



# HNDIT1032

## Computer and Network Systems

### Week4- Karnaugh Maps

# Introduction

- So far we can see that applying Boolean algebra can be awkward in order to simplify expressions.
- The Karnaugh map provides a simple and straight-forward method of minimizing Boolean expressions

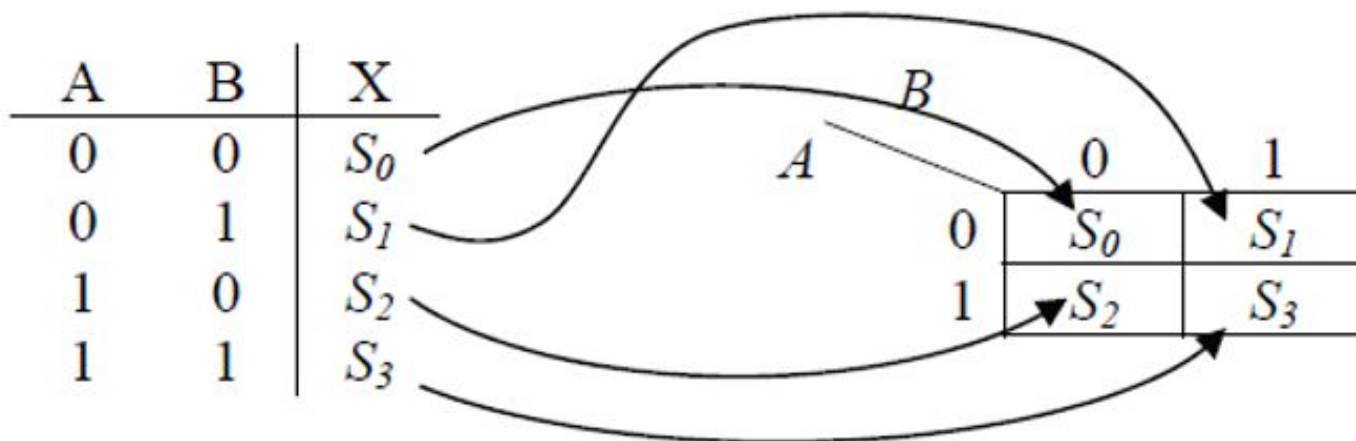
# What is a Karnaugh map?

- A Karnaugh map provides a pictorial method of grouping together expressions with common factors and therefore eliminating unwanted variables.

# Two Variable K Maps

- Two variable K Map is drawn for a boolean expression consisting of two variables.
- The number of cells present in two variable K Map =  $2^2 = 4$  cells.
- So, for a Boolean function consisting of two variables, we draw a 2 x 2 K Map.

# Two Variable K Maps...



# Two Variable K Maps...

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table.

A \ B	0	1
0	0	1
1	1	1

F.

# Three Variable K Maps

- Three variable K Map is drawn for a Boolean expression consisting of three variables.
- The number of cells present in three variable K Map =  $2^3 = 8$  cells.
- So, for a Boolean function consisting of three variables, we draw a 2 x 4 K Map.

