



# Tree

Teaching Team of Algorithm and Data Structure

# Introduction

- *Linked List, Stack, and Queue are linear data structures (Linear List).*
- *Linear List* → used for sequential serial data.
  - Example: queue, name of day of week, name of month of year, etc.
- Tree is a non-linear data structure that has special properties.
- Tree is one form of implementation of many linked lists (double linked lists) that are usually used to describe the hierarchical relationship between the elements that exist.
- *Tree* → used for hierarchical (one to many) data.
  - Example :
    - Family tree
    - The organizational structure of an agency / company.

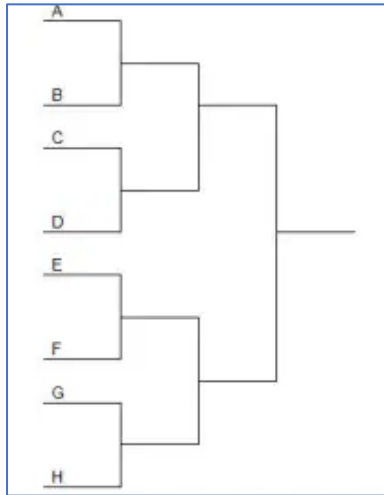
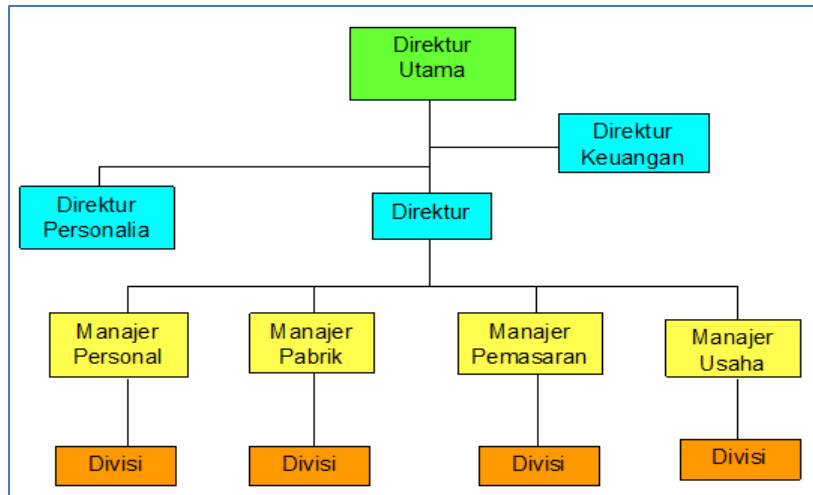
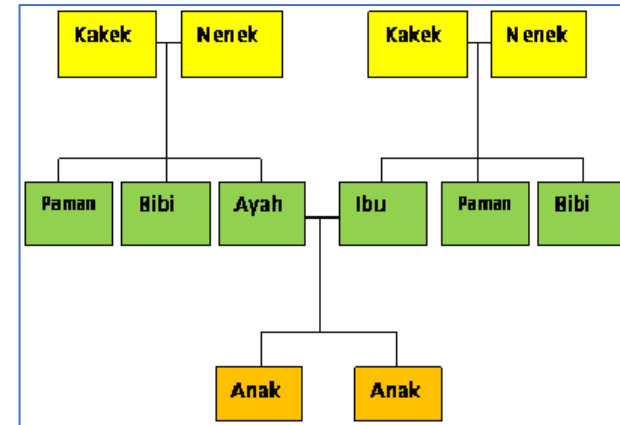


Diagram Pertandingan Sistem Gugur



Struktur Organisasi



Silsilah Keluarga

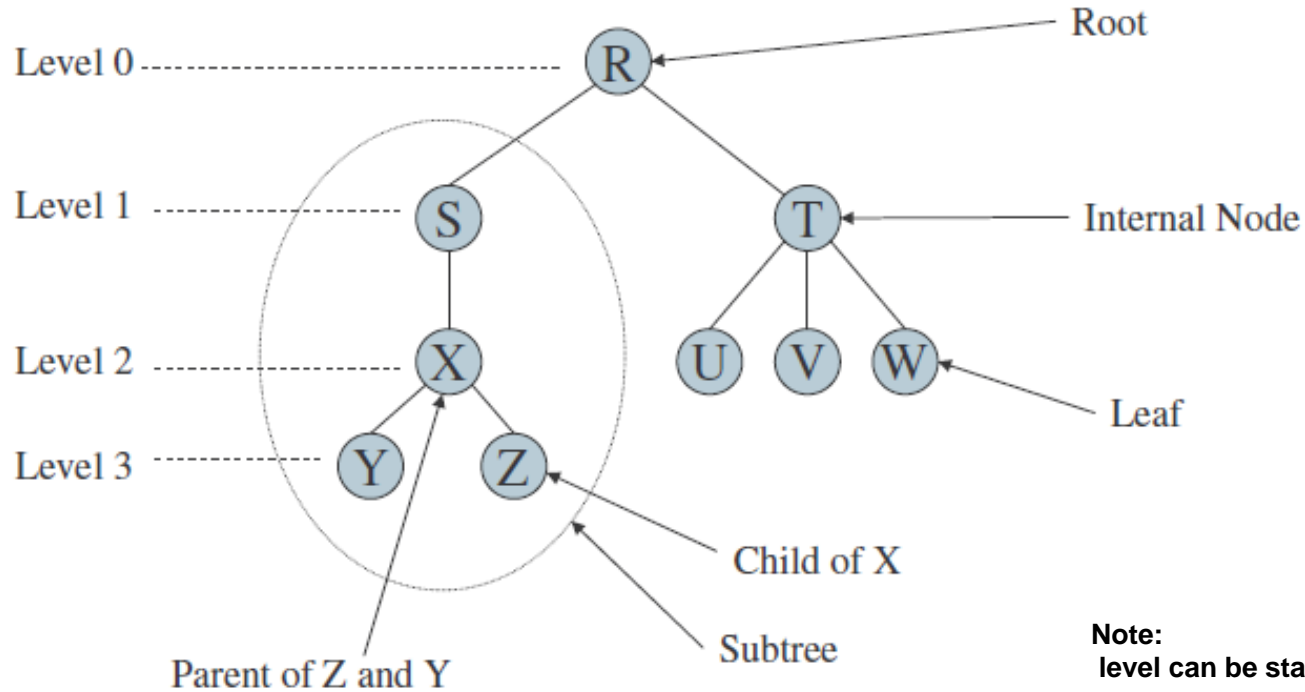
# Definition of Tree

- *Tree is a collection of elements that are interconnected in a hierarchical manner (one to many).*
- *Elements in a tree are called nodes.*

## Rules :

- A node can only have one parent. Except root, it doesn't have a parent.
- Each node can have zero or many children (one to many).
- Nodes that do not have children are called leaf.

# Tree Anatomy



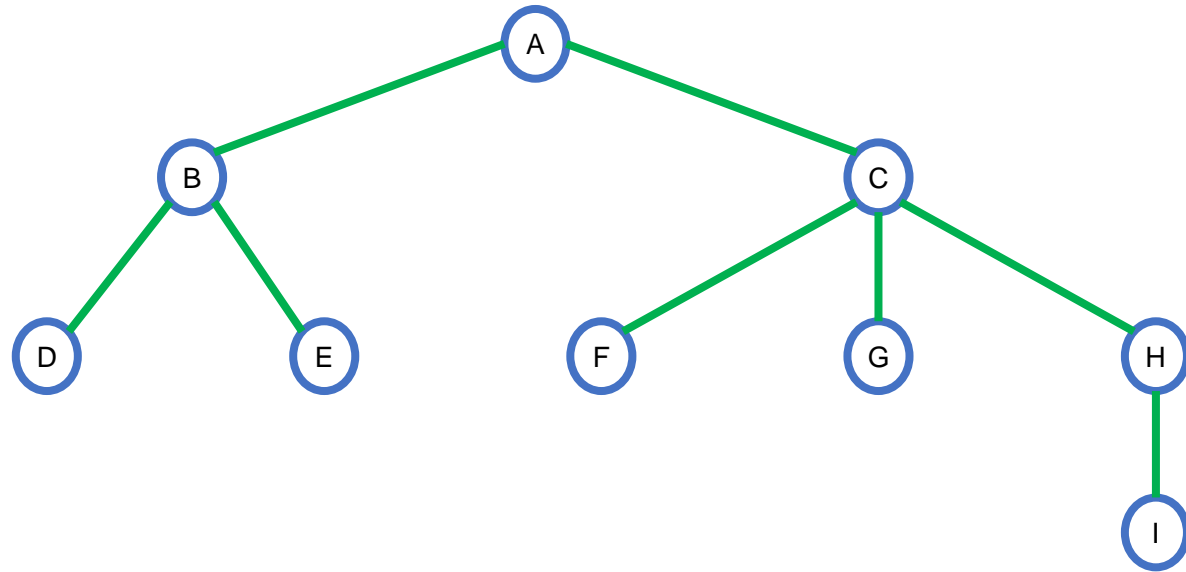
**Note:**  
level can be started from 0 or from 1.

