

CAP 5415 Computer Vision

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Univ. of Central Florida



Filtering

Lecture-2



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- Filtering/Smoothing/Removing Noise

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- Filtering/Smoothing/Removing Noise
- Convolution/Correlation
- Image Derivatives
- Histogram
- Some Matlab Functions

Images: General

The slide features a light green background with a white rounded rectangle in the top-left corner. The title 'Images: General' is written in a dark teal font within this white area. A thick, dark blue horizontal bar spans the width of the slide below the title.

Images: General

Binary



Images: General

Binary



Gray Scale



Images: General

Binary



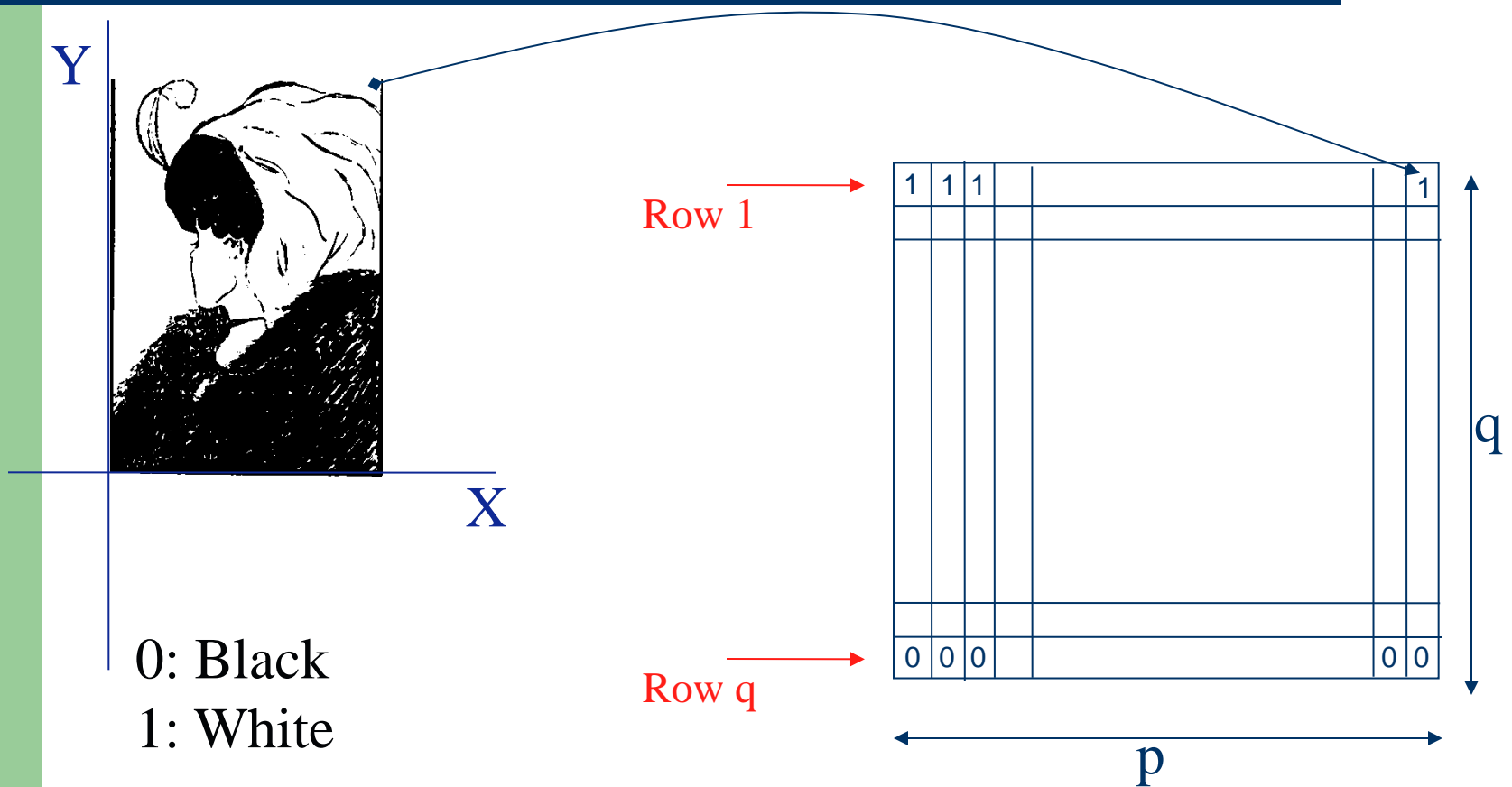
Gray Scale



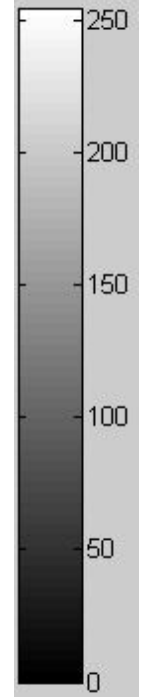
Color



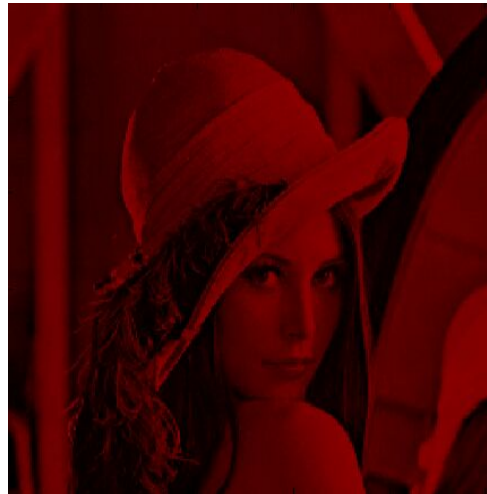
Binary Images



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Gray Scale Image

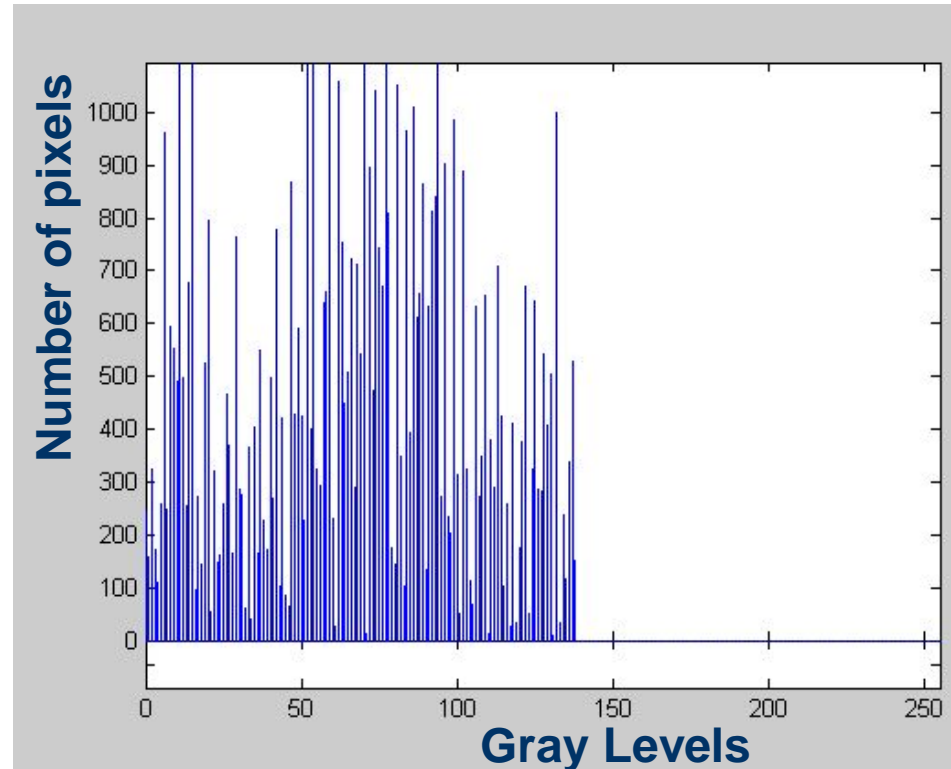


Color Image

Red, Green, Blue Channels



Image Histogram



- Histogram captures the distribution of gray levels in the image.
- How frequently each gray level occurs in the image

Histogram Code

Histogram Code

C Code:
for (i=0;i<m,i++)
 for (j=0;j<n,j++)
 hist[l[i,j]]++;

Histogram Code

C Code:
for (i=0;i<m,i++)
 for (j=0;j<n,j++)
 hist[I[i,j]]++;

MATLAB: imhist(I)

Image Noise



Image Noise

- Light Variations

Image Noise

- Light Variations
- Camera Electronics

Image Noise

- Light Variations
- Camera Electronics
- Surface Reflectance

Image Noise

- Light Variations
- Camera Electronics
- Surface Reflectance
- Lens

Image Noise

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-
- Noise is random, it occurs with some probability

Image Noise

- Light Variations
 - Camera Electronics
 - Surface Reflectance
 - Lens
-
- Noise is random, it occurs with some probability
 - It has a distribution

Image Noise

- $I(x,y)$: the true pixel values



Image Noise

- $I(x,y)$: the true pixel values
- $n(x,y)$: the noise at pixel (x,y)



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Additive noise



Image Noise

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- $n(x,y)$: the noise at pixel (x,y)

$$\hat{I}(x, y) = I(x, y) + n(x, y) \quad \text{Additive noise}$$



Image Noise

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Image Noise



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Multiplicative noise



Image Noise

- $I(x,y)$: the true pixel values
 - $n(x,y)$: the noise at pixel (x,y)
- $\hat{I}(x, y) = I(x, y) \times n(x, y)$ Multiplicative noise



