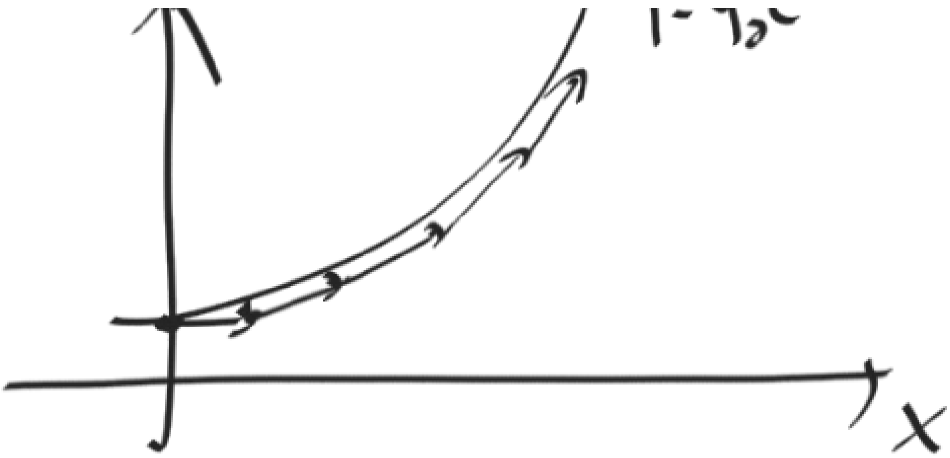
$t+\Delta t \leftarrow$ next time step. $\underline{y(t+\Delta t)}$?

<u>**Euler's Method**</u>

$$y(t+\Delta t) = y(t) + \Delta t \cdot \boxed{2y(t)}$$

slope

$$y = y_0 e^{2t}$$



1. Euler's Method

$$\frac{dy}{dt} = f(t, y)$$

Slope (Rate of Change)

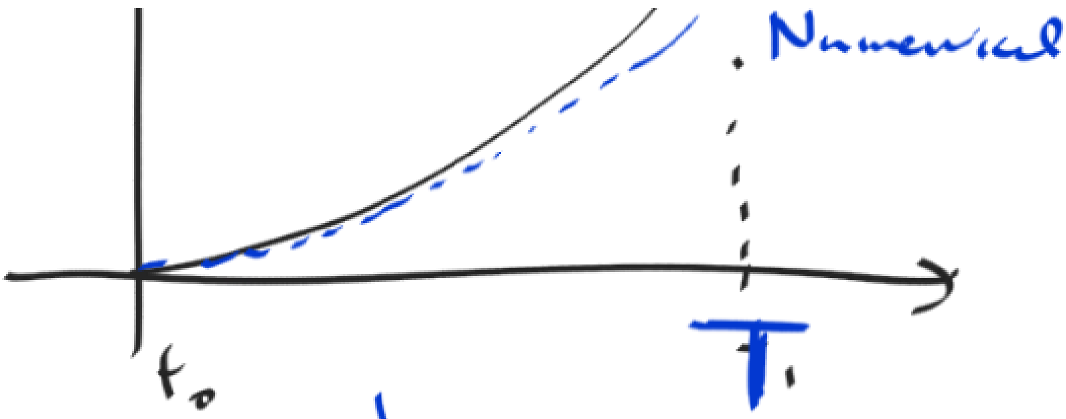
$$\frac{y^{n+1} - y^{n}}{\Delta t} = f(t^{n}, y^{n})$$

Initially.

$$y^{n+1} = y^{\cancel{n}\,0} + \Delta t \cdot f(t^{n}, y^{n})$$

$$y^{n+2} = y^{n+1} + \Delta t \cdot f(t^{n+1}, y^{n+1})$$

$$\vdots \qquad \vdots \qquad \vdots$$



$$\text{Error} = \left| y_{exact}(T) - y_{num}(T) \right|$$

$$\leq C\,(\Delta t)^1 \leftarrow \text{Euler.}$$

$\boxed{2}$ Runge-Kutta

2nd-Order Method (RK2)

$$\begin{cases} \dfrac{dy}{dt} = f(t, y), & t > t_0 \\ y(t_0) = y_0 \end{cases}$$

$$\begin{cases} S_1 = f(t^n, y^n) \\ S_2 = f(t^n + \Delta t, y^n + \Delta t \, S_1) \end{cases}$$

$$y^{n+1} = y^n + \Delta t \, \frac{(S_1 + S_2)}{2}$$

$$\text{Error} = \left| y_{\text{Exact}} - y_{\text{num}} \right| \leq C \, \Delta t^2$$

RK2 is 2^{nd}-order Accurate.