# Three Variable K Maps...

A	B	C	Minterm
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

		BC			
		00	01	11	10
A	0	$m_0$	$m_1$	$m_3$	$m_2$
	1	$m_4$	$m_5$	$m_7$	$m_6$



# Three Variable K Maps...

A	B	C	X			
0	0	0	0			
0	0	1	1	AB	00	0
0	1	0	0			1
0	1	1	1		01	0
1	0	0	1		11	1
1	0	1	0			
1	1	0	1		10	1
1	1	1	1			

	0	1
00	0	1
01	0	
11	1	
10	1	

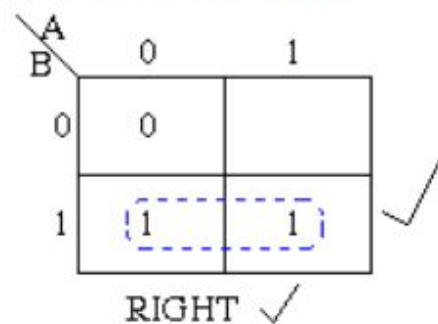
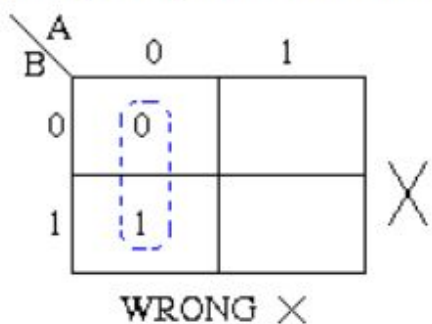
Diagram illustrating the mapping of a three-variable truth table to a Karnaugh map. The truth table on the left shows inputs A, B, and C, and output X. The Karnaugh map on the right shows the output X for each combination of A and B (rows) and C (columns). Arrows indicate the mapping from the truth table rows to the Karnaugh map cells.

# Karnaugh Maps - Rules of Simplification

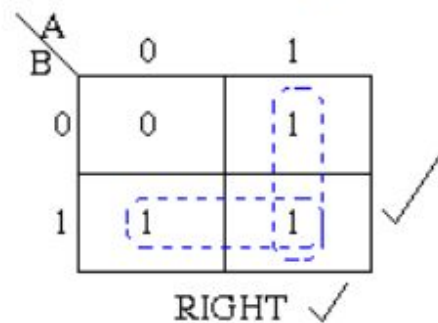
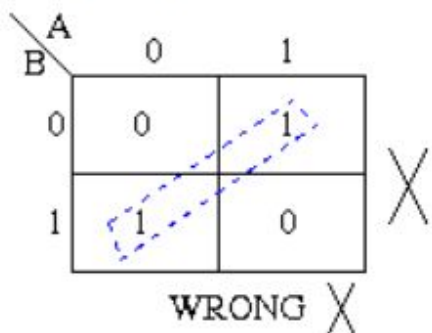
- No zeros allowed.
- No diagonals.
- Only power of 2 number of cells in each group.
- Groups should be as large as possible.
- Every one must be in at least one group.
- Overlapping allowed.
- Wrap around allowed.
- Fewest number of groups possible.

# Karnaugh Maps - Rules of Simplification...

- Groups may not include any cell containing a **zero**



- Groups may be horizontal or vertical, but not diagonal.



# Karnaugh Maps - Rules of Simplification...

- Groups must contain 1, 2, 4, 8, or in general  $2^n$  cells.  
That is if  $n = 1$ , a group will contain two 1's since  $2^1 = 2$ .  
If  $n = 2$ , a group will contain four 1's since  $2^2 = 4$ .