# Tree Terminology



- **Node:** an element in a tree that contains information.
- **Predecessor:** nodes that are above certain nodes
- **Successor:** node under a certain node.
- **Ancestor:** all nodes located before a certain node and located on the same path
- **Descendant:** All nodes located after a certain node and located on the same path.
- **Parent:** Predecessor one level above one node.
- **Child:** Successor one level below one node.
- **Sibling:** A node that has the same parent as one node.
- **Subtree:** The part of a tree in the form of a node and its descendant.
- **Size:** The number of nodes in a tree.
- **Height:** The number of levels in a tree.
- **Root:** A special node in a tree that has no predecessor.
- **Leaf:** Nodes in a tree that have no successor.
- **Degree:** The number of children a node has

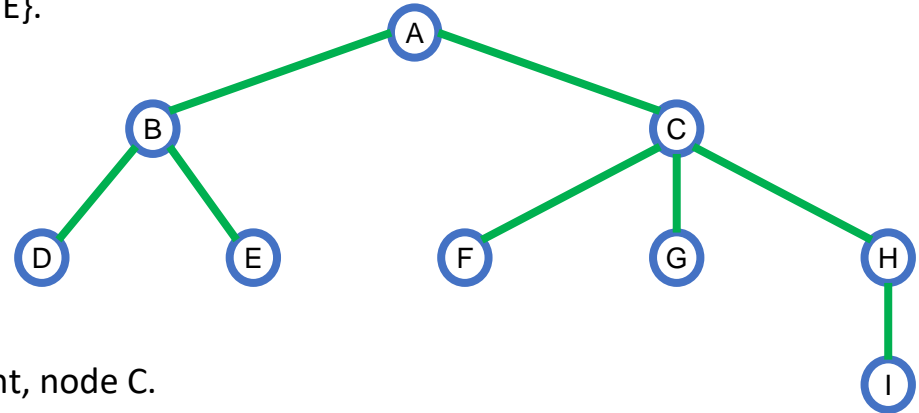
## Tree: an Example



Exercise, which part (node) of the tree above, which is in accordance with the terminology in the tree?

# Answer

- **Node** : There are 9 nodes in the example tree , are node {A, B, C, D,E, F, G, H, I}
- **Predecessor** : node B is the predecessor of node {D, E}.
- **Successor** : node {D, E} is the successor of node B
- **Ancestor**: node {C, A} is the ancestor of node F
- **Descendant**: node {D, E} is descendant of node B
- **Parent**: node C is the parent of node {F, G, H}
- **Child**: node {B, C} is child of node A
- **Sibling**: node {F, G, H} is sibling, has the same parent, node C.
- **Subtree**: there are 2 subtree, namely subtree from node {B, D, E} and from node {C, F, G, H, I}
- **Size**: in the tree there are 9 nodes.
- **Height**: the level of the tree is 4.
- **Root**: node A is the root of the illustration tree.
- **Leaf**: the leaf node is {D, E, F, G, I}.
- **Degree**: node B has degrees 2 {D, E} and node C has degrees 3 {F, G, H}





# *Tree Facts*

- Each node, except root, has ONLY one parent.
- Subtree is a collection of children from a node, which forms a smaller tree structure



# Binary Tree

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# ***Binary Tree***

- In the tree there are types of trees that have special properties, including the **binary tree**.
- Binary Tree is a hierarchical organization of several nodes, where for each node **ONLY** has a maximum of **2 children**.
- Specifically, the 2 children in question, are called left-child and right-child.

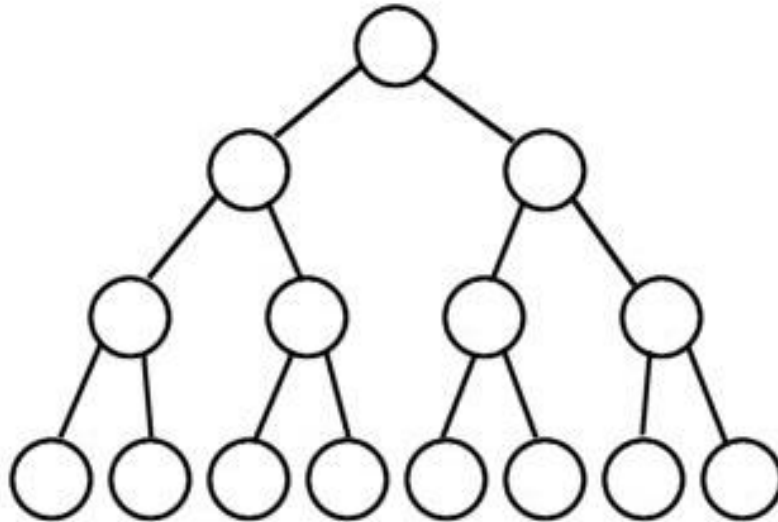
# Type of Binary Tree

There are several types of binary trees based on the structure of the nodes in it, including:

- *Full Binary Tree*
- *Strict Binary Tree*
- *Complete Binary Tree*
- *Incomplete Binary Tree*
- *Skewed Binary Tree*

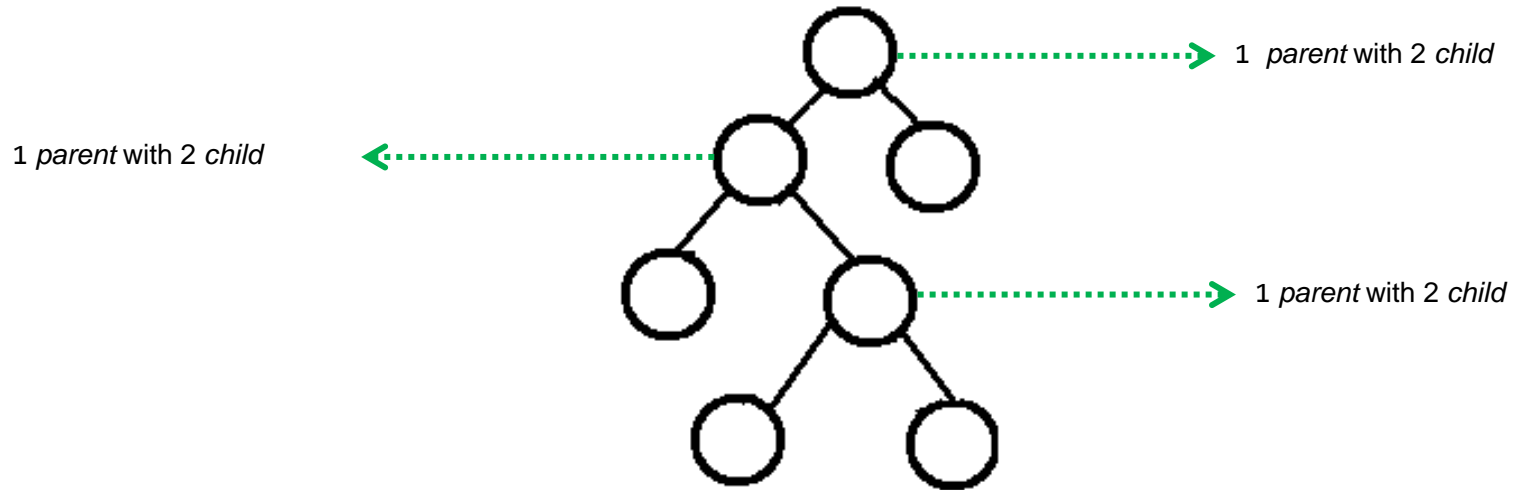
# *Full Binary Tree*

- All nodes (except leaves) have 2 children and each branch has the same segment length. Or, each subtree has the same path length
- Also called maximum binary tree or perfect binary tree.



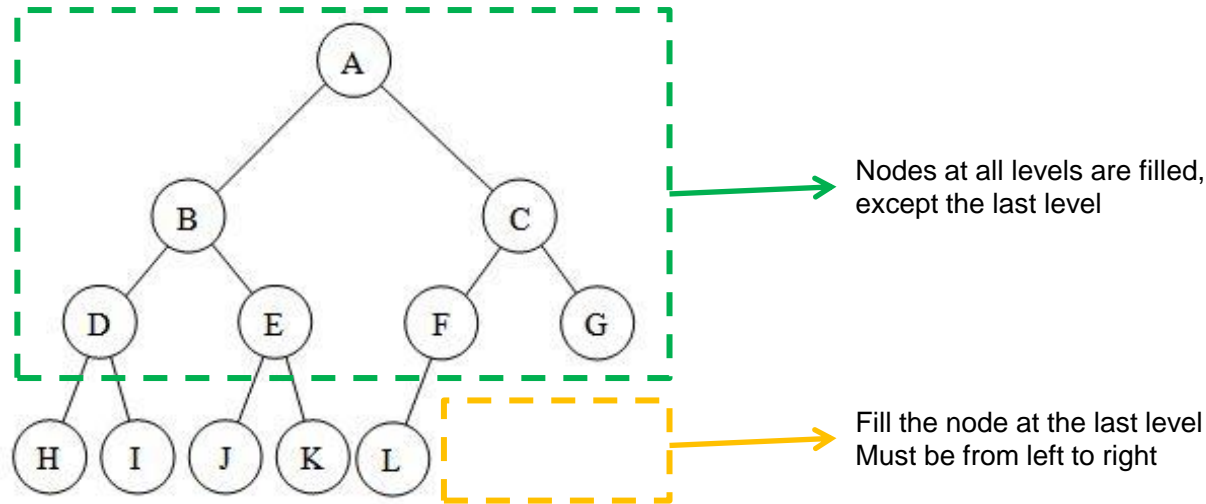
# Strict Binary Tree

- Each parent node **MUST** have **2 children**, no parent node **ONLY** has **one child**.
- Each subtree is not required to have the same path length as the other subtree as in the full binary tree.



