Chapter 8 Optical Properties

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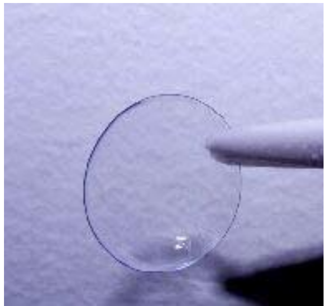


Light waves in a homogeneous medium

Light exhibits typical wave-like properties such as interference and diffraction. We can treat light as an electromagnetic (EM) wave (电磁波) with time-varying electric and magnetic fields $\mathbf{E_x}$ and $\mathbf{B_y}$, respectively, which propagate through space in such a way that they are always perpendicular to each other and the direction of propagation **z**.

The simplest mathematical form of a monochromatic (单色) plane wave:

$$E_x = E_0 \cos(\omega t - kz + \phi_0)$$

E_x: the electric field at position z and time t

E_o: the **amplitude**（振幅） of the wave

ω: the **angular frequency (ω=2πv)**（角频率）

k: the **propagation constant**, **wavevector**（波矢）,

or **wavenumber (k = 2π/λ)** （波数）

(ωt – kz +ϕ_0): is called the phase, ϕ（相位）

Φ_o: a phase constant（相位常数）

Phase velocity （相速度）

The time and space evolution of a given phase φ, is described by

$$\phi = \omega t - kz + \phi_o = \text{constant}$$

During a time interval δt, this constant phase moves a distance δz. The phase velocity of this wave is therefore δz⁄δt.

The **phase velocity** （相速度）v is:

$$v = \frac{dz}{dt} = \frac{\omega}{k} = \upsilon\lambda$$

The above relationship can be obtained by δz/δt from $\phi = \omega t - kz + \phi_0 =$ constant ($\omega\Delta t - k\Delta z = 0$) .

Refractive index（折射率）

EM traveling in a medium: the oscillating **electric field** polarizes the molecules of the medium at the frequency of the wave.

The field and the induced molecular dipoles become coupled. The polarization mechanism delays the propagation of the EM wave.

The stronger the interaction between the field and the dipoles, the slower is the propagation of the wave.

For an EM wave traveling in a non-magnetic dielectric medium of the relative permittivity ε_r（介电常数）, the phase velocity is given by:

$$V = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}}$$

ε_0 = 8.8542x10^{-12} CV^{-1}M^{-1} 真空介电常数

μ_0 = 4πx10^{-7} Hm^{-1} 真空磁导率

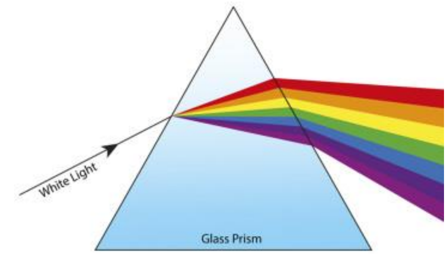
optical frequency range

For an EM wave traveling in free space, ε_r = 1 and $V_{vacuum} = 1/\sqrt{(\varepsilon_0\mu_0)}$ = c = 3x10^8 m/s

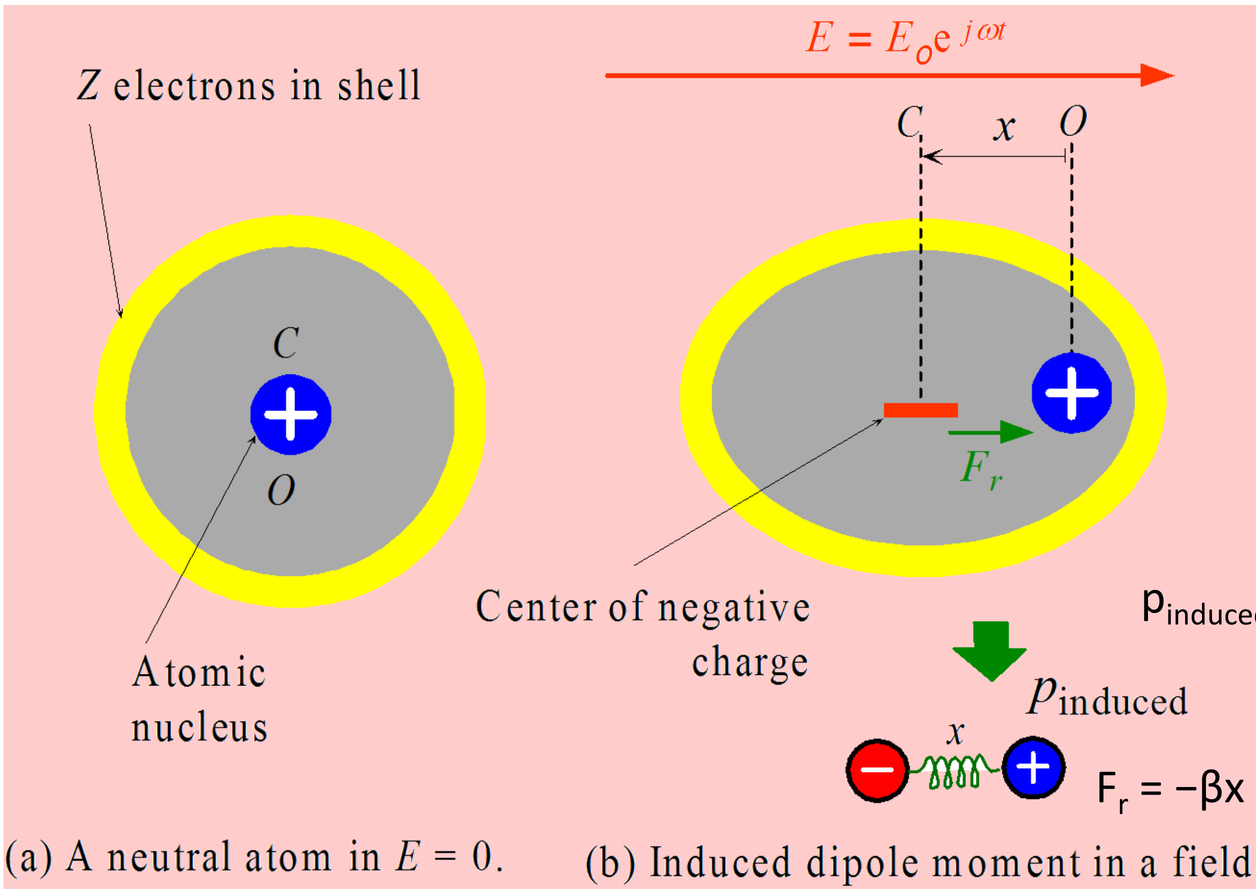
$$V = \frac{c}{\sqrt{\varepsilon_r}}$$

$$\boldsymbol{n} = \frac{c}{v} = \sqrt{\boldsymbol{\varepsilon_r}}$$

Dispersion（色散）: refractive index – wavelength behavior



The refractive index in general depends on the frequency, or the wavelength.