Image Noise

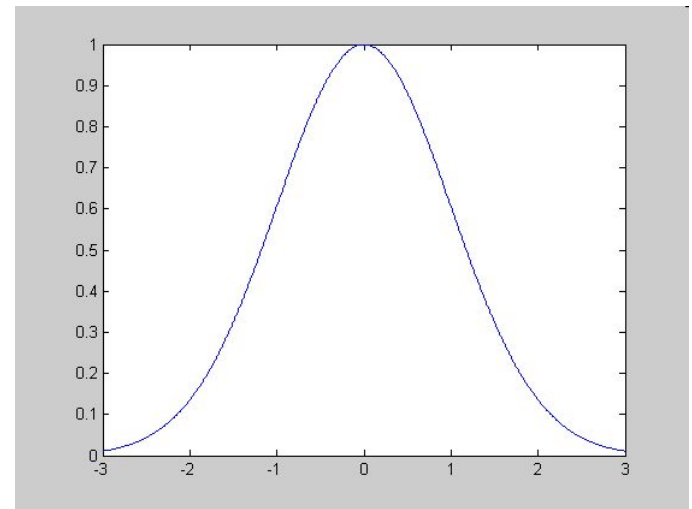
- $I(x,y)$: the true pixel values
 - $n(x,y)$: the noise at pixel (x,y)
- $\hat{I}(x, y) = I(x, y) \times n(x, y)$ Multiplicative noise



Gaussian Noise

$$n(x, y) \approx g(n) = e^{\frac{-n^2}{2\sigma^2}}$$

$g(n)$

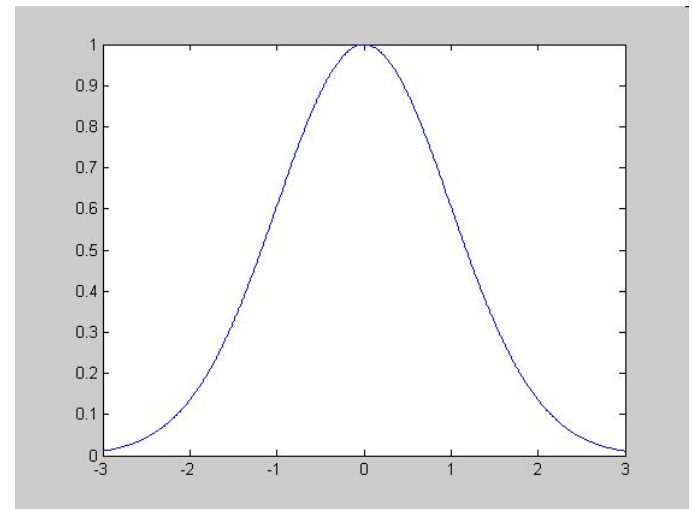


n

Gaussian Noise

$$n(x, y) \approx g(n) = e^{\frac{-n^2}{2\sigma^2}}$$

$g(n)$



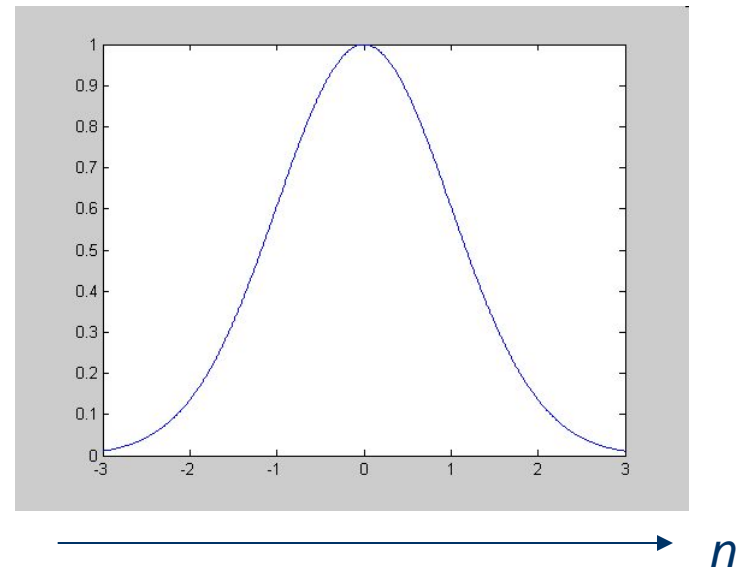
Probability Distribution
 n is a random variable

Gaussian Noise

$$n(x, y) \approx g(n) = e^{\frac{-n^2}{2\sigma^2}}$$

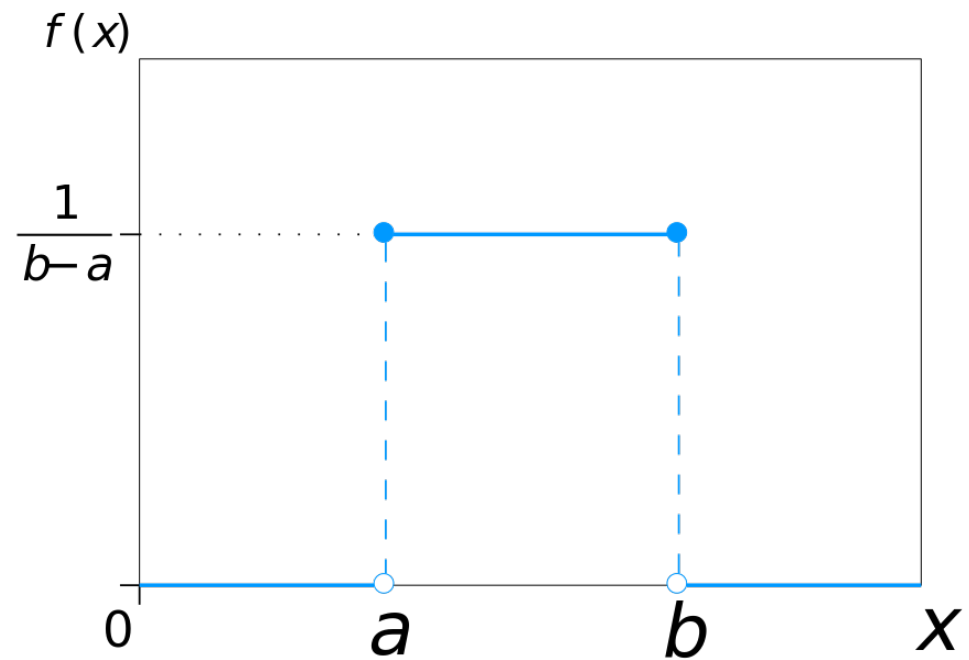


$g(n)$



Probability Distribution
 n is a random variable

Uniform Distribution



Salt and Pepper Noise

- Each pixel is randomly made black or white with a uniform probability distribution



Image filtering

- Image filtering: compute function of local neighborhood at each position

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- Really important!
 - Enhance images
 - Denoise, resize, increase contrast, etc.

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 - Extract information from images
 - Texture, edges, distinctive points, etc.

Image filtering

- Image filtering: compute function of local neighborhood at each position
- Really important!
 - Enhance images
 - Denoise, resize, increase contrast, etc.
 - Extract information from images
 - Texture, edges, distinctive points, etc.
 - Detect patterns
 - Template matching

Image Derivatives & Averages



Definitions

Definitions

- Derivative: Rate of change
 - *Speed* is a rate of change of a *distance*
 - *Acceleration* is a rate of change of *speed*

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- Derivative: Rate of change
 - *Speed* is a rate of change of a *distance*
 - *Acceleration* is a rate of change of *speed*
- Average (Mean)
 - Dividing the sum of N values by N

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$$v = \frac{ds}{dt} \text{ speed} \qquad a = \frac{dv}{dt} \text{ acceleration}$$

Examples: Analytic Derivatives

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$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

Examples: Analytic Derivatives

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Discrete Derivative

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

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Discrete Derivative

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Discrete Derivative

Finite Difference

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Discrete Derivative

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$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

Backward difference

Discrete Derivative

Finite Difference

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

Backward difference

$$\frac{df}{dx} = \frac{f(x) - f(x+1)}{-1} = f'(x)$$

Discrete Derivative

Finite Difference

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Discrete Derivative

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Forward difference

$$\frac{df}{dx} = \frac{f(x+1) - f(x-1)}{2} = f'(x)$$

Discrete Derivative

Finite Difference

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

Backward difference

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Forward difference

$$\frac{df}{dx} = \frac{f(x+1) - f(x-1)}{2} = f'(x)$$

Central difference

Example

Example

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

Example

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Example

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

$$f'(x) = 0 \quad 5 \quad -5 \quad 0 \quad 15 \quad -5 \quad 0 \quad 0$$

Example

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

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Derivative Masks

Example

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$

$$f'(x) = 0 \quad 5 \quad -5 \quad 0 \quad 15 \quad -5 \quad 0 \quad 0$$

$$f''(x) = 0 \quad 5 \quad 10 \quad 5 \quad 15 \quad -20 \quad 5 \quad 0$$

Derivative Masks

Backward difference $[-1 \quad 1]$

Forward difference $[1 \quad -1]$

Central difference $[-1 \quad 0 \quad 1]$

Derivatives in 2 Dimensions

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Given function

$$f(x, y)$$

Derivatives in 2 Dimensions

Given function

$$f(x, y)$$

Gradient vector

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Derivatives in 2 Dimensions

Given function

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Gradient magnitude

$$|\nabla f(x, y)| = \sqrt{f_x^2 + f_y^2}$$

Derivatives in 2 Dimensions

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Gradient magnitude

$$|\nabla f(x, y)| = \sqrt{f_x^2 + f_y^2}$$

Gradient direction

$$\theta = \tan^{-1} \frac{f_x}{f_y}$$

Derivatives of Images

Derivative masks

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Derivative masks

$$f_x \Rightarrow \frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

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$$I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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Derivatives of Images