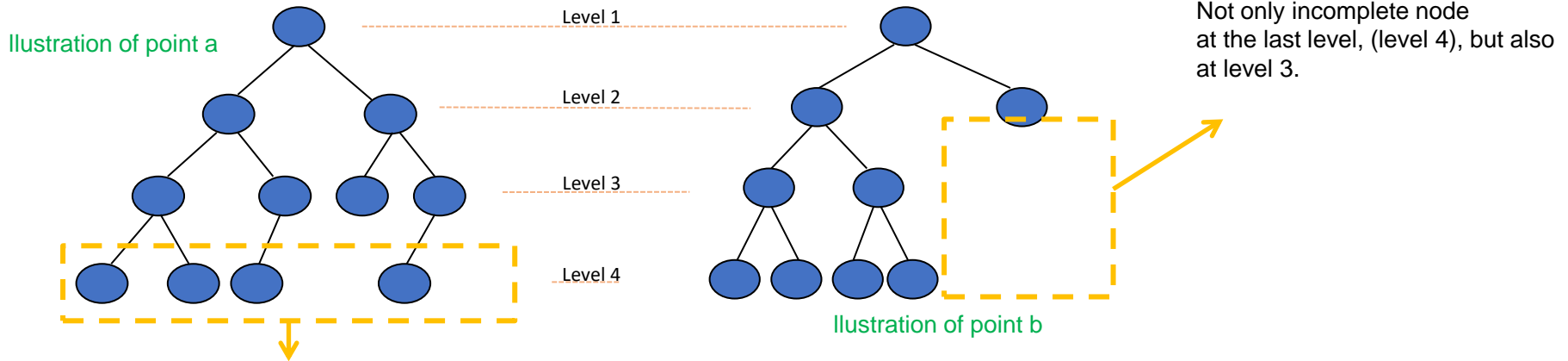
# Complete Binary Tree

- A tree which has **complete nodes at all levels, except for the last level**.
- At the last level, nodes **can be filled in incomplete** but filling from nodes **MUST be ordered from left to right**.



# Incomplete Binary Tree

- a. All nodes at the last level are filled in unevenly from left to right first.
- b. There are empty nodes in the tree in addition to the last level position.

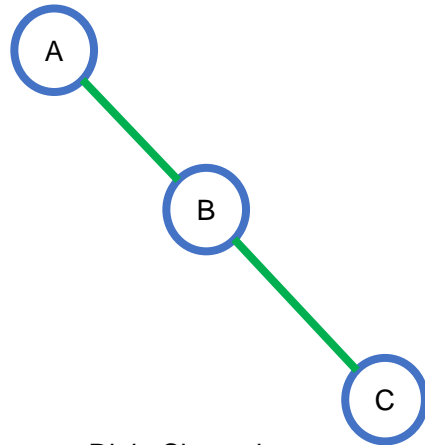


The last level, the loaded node is not coherent from the leftmost node.

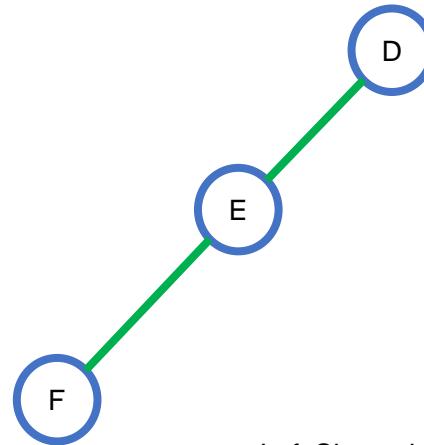


# Skewed Binary Tree

- A binary tree where all nodes (except leaves) have only one child.
- Also called a binary tree minimum.



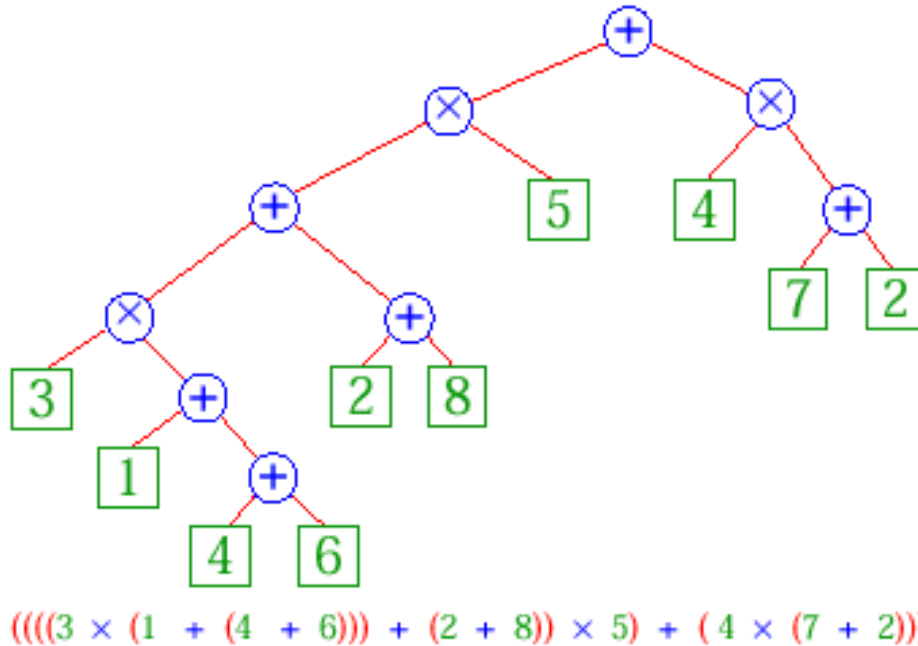
Right Skewed



Left Skewed

# Example of a Binary Tree

- Representation of arithmetic expressions



# Exercise

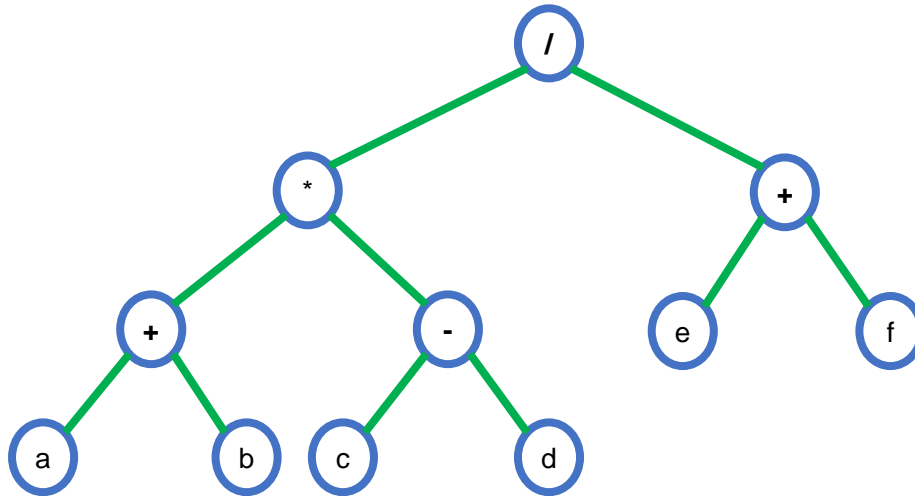
Create a binary tree from the following arithmetic expression:

1.  $(a + b) * (c - d) / (e + f)$
2.  $(a + b) * ((b - c) + d)$

# Exercise – Answer 1

Create a binary tree from the following arithmetic expression:

