2018 Fall

Calculus-2 Midterm-Exam

Dept. or School		proctor	
Student ID		Name	

1. (3pts) Find a 2-dimensional unit vector v such that v is orthogonal to the vector $\langle 1,1\rangle$, and $v \cdot w > 0$ when $w = \langle -2,1\rangle$.

2. (3pts) Is the function $f(x)$ continuous at $(0,0)$? Prove your answer.

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

3. (4pts) Let S be a sphere which is tangent to the two planes $x+y+z=3$ and $x+y+z=9$. Find the center point of S, when the planes $2x-y=0$ and $3x-z=0$ pass through the center point.

4. (4pts) Find the radius of the osculating circle of the curve $\boldsymbol{r}(t) = \langle \cos t, \sin t, t^2 \rangle$ at the point $\boldsymbol{r}(1)$.

5. (4pts) The base radius and height of a right circular cylinder are measured as 10 cm and 25 cm respectively, with a possible error in measurement of as much as 0.01 cm in each. Estimate the maximum error in the calculated volume of the cylinder by using differentials.

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6. (4pts) Evaluate $\frac{\partial^2 u}{\partial y \partial x}$ at $(x,y)=(2,2)$, when

$$u=\ln\sqrt{x^2+y^2}\ .$$

7. (4pts) Find $\frac{\partial z}{\partial y}$, when $yz+x\ln y=z^2$.

8. (4pts) The plane $x+ay+bz+c=0$ is the tangent plane to the surface $xy^2z^3=8$ at the point $(2,2,1)$. Find the value of c.

9. (5pts) Let $f(x,y,z)=x^2+2y^2+z^2$ and $g(x,y,z)=z-xy$. Find a unit vector u satisfying

$$D_u f\,(1,1,\frac{1}{2})=0=D_u\, g\,(1,1,\frac{1}{2}).$$

10. (5pts) Let C be the curve of the intersection of the two surfaces $x^2+y^2=z$ and $4x^2+y^2+z^2=9$. And let L be the tangent line to C at the point $(1,-1,2)$. Calculate the distance from the point $(0,0,0)$ to L.