Electronic polarization of an atom

$p_{induced} = (Ze)x$

Dispersion relation（色散关系）

The relationship between n and λ (or ω) is called the **dispersion relation**.

In a solid, a series of **resonance frequencies** may exist, **Sellmeier dispersion** 塞梅尔色散：

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2}$$

Where A_1, A_2, A_3 and λ_1, λ_2 and λ_3 are constant, called Sellmeier coefficients.
塞梅尔系数

There is another well-known useful **n-λ** dispersion by Cauchy (1836):

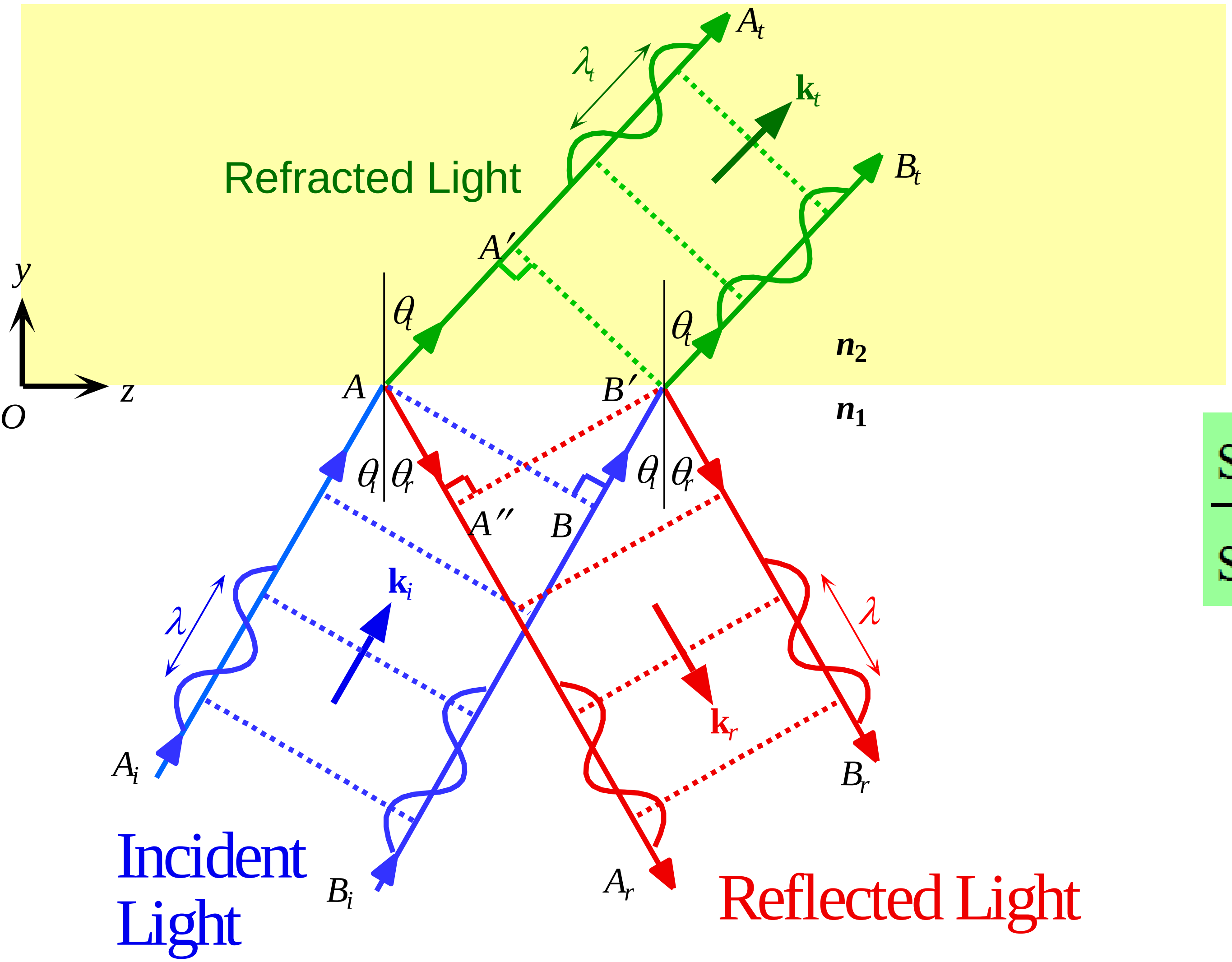
$$n^2 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

Cauchy equation
柯西方程:

$$n = n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4$$

where **hv** is the **photon energy**, and n_0, n_{-2}, n_2, and n_4 are constants

Snell’s law 斯涅尔定律 and total internal reflection (TIR)



$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

A light wave travelling in a medium with a greater refractive index $(n_1 > n_2)$ suffers reflection and refraction at the boundary.