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Derivatives of Images

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Correlation



Correlation

$$f \otimes h = \sum_k \sum_l f(k, l) h(k, l)$$



Correlation

$$f \otimes h = \sum_k \sum_l f(k, l) h(k, l)$$

f = Image

h = Kernel

Correlation

$$f \otimes h = \sum_k \sum_l f(k, l) h(k, l)$$

f = Image

h = Kernel

f

| | | |
|-------|-------|-------|
| f_1 | f_2 | f_3 |
| f_4 | f_5 | f_6 |
| f_7 | f_8 | f_9 |

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\otimes

h

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| | | |
|-------|-------|-------|
| h_1 | h_2 | h_3 |
| h_4 | h_5 | h_6 |
| h_7 | h_8 | h_9 |

$$\begin{aligned} f \otimes h &= f_1 h_1 + f_2 h_2 + f_3 h_3 \\ &\quad + f_4 h_4 + f_5 h_5 + f_6 h_6 \\ &\quad + f_7 h_7 + f_8 h_8 + f_9 h_9 \end{aligned}$$



Convolution



Convolution

$$f * h = \sum_k \sum_l f(k, l) h(-k, -l)$$



Convolution

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Convolution

$$f * h = \sum_k \sum_l f(k, l) h(-k, -l)$$

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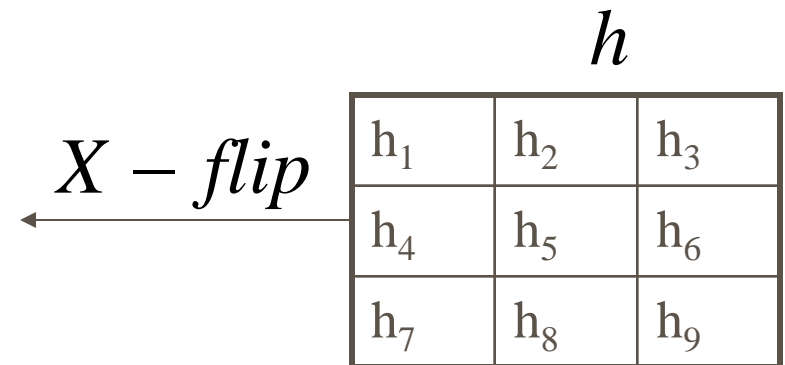
| | | |
|-------|-------|-------|
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| | | |
|-------|-------|-------|
| h_7 | h_8 | h_9 |
| h_4 | h_5 | h_6 |
| h_1 | h_2 | h_3 |

$X - flip$

| | | |
|-------|-------|-------|
| h_1 | h_2 | h_3 |
| h_4 | h_5 | h_6 |
| h_7 | h_8 | h_9 |

h

Convolution

$$f * h = \sum_k \sum_l f(k, l) h(-k, -l)$$

$f = \text{Image}$

$h = \text{Kernel}$

| | | |
|-------|-------|-------|
| h_7 | h_8 | h_9 |
| h_4 | h_5 | h_6 |
| h_1 | h_2 | h_3 |

$X - flip$

| | | |
|-------|-------|-------|
| h_1 | h_2 | h_3 |
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$Y - flip$



Convolution

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| h_7 | h_8 | h_9 |
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$X - flip$

| | | |
|-------|-------|-------|
| h_1 | h_2 | h_3 |
| h_4 | h_5 | h_6 |
| h_7 | h_8 | h_9 |

$Y - flip$

| | | |
|-------|-------|-------|
| h_9 | h_8 | h_7 |
| h_6 | h_5 | h_4 |
| h_3 | h_2 | h_1 |

Convolution

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h

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|-------|-------|-------|
| h_7 | h_8 | h_9 |
| h_4 | h_5 | h_6 |
| h_1 | h_2 | h_3 |

$X - flip$

| | | |
|-------|-------|-------|
| h_1 | h_2 | h_3 |
| h_4 | h_5 | h_6 |
| h_7 | h_8 | h_9 |

h

f

| | | |
|-------|-------|-------|
| f_1 | f_2 | f_3 |
| f_4 | f_5 | f_6 |
| f_7 | f_8 | f_9 |

*

$Y - flip$

| | | |
|-------|-------|-------|
| h_9 | h_8 | h_7 |
| h_6 | h_5 | h_4 |
| h_3 | h_2 | h_1 |

Convolution

$$f * h = \sum_k \sum_l f(k, l) h(-k, -l)$$

f = Image

h = Kernel

$$X - flip$$

| | | |
|-------|-------|-------|
| h_7 | h_8 | h_9 |
| h_4 | h_5 | h_6 |
| h_1 | h_2 | h_3 |

$X - flip$

$$h$$

| | | |
|-------|-------|-------|
| h_1 | h_2 | h_3 |
| h_4 | h_5 | h_6 |
| h_7 | h_8 | h_9 |

f

| | | |
|-------|-------|-------|
| f_1 | f_2 | f_3 |
| f_4 | f_5 | f_6 |
| f_7 | f_8 | f_9 |

*

$Y - flip$

| | | |
|-------|-------|-------|
| h_9 | h_8 | h_7 |
| h_6 | h_5 | h_4 |
| h_3 | h_2 | h_1 |

$$\begin{aligned} f * h = & f_1 h_9 + f_2 h_8 + f_3 h_7 \\ & + f_4 h_6 + f_5 h_5 + f_6 h_4 \\ & + f_7 h_3 + f_8 h_2 + f_9 h_1 \end{aligned}$$

Correlation and Convolution

- Convolution is associative

Correlation and Convolution

- Convolution is associative

$$F * (G * I) = (F * G) * I$$

Averages

- Mean

Averages

- Mean

$$I = \frac{I_1 + I_2 + \dots + I_n}{n} = \frac{\sum_{i=1}^n I_i}{n}$$

Averages

- Mean

$$I = \frac{I_1 + I_2 + \dots + I_n}{n} = \frac{\sum_{i=1}^n I_i}{n}$$

- Weighted mean

Averages

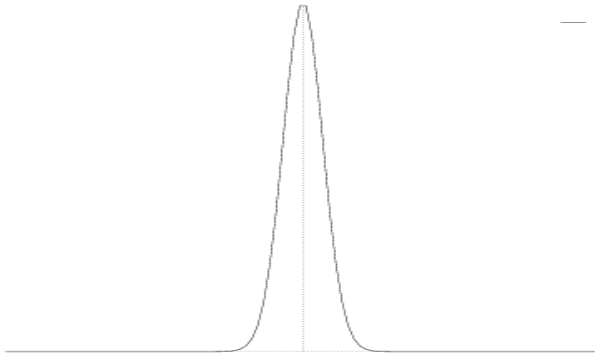
- Mean

$$I = \frac{I_1 + I_2 + \dots + I_n}{n} = \frac{\sum_{i=1}^n I_i}{n}$$

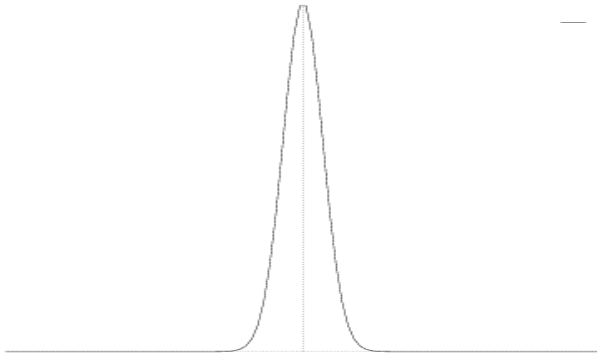
- Weighted mean

$$I = \frac{w_1 I_1 + w_2 I_2 + \dots + w_n I_n}{n} = \frac{\sum_{i=1}^n w_i I_i}{n}$$

Gaussian Filter

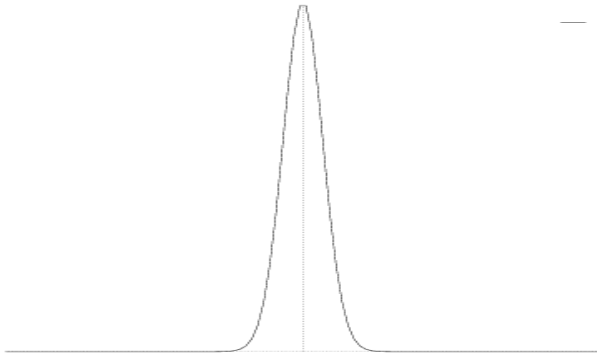


Gaussian Filter

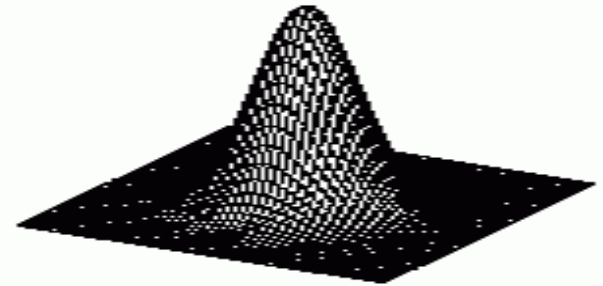


$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

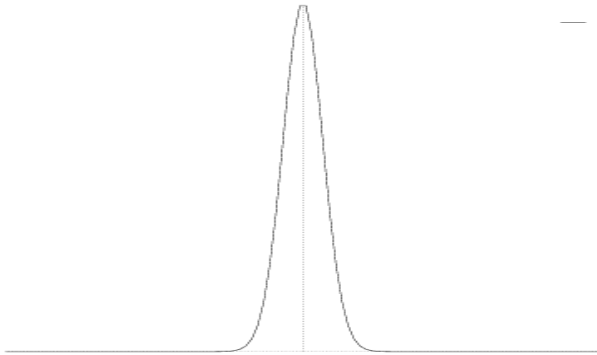
Gaussian Filter



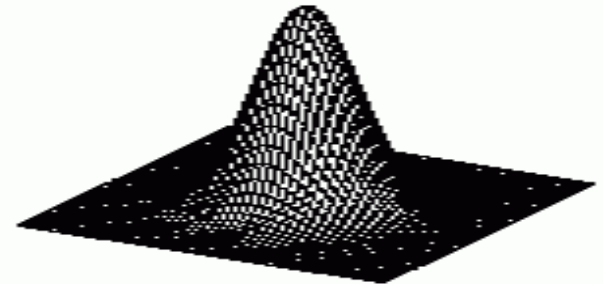
$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$



