CALCULUS II
2017 Fall Semester Final Exam

Dept. or School		proctor	
Student ID		Name	

※ **Your answer must be provided with descriptions how to get the answer.**

1. (4 points) Find the absolute maximum and minimum values of the function $f(x,y) = x^2 + xy + y^2$ on the disk $D = \{(x,y) \mid x^2 + y^2 \leq 9\}$.

2. Evaluate the following double integrals.

(1) (3 points) $\iint_D x^2 y \, dA$

where $D = \{(x,y) : 0 \leq x \leq 1,\ x^2 \leq y \leq \sqrt{x}\}$

(2) (3 points) $\iint_R x^2 y \, dA$ where $R = [0,1] \times [0,1]$

3. (3 points) Evaluate the double integral $\iint_D ye^x \, dA$

where $D = \{(x,y) : x \geq 0, \ y \geq 0, \ x^2 + y^2 \leq 25\}$.

4. (3 points) Find the volume of the solid that lies under $x^2 + y^2 + z^2 = 4a^2$, above the xy-plane, and inside $r = 2a\cos\theta$, $(a > 0)$.

5. (4 points) Find the area of the surface that is enclosed by two paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

CALCULUS II Fall Final 2017 ~~Spring Final~~ Exam	Dept. or School		proctor	
	Student ID		Name	

6. (5 points) Let F be the vector field given by

$$F(x,y) = < 16x^4 + 8x^2y^2 + y^4 + 4x,\ 4x^2 + y^2 + y >.$$

Evaluate $\int_C F \cdot\ dr$, where C consists of the arc C_1 of the curve $4x^2 + y^2 = 4$ oriented in clockwise direction from $(0,2)$ to $(0,-2)$ and the line segment C_2 from $(0,-2)$ to $(-1,0)$. (**Caution:** you must give the parametric equations or vector functions for the arc C_1 and the line segment C_2 exactly).

7. (1) (2 points) Find a potential function f for the field

$$F(x,y) = (ye^x + \sin y)\boldsymbol{i} + (e^x + x\cos y)\boldsymbol{j}.$$

(2) (3 points) The force field

$$F(x,y,z) = \sin(yz)\boldsymbol{i} + xz\cos(yz)\boldsymbol{j} + xy\cos(yz)\boldsymbol{k}$$

moves a particle along a piecewise smooth curve C from $(0,0,0)$ to $(1,\pi,\frac{1}{2})$. Determine the work that is done.

8. Evaluate the line integral $\int_C \frac{y}{x+1}dx + 2xy\,dy$ where C consists of the line segment C_1 from $(1,1)$ to $(0,0)$ followed by the curve C_2: $y=x^2$ from $(0,0)$ to $(1,1)$.

10. (5 points) Find the area of the part of the sphere $x^2+y^2+z^2=a^2$ that lies inside the cylinder $x^2+y^2=ax$ where $a>0$.

9. (4 points) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C consists of the horizontal line segment C_1 from $(0,0)$ to $(2,0)$ followed by the arc C_2 of the semicircle $(x-1)^2+y^2=1,\ y\geq 0$ and

$$\mathbf{F}(x,y)=(y\,e^{\sin x}\cos x+\frac{1}{2}y^2)\mathbf{i}+(2xy+e^{\sin x}+\sqrt{y^3+1})\mathbf{j}.$$