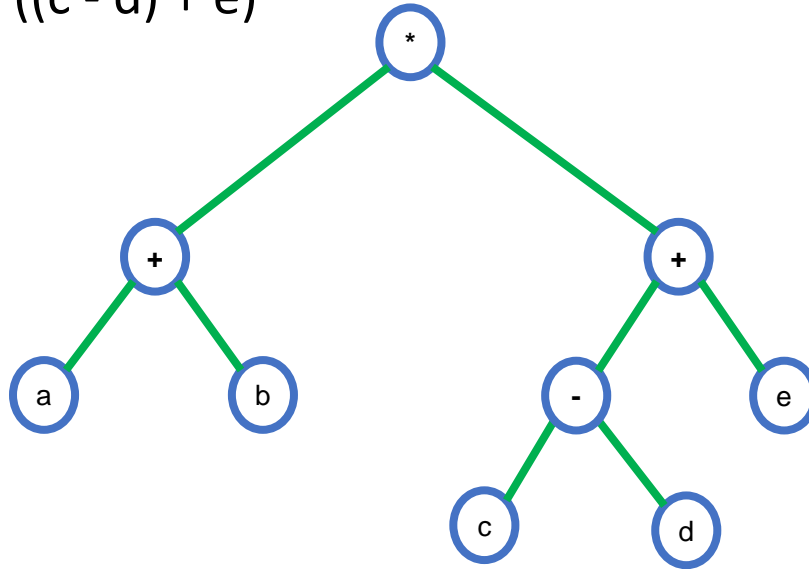
1.  $(a + b) * (c - d) / (e + f)$



# Exercise – Answer 2

Create a binary tree from the following arithmetic expression:

$$2. (a + b) * ((c - d) + e)$$



# Exercise Task 1

Create a binary tree from the following arithmetic expression:

1.  $a * (b + c) / (e + (f - g))$
2.  $((a * b) * c) + (d / e) * f$

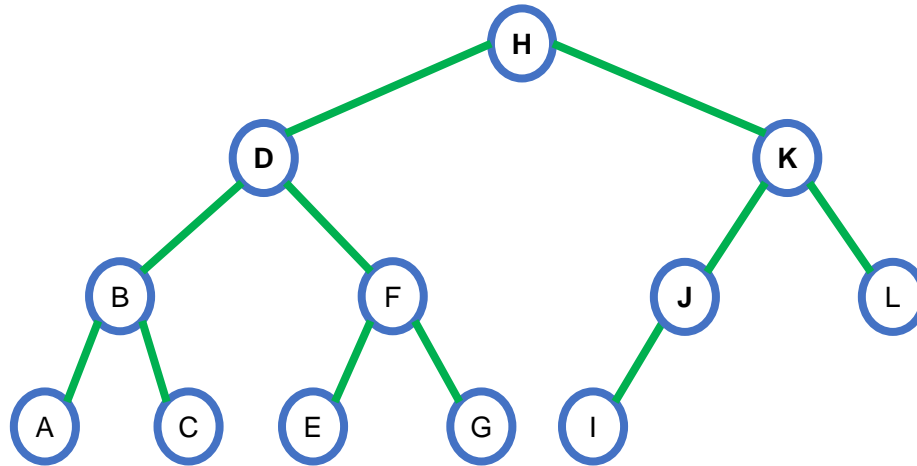


# Binary Tree Representation

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# Binary Tree Representation

- Binary Tree can be represented by using an array or linked list.



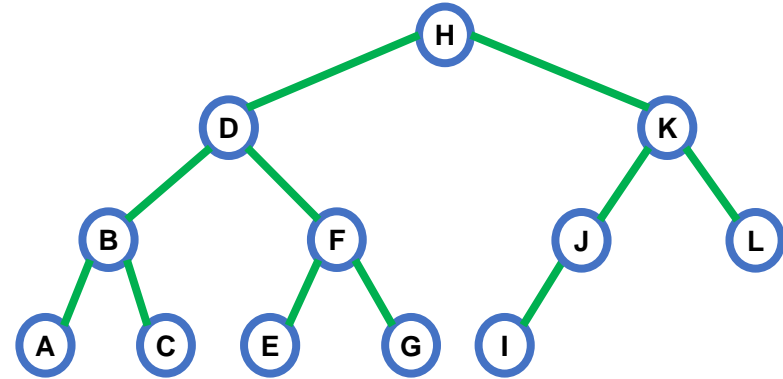


# Tree Representation with Array

The position of the node can be determined based on the following formula:

- The root assumption starts at index-0:
  - The left child of node  $i$  is at index:  $2 * i + 1$
  - Right child of node  $i$  is at index:  $2 * i + 2$
- The root assumption starts at index 1:
  - The left child of node  $i$  is at index:  $2 * i$
  - Right child of node  $i$  is in index:  $2 * i + 1$

# Tree Representation with Array (cont.)



- The root assumption starts at index-0:

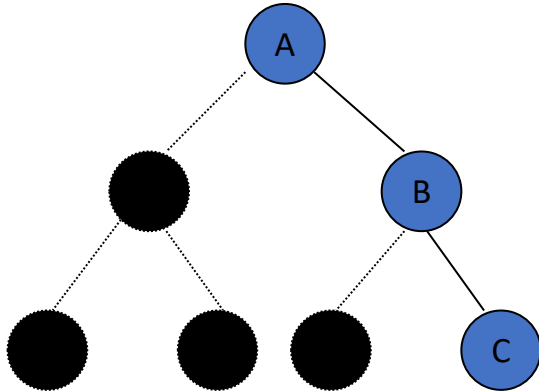
H	D	K	B	F	J	L	A	C	E	G	I
0	1	2	3	4	5	6	7	8	9	10	11

- The root assumption starts at index 1:

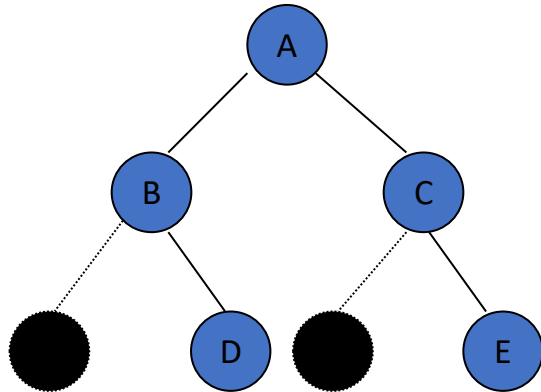
	H	D	K	B	F	J	L	A	C	E	G	I
0	1	2	3	4	5	6	7	8	9	10	11	12

# Tree Representation with Array (cont.)

- The binary tree aside has 3 nodes {A, B, C}
- Black nodes represent missing elements



# Tree Representation with Array (cont.)

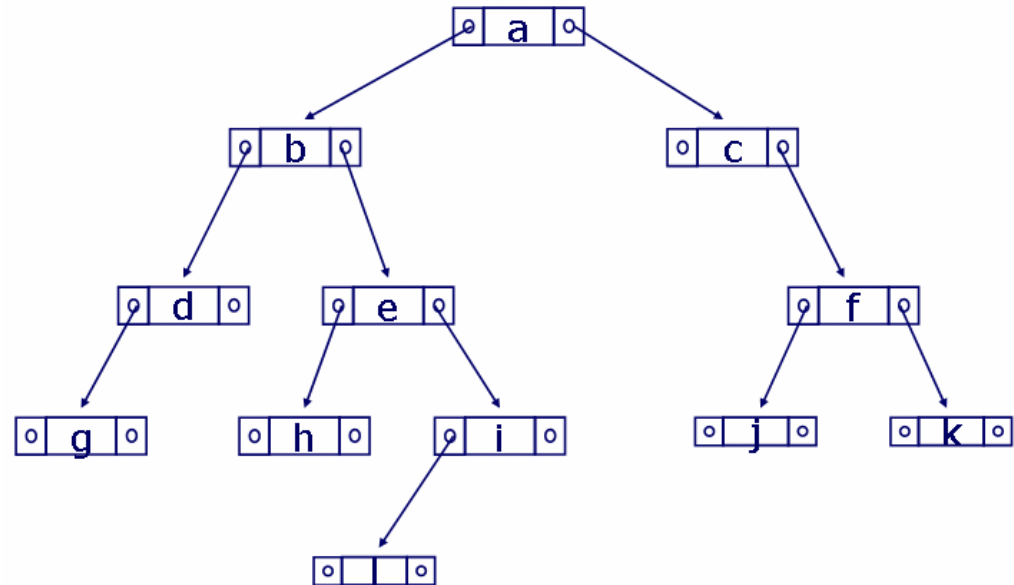


- The binary tree aside has 5 nodes {A, B, C, D, E}
- Black nodes represent missing elements