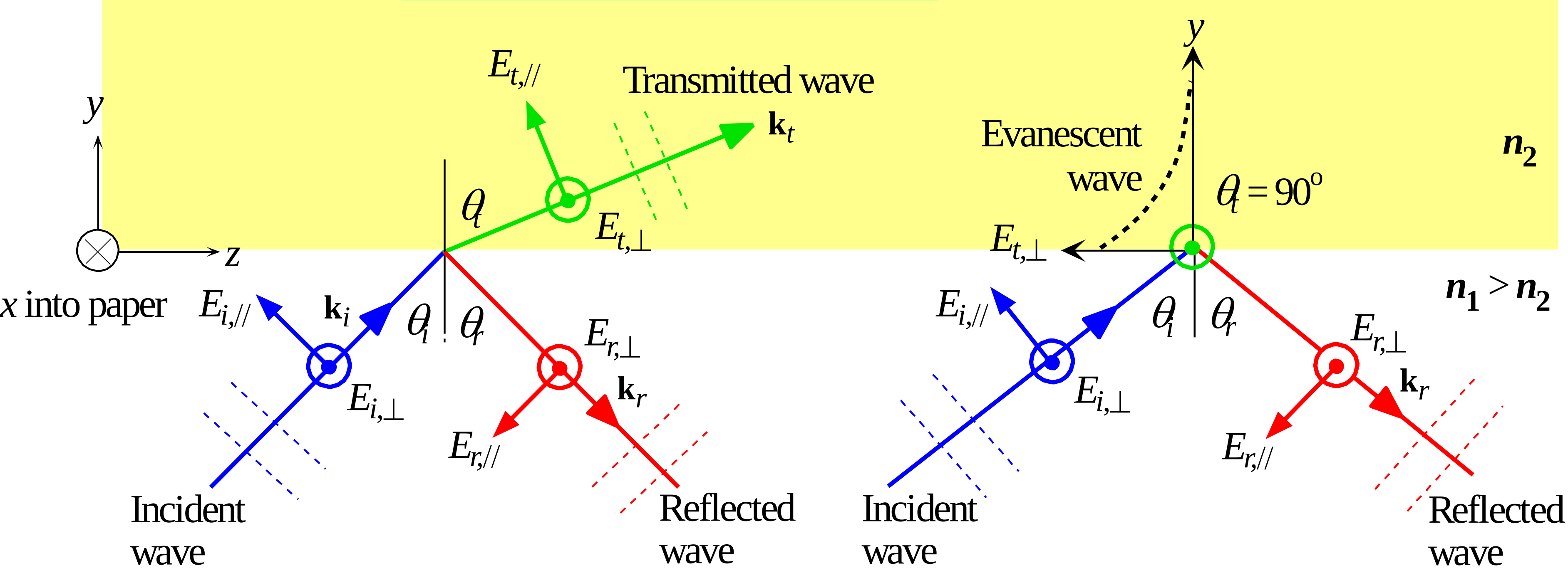
Fresnel's equation（菲涅耳方程）

Incident wave $E_i = \boldsymbol{E}_{i0} \exp j(\omega t - \vec{k}_i \cdot \vec{r}\,)$

Reflected wave $E_r = \boldsymbol{E}_{r0} \exp j(\omega t - \vec{k}_r \cdot \vec{r}\,)$

Transmitted wave $E_t = \boldsymbol{E}_{t0} \exp j(\omega t - \vec{k}_t \cdot \vec{r}\,)$

phase changes ϕ_r and ϕ_t *are* incorporated into the complex amplitudes $\boldsymbol{E}_{ro}$ and $\boldsymbol{E}_{to}$



(a) $\theta_i < \theta_c$ some of the wave transmitted into the less dense medium, some of the wave is reflected.

(b) $\theta_i > \theta_c$ the incident wave suffers total internal reflection. There is a decaying evanescent wave into medium 2

Amplitude reflection and transmission coefficients

Fresnel's equations (obtained from the boundary conditions): If we define **n = n$_2$/n$_1$**, then for $E_\perp$:

reflection coefficient
反射系数

$$r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos\theta_i - (n^2 - \sin^2\theta_i)^{1/2}}{\cos\theta_i + (n^2 - \sin^2\theta_i)^{1/2}}$$

transmission coefficient
折射系数

$$t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2\cos\theta_i}{\cos\theta_i + (n^2 - \sin^2\theta_i)^{1/2}}$$

for E_{II}:

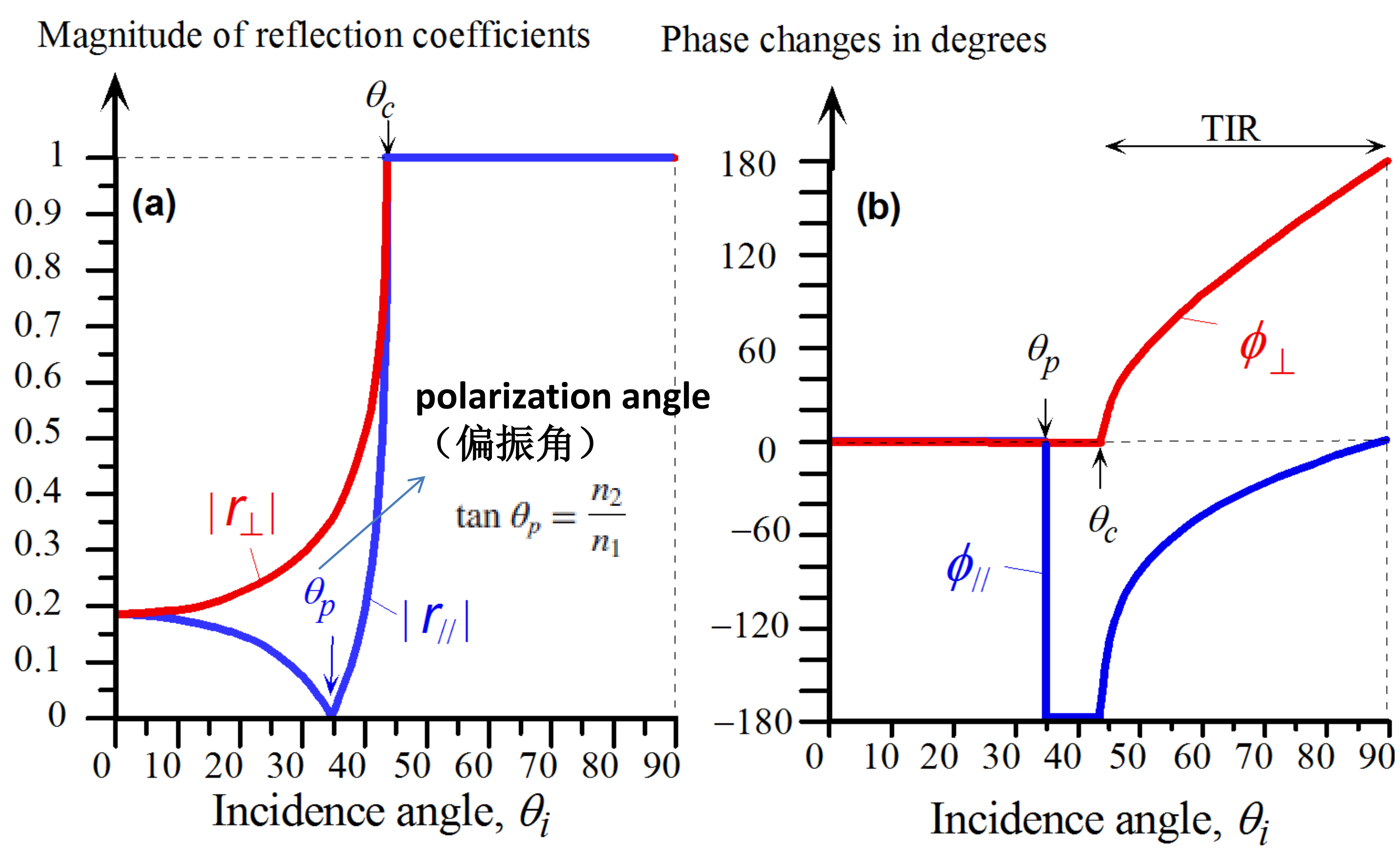
reflection coefficient

$$r_{II} = \frac{E_{r0,II}}{E_{i0,II}} = \frac{(n^2 - \sin^2\theta_i)^{1/2} - n^2\cos\theta_i}{(n^2 - \sin^2\theta_i)^{1/2} + n^2\cos\theta_i}$$

transmission coefficient

$$t_{II} = \frac{E_{t0,II}}{E_{i0,II}} = \frac{2n\cos\theta_i}{(n^2 - \sin^2\theta_i)^{1/2} + n^2\cos\theta_i}$$

Fresnel's equations allows the **amplitudes** and **phases** of the reflected and transmitted waves to be determined from the coefficients r⊥, r‖, t‖, t⊥.



