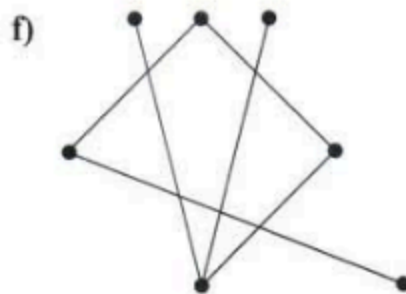
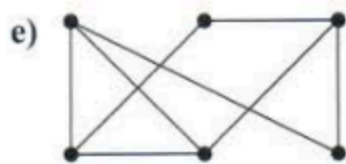
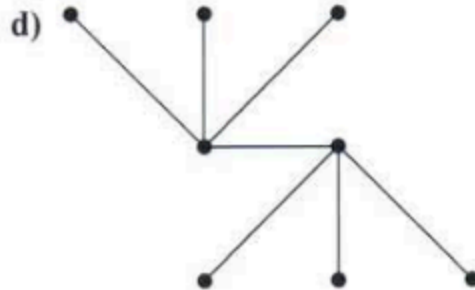
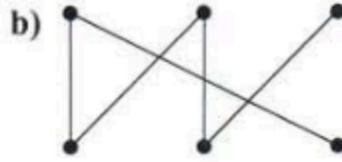
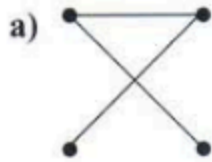


# Discrete

Which of the following is/are tree/s?

Answer [1](#) [2](#) [4](#) [6](#)



Choice [1](#): a

Choice [2](#): b

Choice [3](#): c

Choice [4](#): d

Choice [5](#): e

Choice [6](#): f

Ответы: (a/b/d/f)

---

For the sequences described in the four (4) embedded questions given below, match the first 10 terms of each of their sequences among the choices given here:

A. 1, 3, 7, 15, 31, 63, 127, 255, 511, 1023.

B. 1, 6, 13, 22, 33, 46, 61, 78, 97, 118.

- C. 1, 2, 2, 3, 3, 3, 3, 4, 4, 4.
- D. 1, 2, 2, 4, 8, 11, 33, 37, 148, 153.
- E. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55.
- F. 1, 5, 19, 65, 211, 665, 2059, 6305, 19171, 58025.
- G. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55.
- H. None.

The sequence whose  $n$ th term is the number of bits in the binary expansion of the number  $n$ .

C

The sequence whose  $n$ th term is the largest integer whose binary expansion has  $n$  bits.

A

The sequence whose  $n$ th term is  $3^n - 2^n$ .

F

The sequence whose first two terms are 1 and each succeeding term is the sum of the two preceding terms.

G

Match the following definitions with their terminologies given in the questions:

- A. Vertices that are directly connected by one or more edges.
- B. Multiple edges that connect the same vertices.
- C. No loops or parallel edges.
- D. One with connections between each two vertices.
- E. One in which each vertex has a direct with any other vertex.
- F. A roundtrip returning to the same vertex.

Adjacent vertices

A

Parallel edges

B

Simple graph

C

Complete graph

E

Cycle

F

Connected graph

D

---

For the recursively defined functions in the four (4) embedded questions given below, match the correct values of (  $f(2)$  and  $f(3)$  ) given here, if (  $f(0) = -1$ ,  $f(1) = 2$  for  $n = 1, 2, \dots$  )

A. 8, 176.

B. -0.5, -4.

C. 1, 5

D. -4, 32

E. 3, 13.

F. -1, 5.

G. 3, 5.

H. None.

(  $f(n + 1) = f(n) + 3 f(n - 1)$  )

F

(  $f(n + 1) = f(n)^2 f(n - 1)$  )

D

(  $f(n + 1) = 3f(n)^2 - 4 f(n - 1)^2$  )

A

(  $f(n + 1) = f(n - 1) / f(n)$  )

B

---

**\*\*Select the correct answer/s among the choices given here which correspond to the modulo calculation in the four (4) questions below.**