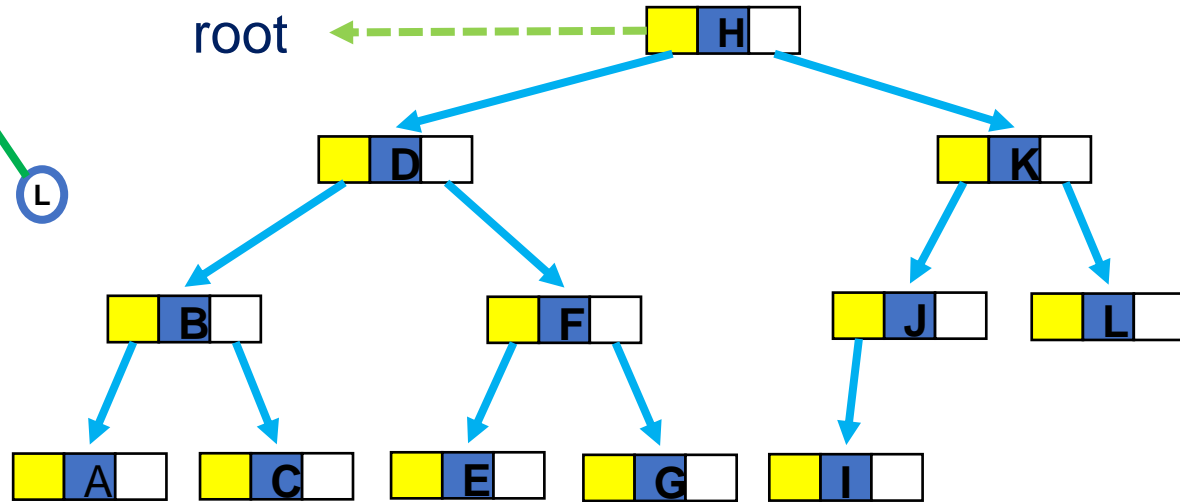
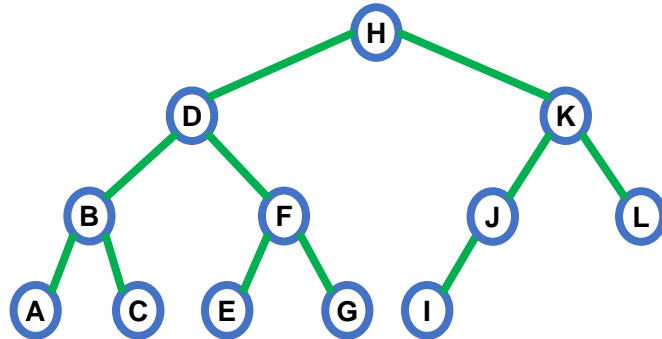


# Tree Representation with *Linked List*

- The binary tree implementation can be done using a data linked list structure; each node consists of 3 parts, i.e
  1. Left Pointer
  2. Data / element info
  3. Right Pointer



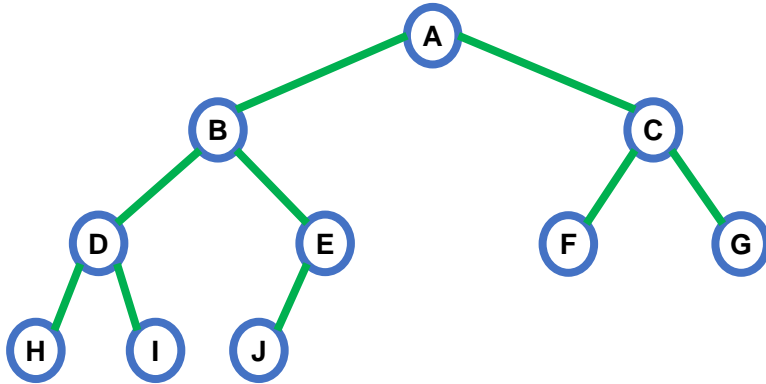
# Tree Representation with *Linked List* (cont.)



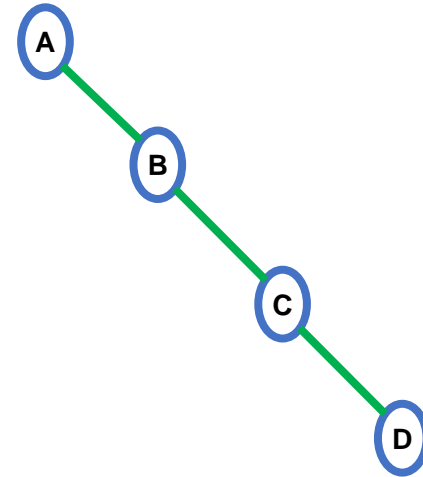
leftChild
  element
  rightChild

# Exercise Task 2

- Represent the following tree with illustration **array** and **linked lists**.



Exercise 1



Exercise 2



# Binary Search Tree

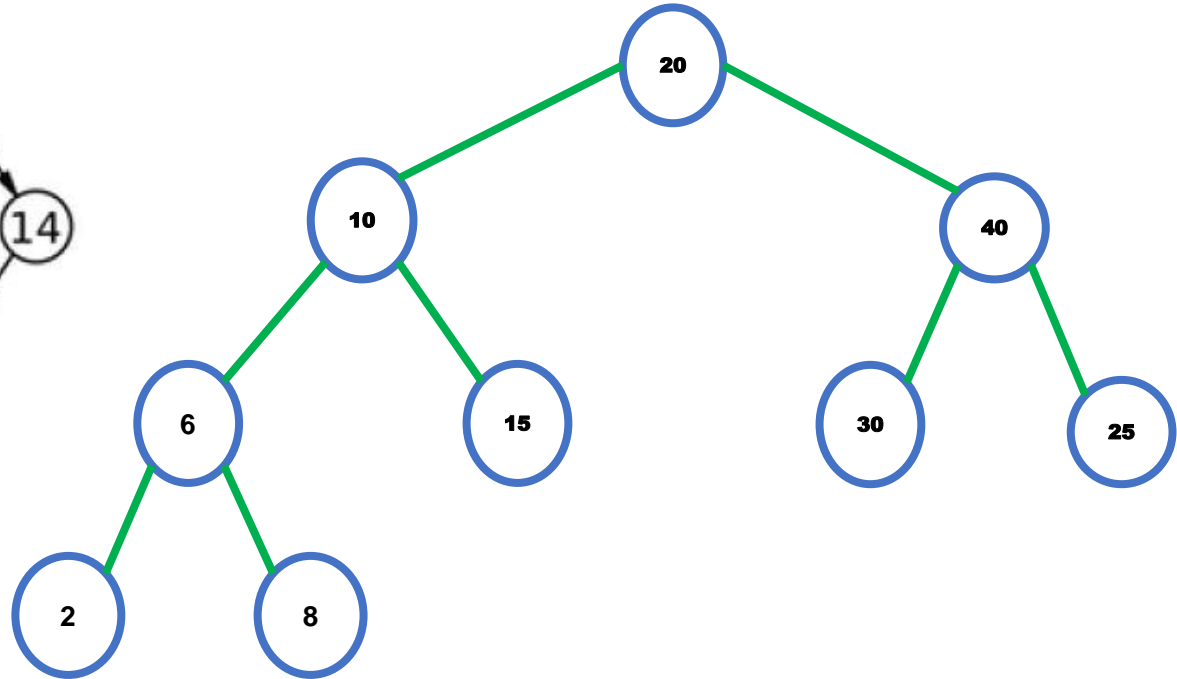
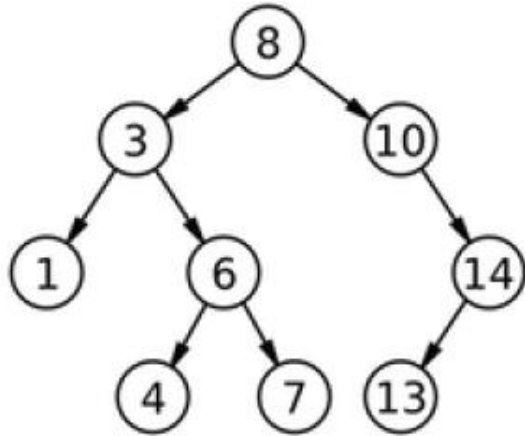
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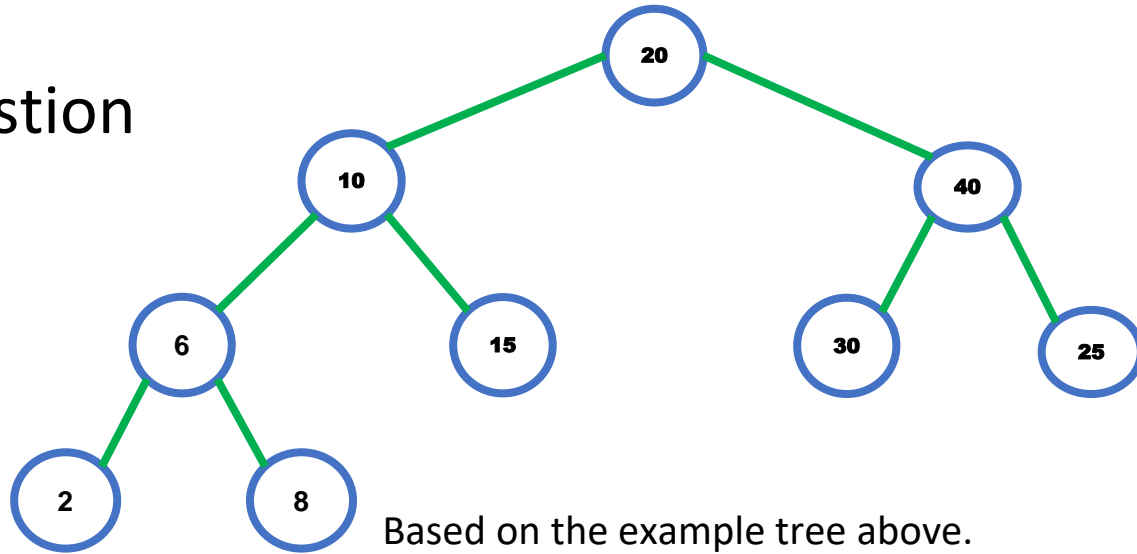
## *Binary Search Tree (BST)*

- Binary Search Tree is a special form of binary tree with the characteristic that all **left-children** **must be smaller** than the **right-child** and its parent.
- Binary search tree is made to overcome **weaknesses** in ordinary binary tree, is the difficulty in searching / searching for certain nodes in the binary tree.
- Also called an **ordered Binary tree**, which is a binary tree that **all children** from each **node are sorted**.

