CALCULUS II
2017 Fall Semester Midterm Exam

Dept. or School		proctor	
Student ID		Name	

※ **Your answer must be provided with descriptions how to get the answer.**

1. Compute the determinants of the following matrices

(1) (2 points) $\begin{pmatrix} 1 & 2 & 4 \\ 3 & 5 & 1 \\ 1 & 0 & 2 \end{pmatrix}$

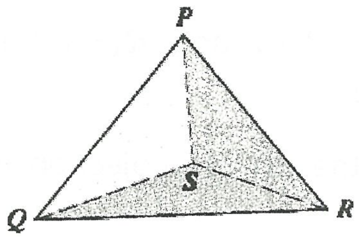
(2) (2 points) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

2. Let $P(1, 1, 0)$, $Q(2, 3, 0)$, and $R(-3, 1, 0)$ be three points in space R^3.

(1) (2 points) Find the vector projection of $\overrightarrow{PR}$ onto $\overrightarrow{PQ}$.

(2) (3 points) Find the value t which P, Q, R and $S(1,\ t,\ 2-t^2)$ are coplanar.

3. (3 points) Let a tetrahedron be given by four vertices, $P(0,0,4)$, $Q(2,1,0)$, $R(0,4,0)$, $S(0,0,0)$ and four triangular faces, as shown in the figure.



If v_1, v_2, v_3, v_4 are vectors with lengths equal to the areas of the faces opposite the vertices P, Q, R, S, respectively, and directions perpendicular to the respective faces and pointing outward. Find the sum $v_1 + v_2 + v_3 + v_4$.

4. (3 points) Find the equation of the plane containing the line $x=1,\ y=t,\ z=t$ which is perpendicular to the plane $x+3y-2z=0$.

5. (1) (2 points) Line L_1 is given by parametric equations,

$$x=2+3t,\quad y=-1+2t,\quad z=3-4t.$$

Line L_2 is given by parametric equations,

$$x=-1-s,\quad y=-7-2s,\quad z=6+s.$$

Determine the point in which these lines intersect.

(2) (3 points) Determine parametric equations for the line that passes through the point in (1) and meets perpendicularly the line L_3 given by symmetric equations,

$$\frac{x+1}{2}=\frac{y+2}{1}=\frac{z+2}{1}.$$

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6. (3 points) Let L be a curve of intersection of the cylinder $x^2+(y-2)^2=4$ and the plane $z=x$.
At what point on this curve is the largest maximum curvature?

7. (3 points) Find the following limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \tan^{-1}\left(\frac{x^2(1-\sin^2 y)+3y^2}{x^2+3y^2}\right)$$

8. (4 points) Let $f(r,\theta)=\int_{r\cos\theta}^{r\sin\theta}\cos(e^t)dt$.

Calculate $\frac{\partial^2 f}{\partial r\,\partial\theta}\left(1,\frac{\pi}{2}\right)$.

9. (3 points) If $z = f(x, y)$ has continuous partial derivatives, $x = s + 2t$, and $y = s - 2t$, find the value of the number c of satisfying the following equation;

$$\frac{\partial z}{\partial s}\frac{\partial z}{\partial t} = 2\left(\frac{\partial z}{\partial x}\right)^2 + c\left(\frac{\partial z}{\partial y}\right)^2.$$

10. (3 points) Find the differential dz of the function $f(x,y) = 4\tan^{-1}(xy)$ at the point $(1,1)$ and use it to estimate the number $4\tan^{-1}[(0.98)^2]$.

11. (4 points) Suppose that the price z of coffee is proportional to the population x divided by the supply y, $z = c\frac{x}{y}$ (c: constant) and x, y depend on time in such a way that

$$\frac{dx}{dt} = 0.01x, \quad \frac{dy}{dt} = -\sqrt{x}.$$

Find the rate of increase in the price z when $x = 1{,}000{,}000$ and $y = 10{,}000$.