Internal reflection: (a) Magnitude of the reflection coefficients $r_{//}$ and $r_\perp$ vs. angle of incidence θ_i for $n_1 = 1.44$ and $n_2 = 1.00$. The critical angle is 44^o. (b) The corresponding phase changes $\phi_{//}$ and $\phi_\perp$ vs. incidence angle.

Intensity, reflectance, and transmittance

For a normal incidence ($\theta_i = 0$):

$$r_{\parallel} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$R = R_{\perp} = R_{II} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Where R is the Reflectance（反射率）

$$T_{\perp} = \frac{n_2 |E_{to,\perp}|^2}{n_1 |E_{io,\perp}|^2} = \left(\frac{n_2}{n_1}\right) |t_{\perp}|^2$$

$$T_{\parallel} = \frac{n_2 |E_{to,\parallel}|^2}{n_1 |E_{io,\parallel}|^2} = \left(\frac{n_2}{n_1}\right) |t_{\parallel}|^2$$

$$T = T_{\perp} = T_{II} = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

Where T is the Transmittance（透光率）

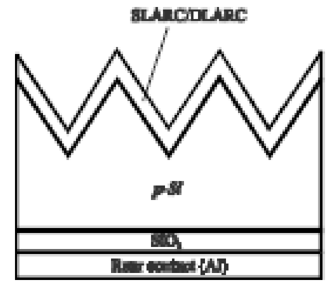
Further:

$$R + T = 1$$

Which one is crystal (SiO_2) with n = 1.46?
Which one is diamond with n = 2.41?

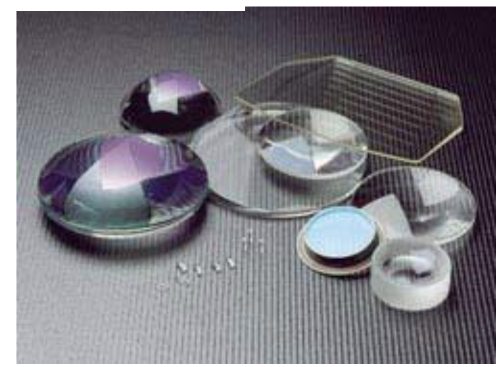
Example (antireflection coating 增透膜)

To reduce the reflected light, A and B must interfere destructively（相消干涉）. This requires the phase difference to be π or odd multiples of π, **mπ** with m = 1, 3, 5,…



from $$E = E_0 \cos(\omega t - kx)$$

$$k_c(2d) = \left(\frac{2\pi n_2}{\lambda}\right)(2d) = m\pi$$ or $$d = m\left(\frac{\lambda}{4n_2}\right)$$



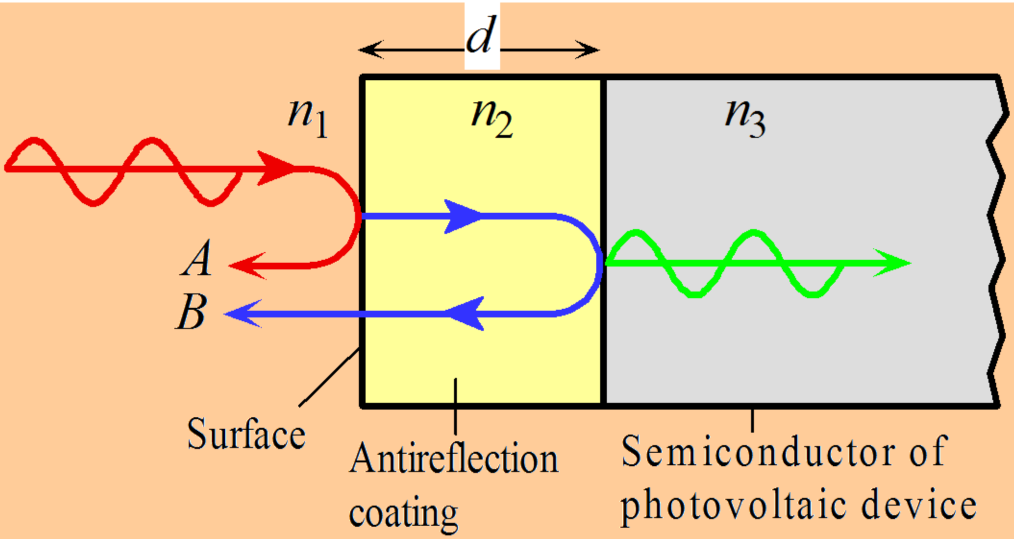
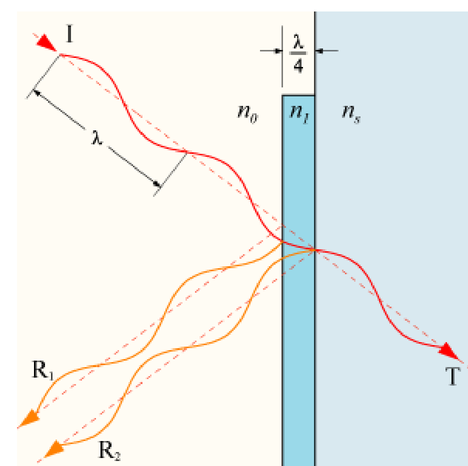


