Assignment 6.3

Question 1:

Sketch the energy diagrams of a *pn* junction, indicating the Fermi energy (E_F), the bottom of the conduction band (E_C), the top of the valence band (E_V), built-in potential (V_o), and the direction of the internal field.

Question 2:

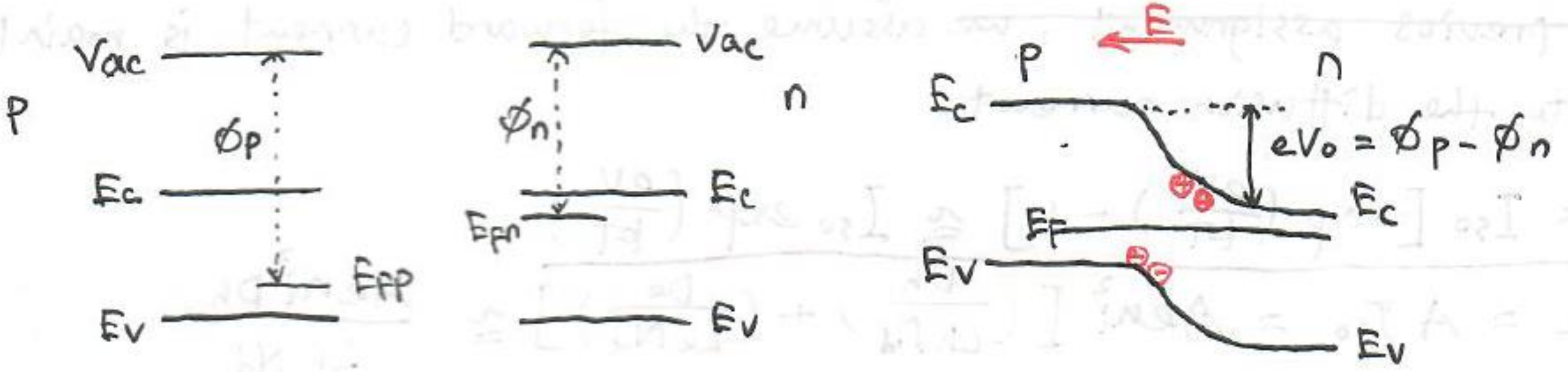
In the lecture, we used Boltzmann statistics to derive the built-in potential, V_o, of a *pn* junction. The energy band treatment allows a simple way to calculate V_o. When the junction is formed, E_{Fp} and E_{Fn} must shift and line up. The shift in E_{Fp} and E_{Fn} to line up is clearly $\Phi_p - \Phi_n$, the work function difference.

Using the energy band diagrams and semiconductor equations, derive an expression for the built-in potential V_o in terms of N_d, N_a, and n_i.

Question 3:

Consider a p^+n junction, which has a heavily doped p-side relative to the n-side, that is, $N_a \gg N_d$. What is your comment on the depletion width on the n-side and the p-side? What is the total depletion width (W_0) for the p^+n junction Si diode that has been doped with 10^{18} acceptor atoms cm^{-3} on the p-side and 10^{16} donor atoms cm^{-3} on the n-side?

$\because$ The shift in E_{Fp} and E_{Fn} to line up is $\phi_p - \phi_n$.



$$\therefore \quad eV_0 = \phi_p - \phi_n = (E_c - E_{Fp}) - (E_c - E_{Fn})$$

$$\because \quad n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right) \Rightarrow n_{no} = N_c \exp\left(-\frac{E_c - E_{Fn}}{kT}\right)$$

$$\Rightarrow n_{po} = N_c \exp\left(-\frac{E_c - E_{Fp}}{kT}\right)$$

$$\Rightarrow \ln\left(\frac{n_{no}}{n_{po}}\right) = \frac{1}{kT}\cdot\left[(E_c - E_{Fp}) - (E_c - E_{Fn})\right] = \frac{eV_0}{kT}$$

$$\because \quad n_{po} = n_i^2 / N_a, \quad n_{no} = N_d$$

$$\therefore \quad V_0 = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right)$$

THE p^+n JUNCTION Consider a p^+n junction, which has a heavily doped p-side relative to the n-side, that is, $N_a \gg N_d$. Since the amount of charge Q on both sides of the metallurgical junction must be the same (so that the junction is overall neutral)

$$Q = eN_aW_p = eN_dW_n$$

it is clear that the depletion region essentially extends into the n-side. According to Equation 6.7, when $N_d \ll N_a$, the width is

$$W_o = \left[\frac{2\varepsilon V_o}{eN_d}\right]^{1/2}$$

What is the depletion width for a pn junction Si diode that has been doped with 10^{18} acceptor atoms cm^{-3} on the p-side and 10^{16} donor atoms cm^{-3} on the n-side?

