

Chapter 1. Preference and Choice

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1.A. Introduction

Two approaches to modeling individual choice behavior:

1. Preference-based Approach: preference as primitive (rationality axioms) \implies consequences on choices
2. Choice-based Approach: choice behavior as primitive (axioms on behavior)

1.B. Preference Relations

X : Set of Alternatives.

- For example, if Alice just graduated from Wuhan University majoring in economics, then her set of alternatives is:
 $X = \{\text{go to graduate school and study economics, go to a Big-4 firm, go to work for the government, ..., run a small business}\}.$

We use capital letters (like X and B) for a set of alternatives, small letters (like x and y) for a specific choice alternative.

Defining Preference Relations

Denote by \succsim the *preference relation* defined on the set X , allowing the comparison of any x and y in X .

- $x \succsim y$: pronounced as “ x is preferred to y ” or “ x is at least as good as y .” The first usage is more common.
- *Strict preference* \succ : $x \succ y \iff x \succsim y$ but not $y \succsim x$ (i.e., $y \not\succsim x$) (“ x is strictly preferred to y .”)
- *Indifference* \sim : $x \sim y \iff x \succsim y$ and $y \succsim x$ (“ x is indifferent to y .”)

Rational Preference

Not all preference relations make sense.

For example, consider Alice's preference:

- “Hot and Dry Noodles” \succ “Doupi” (dòu pí)
- “Doupi” \succ “Xiaolongbao” (xiǎo lóng bāo)
- “Xiaolongbao” \succsim “Hot and Dry Noodles”

Alice must have a hard time choosing her breakfast from

$X = \{\text{Hot and Dry Noodles, Doupi, Xiaolongbao}\}.$

Rational Preference

Definition 1.B.1 (Rational preference). The preference relation \succsim is **rational** if it possesses these two properties:

- (i) Completeness: $\forall x, y \in X$, $x \succsim y$ or $y \succsim x$. (rules out $x \not\succsim y$ and $y \not\succsim x$)
- (ii) Transitivity: $\forall x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

Rational Preference

Question. In the example above, which property does Alice's preference relation violate?

Rational Preference

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Answer: Transitivity.

The first two bullet points

- “Hot and Dry Noodles” \succ “Doupi” (dòu pí)
- “Doupi” \succ “Xiaolongbao” (xiǎo lóng bāo)

implies “Hot and Dry Noodles” \succ “Xiaolongbao”

which contradicts “Xiaolongbao” \succsim “Hot and Dry Noodles”

Implications of Rational Preference on \succ and \sim

The following propositions follow from the definition of *rational preference*.

Proposition 1.B.1. *If \succsim is rational, then:*

- (i) \succ is both irreflexive ($x \succ x$ never holds) and transitive.*
- (ii) \sim is reflexive ($x \sim x$), transitive and symmetric (if $x \sim y$, then $y \sim x$).*
- (iii) if $x \succ y \succsim z$, then $x \succ z$. (slightly stronger than transitivity in (i))*

Utility Functions

Definition 1.B.2. A function $u : X \rightarrow \mathbb{R}$ is a utility function representing preference relation \succsim if

$$x \succsim y \iff u(x) \geq u(y) \text{ for all } x, y \in X. \quad (1)$$

The utility function is nothing but assigning each choice x with a number $u(x)$. Obviously, the function u satisfying Condition (1) is not unique.

Example. $u(x) \geq u(y) \iff \alpha u(x) \geq \alpha u(y)$ for all $\alpha > 0$.

Utility Functions

Exercise. Show that if $f : \mathbb{R} \rightarrow \mathbb{R}$ is a **strictly increasing** function and $u : X \rightarrow \mathbb{R}$ is a utility function representing preference relation \succsim , then the function $v : X \rightarrow \mathbb{R}$ defined by $v(x) = f(u(x))$ is also a utility function representing preference relation \succsim .

Utility Functions

Question. When can a preference relation be represented by a utility function?

Utility Functions

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Answer: Only if the preference relation is rational. See the next proposition.

Utility Functions

Proposition 1.B.2. *If the preference relation \succsim can be represented by a utility function (i.e. $\exists u(\cdot)$ s.t. $u(x) \geq u(y)$ iff $x \succsim y$), then \succsim is rational (i.e. complete & transitive).*

Utility Functions

Question. If \succsim is rational, does there exist a utility function u representing \succsim ?

Utility Functions

Question. If \succsim is rational, does there exist a utility function u representing \succsim ?

Answer: Not always. Rationality is just a necessary condition for the existence of a utility representation, but not sufficient. See the counterexample below.