**A. 1**

**B. -1**

**C. 3**

**D. -5**

**E. -2**

F. 2

G. -3

H. None

-19 mod 5

A

105 mod 26

A

3 mod 13

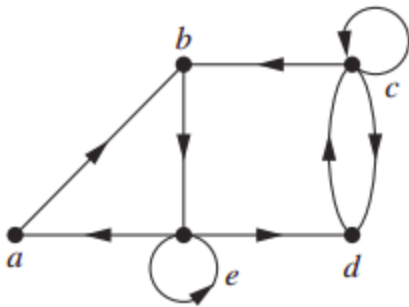
C

172 mod -3

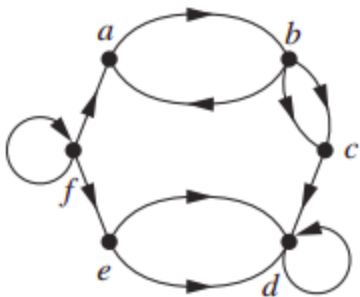
E

---

Which one of the following is true about the two graphs given below?



(Fig. a)



(Fig. b)

Graph given in Fig. b has multiple edges. (100%)

Graph given in Fig. a has multiple edges. (0)

Both graphs have multiple edges. (0)

None (0)

---

Which one of the following is the correct arrangement of the complexity functions  $f(n)$ 's given below in an increasing order as  $n$  grows without bound, where  $n$  is for example the estimate the number of operations needed to solve a problem using a specified procedure or algorithm.

- (  $\sqrt{n}$ ,  $\sqrt{n \log n}$ ,  $\sqrt{2^n}$ ,  $\sqrt{n!}$  )
  - (  $\sqrt{n \log n}$ ,  $\sqrt{n^2}$ ,  $\sqrt{n!}$ ,  $\sqrt{2^n}$  )
  - (  $\sqrt{2^n}$ ,  $\sqrt{n \log n}$ ,  $\sqrt{n^2}$ ,  $\sqrt{n!}$  )
  - (  $\sqrt{n \log n}$ ,  $\sqrt{n^2}$ ,  $\sqrt{2^n}$ ,  $\sqrt{n!}$  ) (100%)
- 

From the definition of the fact that  **$f(x)$  is  $O(x)$**  which of the following are the **correct pairs of witnesses "C" and "k"** fulfilling (  $f(x) = 2^{\sqrt{x}} + 15$  ) as (  $O(3^{\sqrt{x}})$  ) , respectively.

- 5, 2 (50%)
  - 2, 2
  - 2, 4 (50%)
  - None
- 

Which one of the following will be the most efficient algorithm in finding the  $n$ th term of the sequence defined by

$$a_0 = 1, a_1 = 2, \text{ and } a_n = a_{n-1}a_{n-2}, \text{ for } n = 2, 3, 4, \dots?$$

- a recursive algorithm.
  - an iterative algorithm. (100%)
- 

Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to five other people. Some people do this, but others do not send any letters.

How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 253 people who read it but did not send it out?

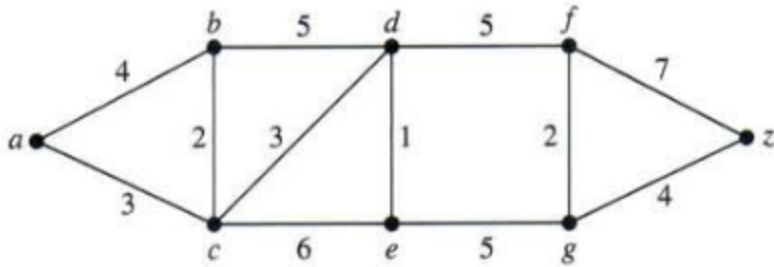
- 316
  - 3160
-

There are 110 students at a college who have taken a course in calculus, 97 who have taken a course in discrete mathematics, and 42 who have taken courses in both calculus and discrete mathematics. How many students have taken a course in either calculus or discrete mathematics?

165

---

What is the length of the shortest path in the weighted graph below between point a and z?