SOLUTION

To apply the above equation for W_o, we need the built-in potential, which is

$$V_o = \left(\frac{kT}{e}\right)\ln\left(\frac{N_dN_a}{n_i^2}\right) = (0.0259\ \text{V})\ln\left[\frac{(10^{16})(10^{18})}{(1.0\times 10^{10})^2}\right] = 0.835\ \text{V}$$

Then with $N_d = 10^{16}$ cm^{-3}, that is, 10^{22} m^{-3}, $V_o = 0.835$ V, and $\varepsilon_r = 11.9$ in the equation for W_o

$$W_o = \left[\frac{2\varepsilon V_o}{eN_d}\right]^{1/2} = \left[\frac{2(11.9)(8.85\times 10^{-12})(0.835)}{(1.6\times 10^{-19})(10^{22})}\right]^{1/2}$$

$$= 3.32\times 10^{-7}\ \text{m} \qquad \text{or} \qquad 0.33\ \mu\text{m}$$

Nearly all of this region (99 percent of it) is on the n-side.

Assignment 6.3 (2)

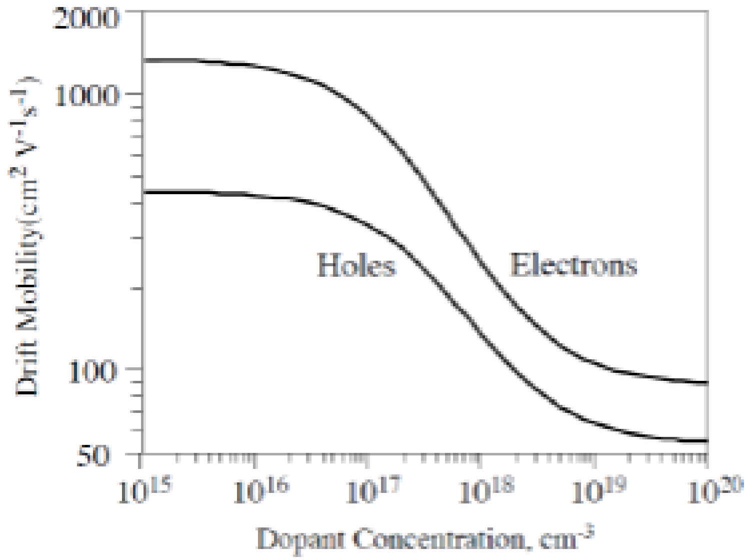


Figure 1. The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K

Question 1:

Consider a Si ($n_i = 1.45\times10^{10}$ cm^3, ε_r is 11.9) *pn* junction diode, with an acceptor concentration N_a of 10^{18} cm^{-3} on the p-side and donor concentration N_d of 10^{15} cm^{-3} on the n-side. The drift mobility refers to Figure 1. The diode is forward biased and has a voltage of 0.6 V across it. The diode cross-sectional area is 1 mm^2. The minority carrier recombination time, τ, depends on the dopant concentration, N_{dopant} (cm^{-3}), through the following approximate relation

$$\tau = \frac{5\times10^{-7}}{\left(1+2\times10^{-17}N_{dopant}\right)}$$

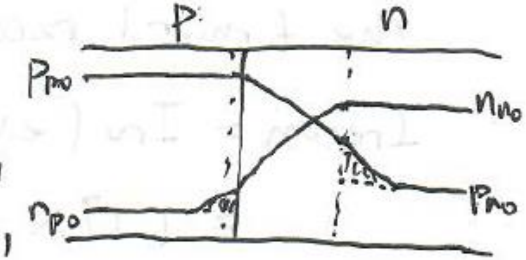
Calculate the diffusion current and the recombination current. What is your conclusion on the contributions to the total diode current?

Q1. This is a p^+n diode: $N_d = 10^{15}\,cm^{-3}$. Hole recombination lifetime τ_h:

$$\tau_h = \frac{5\times 10^{-7}}{(1+2\times10^{-17} N_{dopant})} = \frac{5\times10^{-7}}{1+2\times10^{-17}\times10^{15}\,cm^{-3}} = 490.2\,ns$$

similarly: $\tau_e = 23.81\,ns$

1) Diffusion diode current:



From Figure 1, $N_a = 10^{18}\,cm^{-3} \Rightarrow \mu_e \cong 250\,cm^2V^{-1}s^{-1}$

$N_d = 10^{15}\,cm^{-3} \Rightarrow \mu_h \cong 450\,cm^2V^{-1}s^{-1}$

Einstein Relation: $D_e = \frac{kT}{e}\cdot\mu_e = (0.02586\,V)(250\,cm^2V^{-1}s^{-1}) = 6.465\,cm^2s^{-1}$

$D_h = \frac{kT}{e}\cdot\mu_h = (0.02586\,V)(450\,cm^2V^{-1}s^{-1}) = 11.64\,cm^2s^{-1}$