## Example of a Binary Search Tree



## Question



Based on the example tree above.

- What if there are additional nodes with a value of 13?
- What if there are additional nodes with a value of 50?
- What if there are additional nodes with a value of 27?

**Answer:** By tracing each node element in the tree before adding it by traversal.

**Binary Tree Traversal**



# Binary Tree Traversal

# Binary Tree Traversal

- A method of tracing all nodes in the binary tree.
- There are several methods, including:
  - Preoder
  - Inorder
  - Postorder
  - Level order

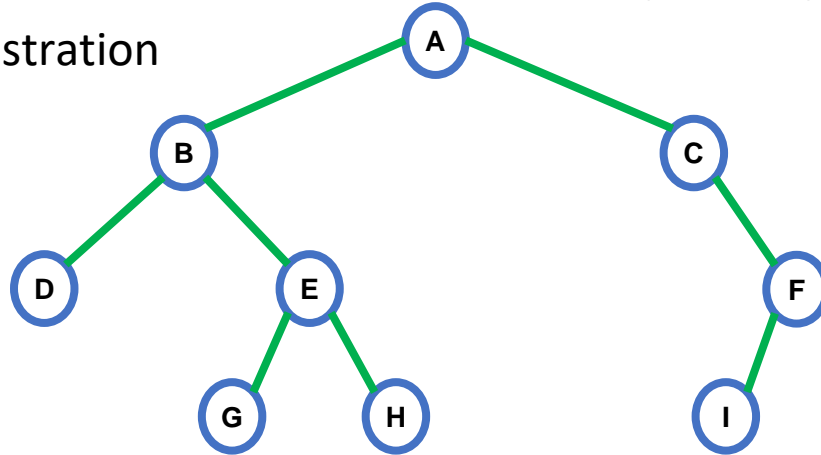


## *Binary Tree Traversal - Preorder*

- *Preorder Traversal Steps:*
  1. Visit and print data on root
  2. Recursively visit and print all data in the **left** subtree starting from the **left-child**.
  3. Recursively visit and print all data in the **right** subtree starting from the **left-child**.

## Binary Tree Traversal - Preorder (cont.)

- Example illustration



Preorder Traversal → A, B, D, E, G, H, C, F, I

Video illustration: <https://youtu.be/1WxLM2hwL-U>

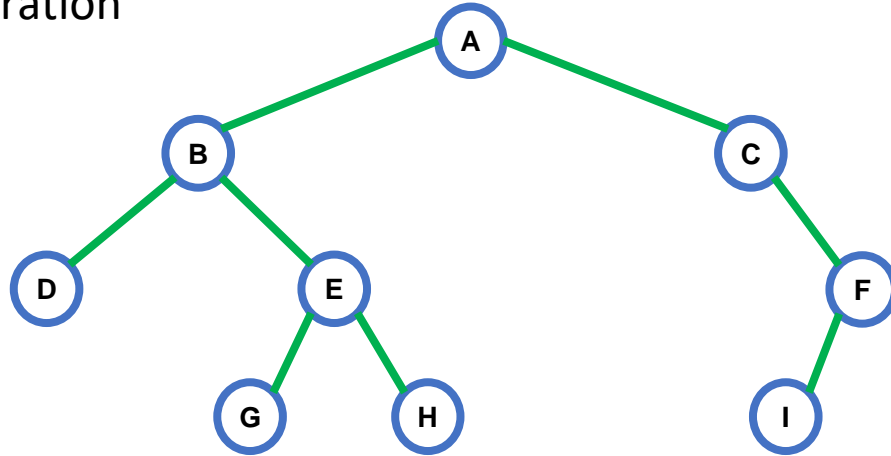
## *Binary Tree Traversal - Inorder*

- *Inorder Traversal Steps:*
  1. Recursively visit and print all data in the **left** subtree.
  2. Visit and print data on root
  3. Recursively visit and print all data in the **right** subtree.



## Binary Tree Traversal - Inorder (cont.)

- Example illustration



Inorder Traversal → D, B, G, E, H, A, C, I, F

Video illustration : <https://youtu.be/5dySuyZf9Qg>

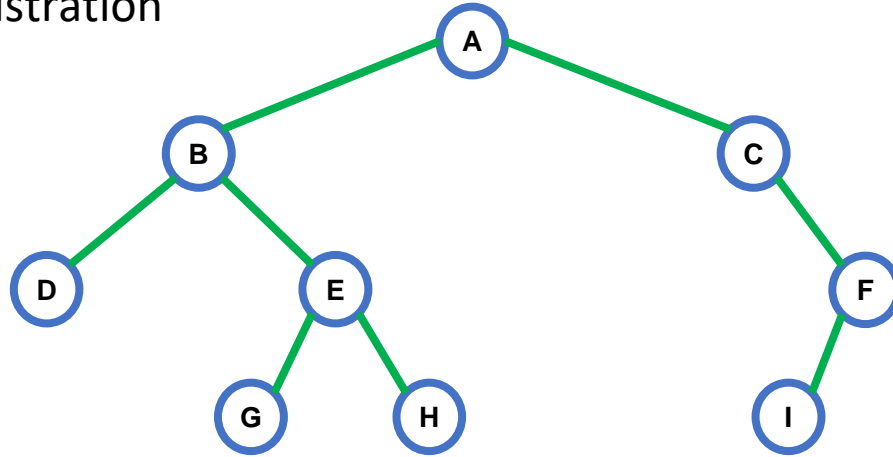


## *Binary Tree Traversal - Postorder*

- *Postorder Traversal Steps:*
  1. Recursively visit and print all data in the **left** subtree.
  2. Recursively visit and print all data in the **right** subtree.
  3. Visit and print data on root

## Binary Tree Traversal - Postorder (cont.)

- Example illustration



Postorder Traversal → D, G, H, E, B, I, F, C, A

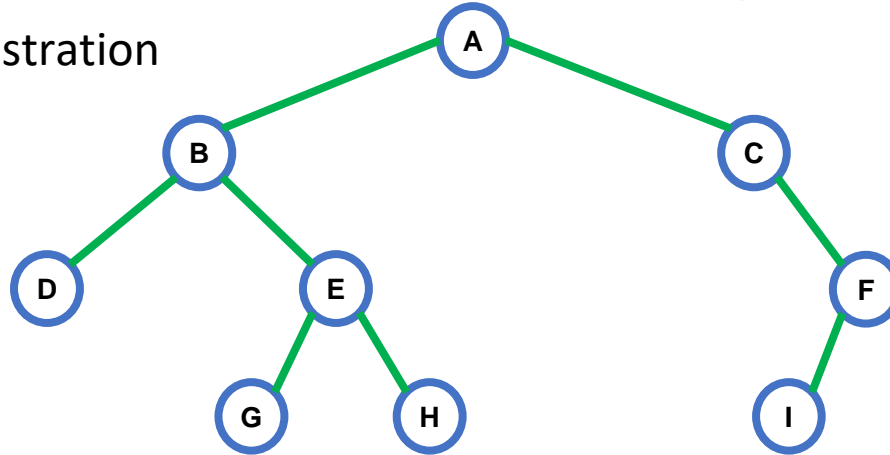
Video illustration : <https://youtu.be/4zVdfkpcT6U>

## *Binary Tree Traversal – Level order*

- The order level is also called the Breath First Order
- Postorder Traversal Steps:
  1. Visit and print data on root
  2. Recursively visit the nodes under root from the leftmost to the rightmost
  3. Go back to 2nd point to the lowest level.