16

---

Find the value of the double sum given below:

$$\sum_{i=0}^3 \sum_{j=0}^2 (5i - 3j).$$

54

---

What is the original message encrypted using the RSA system with  $n = 43 \cdot 59$  and  $e = 13$  if the encrypted message is 0667 1947 0671? (To decrypt, first find the decryption exponent  $d$  which is the inverse of  $e = 13$  modulo  $42 \cdot 58$ .)

You should provide your answer in capital letter (upper case) with out space.

SILVER

---

If  $n$  is an integer not divisible by 2 or 3, then  $(n - 1) (n + 1)$  is divisible by 24.

True

---

To reason backward to prove a statement **q**, we find a statement **p** that we can prove with the property that **( p  $\rightarrow$  q )**.

True

---

Question summary: What does the recursive formula  $a_{n+1} = 2a_n + 3$ , where  $a_0 = 3$ , represent?: Linear recursion; Non-linear recursion; Arithmetic progression; Geometric progression  
Right answer summary: Non-linear recursion

Question summary: What is the base case in a proof by mathematical induction?: The assumption that the statement holds for  $k$ ; The recursive definition of the problem.; The conclusion drawn from the inductive step.; The proof that the statement holds for  $n = 0$  or  $n = 1$ .  
Right answer summary: The proof that the statement holds for  $n = 0$  or  $n = 1$ .

Question summary: What is the closed-form expression for the arithmetic sequence  $(5, 11, 17, \dots)$ ?:  $a_n = 6n$ ;  $a_n = 11n - 6$ ;  $a_n = 6n - 1$ ;  $a_n = 5n + 1$   
Right answer summary:  $a_n = 6n - 1$

Question summary: What is the factorial function recursively defined as?:  $F(n) = n \cdot F(n-1), \text{quad } F(0) = 1$ ;  $F(n) = n!$ ;  $F(n) = n + F(n-1)$ ;  $F(n) = \frac{F(n-1)}{n}, \text{quad } F(0) = 0$   
Right answer summary:  $F(n) = n \cdot F(n-1), \text{quad } F(0) = 1$

Question summary: What is the purpose of a base case in a recursive function?: To call another recursive function; To reduce memory usage; To stop the function from calling itself infinitely; To ensure faster execution  
Right answer summary: To stop the function from calling itself infinitely

Question summary: What is the value of  $a_5$  in the sequence defined by  $a_n = 5 + 6(n - 1)$ ? : 47 ; 41 ; 29 ; 35  
Right answer summary: 29

Question summary: What type of induction assumes that the proposition holds for all values up to  $(n)$ ? Complete induction; Simple induction; Strong induction; Logical induction

Right answer summary: Strong induction

Question summary: Which algorithm is defined by  $(\gcd(a, b) = \gcd(b \bmod a, a))$ ? Binary Search; Factorial Algorithm; Iterative Fibonacci; Euclidean Algorithm

Right answer summary: Euclidean Algorithm

Question summary: Which algorithm would you use to search an element in a sorted array? Linear search; Recursive search; Binary search; Jump search

Right answer summary: Binary search

Question summary: Which of the following defines the Fibonacci sequence?  $(f_n = 2f_{n-1}, \text{quad } f_0 = 1)$ ;  $(f_n = f_{n-1} + f_{n-2}, \text{quad } f_0 = 0, f_1 = 1)$ ;  $(f_n = f_{n-1} \cdot f_{n-2})$ ;  $(f_n = f_{n-1} - f_{n-2})$

Right answer summary:  $(f_n = f_{n-1} + f_{n-2}, \text{quad } f_0 = 0, f_1 = 1)$

Question summary: Which of the following is true about Ackermann's function? It grows faster than any primitive recursive function.; It is a linear recursive function.; It is only defined for even numbers.; It is used for sorting arrays.

Right answer summary: It grows faster than any primitive recursive function.