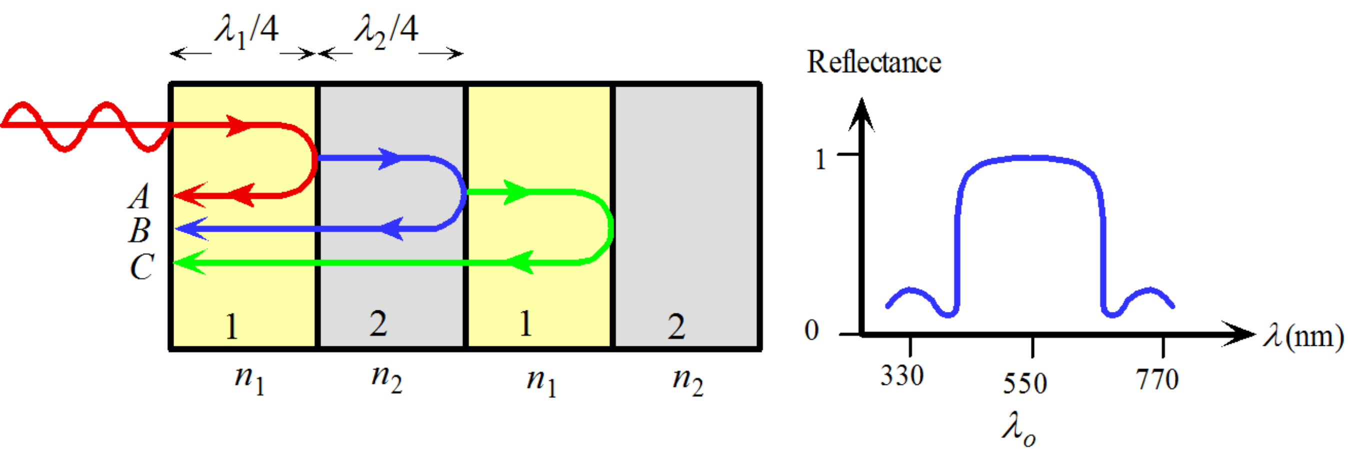
Illustration of how an antireflection coating reduces the reflected light intensity



Example (dielectric mirrors 介质镜)

The reflected intensity is enhanced, if waves A and B are in phase and interfere constructively（相长干涉）.

If there are sufficient number of layers, the reflectance can approach unity at the wavelength λ_0. ($\lambda_1 = \lambda_0/n_1$ & $\lambda_2 = \lambda_0/n_2$)

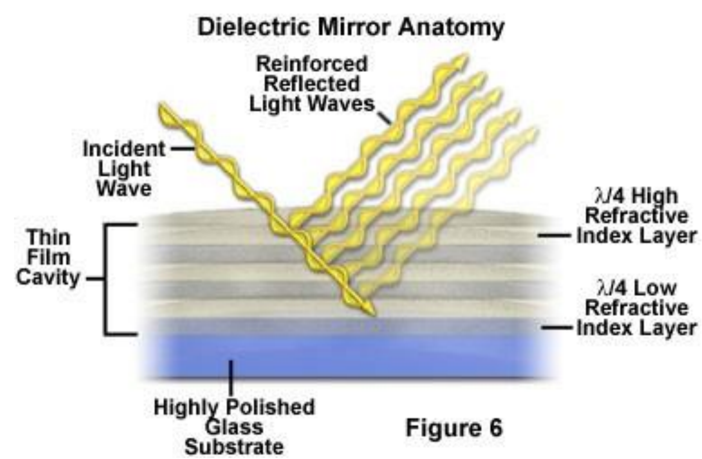


Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers and its reflectance.

quarter-wave dielectric stack ***(n1 > n2)***

r12 = ($n1 - n2$)/($n1 + n2$) *positive*: no phase change

r21 = ($n2 - n1$)/($n2 + n1$) *negative*: π phase change

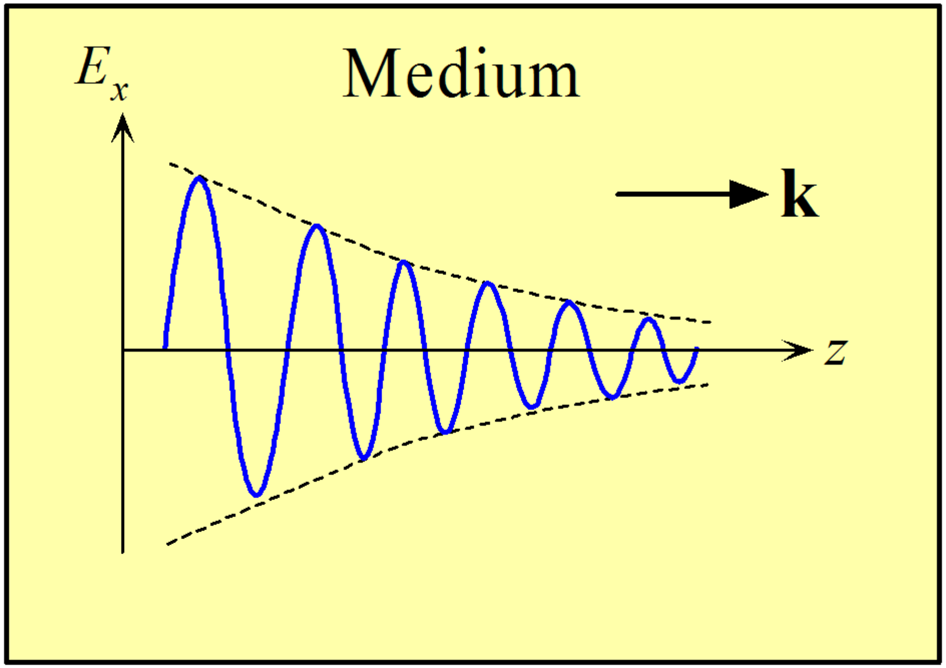


Complex refractive index and light absorption

When light propagates through a material, it becomes attenuated, due to absorption and scattering.

Absorption（吸收）: the loss of light energy to other forms of energy (lattice vibration, electron excitation etc)