A \ B	0	1
0	1	1
1	0	0

Group of 2

RIGHT ✓

AB \ C	00	01	11	10
0	0	1	1	1
1	0	0	0	0

Group of 3

WRONG ✗

A \ B	0	1
0	1	1
1	1	1

Group of 4

RIGHT ✓

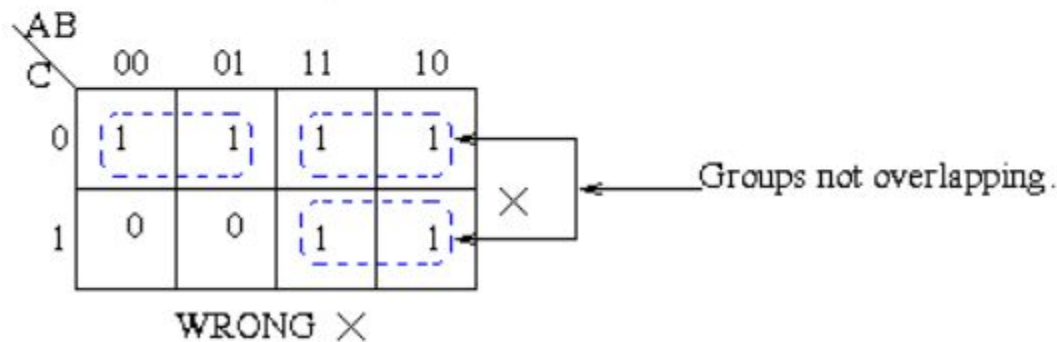
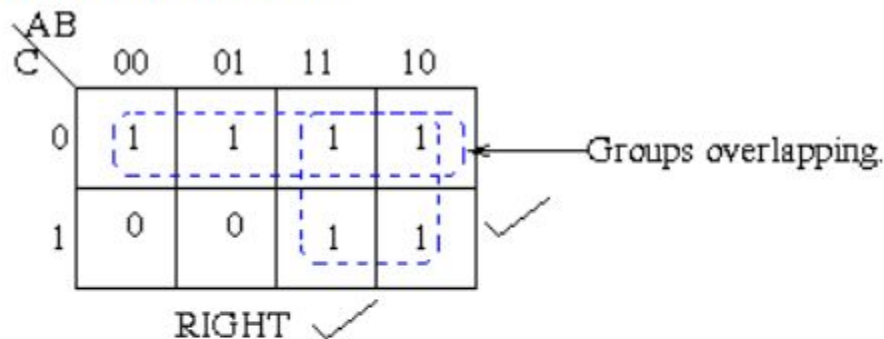
AB \ C	00	01	11	10
0	1	1	1	1
1	0	0	0	1

Group of 5

WRONG ✗

# Karnaugh Maps - Rules of Simplification...

- Groups may overlap.



# Example 01

- $F(A, B) = A\bar{B} + AB$

B \ A	0	1
0		1
1		1

## Example 01...

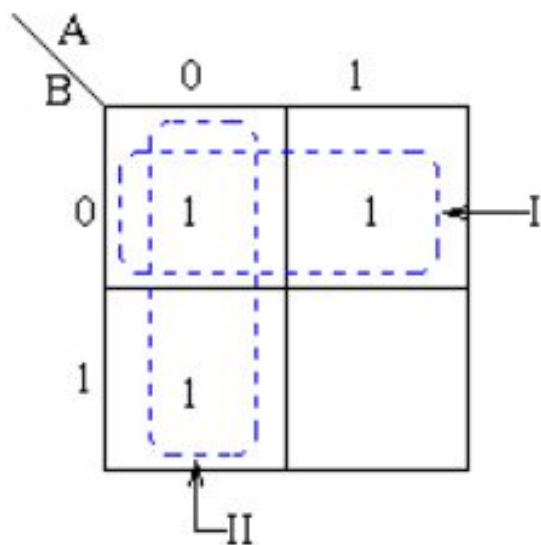
- The two adjacent 1's are grouped together. Through inspection it can be seen that variable B has its true and false form within the group.
- This eliminates variable B leaving only variable A which only has its true form. The minimized answer therefore is  $Z = A$ .

## Example 02

- $F(A, B) = \bar{A}\bar{B} + A\bar{B} + \bar{A}B$



# Example 02



## Example 03

- $F(A, B) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$

## Example 03

- $F(A, B) = \bar{A}\bar{B} + A\bar{B} + \bar{A}B$

Out =  $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C$

BC \ A	00	01	11	10
0	1	1		
1				

Out =  $\bar{A}\bar{B}$

## Example 04

- $F(A, B) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$

## Example 04

- $F(A, B) = \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$

$$\text{Out} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C}$$

		BC			
		00	01	11	10
A	0	1	1	1	1
	1				

$$\text{Out} = \bar{A}$$

## Example 05

- $F(A, B) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$

## Example 05

- $F(A, B) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$

$$\text{Out} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + ABC$$

		BC			
		00	01	11	10
A	0		1	1	
	1		1	1	

$$\text{Out} = C$$

# Next Week Discussion

- How to draw circuits?