Lexicographic Preference

Definition (Lexicographic Preference). Let $X = \mathbb{R}^2$. The preference relation \succsim is a *lexicographic preference* if for all $x, y \in X$, $x \succsim y$ whenever (i) $x_1 > y_1$ or (ii) $x_1 = y_1$ and $x_2 \geq y_2$.

Claim. The lexicographic preference on \mathbb{R}^2 do *not* have a utility representation.

Example of Lexicographic Preference

Alice is considering buying a new phone. The relevant attributes include brand name, price, CPU, and so on. For simplicity, suppose Alice only cares about the brand (Apple or Huawei) and price. Alice is a Apple fan and strictly prefers an iPhone to a Huawei Phone regardless of the price. For Alice,

$$(\text{Apple}, 5000) \succ (\text{Apple}, 8000) \succ (\text{Huawei}, 5000).$$

Utility Function

Remark. If X is **finite** and \succsim is a rational preference relation on X , then there is a utility function $u : X \rightarrow \mathbb{R}$ that represents \succsim .

1.C. Choice Rules

A *choice structure* $(\mathcal{B}, C(\cdot))$ consists of two ingredients:

(i) \mathcal{B} is a family (a set) of nonempty subsets of X : that is, every $B \in \mathcal{B}$ is a set $B \subset X$.

- In consumer theory, B are budget sets.
- \mathcal{B} needs NOT to include all possible subsets of X .

(ii) $C(\cdot)$ is a choice rule that assigns a nonempty subset of chosen elements $C(B) \subset B$ for every $B \in \mathcal{B}$.

- $C(B)$ is a set of *acceptable alternatives*.

Choice Rules

Example 1.C.1. $X = \{x, y, z\}, \mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$

Choice Structure 1 $(\mathcal{B}, C_1(\cdot))$:

$$C_1(\{x, y\}) = \{x\}, C_1(\{x, y, z\}) = \{x\}$$

Choice Structure 2 $(\mathcal{B}, C_2(\cdot))$:

$$C_2(\{x, y\}) = \{x\}, C_2(\{x, y, z\}) = \{x, y\}$$

Under $(\mathcal{B}, C_2(\cdot))$, y is acceptable only if z is available.

Choice Rules

You might find the choice structure 2 unreasonable.

Consider the following conversation.

Waiter: Coffee or Tea?

Customer: Coffee, please.

Waiter: Sure. Oh sorry, actually we also serve coke. Do you want some coke?

Customer: *Since coke is available, I'd prefer tea rather than coffee.*

Weak Axiom of Revealed Preference (W.A.R.P)

Definition 1.C.1. The choice structure $(\mathcal{B}, C(\cdot))$ satisfies the weak axiom of revealed preference (W.A.R.P) if the following property holds:

If for some $B \in \mathcal{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathcal{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$.

Weak Axiom of Revealed Preference (W.A.R.P)

In the last example, $(\mathcal{B}, C_2(\cdot))$ violates W.A.R.P since

$y \in C_2(\{x, y, z\})$, $x, y \in \{x, y\}$, $x \in C_2(\{x, y\})$ but

$y \notin C_2(\{x, y\})$.

[Think of $\{x, y, z\}$ as B and $\{x, y\}$ as B' in Definition [1.C.1](#).]

IDEA: Agent's choice between x and y should not be affected by irrelevant options/alternatives.

Revealed Preference: Preference inferred from/ revealed through Choice

Definition 1.C.2. Given a choice structure $(\mathcal{B}, C(\cdot))$, the **revealed preference relation** \succsim^* is defined by

$$x \succsim^* y \iff \exists B \in \mathcal{B} \text{ s.t. } x, y \in B \text{ and } x \in C(B).$$

$x \succsim^* y$ reads “ x is revealed at least as good as y ”

Revealed Preference

- $x \succ^* y$:

$\exists B \in \mathcal{B}$ s.t. $x, y \in B$ and $x \in C(B)$, and $y \notin C(B)$. (

“ x is revealed preferred to y ”)

- \succ^* needs not to be complete or transitive.
- “Revealed preference” is defined reference to B .
(Compare with “preference”)
- **Restatement of W.A.R.P:** If $x \succ^* y$, then $y \not\succ^* x$.
(only imposed on $B \in \mathcal{B}$)

Revealed Preference

Example 1.C.2. Recall Example 1.C.1.