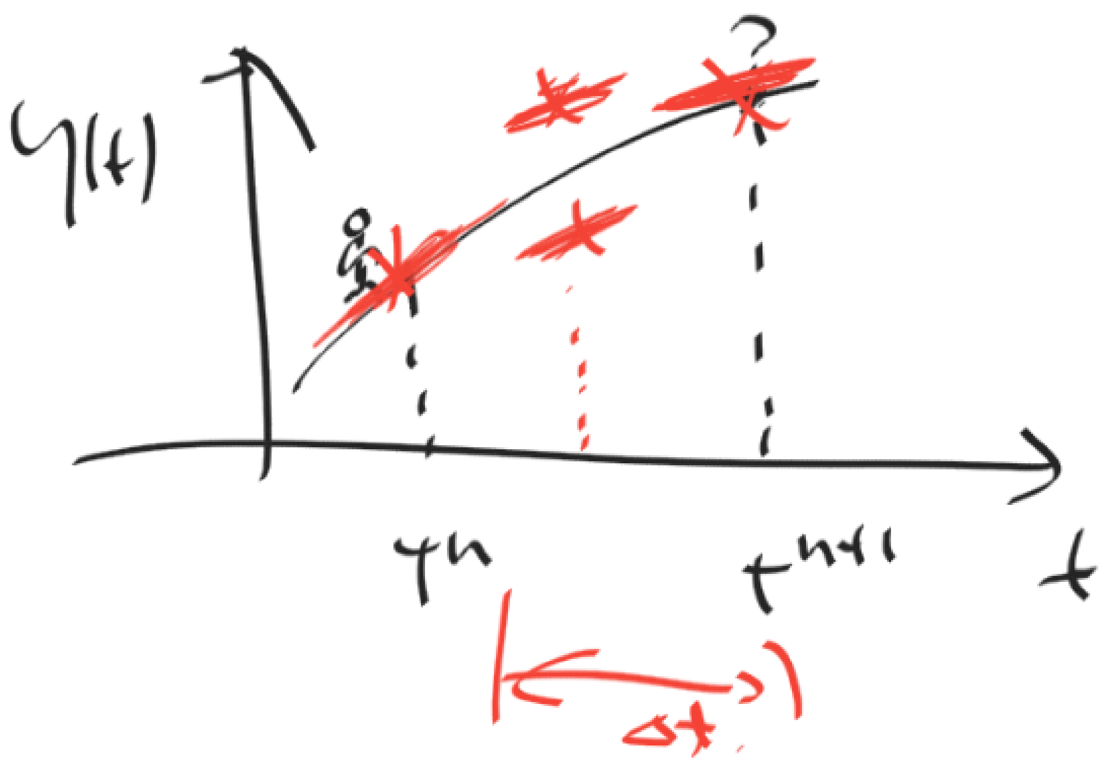
3. Runge Kutta 4^{th}-Order Method (**RK4**)

$$\frac{dy}{dt} = f(t, y)$$

o.DE.

t) t_0.

Slope.



$$\begin{cases} s_1 = f(t^n, y^n) \\ s_2 = f\left(t^n + \frac{\Delta t}{2}, y^n + \frac{1}{2}\Delta t \cdot s_1\right) \\ s_3 = f\left(t^n + \frac{\Delta t}{2}, y^n + \frac{1}{2}\Delta t \cdot s_2\right) \\ s_4 = f(t^n + \Delta t, y^n + \Delta t \cdot s_3) \end{cases}$$

$$y^{n+1} = y^n + \Delta t \cdot \left(\frac{s_1 + 2s_2 + 2s_3 + s_4}{6} \right)$$

Multiple Eq. (Sys. of Eq.)