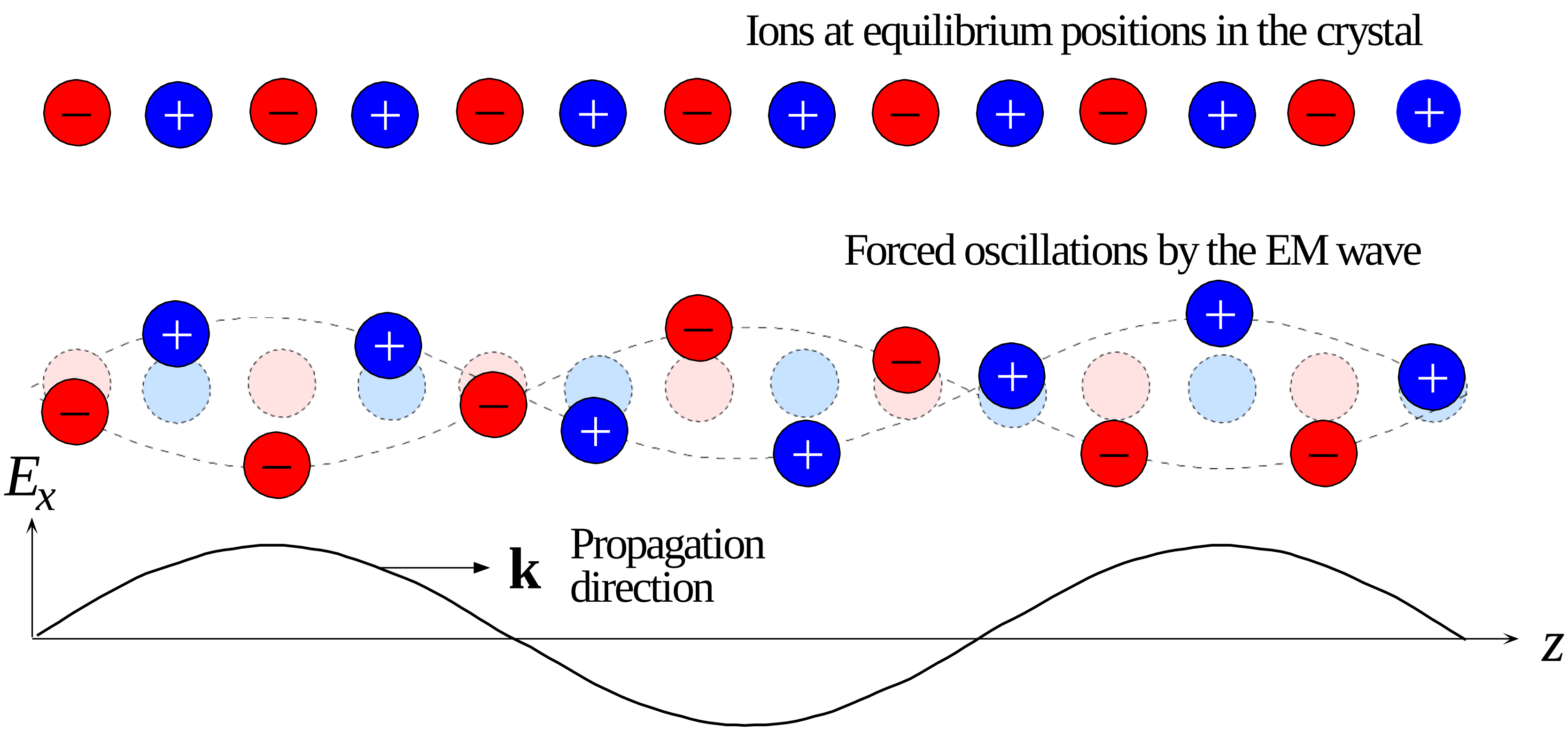
Scattering（ 散射 ）: Redirection as secondary EM waves.



Attenuation of light in the direction of propagation.

Light absorption and scattering

Absorption through lattice vibration



Lattice absorption through a crystal. The field in the EM wave oscillates the ions, which consequently generate "mechanical" waves in the crystal; energy is thereby transferred from the wave to lattice vibrations.

Band-to-band absorption

From $E_g = h(c/\lambda_g)$, the upper cut-off-wavelength:

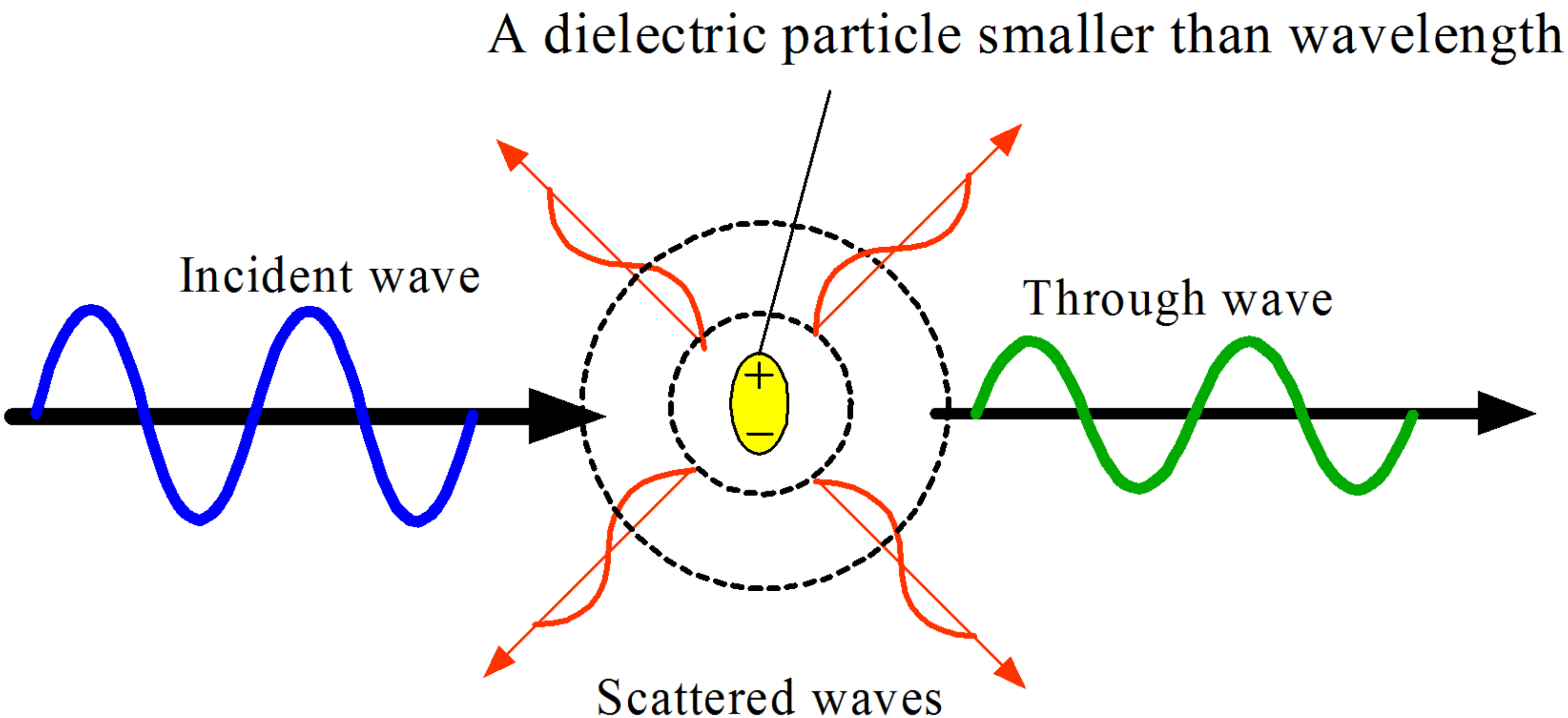
$$\lambda_g(\mu m) = \frac{1.24}{E_g(eV)}$$

Table 9.3 Bandgap energy E_g at 300 K, cut-off wavelength λ_g, and type of bandgap (D = direct and I = indirect) for some photodetector materials

Semiconductor	E_g (eV)	λ_g (µm)	Type
InP	1.35	0.91	D
$GaAs_{0.88}Sb_{0.12}$	1.15	1.08	D
Si	1.12	1.11	I
$In_{0.7}Ga_{0.3}As_{0.64}P_{0.36}$	0.89	1.4	D
$In_{0.53}Ga_{0.47}As$	0.75	1.65	D
Ge	0.66	1.87	I
InAs	0.35	3.5	D
InSb	0.18	7	D

Incident photons with wavelengths shorter than λg become absorbed

Light scattering in materials



Rayleigh scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.

