

# Advanced Microeconomics

## Assignment 4

The exercises are embedded in the Chapter 5 lecture notes (red boxes). You are advised to read the relevant sections when you work on the exercises.

The same set of exercises are provided below:

**5.B.2** Suppose that  $f(\cdot)$  is the production function associated with a single-output technology, and let  $Y$  be the production set of this technology. Show that  $Y$  satisfies constant returns to scale if and only if  $f(\cdot)$  is homogeneous of degree one.

**5.B.3** Show that for a single-output technology,  $Y$  is convex if and only if the production function  $f(\cdot)$  is concave.

**5.C.9** Derive the profit function  $\pi(p)$  and supply function (or correspondence)  $y(p)$  for the single-output technologies whose production functions  $f(z)$  are given by

(a)  $f(z) = \sqrt{z_1 + z_2}$ .

(b)  $f(z) = \sqrt{\min\{z_1, z_2\}}$ .

(c)  $f(z) = (z_1^\rho + z_2^\rho)^{1/\rho}$ , for  $\rho \leq 1$ .

**5.C.10** Derive the cost function  $c(w, q)$  and conditional factor demand functions (or correspondences)  $z(w, q)$  for each of the following single-output constant return technologies with production functions given by

(a)  $f(z) = z_1 + z_2$  (perfect substitutable inputs)

(b)  $f(z) = \min\{z_1, z_2\}$  (leontief technology)

(c)  $f(z) = (z_1^\rho + z_2^\rho)^{1/\rho}$ ,  $\rho \leq 1$  (constant elasticity of substitution technology)

**5.C.11** Show that  $\partial z_l(w, q)/\partial q > 0$  if and only if marginal cost at  $q$  is increasing in  $w_l$ .

**5.D.1** Show that  $AC(\bar{q}) = C'(\bar{q})$  at any  $\bar{q}$  satisfying  $AC(\bar{q}) \leq AC(q)$  for all  $q$ . Does this result depend on the differentiability of  $C(\cdot)$  everywhere?

**5.D.2** Depict the supply locus for a case with partially sunk costs, that is, where  $C(q) = K + C_v(q)$  if  $q > 0$  and  $0 < C(0) < K$ .

**5.D.3** Suppose that a firm can produce good  $L$  from  $L - 1$  factor inputs ( $L > 2$ ). Factor prices are  $w \in \mathbb{R}^{L-1}$  and the price of output is  $p$ . The firm's differentiable cost function is  $c(w, q)$ . Assume that this function is strictly convex in  $q$ . However, although  $c(w, q)$  is the cost function when all factors can be freely adjusted, factor 1 cannot be adjusted in the short run.

Suppose that the firm is initially at a point where it is producing its long-run profit-maximizing output level of good  $L$  given prices  $w$  and  $p$ ,  $q(w, p)$  [i.e., the level that is optimal under the long-run cost conditions described by  $c(w, q)$ ], and that all inputs are optimally adjusted [i.e.,  $z_l = z_l(w, q(w, p))$  for all  $l = 1, \dots, L - 1$ , where  $z_l(\cdot, \cdot)$  is the long-run input demand function]. Show that the firm's profit-maximizing output response to a marginal increase in the price of good  $L$  is larger in the long run than in the short run. [*Hint*: Define a short-run cost function  $c_s(w, q|z_1)$  that gives the minimized costs of producing output level  $q$  given that input 1 is fixed at level  $z_1$ .]