Introduction
Special issue in memory of Patrick Dehornoy

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This issue of the Journal of Algebra is dedicated to the memory of Patrick Dehornoy, and we are grateful to Gunter Malle for inviting us to act as guest editors. With one or two exceptions, the papers in this issue concern braid groups, Artin groups, Garside groups and related subjects, that were an important area of interest for Patrick, but not the only one. Actually, he had two main areas of interest: set theory, on the one hand, and theory of braid groups in the broad sense, on the other hand. These two areas are *a priori* distant, and making the link between them was one of the amazing feats of Patrick's story that we will not repeat in this volume.

Patrick started his career as mathematician in the second half of the 1970s in logic and set theory, with works that are still references in the domain. Although his most famous and cited papers concern braid groups in the broad sense, he remained passionate about this field and continued all his life to have exchanges with experts and to publish in that field. He also always tried to share with everyone his passion for large cardinals and their interest for addressing some more standard mathematical questions. It is with this passion and enthusiasm that he gave many wonderful lectures for a large public and wrote two books on set theory. He described it as "a magnificent theory, but subject to many misunderstandings" in the preamble of his latest book.

The magic of Patrick's ideas led him from set theory to self-distributive sets, and from self-distributive sets to braid groups. In the theory of large cardinals the equality $\psi(\varphi x) = (\psi\varphi)(\psi x)$ appears, where ψ and φ are embeddings between models. Patrick's incredible idea to study self-distributive sets, that is, sets endowed with an operation $*$ that satisfies $x*(y*z) = (x*y)*(x*z)$ for all x, y, z, came from there.

Most researchers working on braid groups come from low-dimensional topology, group theory, singularities, or even mathematical physics. Patrick did not come from any of these fields, and he never did think like the others. He arrived at the theory of braid groups in a curious way.

We believe that some of the results of which Patrick was most proud, and in our opinion some of the most original, are those published in 1994 in the Transactions of the American Mathematical Society. In that paper he proved three results that marked his career from that date.

(1) He proved the existence and uniqueness of a free self-distributive set generated by a given set.

(2) He proved that each free self-distributive set can be endowed with a total ordering invariant by left multiplication.

(3) By using a partial action of the infinitely generated braid group B_∞ on the free self-distributive set on one generator, he proved that any braid group is orderable.

There is nothing easy in the proofs of these three results, and no other proof in the literature looks like them. For instance, the proof of the existence of the order on a free self-distributive set is based on an unprovable assertion of the theory of large cardinals, which puzzled the mathematical community before another more classical proof was proposed.

This paper was a turning point in Patrick's career, and a large part of the mathematics he produced afterwards is a more or less direct development of it. From that time Patrick found a passion for braid groups that never left him and he quickly become a leader in the field. He put all his energy into this subject and he brought together around him most of the French mathematicians working on braid groups, as well as many others around the world.

In his career as a mathematician Frank Arnold Garside (1915–1988) published only one paper, in 1969, but Patrick made it the leitmotif what is now called Garside theory. In order to understand his partial action of the infinitely generated braid group B_∞ on the free self-distributive set, Patrick studied many known techniques for solving algorithmic problems on braid groups. He also invented new techniques such as his famous "handle reduction" process which he claimed to be the fastest and most economical algorithm for solving the word problem in braid groups. Patrick's work was particularly influenced by Garside's paper, which he hurried to improve and generalize. His first attempt was the theory of "reversing processes" introduced in a paper in 1997. His second attempt, in collaboration with several other mathematicians, was the ground-breaking theory of Garside groups, which has continued to progress for almost 20 years and which has given rise to a monumental book of about 700 pages in which Patrick and his coauthors explain the foundations of Garside theory – although Patrick always said that it was not (yet) a theory; as he stated in the introduction of that book, "such a title is certainly too ambitious for what we can offer".

This is only a fraction of Patrick's work, which also concerns subjects such as Thompson's groups, cryptography, algorithms, Artin groups, and so on. But our goal is not just to recall the large volume of contributions Patrick made to mathematics (he wrote more than 100 papers and more than 8 books). Beyond the numbers we wish also to underline Patrick's leadership role in the development of the theory of braid groups in the broad sense. This role takes different forms.

The first was his innate sense of organization and his ability to bring people together from all sides. At the end of the 1990s he created a research network called "GDR Tresses" which brought together almost all the French mathematicians and many others from abroad having a more or less direct link with braid groups. These researchers came from many different backgrounds such as group theory, combinatorics, algebraic geometry, hyperplane arrangements, dynamical systems, etc., and he managed to get them to work together. This "GDR Tresses" and its many meetings, not only in France, was a springboard for the career of many young mathematicians, and not so young, who keep a lifelong memory of it and are deeply grateful to Patrick.

His influence also came from his approach to mathematics which consisted not only in proving theorems but also in developing new theories. He required both completeness and depth for the theories he wished to develop, and breadth for the dissemination of the theories in the most educational way possible in order to find adherents. This approach is present in all his books whether on set theory, on self-distributive sets, on ordering on braid groups, or on Garside structures.

Patrick is now somewhere in the clouds. He is observing us through his telescope and he is delighted to see us studying with passion his book “Braids and self-distributivity” which was so close to his heart.