$$\dot{y}_1 = f_1(t, y_1, y_2)$$

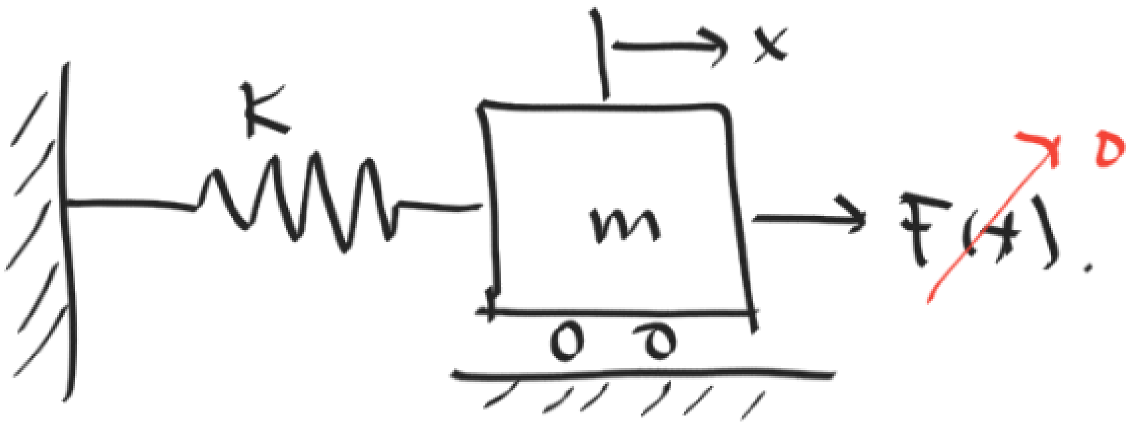
$$\dot{y}_2 = f_2(\ldots)$$

$$\boxed{y_2' = f_2(t, y_1, y_2)}$$

$$\Updownarrow$$

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{pmatrix}$$

H.W.



$$\boxed{m\ddot{x} + kx = \cancel{F(t)}\ 0}$$

I.C. $t > 0$

$x(0) = 1, \quad \dot{x}(0) = 0$

OR

I.C.

$x(0) = 2 \qquad \dot{x}(0) = 1$

$x(0)=0$, $\dot{x}(0)=1$

ODE : $m\ddot{x}+kx=0$

2nd-order ODE $\boxed{m\ddot{y}+ky=0}$

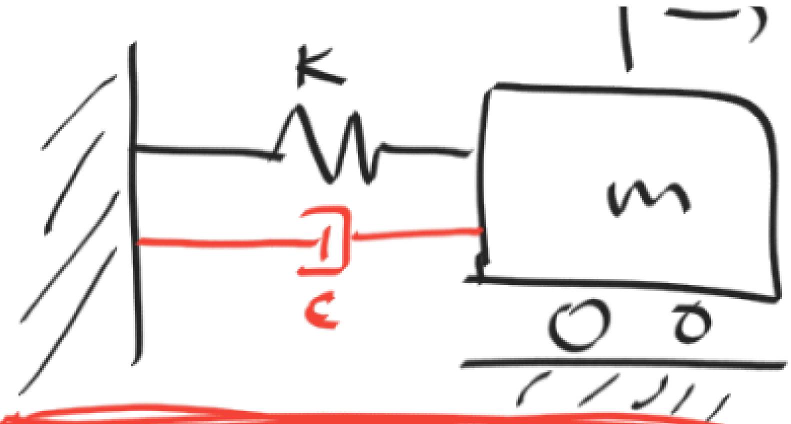
Let

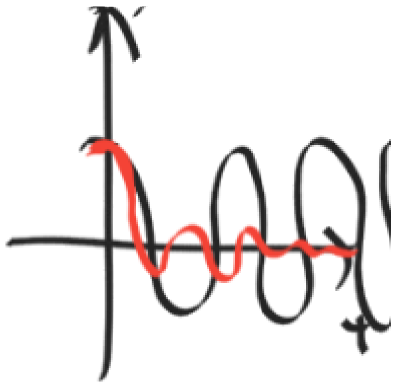
$$\boxed{\begin{aligned} y_1 &= y \\ y_2 &= \dot{y} \end{aligned}} \Rightarrow \boxed{\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -\frac{k}{m}y_1 \end{aligned}}$$

2 1st-order ODE.

$$\frac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_2 \\ -k/m \cdot y_1 \end{pmatrix}$$

H.W Last





$$\boxed{m\ddot{x} + c\dot{x} + kx = 0.}$$

$m = 1,$
$c = 1,$
$k = 1,$

I.C. $\boxed{\begin{array}{l} x(0) = 1 \\ \dot{x}(0) = 0 \end{array}}$

Find $\underline{x(t) = ?}$, $t > 0.$

$t \in [0, 10].$

$\Delta t = 1/100.$

1) Euler
2) RK2
3) RK4

마지막 수정: 오후 3:45