Rayleigh scattering process decreases with wavelength and, according to Lord Rayleigh, it is inversely proportional to λ^4.

Luminescence, phosphors, and white LED’s

Luminescence 冷光 is the emission of light by a material, called **phosphor** 荧光体, due to the absorption and conversion of energy into EM radiation.

Luminescence is light emitted by a non-thermal source when it is excited (Note: the emission of radiation from a heated object such as the tungsten filament is called **incandescence** 白炽光).

Typically, the emission of light occurs from certain dopants, impurities, or defects, called **luminescent** or **luminescence center** (**an activator**). For example, in ruby, the Cr^{3+} ions are the luminescent centers in the sapphire (Al_2O_3) **crystal host**.

<u>Luminescence is normally categorized according to the source of excitation energy:</u>

-**Photoluminescence** 光致发光 (photons)

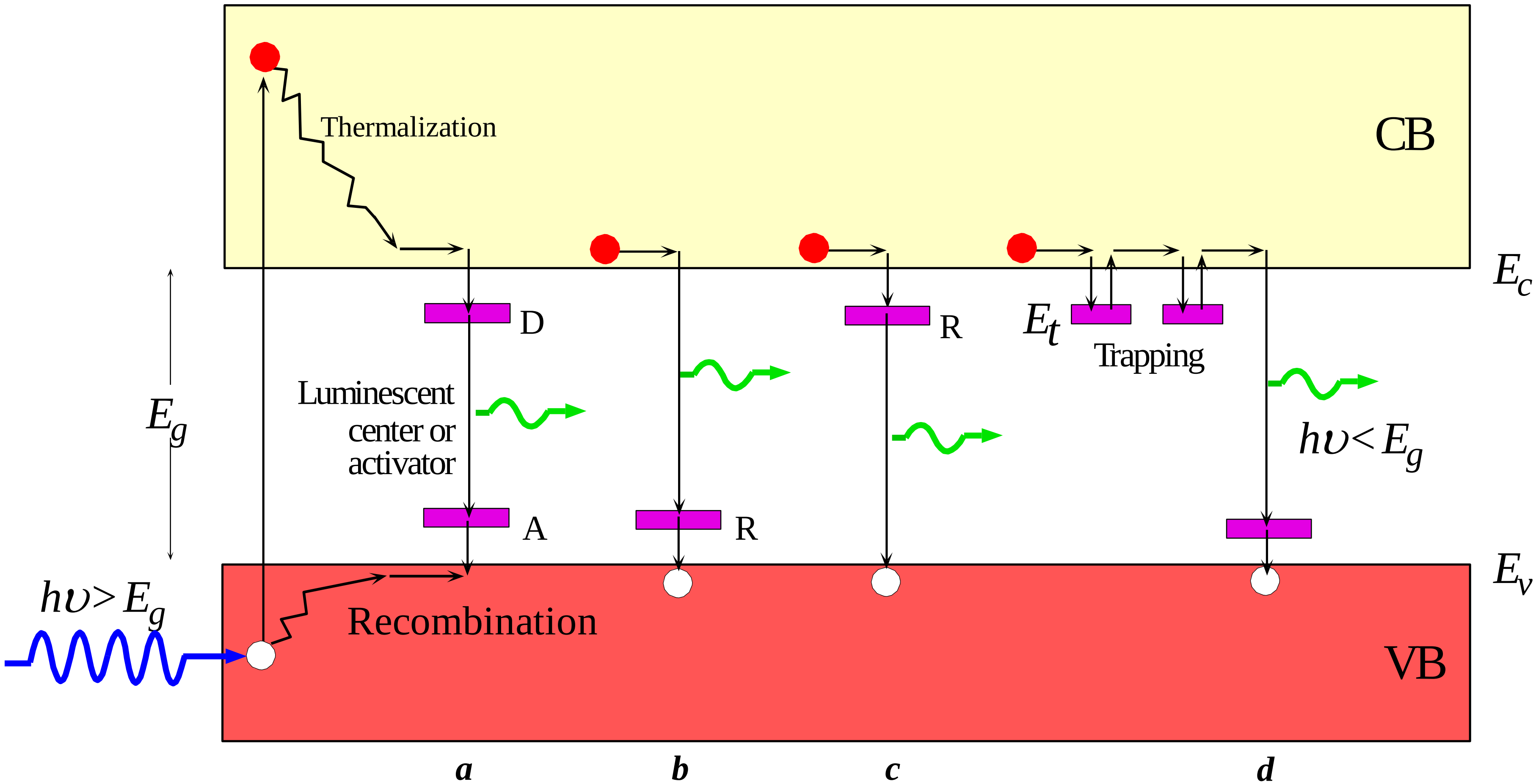
-**Cathodoluminescence** 阴极射线致发光 (energetic electrons)

-**Electroluminescence** 电致发光 (an electric current)

- **Fluorescence** 荧光**:** quick luminenscence process (in the range of nanoseconds).

- **Phosphorescence** 磷光**:** light emission may continue for milliseconds to hours.

Example (ZnS-based phosphors)



Optical absorption generates an EHP. There are a number of recombination processes via a dopant that can result in a luminescent emission.