Thus diffusion length: $L_e = \sqrt{D_e\tau_e} = \sqrt{(6.465\,cm^2s^{-1})(23.81\times10^{-9}\,s)}$

$= 3.923\times10^{-4}\,cm$

$L_h = \sqrt{D_h\tau_h} = \sqrt{(11.64\,cm^2s^{-1})(490.2\times10^{-9})}$

$= 2.389\times10^{-3}\,cm$

Diffusion current : $I_{diff} = A \cdot J_{diff} = A \cdot J_{so} [\exp(eV/kT - 1]$

$\cong A \cdot J_{so} \exp(eV/kT)$ $\quad \because V \gg kT/e = 0.0258\overset{V}{6}$

where $I_{so} = A \cdot J_{so} = Aen_i^2 [D_h/(L_h N_d) + D_e/(L_e N_a)] \cong Aen_i^2 D_h/(L_h N_d)$

$\because N_a \gg N_d$. In other words, the current is mainly due to the diffusion of the holes in n-region.

$$\therefore \quad I_{so} = \frac{(1\times10^{-2}cm^2)(1.602\times10^{-19}C)(1.45\times10^{10}cm^{-3})^2(11.64cm^2s^{-1})}{(2.389\times10^{-3}cm)(1\times10^{15}cm^{-3})}$$

$$= 1.641\times10^{-12}A$$

$\therefore$ Forward current due to diffusion :

$$I_{diff} = I_{so}\exp(eV/kT) = (1.641\times10^{-12}A)\exp(0.6V/0.02586V)$$

$$= 0.0196\ A \quad \text{or} \quad 19.6\ mA$$

2) Recombination current.

Buit-in potential : $V_o = (kT/e)\ln(N_a N_d/n_i^2) = (0.02586V)\ln(10^{18}cm^{-3}\times 10^{15}cm^{-3})/(1.45\times10^{10}cm^{-3})^2] = 0.7549V$

depletion region width W is mainly on the n-side :

$$W = \left[\frac{2\varepsilon(N_a+N_d)(V_o-V)}{eN_aN_d}\right]^{1/2} \cong \left[\frac{2\varepsilon(V_o-V)}{eN_d}\right]^{1/2} (= W_n)$$

$$= \left[\frac{2(11.9)(8.854\times10^{-12}Fm^{-1})(0.7549V-0.6V)}{(1.602\times10^{-19}C)(10^{21}m^{-3})}\right]^{1/2} = 0.4514\times10^{-4}cm$$

$$J_{recom} = J_{ro}\left[\exp\left(\frac{eV}{2kT}\right) - 1\right] \quad \text{where } J_{ro} = \frac{en_i}{2}\left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h}\right)$$

$$I_{ro} = A \cdot \frac{en_i}{2} \cdot W_n/\tau_h = \frac{(1\times10^{-2}\,cm^2)(1.602\times10^{-19}\,C)(0.4514\times10^{-4}\,cm)(1.45\times10^{10}\,cm^{-3})}{2(490\times10^{-9}\,s)}$$

$$= 1.070\times10^{-9}\,A$$

∴ The forward recombination current is:

$$I_{recom} = I_{ro}(eV/2kT) = (1.070\times10^{-9}A)\exp\left[(0.6V)/2(0.02586V)\right]$$

$$= 1.17\times10^{-4}\,A \text{ or } 0.117\,mA$$

Conclusion: the diffusion current dominates the total diode current in this diode.

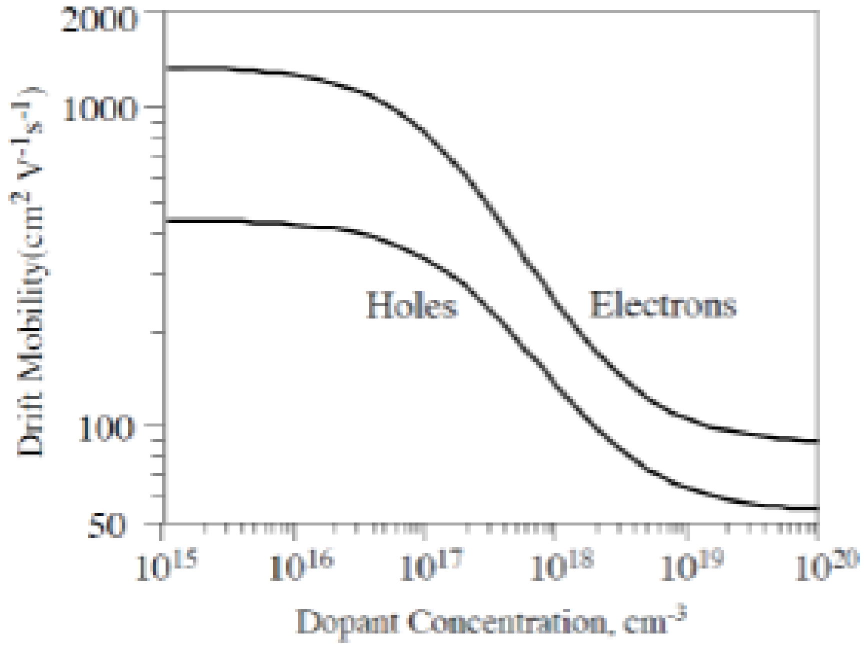


Figure 1. The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K

Question 2:

