CALCULUS II
2019 Fall Midterm Exam

Dept. or School		Year		proctor	
Student ID		Name			

※ **Your answer must be provided with descriptions how to get the answer.**

1. Let L_1 be the line through the point $(2,4,8)$ and parallel to the vector $<1,2,2>$. Let L_2 be the line of intersection of the planes α_1 and α_2, where α_1 is the plane $3x-3y+6z+3=0$ and α_2 is the plane through points $(3,6,1)$, $(0,-2,0)$ and $(-1,2,3)$.

(a)(5 points) Find parametric equation for the line L_2.

(b)(5 points) Find the distance between L_1 and L_2.

2. Let C be a curve given by the vector equation $\mathbf{r}(t) = < t^2 + t + 2,\ 1 - 2t,\ \frac{1}{2}t^2 >$ and $P(2,1,0)$ be a point represented by rectangular coordinate.

(a)(5 points) Find the curvature of the curve C at $P(2,1,0)$.

(b)(5 points) Find the center of the osculating circle of the curve C at $P(2,1,0)$.

CALCULUS II 2019 Fall Midterm Exam	Dept. or School		Year		proctor	
	Student ID		Name			

3. (6 points) Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2x^2 y \sin x}{x^4+y^2}.$$

4. (6 points) Let a function f defined on R^2 by

$$f(x,y)=\begin{cases} 2x-y+\dfrac{x^2y^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0). \end{cases}$$

and $f(x,y)$ is differentiable at $(0,0)$. Find the linear approximation of the functin $f(x,y)$ at $(0,0)$ and use it to approximate $f(0.01,-0.10)$.

5. **(8 points)** If a differentiable function $s = f(x, y, z)$ decrease fastest in the direction of the vector $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ at the point $P(1, 1, 1)$ and the minimum rate of change is $-3\sqrt{3}$.
Find the maximum rate of change of

$$g(x, y, z) = f(x^2 + yz,\ y^2 + zx,\ z^2 - xy)$$

at the point $Q(2, 2, 0)$ and the direction in which it occurs.