Gaussian Filter

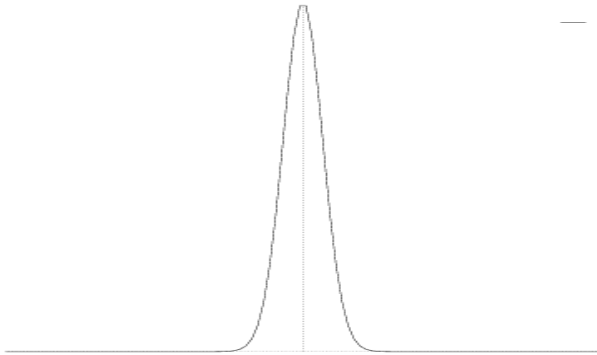


$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

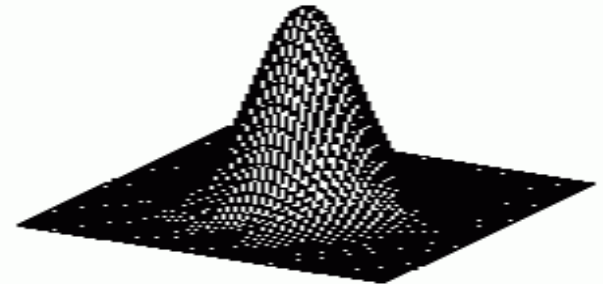


$$g(x, y) = e^{\frac{-(x^2 + y^2)}{2\sigma^2}}$$

Gaussian Filter



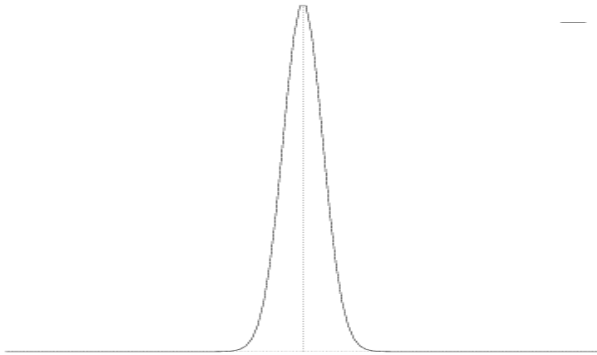
$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$



$$g(x, y) = e^{\frac{-(x^2 + y^2)}{2\sigma^2}}$$

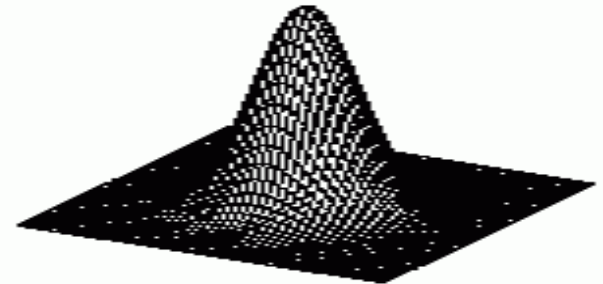
$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

Gaussian Filter



$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$



$$g(x, y) = e^{\frac{-(x^2 + y^2)}{2\sigma^2}}$$

$$\sigma = 1$$

Properties of Gaussian

Properties of Gaussian

- Most common natural model

Properties of Gaussian

- Most common natural model
- Smooth function, it has infinite number of derivatives

Properties of Gaussian

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- Most common natural model
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- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.

Properties of Gaussian

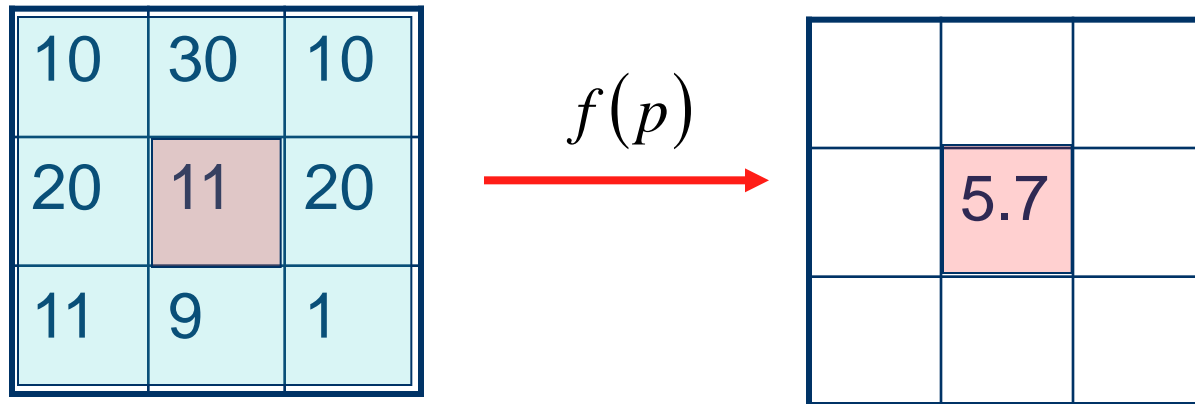
- Most common natural model
- Smooth function, it has infinite number of derivatives
- It is Symmetric
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- Gaussian is separable; 2D convolution can be performed by two 1-D convolutions

Properties of Gaussian

- Most common natural model
- Smooth function, it has infinite number of derivatives
- It is Symmetric
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- Gaussian is separable; 2D convolution can be performed by two 1-D convolutions
- There are cells in eye that perform Gaussian filtering.

Filtering

- Modify pixels based on some function of the neighborhood



Linear Filtering

- The output is the linear combination of the neighborhood pixels

| | | |
|---|----|---|
| 1 | 3 | 0 |
| 2 | 10 | 2 |
| 4 | 1 | 1 |

Image

\otimes

| | | |
|---|-----|----|
| 1 | 0 | -1 |
| 1 | 0.1 | -1 |
| 1 | 0 | -1 |

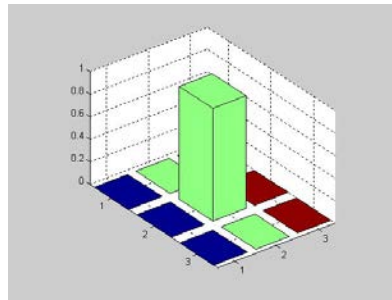
Kernel

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| | | |
|--|---|--|
| | | |
| | 5 | |
| | | |

Filter Output

Filtering Examples



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| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

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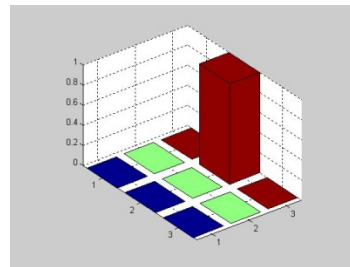
Filtering Examples



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| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

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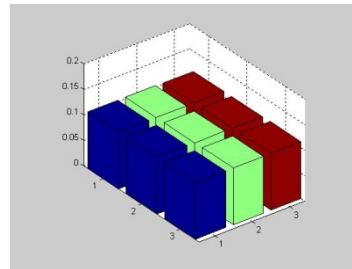
Filtering Examples



$\ast \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

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Filtering Examples



$\ast \frac{1}{25}$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

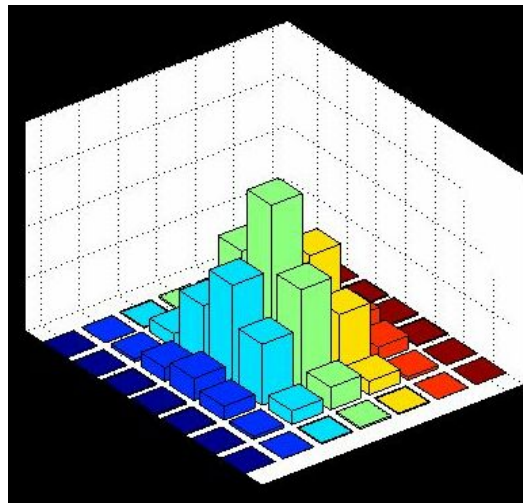
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Filtering Gaussian



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Gaussian vs. Averaging



Gaussian Smoothing



Smoothing by Averaging

Noise Filtering



After additive Gaussian Noise



After Averaging



After Gaussian Smoothing
Alper Yilmaz, Mubarak Shah, UCF

Example: box filter

$$\frac{1}{9} g[\cdot, \cdot]$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |



Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

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$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Image filtering

$$g[\cdot, \cdot] \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

$h[\cdot, \cdot]$

| | | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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|--|---|--|--|--|--|--|--|--|--|--|
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$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

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| | | | | | | | | | |
| | 0 | 10 | | | | | | | |
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| | | | | | | | | | |