Assignment 8.1

Question 1:
Using the Cauchy dispersion, calculate the refractive index of diamond at 450 nm. (Cauchy coefficients from Table 9.2)

Q1. Cauchy dispersion : $n = n_{-2}(h\nu)^{-2} + n_0 + n_2(h\nu)^2 + n_4(h\nu)^4$

$$\lambda = 450\,\text{nm} \Rightarrow h\nu = h\frac{c}{\lambda} = (6.626\times10^{-34})\left(\frac{3\times10^8}{450\times10^{-9}}\right)$$

$$= 4.417\times10^{-19}\,\text{J} \times \frac{1\,\text{eV}}{1.6\times10^{-19}\,\text{J}} = 2.76\,\text{eV}$$

$$n = (-1.07\times10^{-5})(2.76)^{-2} + (2.378) + (8.01\times10^{-3})(2.76)^2$$
$$+ (1.04\times10^{-4})(2.76)^4 = 2.45$$

Table 9.2 Sellmeier and Cauchy coefficients

	Sellmeier					
	A_1	A_2	A_3	λ_1 (μm)	λ_2 (μm)	λ_3 (μm)
SiO_2 (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5% SiO_2–13.5% GeO_2	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO_2	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	—	0.1750	0.1060	

	Cauchy				
	Range of $h\nu$ (eV)	n_{-2} (eV^2)	n_0	n_2 (eV^{-2})	n_{-4} (eV^{-4})
Diamond	0.05–5.47	-1.07×10^{-5}	2.378	8.01×10^{-3}	1.04×10^{-4}
Silicon	0.002–1.08	-2.04×10^{-8}	3.4189	8.15×10^{-2}	1.25×10^{-2}
Germanium	0.002–0.75	-1.0×10^{-8}	4.003	2.2×10^{-1}	1.4×10^{-1}

Question 2:

Using n = $\sqrt{\varepsilon_r}$, calculate the refractive index n of the crystals in the table given their low frequency permittivities ε_r (LF). What is your conclusion compared to the measured n values?

	Crystal			
	a-Se	Ge	NaCl	MgO
ε_r (LF)	6.4	16.2	5.90	9.83
n (~1-5 μm)	2.45	4.0	1.54	1.71

Q2.

	$\sqrt{\varepsilon_r(LF)}$	n (~1-5 μm)	
a-Se	2.53	2.45	} electronic polarization
Ge	4.02	4.0	
NaCl	2.43	1.54	} ionic polarization, only contribute to $\sqrt{\varepsilon_r(LF)}$
MgO	3.74	1.71	

For a-Se, Ge, good agreement. Both are covalent solids ⇒ only electronic bond polarization at low and high frequencies. Electronic polarization involes the displacement of electrons with respect to positive ion. This displacement can readily respond to optical frequencies.

For NaCl, MgO, $\sqrt{\varepsilon_r(LF)}$ is larger than n ⇒ at low frequencies, both solids possess ionic polarization and contribute to ε_r. The ionic polarization can't catch up the field oscillation at optical frequencies.

Question 3:

Optical fibers for long-haul applications usually have a core region that has a diameter of about 10 μm and the whole fiber would be about 125 μm in diameter. The core and cladding refractive indices, n_1 and n_2, respectively, are normally only 0.3-0.4 percent different. Consider a fiber with n_1 (core) = 1.4510 and n_2 (cladding) = 1.4477, both at 1550 nm. What is the maximum angle that a light ray can take with the fiber axis if it is still to propagate along the fiber?

Q3. n_1(core) = 1.4510, n_2(cladding) = 1.4477

$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.4477}{1.4510} = 0.9977$. $\theta_c = 86.13°$

During bending, maximum angle can take, $\theta_{max} = 90 - 86.13 = 3.87°$

Assignment 8.2

Question 1:

Consider the reflection of light at normal incidence on a boundary between GaAs crystal of refractive index 3.6 and air of refractive index 1.

a) If the light is traveling from air to GaAs, what is the reflection coefficient and the intensity of the reflected light in terms of the incident light? Comment on the phase change.

b) If the light is traveling from GaAs to air, what is the reflection coefficient and the intensity of the reflected light in terms of the incident light? Comment on the phase change.

Q1. a) normal incidence : $\theta_i = 0$. air → GaAs. $n_1 = 1$, $n_2 = 3.6$

$$r_{\parallel} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - 3.6}{1 + 3.6} = -0.565$$

r negative, 180° phase change

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = (-0.565)^2 = 31.9\%$$

b) GaAs → air. $n_1 = 3.6$, $n_2 = 1$

$$r_{\parallel} = r_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} = 0.565$$

r positive, no phase change

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = 31.9\%$$

Question 2:

a) Consider three dielectric media with flat and parallel boundaries with refractive indices n_1, n_2 and n_3. Show that for normal incidence the reflection coefficient between 1 and 2 is the same as that between layers 2 and 3 if $n_2 = \sqrt{(n_1 n_3)}$.

b) Consider a Si photodiode that is designed at 900 nm. Given a choice of two possible antireflection coatings, SiO_2 with a refractive index of 1.5 and TiO_2 with a refractive index of 2.3, which would you use and what would be the thickness of the antireflection coating you chose? The refractive index of Si is 3.5.

Q2. normal incidence $\Rightarrow \theta_i = 0$

a)
$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2} = \frac{n_1 - \sqrt{n_1 n_3}}{n_1 + \sqrt{n_1 n_3}} = \frac{1 - \sqrt{\frac{n_3}{n_1}}}{1 + \sqrt{\frac{n_3}{n_1}}}$$

$$r_{23} = \frac{n_2 - n_3}{n_2 + n_3} = \frac{\sqrt{n_1 n_3} - n_3}{\sqrt{n_1 n_3} + n_3} = \frac{1 - \sqrt{\frac{n_3}{n_1}}}{1 + \sqrt{\frac{n_3}{n_1}}}$$

$\therefore r_{12} = r_{23}$

b) For antireflection coating: refection wave A and B $\Rightarrow$ interfere destructively and have comparable amplitude.

$\Rightarrow r_{12} = r_{23}$

$\Rightarrow$ best coating: $n_2 = \sqrt{n_1 n_3} = \sqrt{1 \times 3.5} = 1.87$

Both SiO_2 (1.5) and TiO_2 (2.3) are close

$$d = m\left(\frac{\lambda}{4n_2}\right) \qquad m = 1, 3, 5 \cdots$$

For SiO_2: $d = \dfrac{900\,nm}{4 \times 1.5} = 150nm$

TiO_2: $d = \dfrac{900\,nm}{4 \times 2.3} = 97.8nm$

2020年暑期学校海外师资课程

先进材料与器件（Compound Semiconductor Materials and Devices）

学时：16，学分：1，授课人：新加坡国立大学**蔡树仁教授**

授课日期：2020年7月10日-7月17日，14：00-15：50

授课形式：线上教学，授课对象：信息学院本科生

选课日期：正选时间2020年6月4日9:00-6月8日16:00

课程介绍：先进化合物半导体主要用于光的激发、调制和探测，如InGaAsP用于光通讯、GaAs用于安保探测、InGaN和AlInGaP用于显示和照明等。被广泛应用于半导体激光器、发光二极管（LED）、高电子迁移率晶体管（HEMT）、异质结双极晶体管（HBT）等。本课程兼顾理论与应用，讲授先进化合物半导体的基础原理、器件机制、以及在光电子产业中的应用。课程由新加坡国立大学（NUS）电子工程系（ECE）教授、前新加坡科技局材料研究与工程研究院院长，蔡树仁教授主讲。