Assignment 7.1

Question 1:

Consider a long solenoid with a core that is an iron alloy. Suppose that the diameter of the solenoid is 2 cm and the length of the solenoid is 20 cm. The number of turns on the solenoid is 200. The current is increased until the core is magnetized to saturation at about $I = 2$ A and the saturated magnetic field B_{sat} is 1.6 T.

a. What is the magnetic field intensity, B_0 at the center of the solenoid and the applied magnetic field, H?

b. What is the saturation magnetization M_{sat} of this iron alloy?

c. If we were to have the same magnetic field of 1.6 T inside the solenoid *without* the iron-alloy core, how much current would we need? Is there a practical way of doing this?

Q1. (a)
$$B_0 = \mu_0 n I = \frac{\mu_0 N I}{L} = \frac{(4\pi \times 10^{-7})(200)(2)}{0.2} = 2.51 \times 10^{-3}\ \mathrm{T}$$

$$B_0 = \mu_0 H \Rightarrow H = \frac{2.51 \times 10^{-3}}{4\pi \times 10^{-7}} = 2000\ \mathrm{A/m}$$

(b)
$$B = \mu_0 H + \mu_0 M \Rightarrow M_{sat} = \frac{B_{sat} - \mu_0 H_{sat}}{\mu_0}$$

$$= \frac{1.6 - 2.51 \times 10^{-3}}{4\pi \times 10^{-7}} = 1.27 \times 10^{6}\ \mathrm{A/m}$$

(c)
$$B_0 = \mu_0 n I = \frac{\mu_0 N I}{L}$$

$$I = \frac{B_0 L}{\mu_0 N} = \frac{1.6 \times 0.2}{4\pi \times 10^{-7} \times 200} = 1273\ \mathrm{A}$$

Not practical! Can be achieved using a superconducting solenoid.

Question 2:

Sometimes magnetic susceptibilities are reported as molar or mass susceptibilities. **Mass susceptibility** (in $m^3\ kg^{-1}$) is χ_m/ρ where ρ is the density. **Molar susceptibility** (in $m^3\ mol^{-1}$) is $\chi_m(M_{at}/\rho)$ where M_{at} is the atomic mass. Terbium (Tb) has a magnetic molar susceptibility of 2 $cm^3\ mol^{-1}$. Tb has a density of 8.2 g cm^{-3} and an atomic mass of 158.93 g mol^{-1}. What is its susceptibility, mass susceptibility and relative permeability? What is the magnetization in the sample in an applied magnetic field of 2 T?

Q2. Molar susceptibility : $\dfrac{\chi_m M_{at}}{\rho} = 2\,cm^3/mol$

$\Rightarrow \chi_m = 2\left(\dfrac{\rho}{M_{at}}\right) = 2\left(\dfrac{8.2}{158.93}\right) = 0.1032$ susceptibility

$\dfrac{\chi_m}{\rho} = \dfrac{0.1032}{8.2} = 1.26 \times 10^{-5}\ m^3/kg$ mass susceptibility

$\mu_r = 1 + \chi_m = 1 + 0.1032 = 1.1032$ relative permeability

$M = \chi_m H = \chi_m\left(\dfrac{B_0}{\mu_0}\right) = \dfrac{0.1032 \times 2}{4\pi \times 10^{-7}} = 1.64 \times 10^5\ A/m$

Question 3:

Consider bismuth with χ_m =-17×10^{-5} and aluminum with χ_m = 2×10^{-5}. Suppose that we subject each sample to an applied magnetic field B_o of 1 T applied in the +x direction. What is the magnetization **M** and the equivalent magnetic field $\mu_o M$ in each sample? Which is paramagnetic and which is diamagnetic?

Q3. $M = \chi_m H = \chi_m \left(\frac{B_o}{\mu_o}\right)$

Bismuth: $\chi_m = -17 \times 10^{-5}$, $M = (-17 \times 10^{-5})\left(\frac{1}{4\pi \times 10^{-7}}\right) = -135.28 \, A/m$

χ_m negative and small $\Rightarrow$ diamagnetic. $\mu_o M = \chi_m B_o = -17 \times 10^{-5}$ T.