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

| | | | | | | | | | |
|--|---|----|----|--|--|--|--|--|--|
| | | | | | | | | | |
| | 0 | 10 | 20 | | | | | | |
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| | | | | | | | | | |

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

| | | | | | | | | | |
|--|---|----|----|----|--|--|--|--|--|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | | | | | |
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| | | | | | | | | | |

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

| | | | | | | | | | |
|--|---|----|----|----|----|--|--|--|--|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$



$$h[\cdot, \cdot]$$

[illegible]

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l]$$



Image filtering

$$g[\cdot, \cdot] \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

| | | | | | | | | | |
|--|---|----|----|----|----|---|--|--|--|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
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| | | | | | | | | | |

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Image filtering

$$g[\cdot, \cdot] \quad \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$f[\cdot, \cdot]$

| | | | | | | | | | |
|---|---|----|----|----|----|----|----|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[\cdot, \cdot]$

| | | | | | | | | | |
|--|----|----|----|----|----|----|----|----|--|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 60 | 90 | 90 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 10 | 20 | 30 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l]$$

Box Filter

What does it do?

$$\frac{1}{9} g[\cdot, \cdot]$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Box Filter

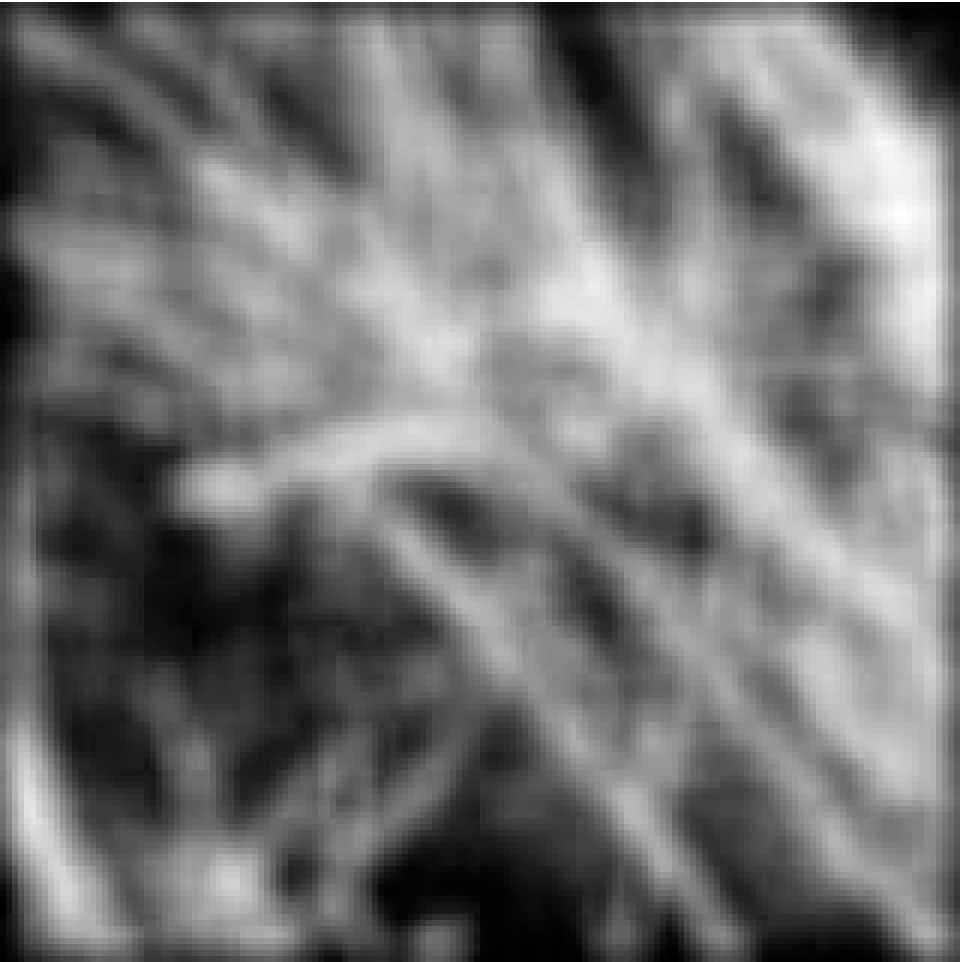
What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)

$$\frac{1}{9} g[\cdot, \cdot]$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Smoothing with box filter



Practice with linear filters



Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

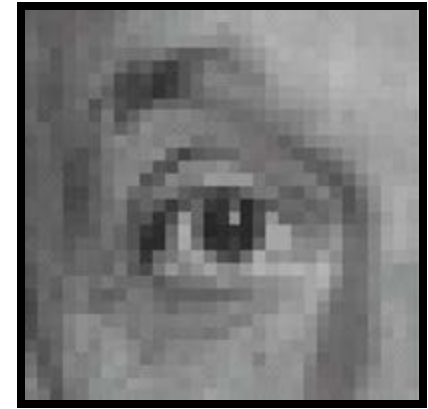
?

Practice with linear filters



Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |



Filtered
(no change)

Practice with linear filters



Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

?

Practice with linear filters



Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |



Shifted left
By 1 pixel

Practice with linear filters



Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 2 | 0 |
| 0 | 0 | 0 |

—

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

?

(Note that filter sums to 1)

Practice with linear filters



Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 2 | 0 |
| 0 | 0 | 0 |

−

$\frac{1}{9}$

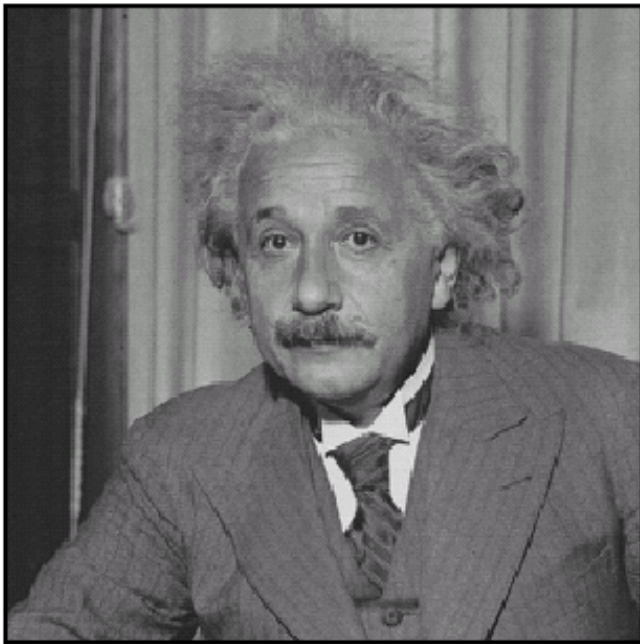
| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |



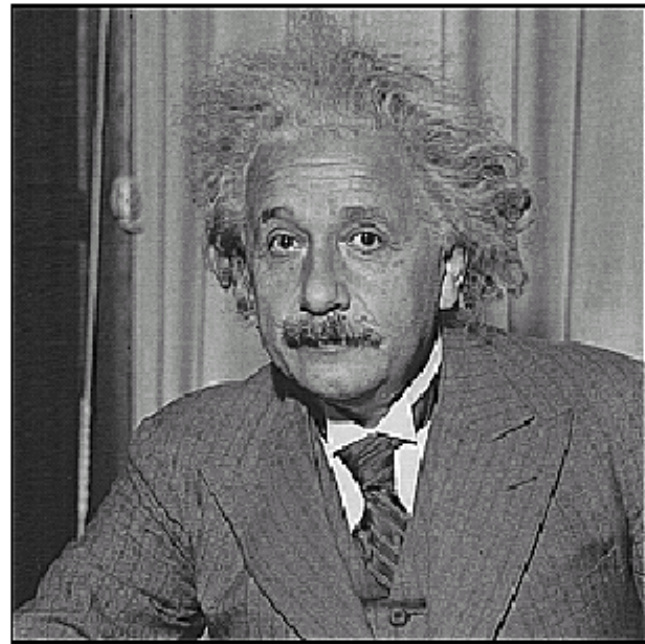
Sharpening filter

- Accentuates differences with local average

Sharpening

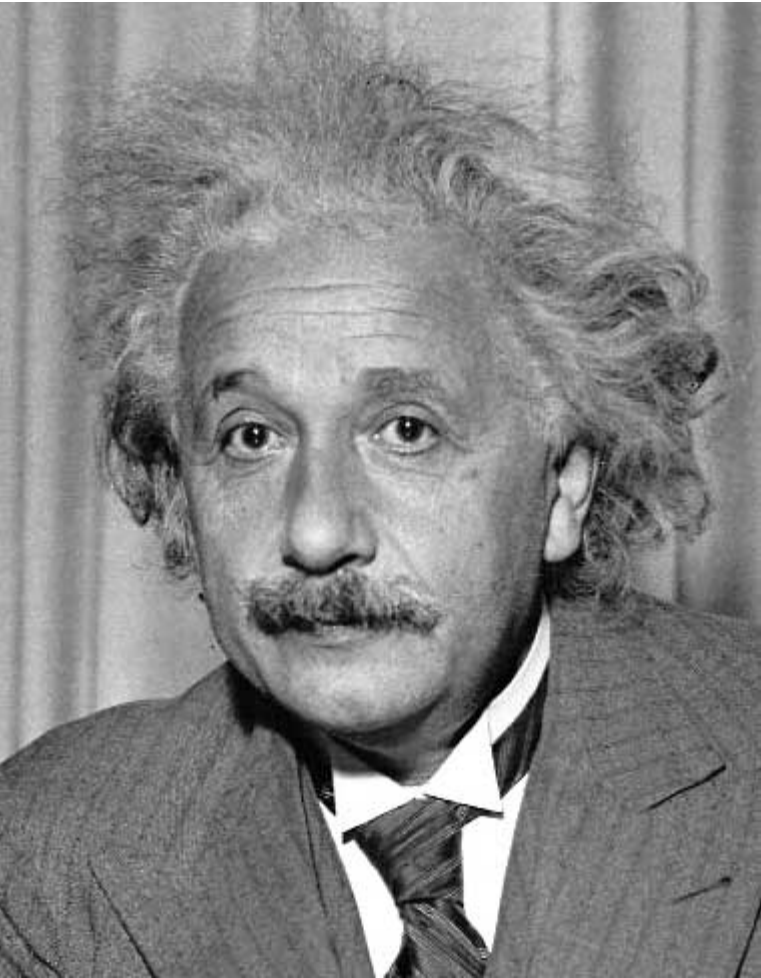


before



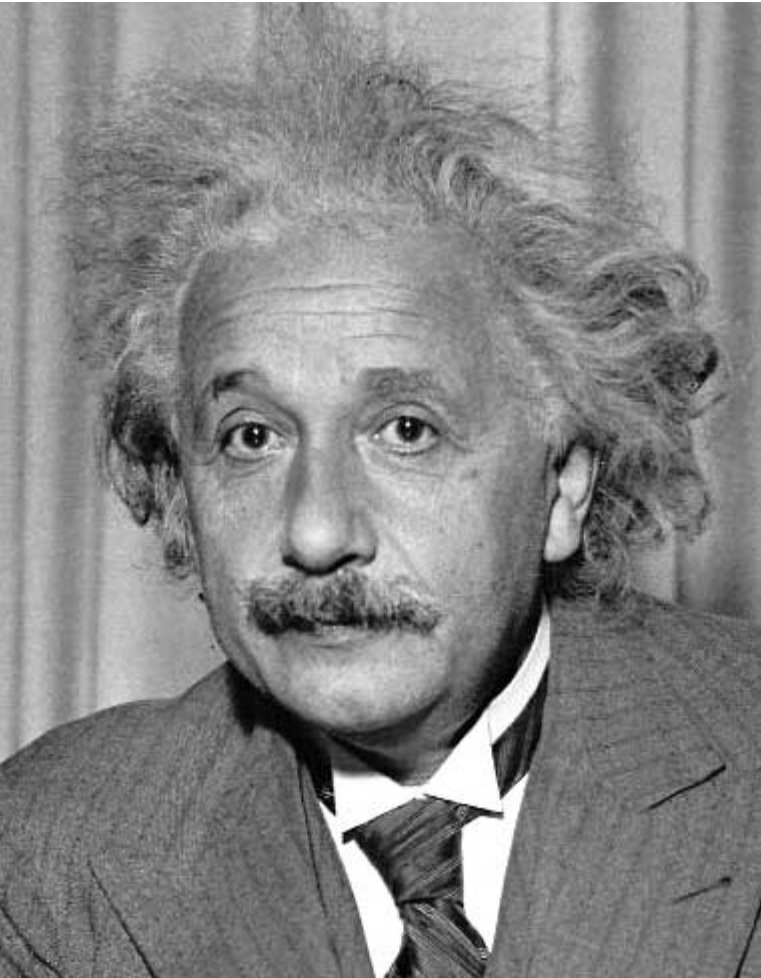
after

Other filters



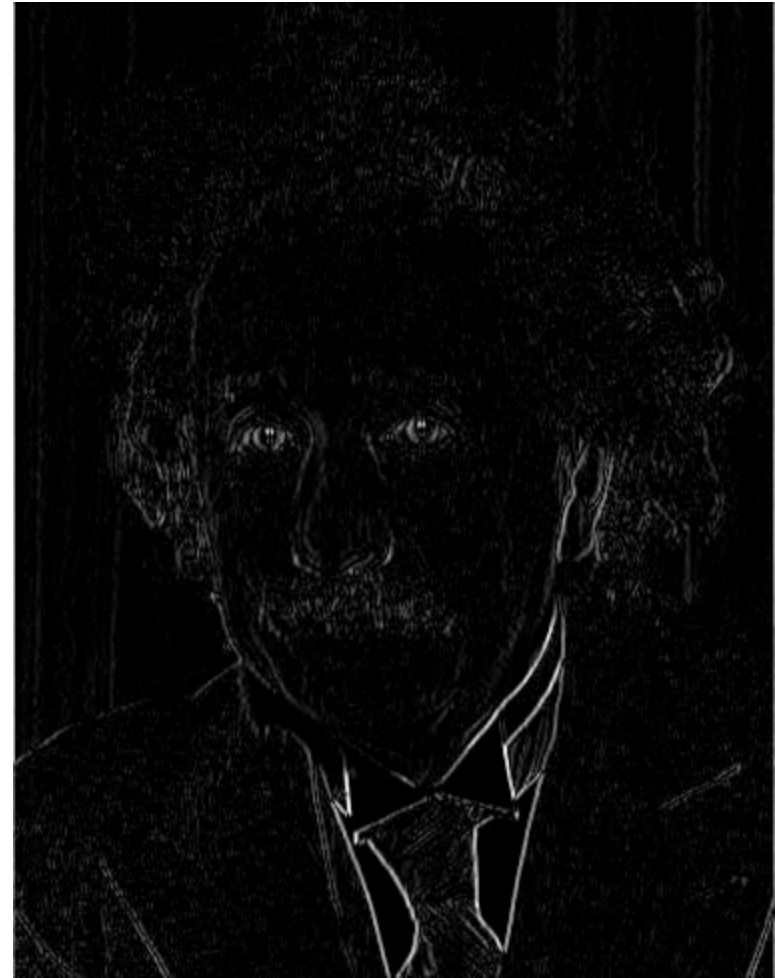
| | | |
|---|---|----|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

Other filters



| | | |
|---|---|----|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

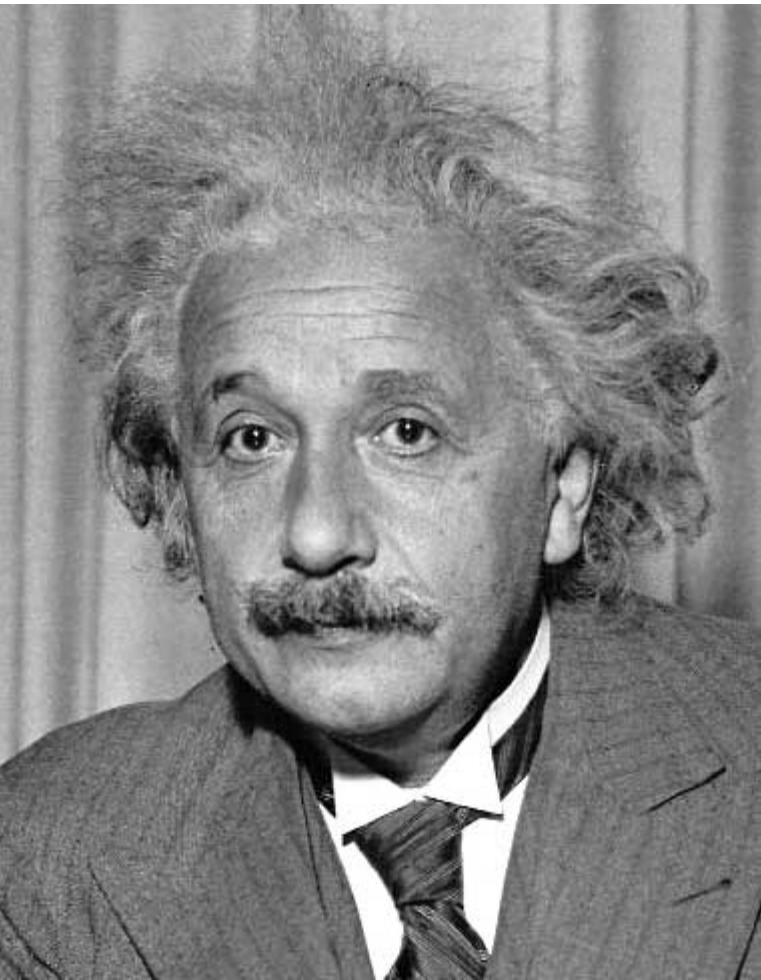
Sobel



Vertical Edge
(absolute value)