Question summary: Which of the following sequences is defined recursively?  $(a_n = \frac{n(n+1)}{2})$ ;  $(a_n = a_{n-1} + 3)$ ;  $(a_n = 2^n)$ ;  $(a_n = 2n)$

Right answer summary:  $(a_n = a_{n-1} + 3)$

Question summary: Which one of these is NOT a characteristic of a recursive algorithm? Has a base case; Solves smaller subproblems; Calls itself; Uses loops only

Right answer summary: Uses loops only

Question summary: Which statement is TRUE about countable sets? A set is countable if it has more elements than  $(\mathbb{Z}^+)$ .; All uncountable sets are subsets of  $(\mathbb{Z}^+)$ .; A set is countable only if it is finite.; Every finite set is countable.

Right answer summary: Every finite set is countable.



Question summary: Which summation formula correctly evaluates  $\sum_{k=1}^n k$ ? :  $\frac{n^2}{2}$ ;  $\frac{n(n-1)}{2}$ ;  $\frac{n(n+1)}{2}$ ;  $2n + 1$

Right answer summary:  $\frac{n(n+1)}{2}$

-----

Question summary: Each hexadecimal digit corresponds to how many bits?: 5; 6; 3; 4

Right answer summary: 4

Question summary: How many bits are there in a byte?: 32; 16; 4; 8

Right answer summary: 8

Question summary: In binary addition, what does a carry bit of 1 mean?: Overflow must be handled; Propagate carry to the next bit; Start over; You subtract the bit

Right answer summary: Propagate carry to the next bit

Question summary: In binary multiplication, a term is included in the product only if:: The result is even; The corresponding multiplier bit is 1; The bit is 0; It's a divisor

Right answer summary: The corresponding multiplier bit is 1

Question summary: Multiplying a binary number by  $2^3$  is equivalent to:: Left shift by 3 bits; Inverting the bits; Rotating bits; Right shift by 3 bits

Right answer summary: Left shift by 3 bits

Question summary: The base  $b$  expansion of an integer is:: Not unique; Used only in binary computations; A unique representation as a sum of powers of  $b$ ; Always decimal

Right answer summary: A unique representation as a sum of powers of  $b$

Question summary: The binary expansion of a number is its representation using:: Base 16; Base 10; Base 8; Base 2

Right answer summary: Base 2

Question summary: Using the Euclidean algorithm, what is the gcd of 528 and

396?: 12; 264; 44; 132

Right answer summary: 132

Question summary: What decimal value does the hexadecimal digit 'E' represent?: 10; 14; 16; 12

Right answer summary: 14

Question summary: What is the base-8 (octal) representation of the decimal number 932?: 1516; 1444; 1364; 1654

Right answer summary: 1364

Question summary: What is the decimal value of  $(100110111)_2$ ?: 298; 311; 253; 271

Right answer summary: 311

Question summary: What is the gcd of 104 and 728?: 4; 12; 8; 52

Right answer summary: 8

Question summary: What is the goal of the binary addition algorithm?: Convert binary to decimal; Compare two values; Generate pseudorandom numbers; Compute the sum of binary integers

Right answer summary: Compute the sum of binary integers

Question summary: What is the hexadecimal equivalent of  $(111010111001)_2$ ?: EB9; 3B9; DE9; FA9

Right answer summary: EB9

Question summary: What is the main use of the Euclidean algorithm?: Finding the least common multiple; Calculating the greatest common divisor; Simplifying fractions; Factorizing numbers

Right answer summary: Calculating the greatest common divisor

Question summary: What is the product of  $(101)_2$  times  $(111)_2$ ?:  $(110001)_2$ ;  $(100011)_2$ ;  $(101101)_2$ ;  $(100101)_2$

Right answer summary:  $(100101)_2$

Question summary: What is the result of adding  $(1011)_2$  +  $(1101)_2$ ?:  $(11000)_2$ ;  $(11100)_2$ ;  $(10110)_2$ ;  $(10010)_2$

Right answer summary:  $(11000)_2$

Question summary: Which is the hexadecimal representation of the binary number  $(11010100)_2$ ? B7; A3; C2; D4