## Binary Tree Traversal – Level order (cont.)

- Example illustration

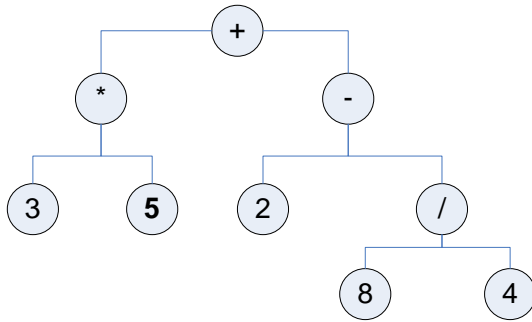


Level order Traversal → A, B, C, D, E, F, G, H, I

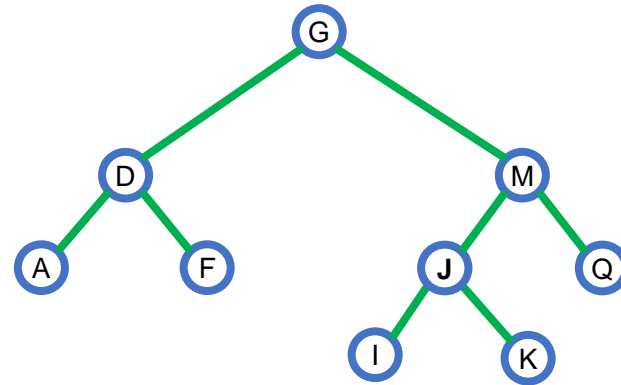
Video illustration: <https://youtu.be/lozGo2kwRYE>

# Exercise 3

- Browse the following binary trees using the preorder, inorder, postorder, and order traversal level methods.



Question 1



Question 2



# Operations on the Binary Search Tree

# Operations on BST

In BST there are several operations that are used to manage binary trees, including:

- **Find:** data search operation in the same tree as  $n$  (the value to be searched for).
- **Insert:** Operation of adding data / nodes to a tree
- **Delete:** The operation of deleting data / nodes in a tree
- **Display:** Operation display data

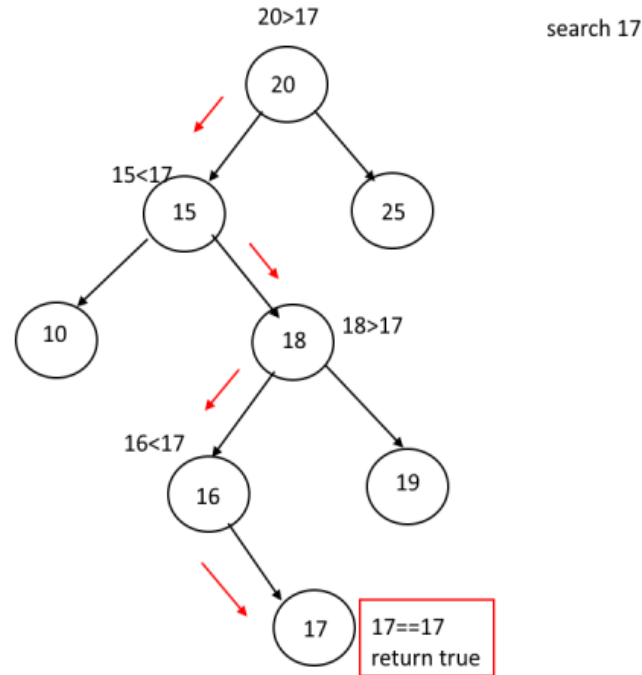


## Operation - Find

1. Start from root and compare the root value with  $n$  (the value sought).
2. If the value of  $n$  is smaller than root, do a search on the left subtree of root.
3. If the value of  $n$  is greater than root, search the right subtree of the root.
4. Search nodes under root in the same manner as steps number 2 & 3.
5. If found, return true.
6. If at the end of the node (leaf) is not found, return false.

# Operation - Find(*cont.*)

## ► Illustration



Video illustration Find/search element : <https://youtu.be/a7xOXL7hn94>

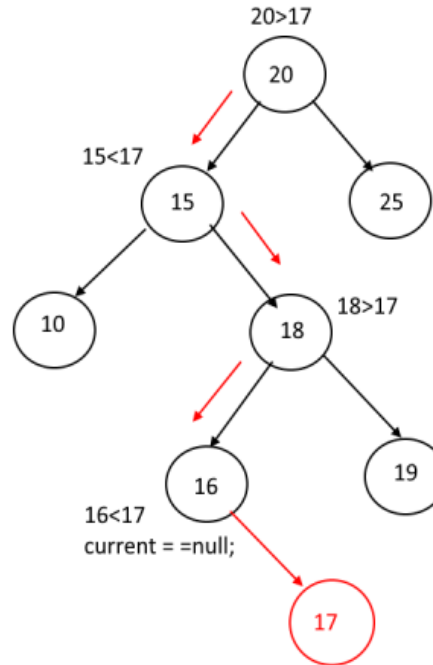
## Operation – *Insert*

The insert operation is similar to the find operation. To insert a node, we must first find the right place to put the node. The steps:

1. Create a variable of type current node, set current with root.
2. Compare the current value with n (the value to be entered).
3. If the current value is greater than n, look for a place to the left of the root.
4. If the current value is smaller and n, search for a place to the right of the root.
5. If you find the current null value, which means that the search has reached the leaf (end of the node), place the new node containing the n into the current position.

# Operation – *Insert (cont.)*

## ► Illustration



Insert 17

Video illustration *Insert element* : <https://youtu.be/WoRa9vnHkDM>

# Operation – *Delete*

Possible delete scenario that can be done:

1. The node to be deleted is the leaf node (no children).
2. The node to be deleted only has one child.
3. The node to be deleted has two children.

Video illustration *Delete element* : <https://youtu.be/c1zKNiABiHk>

## Operation – *Delete* (scenario 1)

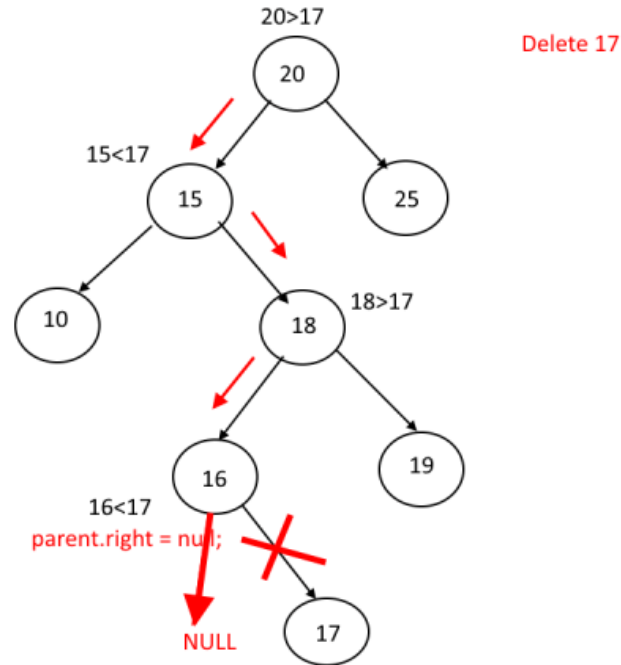
**The node to be deleted is the leaf node (no children).**

Steps:

1. Traverse the tree by comparing each node visited.
2. If the node visited is greater than  $n$  (the value of the node to be deleted), continue traversing to the left.
3. If the node visited is smaller than  $n$ , continue traversing to the right.
4. If you find it, set the pointer
  - $\text{left} = \text{null}$   $\rightarrow$  if the value of  $n$  (the node to be deleted or child) is smaller than the parent, or
  - $\text{right} = \text{null}$   $\rightarrow$  if the value of  $n$  (the node to be deleted or child) is greater than the parent.

# Operation – *Delete* (scenario 1) (cont.)

## Illustration



Case 1 : Node to be deleted is a leaf node ( No Children).

## Operation – *Delete* (scenario 2)

**The node to be deleted has one child.**

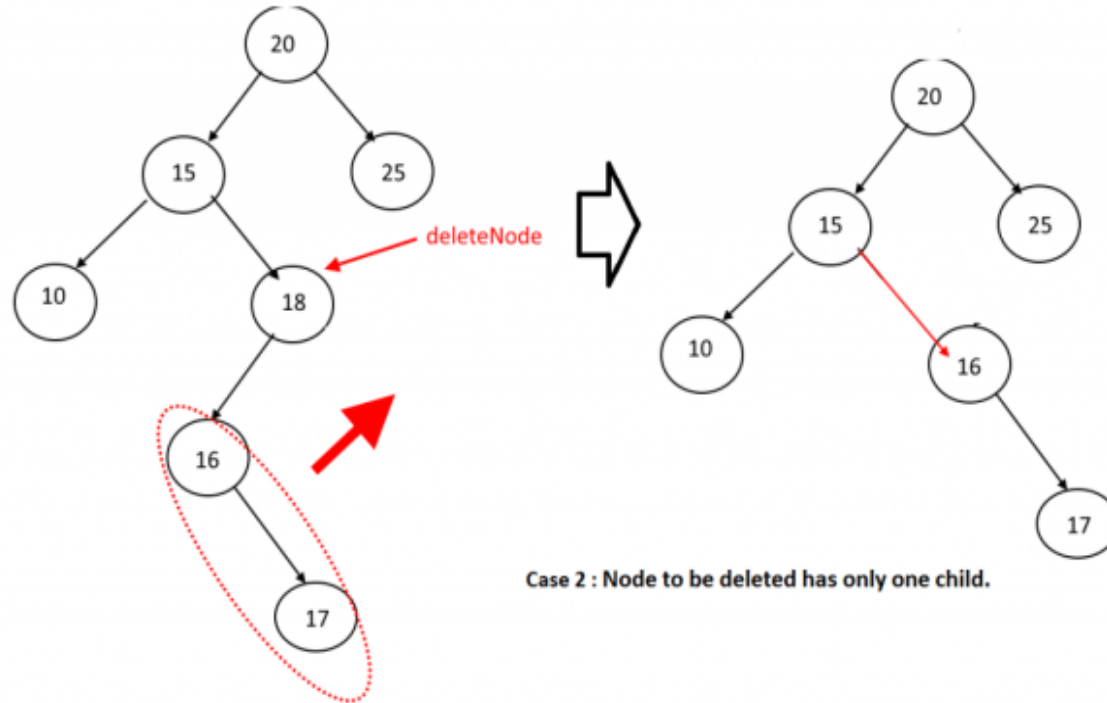
Steps:

1. Traverse the node to be deleted.
2. If found, note the parent of the node, and there is a node next to the parent node (right or left).
3. From that node, check which side is null (because that node only has one child).
4. For example, the node to be deleted has a child on the left. Then set the child of the node (along with its sub tree) and place it on the parent side to replace the position of the node to be deleted earlier.