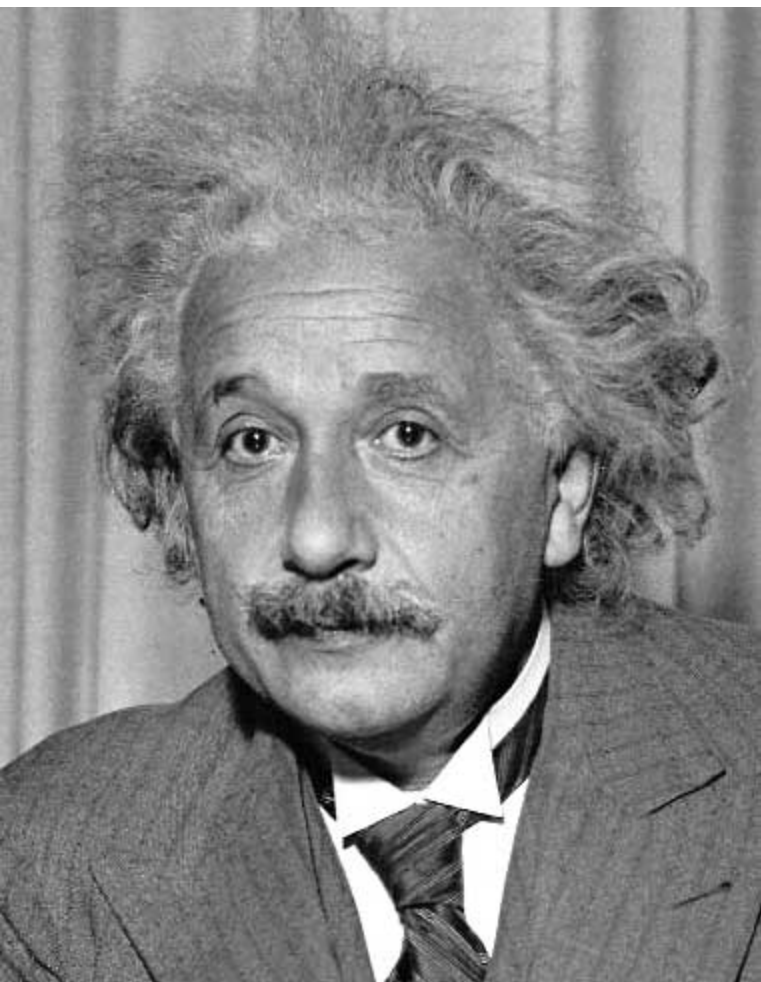
Other filters



| | | |
|----|----|----|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

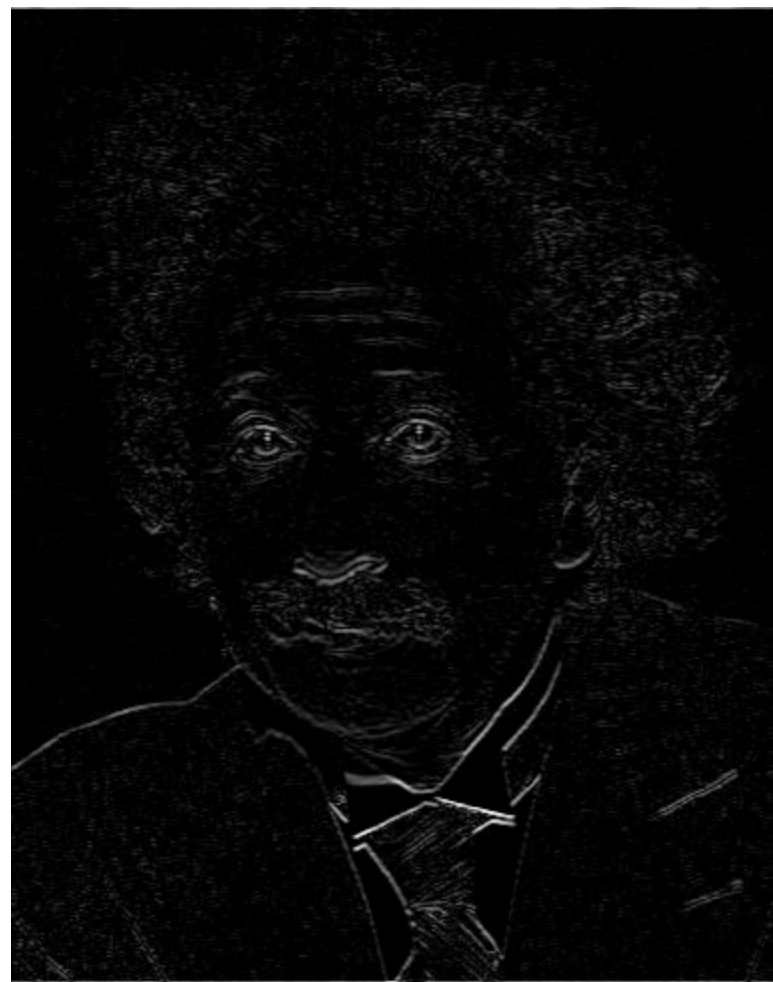


Other filters



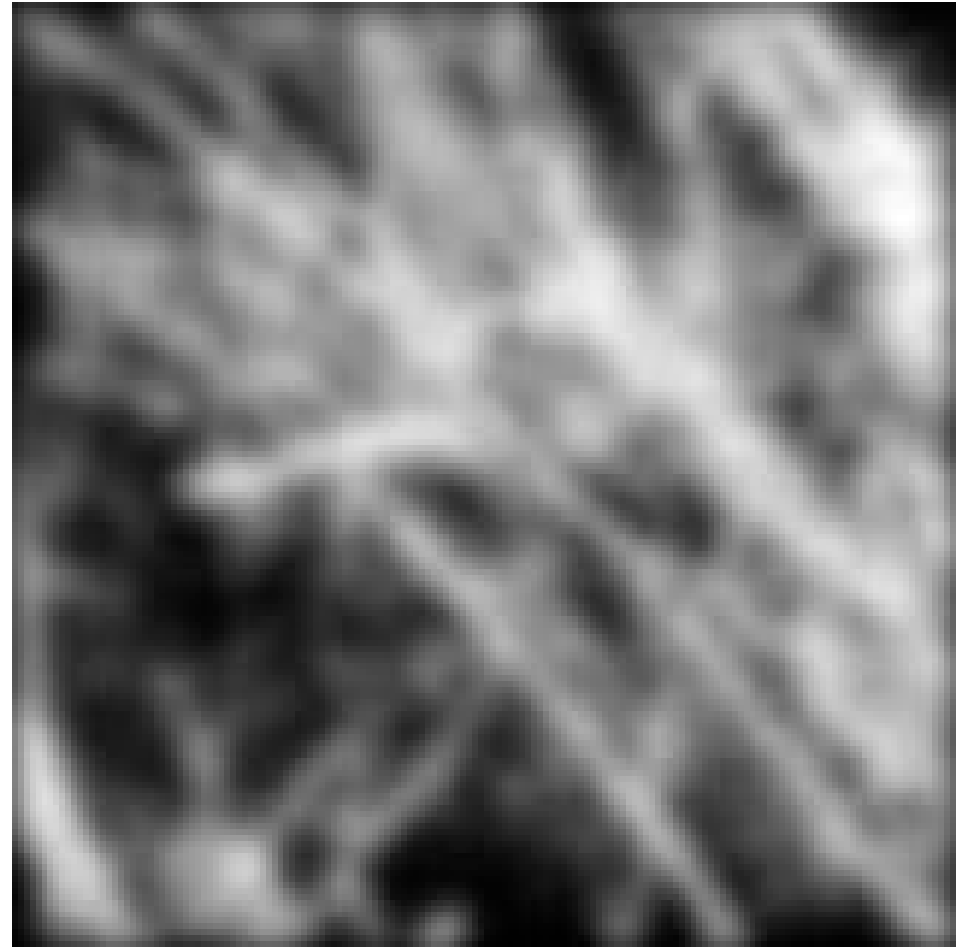
| | | |
|----|----|----|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Sobel

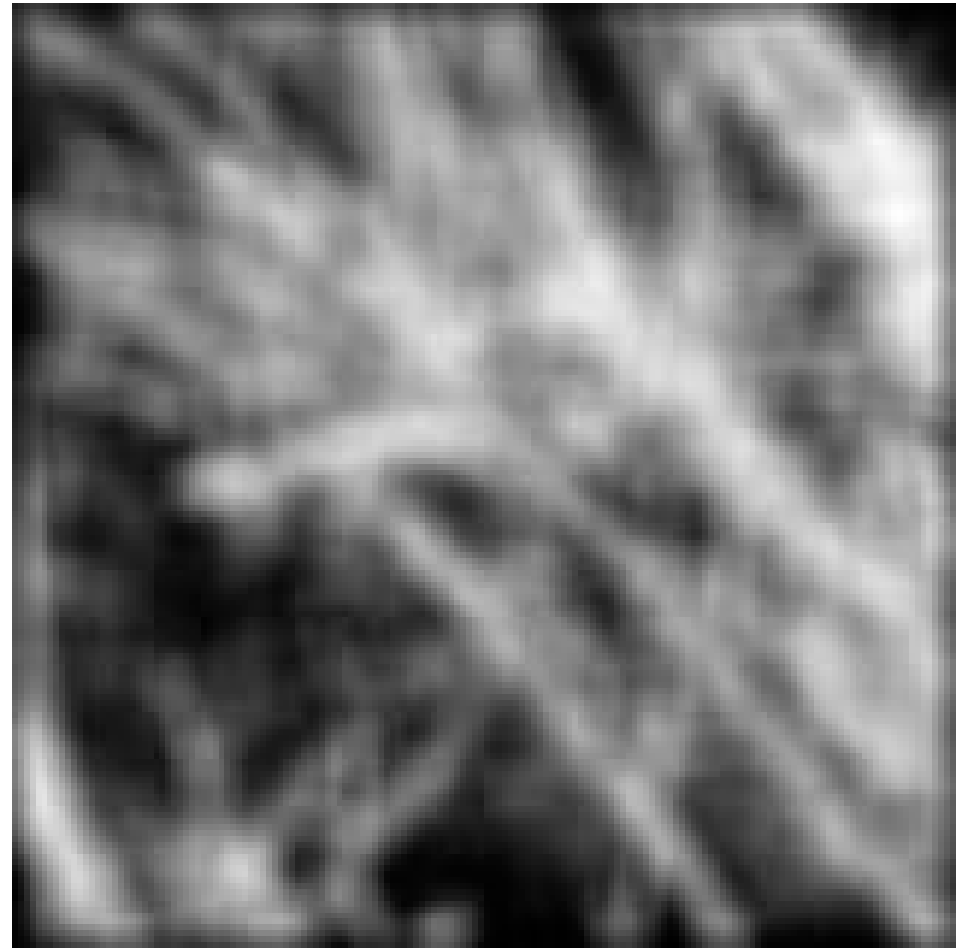


Horizontal Edge
(absolute value)