Right answer summary: D4

Question summary: Which of the following correctly states Lemma 1 of the Euclidean algorithm?:  $\gcd(a, b) = \gcd(b, r)$ , where  $a = bq + r$ ;  $\gcd(a, b) = a \div b$ ;  $\gcd(a, b) = \gcd(b, a)$ ;  $\gcd(a, b) = \gcd(a, a \bmod b)$

Right answer summary:  $\gcd(a, b) = \gcd(b, r)$ , where  $a = bq + r$

Question summary: Which of the following is a fundamental property of an algorithm?: It must terminate after a finite number of steps; It must give different results each time; It should be ambiguous; It should run indefinitely

Right answer summary: It must terminate after a finite number of steps

-----

Question summary: Compute the Boolean product of  $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ :  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Question summary: Find the Boolean product of  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ :  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Question summary: Given  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ , compute  $A \vee B$ :  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Question summary: Given  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ , what is  $A \wedge A$ ?  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ;  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ;  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Question summary: Let  $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ . What is  $A \odot B$  (Boolean product)?:  $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}; \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

Question summary: Let  $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ , compute the Boolean cube  $A[3]$ .:  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Question summary: What is the Boolean square ( $A[2]$ ) of matrix  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ?:  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Question summary: What is the result of the join of the zero-one matrices  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ?:  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Question summary: What is the result of the meet of  $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ?:  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}; \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Question summary: What is the result of the meet operation for  $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ ?:  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}; \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Right answer summary:  $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

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Question summary: According to the generalized pigeonhole principle, if 20 pigeons are placed into 6 holes, then at least one hole contains at least how many pigeons?: 3; 5; 6; 4

Right answer summary: 4

Question summary: How many 3-letter passwords can be formed using the 26-letter English alphabet, if repetition is allowed?:  $\{( 26 \}$ ;  $\{( 26^2 \}$ ;  $\{( 26^3 \}$ ;  $\{( 3 \cdot 26 \}$

Right answer summary:  $\{( 26^3 \}$

Question summary: How many ways are there to assign 5 different jobs to 3 workers if each worker can do multiple jobs?:  $\{( 3! \}$ ;  $\{( 5! \}$ ;  $\{( 3^5 \}$ ;  $\{( 5^3 \}$

Right answer summary:  $\{( 3^5 \}$

Question summary: If a set  $\{( A \}$  has 5 elements and a set  $\{( B \}$  has 4 elements, how many elements are in  $\{( A \cup B \}$  if  $\{( A \cap B \}$  has 2 elements?: 7; 9; 10; 11

Right answer summary: 7

Question summary: If a task can be done in  $\{( m \}$  ways and another independent task in  $\{( n \}$  ways, how many ways can both tasks be done?:  $\{( m \cdot n \}$ ;  $\{( m + n \}$ ;  $\{( \max(m, n) \}$ ;  $\{( m! + n! \}$

Right answer summary:  $\{( m \cdot n \}$

Question summary: If a task can be done in either  $\{( m \}$  or  $\{( n \}$  ways (but not both), how many total ways are there to perform the task?:  $\{( m + n \}$ ;  $\{( m \cdot n \}$ ;  $\{( m! + n! \}$ ;  $\{( \min(m, n) \}$

Right answer summary:  $\{( m + n \}$

Question summary: Using inclusion-exclusion, find the number of integers between 1 and 100 divisible by 3 or 5.: 67; 53; 33; 47

Right answer summary: 47

Question summary: What is the minimum number of students required in a class to guarantee that at least two students have the same birthday (assuming 365 days in a year)?: 364; 365; 367; 366

Right answer summary: 367

Question summary: What principle states that if  $\{( n \}$  items are placed into  $\{( m \}$  containers, and  $\{( n > m \}$ , then at least one container has more than one item?: Inclusion-Exclusion Principle; Product Rule; Pigeonhole Principle; Sum Rule

Right answer summary: Pigeonhole Principle

Question summary: You are choosing a meal. There are 3 appetizers and 4 main courses. Using the product rule, how many different meal combinations can you choose?: 1; 12; 3; 7

Right answer summary: 12