Advanced Microeconomics
Assignment 3

Due date: November 11, 2019 (before class)
Submission method: Please submit your assignment to me in class, or via E-mail: sherryecon@qq.com.

- 纸质版：要求字迹工整，可辨认。
- 电子版：附件要求 .pdf格式。邮件标题格式为“作业编号-学号-姓名”，如：作业1-201901010101-张三。

Grading: Your assignment will be graded based on your effort, not the accuracy of your answers.

The exercises are embedded in the Chapter 3 lecture notes (red boxes). You are advised to read the relevant sections when you work on the exercises.

The same set of exercises are provided below:

3.B.2 The preference relation $\succsim$ defined on the consumption set $X = \mathbb{R}^L_+$ is said to be *weakly monotone* if and only if $x \geq y$ implies that $x \succsim y$. Show that if $\succsim$ is transitive, locally nonsatiated, and weakly monotone, then it is monotone.

3.C.6 Suppose that in a two-commodity world, the consumer's utility function takes the form $u(x) = [\alpha_1 x_1^\rho + \alpha_2 x_2^\rho]^{1/\rho}$. This utility function is known as the *constant elasticity of substitution* (or *CES*) utility function.

(a) Show that when $\rho = 1$, indifference curves become linear.

(b) Show that as $\rho \to 0$, this utility function comes to represent the same preference as the (generalized) Cobb-Douglas utility function $u(x) = x_1^{\alpha_1} x_2^{\alpha_2}$.

(c) Show that as $\rho \to -\infty$, indifference curves become "right angles"; that is, this utility function has in the limit the indifference map of the Leontief utility function $u(x_1, x_2) = \min\{x_1, x_2\}$.

3.D.5 Consider again the CES utility function of Exercise 3.C.6, and assume that $\alpha_1 = \alpha_2 = 1$.

(a) Compute Walrasian demand and indirect utility functions for this utility function.

(b) Verify that these functions satisfy all the properties of Propositions 3.D.2 and 3.D.3.

(c) Derive the Walrasian demand correspondence and indirect utility function for the case of linear utility and the case of Leontief utility (see Exercise 3.C.6). Show that the CES Walrasian demand and indirect utility functions approach these as ρ approaches 1 and $-\infty$, respectively.

(d) The *elasticity of substitution between goods* 1 and 2 is defined as

$$\xi_{12}(p, w) = -\frac{\partial[x_1(p, w)/x_2(p, w)]}{\partial[p_1/p_2]} \frac{p_1/p_2}{x_1(p, w)/x_2(p, w)}.$$

Show that for CES utility function, $\xi_{12}(p, w) = 1/(1 - \rho)$, thus justifying the name. What is $\xi_{12}(p, w)$ for the linear, Leontief, and Cobb-Douglas utility functions?

3.E.6 Consider the constant elasticity of substitution utility function studied in Exercises 3.C.6 and 3.D.5 with $\alpha_1 = \alpha_2 = 1$. Derive its Hicksian demand function and expenditure function. Verify the properties of Propositions 3.E.2 and 3.E.3.

3.E.9 Use the relations in (3.E.1) to show that the properties of the indirect utility function identified in Proposition 3.D.3 imply Proposition 3.E.2. Likewise, use the relations in (3.E.1) to prove that Proposition 3.E.2 implies Proposition 3.D.3.

3.G.1 Prove that Proposition 3.G.1 is implied by Roy's identity (Proposition 3.G.4).

3.G.8 The indirect utility function $v(p, w)$ is logarithmically homogeneous if $v(p, \alpha w) = v(p, w) + \ln \alpha$ for $\alpha > 0$ [in other words, $v(p, w) = \ln(v^*(p, w))$, where $v^*(p, w)$ is homogeneous of degree one in w]. Show that if $v(\cdot, \cdot)$ is logarithmically homogeneous, then $x(p, 1) = -\nabla_p v(p, 1)$.

3.G.15 Consider the utility function

$$u = 2x_1^{1/2} + 4x_2^{1/2}.$$

(a) Find the demand functions for goods 1 and 2 as they depend on prices and wealth.

(b) Find the compensated demand function $h(\cdot)$.

(c) Find the expenditure function, and verify that $h(p, u) = \nabla_p e(p, u)$.

(d) Find the indirect utility function, and verify Roy's identity.