Smoothing with Gaussian filter



Smoothing with box filter





Key properties of linear filters



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Linearity:

$$\text{filter}(f_1 + f_2) = \text{filter}(f_1) + \text{filter}(f_2)$$



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$$\text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f))$$



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Shift invariance: same behavior regardless of pixel location

$$\text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f))$$

Any linear, shift-invariant operator can be represented as a convolution

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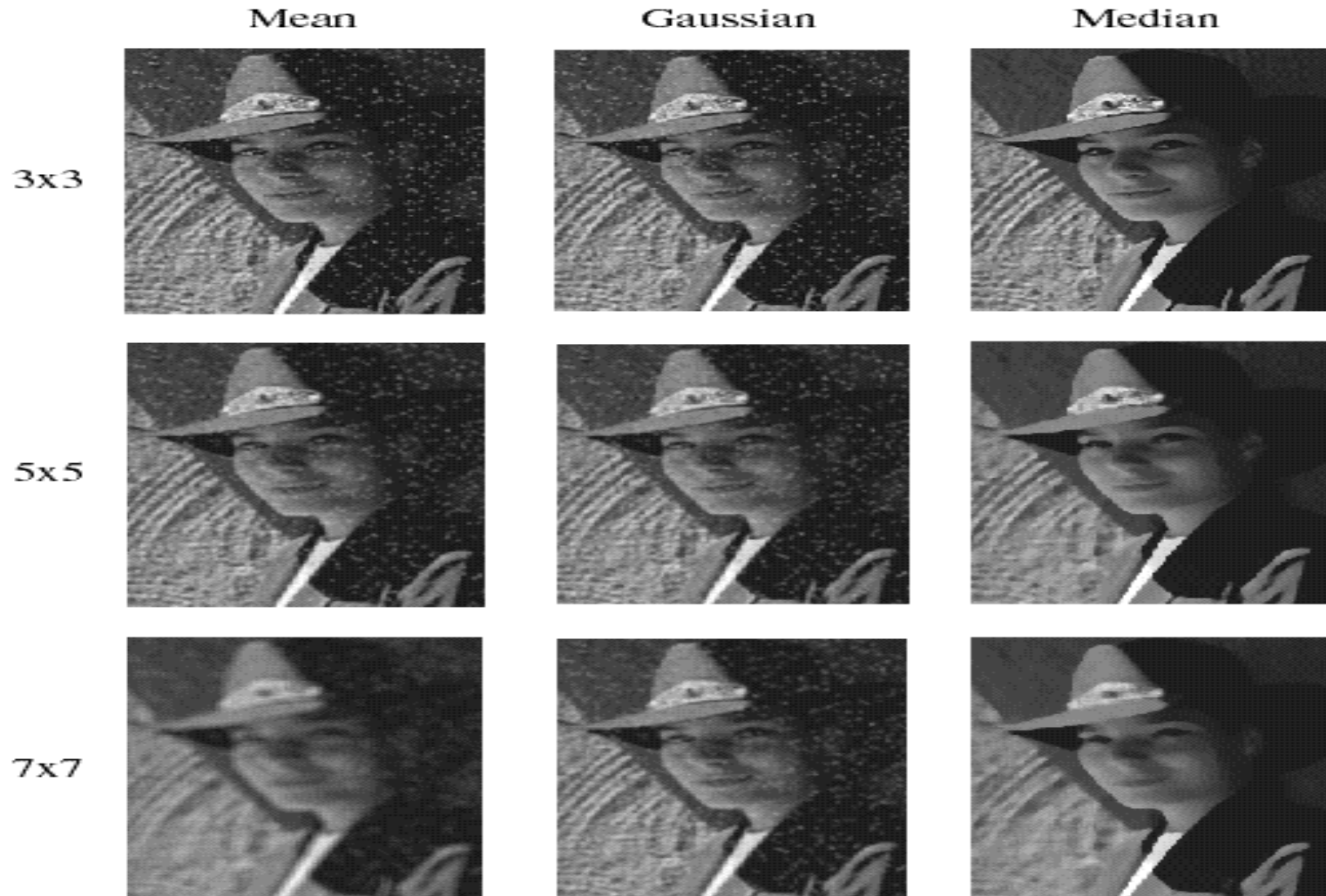
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- Identity: unit impulse $e = [0, 0, 1, 0, 0]$,
 $a * e = a$



Median filters

- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?

Comparison: salt and pepper noise



MATLAB Functions

The slide features a light green background with a white rounded rectangle in the top-left corner. The title "MATLAB Functions" is written in a bold, dark teal font within this white area. A thick, dark blue horizontal bar spans the width of the slide below the title.

MATLAB Functions

- **conv:** 1-D Convolution.

$C = \text{conv}(A, B)$ convolves vectors A and B .

MATLAB Functions

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 $C = \text{conv}(A, B)$ convolves vectors A and B .
- **conv2:** Two dimensional convolution.
 - $C = \text{conv2}(A, B)$ performs the 2-D convolution of matrices A and B .

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 - The result, Y , is computed using 2-D correlation and is the same size as X .
 - `filter2` uses `CONV2` to do most of the work. 2-D correlation is related to 2-D convolution by a 180 degree rotation of the filter matrix.

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 - For vectors, $\text{mean}(X)$ is the mean value (average) of the elements in X .

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Example: $H = \text{fspecial}(\text{'gaussian'}, 7, 1)$ creates a 7x7 Gaussian filter with variance 1.

Some practical matters

Practical matters

How big should the filter be?

- Values at edges should be near zero
- Rule of thumb for Gaussian: set filter half-width to about 3σ

Practical matters

- What about near the edge?
 - the filter window falls off the edge of the image
 - need to extrapolate
 - methods:
 - clip filter (black)
 - wrap around
 - copy edge
 - reflect across edge



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Practical matters

– methods (MATLAB):

- clip filter (black): `imfilter(f, g, 0)`
- wrap around: `imfilter(f, g, 'circular')`
- copy edge: `imfilter(f, g, 'replicate')`
- reflect across edge: `imfilter(f, g, 'symmetric')`

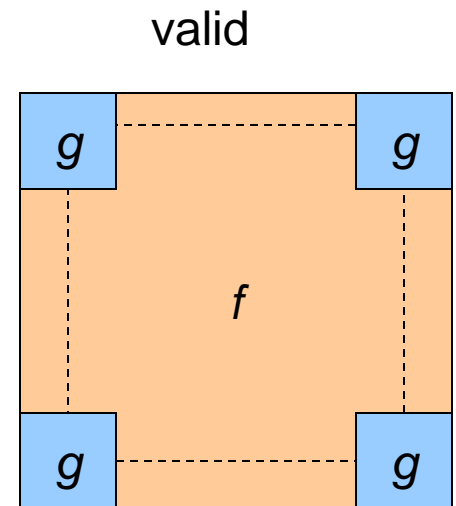
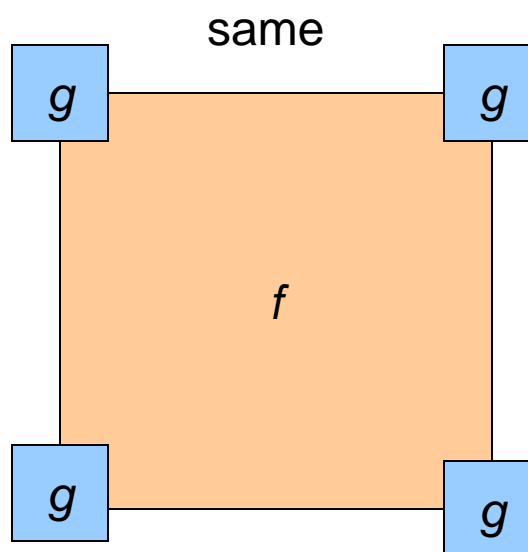
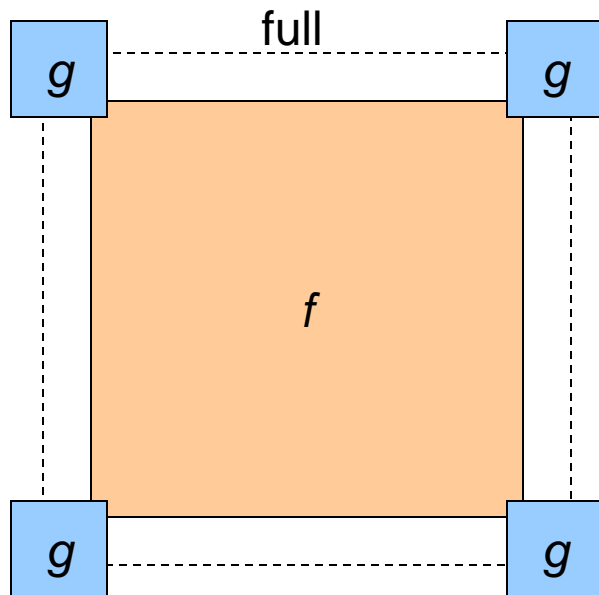
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Practical matters

- What is the size of the output?
- MATLAB: `filter2(g, f, shape)`
 - *shape* = 'full': output size is sum of sizes of *f* and *g*
 - *shape* = 'same': output size is same as *f*
 - *shape* = 'valid': output size is difference of sizes of *f* and *g*



Reading Material



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- Mubarak Shah, "Fundamentals of Computer Vision".
 - Chapter, 2

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- Mubarak Shah, "Fundamentals of Computer Vision".
 - Chapter, 2
- Richard Szeliski, "Computer Vision: Algorithms and Applications".
 - Section 3.1 and 3.2