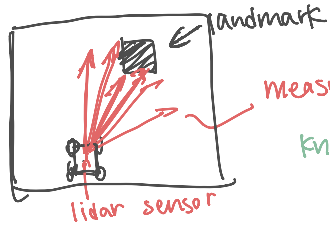
Bayes filter / particle filtering for robotic localization.

↳ localizing from a known map

state $x_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix}$



measurement y_k → $y_k = \begin{bmatrix} \ell_k \\ \alpha_k \end{bmatrix}$ ← range, ← angle

known map ↓ robot's localization

$v_k = \begin{bmatrix} u \\ \omega \end{bmatrix}$

Bayes filter

problem setup:

motion model $x_k = f(\underbrace{x_{k-1}}_{\text{state}}, \underbrace{v_k}_{\text{velocity}}, \underbrace{w_k}_{\text{system noise}})$ $k = 1, \dots K$

observation model $y_k = g(x_k, \underbrace{n_k}_{\text{sensor noise}})$, $k = 1, \dots K$

find x_k belief function, initial pos, action series, observation, Bayes Rule

$$p(x_k \mid \check{x}_0, v_{1:k}, y_{0:k}) = \underbrace{\eta}_{\text{scaling factor}} \underbrace{p(y_k \mid x_k)}_{\text{likelihood}} \underbrace{p(x_k \mid \check{x}_0, v_{1:k}, y_{0:k-1})}_{\text{prior}} \quad ①$$

Bayes Rule

$P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

$P(A) = P(x_k \mid \check{x}_0, v_{1:k}, y_{0:k-1})$

$P(B) = P(y_k)$

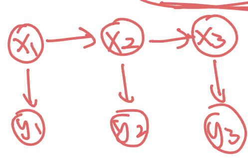
$P(B|A) = P(y_k|x_k)$

$\eta = P(y_k)$

new variable: x_{k-1}

$$p(x_k \mid \check{x}_0, v_{1:k}, y_{0:k-1}) = \int p(x_k, x_{k-1} \mid \check{x}_0, v_{1:k}, y_{0:k-1})\, dx_{k-1}$$

Markov property

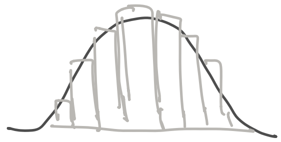


$$= \int \underbrace{p(x_k \mid x_{k-1}, \cancel{\check{x}_0}, \underbrace{\cancel{v_{1:k}}}_{v_k}, \cancel{y_{0:k-1}})}_{p(x_k \mid x_{k-1}, v_k)\ ②} \underbrace{p(x_{k-1} \mid \check{x}_0, \underbrace{v_{1:k}}_{v_{1:k-1}}, y_{0:k-1})}_{③\ p(x_{k-1} \mid \check{x}_0, v_{1:k-1}, y_{0:k-1})}$$

Sub ② & ③ into ①

$$\underbrace{p(x_k \mid \check{x}_0, v_{1:k}, y_{0:k})}_{\text{posterior}} = \eta\, p(y_k \mid x_k) \int p(x_k \mid x_{k-1}, v_k)\, p(x_{k-1} \mid \check{x}_0, v_{1:k}, y_{0:k-1})\, dx_{k-1}$$

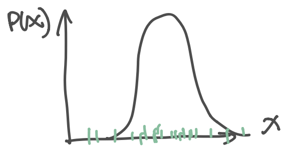
limitation: intractable



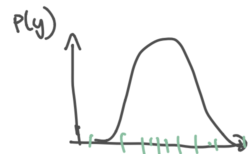
histogram

particle

Monte Carlo Sampling



random samples from input density $\longrightarrow$ $\boxed{y_i = g(x_i)}$ $\longrightarrow$



combine samples to form output density

prior

$$\begin{bmatrix}\hat{x}_{k-1,m}\\ w_{k,m}\end{bmatrix} \xleftarrow{\text{M samples}} p(x_{k-1}|\check{x}_0, v_{1:k-1}, y_{1:k-1})\, p(w_k)$$

particle ind

Prediction step

$$\check{x}_{k,m} = f(\hat{x}_{k-1,m}, v_k, w_{k,m})$$

$\Downarrow$

approximate PDF

correction step

desired

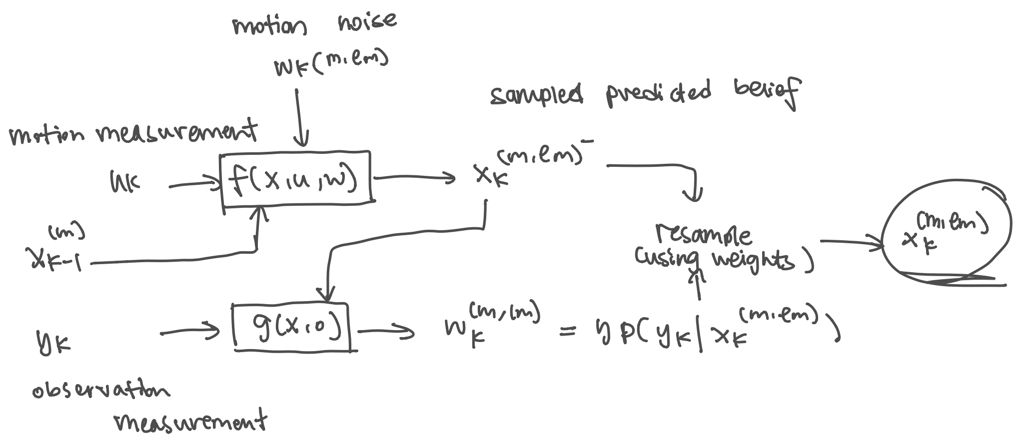
$$w_{k,m} = \frac{p(\check{x}_{k,m}|\check{x}_0, v_{1:k}, y_{1:k})}{p(\check{x}_{k,m}|\check{x}_0, v_{1:k}, y_{1:k-1})} = \eta\, p(y_k|\check{x}_{k,m})$$

predicted

$$\check{y}_{k,m} = g(\check{x}_{k,m}, 0)$$

assume: $p(y_k|\check{x}_{k,m}) = \underbrace{p(y_k|\check{y}_{k,m})}_{\text{gaussian}}$

$$\hat{x}_{k,m} \Longleftarrow \{\check{x}_{k,m}, w_{k,m}\}$$


motion noise
wk(m,em)
sampled predicted belief
motion measurement
uk
f(x,u,w)
xk(m,em)-
xk-1(m)
resample
(using weights)
xk(m,em)
yk
g(x,o)
wk(m,(m)) = y p(yk | xk(m,em))
observation
measurement