$(\mathcal{B}, C_1(\cdot))$: $x \succ^* y$ and $x \succ^* y, x \succ^* z$

$(\mathcal{B}, C_2(\cdot))$: $x \succ^* y$ and $y \succsim^* x \implies$ contradicts W.A.R.P

Useful alternative statements of W.A.R.P

Restatement of W.A.R.P 1. $x, y \in B, x \in C(B),$
 $y \in C(B') \ \& \ x \notin C(B'),$ then $x \notin B'.$

Proof. Proof by contradiction. If $x \in B' \ \& \ y \in C(B'),$

W.A.R.P $\implies x \in C(B').$ □

Restatement of W.A.R.P 2. Suppose that $B, B' \in \mathcal{B},$ that
 $x, y \in B,$ and that $x, y \in B'.$ Then if $x \in C(B)$ and
 $y \in C(B'),$ we must have $\{x, y\} \subset C(B)$ and $\{x, y\} \in C(B').$

The proof is left as an exercise.

1.D. Relationship between Preference Relations & Choice Rules

More precisely, we want to know the relationship between *rational preference* and *W.A.R.P.*

(i) Does Rational Preference imply W.A.R.P?

(ii) Does W.A.R.P imply Rational Preference?

1.D. Relationship between Preference Relations & Choice Rules

More precisely, we want to know the relationship between *rational preference* and *W.A.R.P.*

(i) Does Rational Preference imply W.A.R.P? (Yes)

(ii) Does W.A.R.P imply Rational Preference? (Maybe)

Preference Generated Choice Structure

Consider rational preference \succsim on X .

Define: $C^*(B, \succsim) = \{x \in B: x \succsim y \text{ for every } y \in B\}$

- Elements of $C^*(B, \succsim)$ are DM's most preferred alternatives in B .
- Assumption: $C^*(B, \succsim)$ is nonempty for all $B \in \mathcal{B}$.

Remark. If X is **finite**, then any rational preference relation generates a nonempty choice rule.

The proof is left as an exercise.

Preference Generated Choice Structure

We say that the preference \succsim *generates* the choice structure $(\mathcal{B}, C^*(\cdot, \succsim))$.

Proposition 1.D.1. *Suppose \succsim is a rational preference relation. Then the choice structure generated by \succsim , $(\mathcal{B}, C^*(\cdot, \succsim))$ satisfies W.A.R.P.*

Preference Generated Choice Structure

Definition 1.D.1. Given a choice structure $(\mathcal{B}, C(\cdot))$, we say that the rational preference relation \succsim rationalizes $C(\cdot)$ relative to \mathcal{B} if $C(B) = C^*(B, \succsim)$ for all $B \in \mathcal{B}$, that is, if \succsim generates the choice structure $(\mathcal{B}, C(\cdot))$.

Preference Generated Choice Structure

1. If a rational preference relation rationalizes the choice rule, we can interpret the DM's choices as if she were a preference maximizer.
2. In general, there may be more than one rationalizing preference relation \succsim for a given choice structure $(\mathcal{B}, C(\cdot))$.

Example. $X = \{x, y\}, \mathcal{B} = \{\{x\}, \{y\}\},$

$$C(\{x\}) = \{x\}, C(\{y\}) = \{y\}.$$

Preference Generated Choice Structure

Example 1.D.1. $X = \{x, y, z\}$,

$$\mathcal{B} = \{\{x, y\}, \{y, z\}, \{x, z\}\}^1,$$

$$C(\{x, y\}) = \{x\}, C(\{y, z\}) = \{y\}, C(\{x, z\}) = \{z\}.$$

This choice structure satisfies the W.A.R.P.

However, it cannot be rationalized by a rational preference.

Remark. W.A.R.P is defined by \mathcal{B} . And the choice is not challenged by having to choose from $\{x, y, z\}$.

¹ $\{x, y, z\}$ is not empirically relevant.

Preference Generated Choice Structure

Proposition 1.D.2. *If $(\mathcal{B}, C(\cdot))$ is a choice structure such that*

(i) the W.A.R.P is satisfied, $[x \succsim^ y, \text{ then } y \not\succ^* x]$*

(ii) \mathcal{B} includes all subsets of X of up to three elements,

then \exists rational \succsim that rationalizes $C(\cdot)$ relative to \mathcal{B} , i.e.,

$$C(B) = C^*(B, \succsim), \forall B \in \mathcal{B}.$$

Furthermore, this rational preference relation is unique.

Summary of Chapter 1

- Preference relation \succsim is binary relation on choice set X .
- \succsim is rational if Completeness & Transitivity.
- Choice function $C(\cdot)$ is defined on \mathcal{B} , NOT on X .
(Assumptions: W.A.R.P & $C(\cdot) \neq \emptyset$)
- Rational Preference implies W.A.R.P.

But for W.A.R.P to imply Rational Preference, it requires $C(\cdot) \neq \emptyset$ and that \mathcal{B} includes all 2 & 3-element subsets of X .