Aluminum: $\chi_m = 2 \times 10^{-5}$

$M = \chi_m H = \chi_m \left(\frac{B_o}{\mu_o}\right) = (2 \times 10^{-5})\left(\frac{1}{4\pi \times 10^{-7}}\right) = 15.92 \, A/m$

$\mu_o M = \chi_m B_o = 2 \times 10^{-5}$ T.

Assignment 7.2

Question 1:

Consider dysprosium (Dy), which is a rare earth metal with a density of 8.54 g cm^{-3} and atomic mass of 162.50 g mol^{-1}. If the saturation magnetization of Dy near absolute zero of temperature is 2.4×10^{6} Am^{-1}, what is the magnetic moment per atom in Bohr-magneton μ_B? What is the exchange energy E_{ex} in eV per atom in Dy if the Curie temperature is 85 K?

Q1. $M_{sat} = n_{at} \vec{\mu}_{at}$

$$n_{at} = \frac{\rho N_A}{M_{at}} = \frac{(8.54 \times 10^{6})(6.022 \times 10^{23})}{(162.50)} = 3.16 \times 10^{28} \text{ atoms/m}^3$$

$$\vec{\mu}_{at} = \frac{M_{sat}}{n_{at}} = \frac{2.4 \times 10^{6}}{3.16 \times 10^{28}} \times \frac{\mu_B}{9.27 \times 10^{-24} Am^2} = 8.18\, \mu_B$$

$$E_{ex} = kT_c = (1.38 \times 10^{-23}) \cdot 85 = 1.173 \times 10^{-21} J.$$

Question 2:

The energy of a domain wall depends on two main factors: the exchange energy E_{ex} (J/atom) and magnetocrystalline energy K (J m^{-3}). If a is the interatomic distance, δ' is the wall thickness, then it can be shown that the potential energy per unit area of the wall is

$$U_{\text{wall}} = \frac{\pi^2 E_{ex}}{2a\delta} + K\delta \qquad \textit{Potential energy of a domain wall}$$

Show that the minimum energy occurs when the wall has the thickness

$$\delta' = \left(\frac{\pi^2 E_{ex}}{2aK}\right)^{1/2} \qquad \textit{domain wall thickness}$$

and show that when $\delta = \delta'$, the exchange and anisotropy energy contributions are *equal*.

Q2. $U_{wall} = \dfrac{\pi^2 E_{ex}}{2a\delta} + k\delta$

$\therefore \dfrac{dU_{wall}}{d\delta} = -\dfrac{\pi^2 E_{ex}}{2a\delta^2} + k = 0 \qquad \therefore \delta' = \left(\dfrac{\pi^2 E_{ex}}{2ak}\right)^{1/2}$

$U_{wall} = \dfrac{\pi^2 E_{ex}}{2a\delta} + k\delta = \left(\dfrac{\pi^2 E_{ex}}{2a}\right)\left(\dfrac{2ak}{\pi^2 E_{ex}}\right)^{1/2} + k\left(\dfrac{\pi^2 E_{ex}}{2ak}\right)^{1/2}$

$= \left(\dfrac{\pi^2 E_{ex} k}{2a}\right)^{1/2} + \left(\dfrac{\pi^2 E_{ex} k}{2a}\right)^{1/2}$

$\therefore$ when $\delta = \delta'$, $\dfrac{\pi^2 E_{ex}}{2a\delta} = k\delta$

Question 3:
Estimate the potential energy and wall thickness of a domain wall for Ni. The properties of Ni are given in Table 8.4 from lecture notes.

Q3. For Ni: $a = 0.3\,nm$, $E_{ex} = 50\,meV$, $K = 5\,mJ\,cm^{-3}$

$$\delta' = \left(\frac{\pi^2 E_{ex}}{2aK}\right)^{1/2} = \left[\frac{\pi^2 (50\times10^{-3})(1.6\times10^{-19})}{2(0.3\times10^{-9})(5\times10^{-3}\times10^{6})}\right]^{1/2}$$

$$= 1.62\times10^{-7}\,m = 162\,nm$$

$$U_{wall} = \frac{\pi^2 E_{ex}}{2a\delta} + K\delta = \frac{\pi^2 (50\times10^{-3})(1.6\times10^{-19})}{2(0.3\times10^{-9})(1.62\times10^{-7})} + (5\times10^{-3}\times10^{6})\cdot(1.62\times10^{-7})$$

$$= 1.62\times10^{-3}\,J/m^2$$