

# Advanced Microeconomics

## Assignment 2

**Due date:** October 12, 2019 (before class)

**Submission method:** Please submit your assignment to me in class, or via E-mail: [sherryecon@qq.com](mailto:sherryecon@qq.com).

- 纸质版：要求字迹工整，可辨认。
- 电子版：附件要求 .pdf格式。邮件标题格式为“作业编号-学号-姓名”，如：作业1-201901010101-张三。

**Grading:** Your assignment will be graded based on your effort, not the accuracy of your answers.

The exercises are embedded in the Chapter 2 lecture notes (red boxes). You are advised to read the relevant sections when you work on the exercises.

The same set of exercises are provided below:

**2.D.1** A consumer lives for two periods, denoted 1 and 2, and consumes a single consumption good in each period. His wealth when born is  $w > 0$ . What is his (lifetime) Walrasian budget set?

**2.D.2** A consumer consumes one consumption good  $x$  and hours of leisure  $h$ . The price of the consumption good is  $p$ , and the consumer can work at a wage rate of  $s = 1$ . What is the consumer's Walrasian budget set?

**2.E.1** Suppose  $L = 3$ , and consider the demand function  $x(p, w)$  defined by

$$\begin{aligned}x_1(p, w) &= \frac{p_2}{p_1 + p_2 + p_3} \frac{w}{p_1} \\x_2(p, w) &= \frac{p_3}{p_1 + p_2 + p_3} \frac{w}{p_2} \\x_3(p, w) &= \frac{\beta p_1}{p_1 + p_2 + p_3} \frac{w}{p_3}\end{aligned}$$

Does this demand function satisfy homogeneity of degree zero and Walras' law when  $\beta = 1$ ? What about when  $\beta \in (0, 1)$ ?

**2.E.3** Use Proposition 2.E.1 to 2.E.3 to show that  $p \cdot D_p x(p, w) p = -w$ .

**2.E.5** Suppose that  $x(p, w)$  is a demand function which is homogeneous of degree one with respect to  $w$  and satisfies Walras' law and homogeneity of degree zero. Suppose also that all the cross-price effects are zero, that is  $\partial x_l(p, w) / \partial p_k = 0$  whenever  $k \neq l$ . Show that this implies that for every  $l$ ,  $x_l(p, w) = \alpha_l w / p_l$ , where  $\alpha_l > 0$  is a constant independent of  $(p, w)$ .

**2.E.7** A consumer in a two-good economy has a demand function  $x(p, w)$  that satisfies Walras' law. His demand function for the first good is  $x_1(p, w) = \alpha w / p_1$ . Derive his demand function for the second good. Is his demand function homogeneous of degree zero?

**2.E.8** Show that the elasticity of demand for good  $l$  with respect to price  $p_k$ ,  $\varepsilon_{lk}(p, w)$ , can be written as  $\varepsilon_{lk}(p, w) = d \ln(x_l(p, w)) / d \ln(p_k)$ , where  $\ln(\cdot)$  is the natural logarithm function. Derive a similar expression for  $\varepsilon_{lw}(p, w)$ . Conclude that if we estimate the parameters  $(\alpha_0, \alpha_1, \alpha_2, \gamma)$  of the equation  $\ln(x_l(p, w)) = \alpha_0 + \alpha_1 \ln p_1 + \alpha_2 \ln p_2 + \gamma \ln w$ , these parameter estimates provide us with estimates of the elasticities  $\varepsilon_{l1}(p, w)$ ,  $\varepsilon_{l2}(p, w)$ , and  $\varepsilon_{lw}(p, w)$ .

**2.F.11** Show that for  $L = 2$ ,  $S(p, w)$  is always symmetric. [Hint: Use Proposition 2.F.3.]

**2.F.17** In an  $L$ -commodity world, a consumer's Walrasian demand function is

$$x_k(p, w) = \frac{w}{\sum_{l=1}^L p_l} \text{ for } k = 1, \dots, L.$$

- (a) In this demand function homogeneous of degree zero in  $(p, w)$ ?
- (b) Does it satisfy Walras' law?
- (c) Does it satisfy the weak axiom?
- (d) Compute the Slutsky substitution matrix for this demand function. Is it negative semidefinite? Negative definite? Symmetric?