# Operation – *Delete* (scenario 2) (cont.)

## Illustration





## Operation – *Delete* (scenario 3)

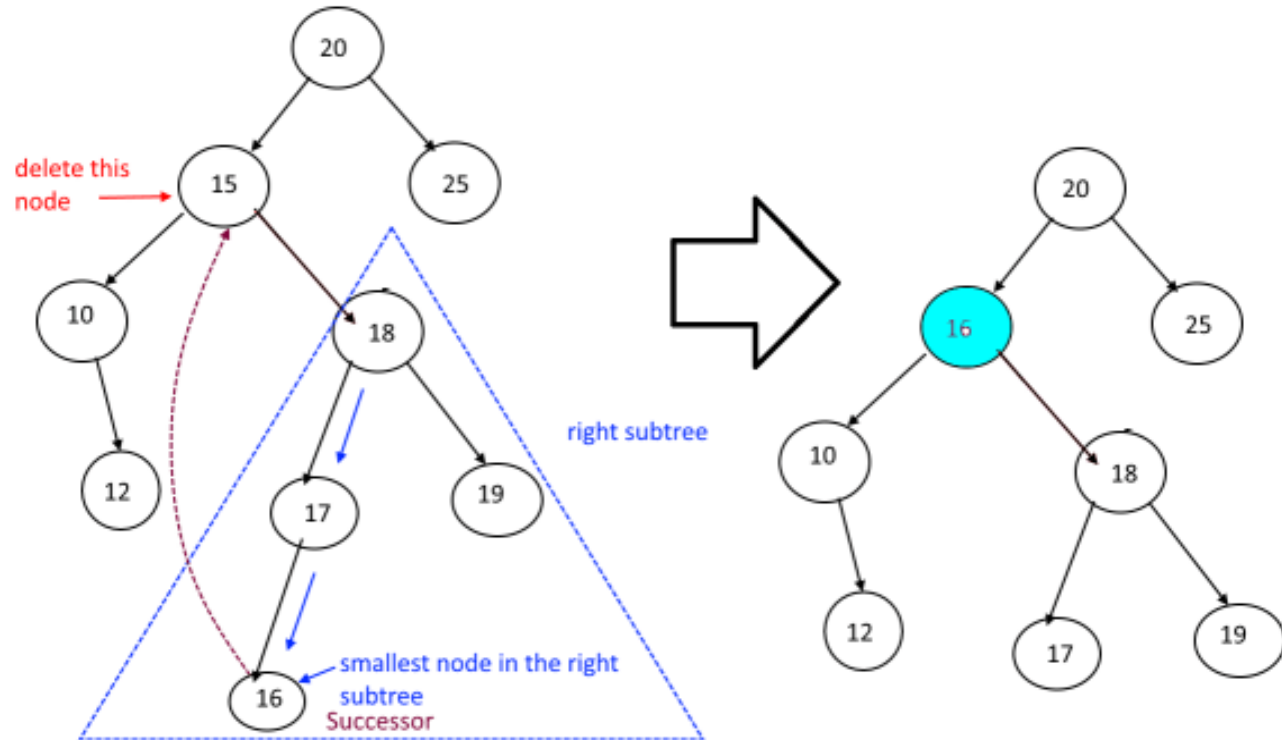
**The node to be deleted has two children.**

Steps:

1. Look for the successor of the node to be deleted, to replace the node to be deleted.
2. Successor is the smallest node from the right subtree on the node to be deleted.

# Operation – *Delete* (scenario 3) (cont.)

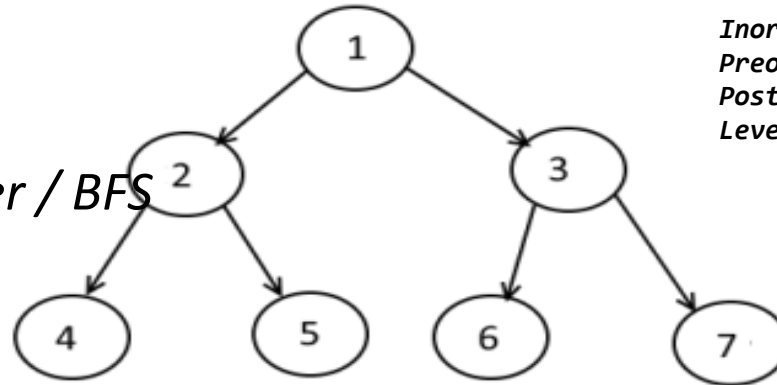
## Illustration



## Operation – *Display*

Prints all the nodes in the tree. The process of printing data uses the traversal method that has been studied, including the method

- *Preorder*
- *Inorder*
- *Postorder*
- *Level order / BFS*

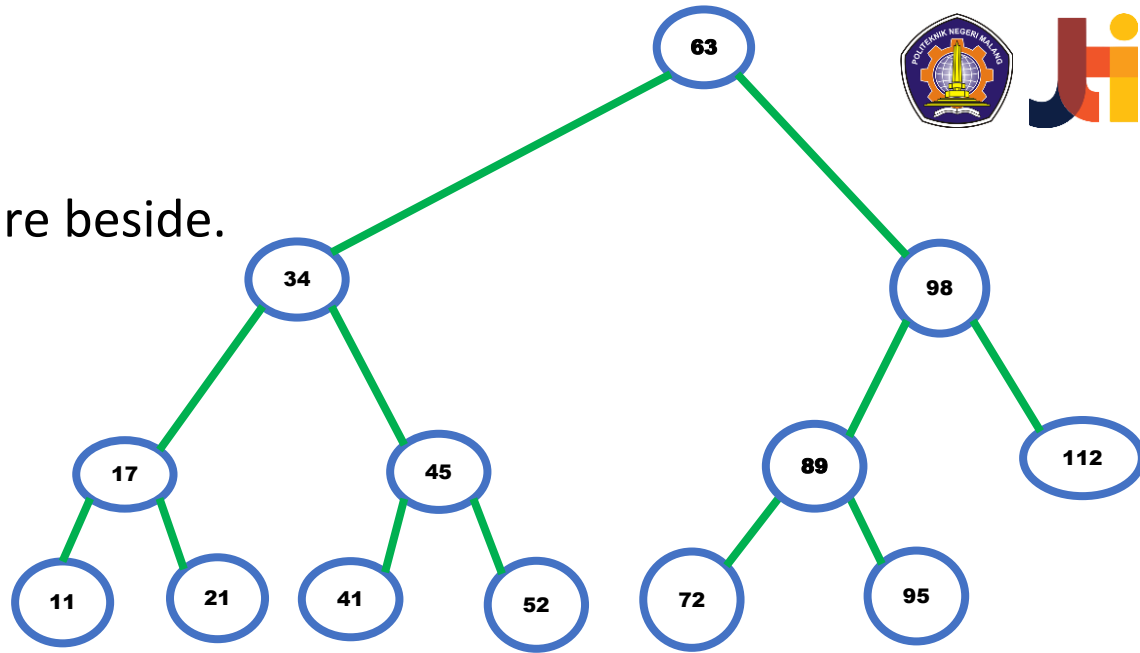


*Inorder* : 4, 2, 5, 1, 6, 3, 7  
*Preorder*: 1, 2, 4, 5, 3, 6, 7  
*Postorder*: 4, 5, 2, 6, 7, 3, 1  
*Level order*: 1, 2, 3, 4, 5, 6, 7

For the illustration video the Display element follows the Binary Tree Traverse illustration video

## Exercise 4

There is a tree like the picture beside.



There is new data (40) to be added and old data (98) to be deleted.

Illustrate the operations (find, insert, delete, display) that will be performed to overcome the addition and deletion of that data.