An Si p^+n junction diode has a cross-sectional area of 1 mm^2, an acceptor concentration of 5×10^{18} cm^{-3} on the p-side, and a donor concentration of 10^{16} cm^{-3} on the n-side. The recombination lifetime of holes in the n-region is 420 ns, whereas that of electrons in the p-region is 5 ns due to a greater concentration of impurities (recombination centers) on that side. Mean thermal generation lifetime (τ_g) is about 1 µs.

(a) Calculate the minority carrier diffusion lengths.

(b) What is the built-in potential across the junction?

(c) What is the current when there is a forward bias of 0.6 V across the diode at 27 °C?

Assume that the current is by minority carrier diffusion.

(d) What is the reverse current when the diode is reverse biased by a voltage $V_r = 5$ V?

a) From Figure 1. $N_a = 5\times10^{18}\,cm^{-3} \Rightarrow \mu_e \approx 150\,cm^2V^{-1}s^{-1}$

$N_d = 10^{16}\,cm^{-3} \Rightarrow \mu_h \approx 430\,cm^2V^{-1}s^{-1}$

Einstein relation: $D_e = \frac{kT}{e}\cdot\mu_e = (0.02586V)(150\,cm^2V^{-1}s^{-1}) = 3.88\,cm^2s^{-1}$

$D_h = \frac{kT}{e}\cdot\mu_h = (0.02586V)(430\,cm^2V^{-1}s^{-1}) = 11.12\,cm^2s^{-1}$

Diffusion lengths: $L_e = \sqrt{D_e\tau_e} = \sqrt{(3.88\,cm^2s^{-1})(5\times10^{-9}s)} = 1.39\times10^{-4}cm$

$L_h = \sqrt{D_h\tau_h} = \sqrt{(11.12cm^2s^{-1})(420\times10^{-9}s)} = 21.6\times10^{-4}cm$

(b) Built-in potential:

$$V_o = \left(\frac{kT}{e}\right)\ln\left(\frac{N_dN_a}{n_i^2}\right) = (0.02586V)\ln\left[\frac{10^{16}\times5\times10^{18}}{(1.45\times10^{10})^2}\right] = 0.875V$$

(c) ~~From previos assignment~~, we assume the forward current is mainly due to the diffusion current.

$$I = I_{s0}\left[\exp\left(\frac{eV}{kT}\right) - 1\right] \cong I_{s0}\exp\left(\frac{eV}{kT}\right)$$

where $I_{s0} = A J_{s0} = Aen_i^2\left[\left(\frac{D_h}{L_h N_d}\right) + \left(\frac{D_e}{L_e N_a}\right)\right] \cong \frac{Aen_i^2 D_h}{L_h N_d}$

as $N_a >> N_d$, the diffusion current mainly due to holes diffusing in n-region.

$$I_{s0} = \frac{(0.01\,cm^2)(1.602\times10^{-19}C)(1.45\times10^{10}\,cm^{-3})^2(11.12\,cm^2 s^{-1})}{(21.6\times10^{-4}\,cm)(10^{16}\,cm^{-3})}$$

$$= 8.24\times10^{-14} A$$

$$I \cong I_{s0}\exp\left(\frac{eV}{kT}\right) = (8.24\times10^{-14} A)\exp\left(\frac{0.6V}{0.02586V}\right)$$

$$= 0.99\times10^{-3} A = 1.0\,mA$$

(d) Reverse saturation current: $I = I_{s0} = 8.24\times10^{-14} A$

Thermal generation current:

$$I_{gen} = A\cdot J_{gen} = A\cdot\frac{eWn_i}{\tau_g}$$

$$W = \left[\frac{2\varepsilon(V_0+V_r)}{eN_d}\right]^{1/2} = \left[\frac{2(11.9)(8.85\times10^{-12})(0.875+5)}{(1.6\times10^{-19})(10^{22})}\right]^{1/2}$$

$$= 0.88\times10^{-4}\,cm$$

$$\therefore I_{gen} = (0.01\,cm^2)\frac{(1.602\times10^{-19}c)(0.88\times10^{-4}\,cm)(1.45\times10^{10}\,cm^{-3})}{(10^{-6}s)}$$

$$= 1.41\times10^{-9} A$$

$I_{gen} >> I_{s0}$ $\Rightarrow$ The reverse current is dominated by thermal generation current.