Advanced Microeconomics
Assignment 5

Due date: December 9, 2019 (before class)
Submission method: Please submit your assignment to me in class, or via E-mail: sherryecon@qq.com.

- 纸质版：要求字迹工整，可辨认。
- 电子版：附件要求 .pdf格式。邮件标题格式为“作业编号-学号-姓名”，如：作业1-201901010101-张三。

Grading: Your assignment will be graded based on your effort, not the accuracy of your answers.

The exercises are embedded in the Chapter 5 lecture notes (red boxes). You are advised to read the relevant sections when you work on the exercises.

The same set of exercises are provided below:

5.B.2 Suppose that $f(\cdot)$ is the production function associated with a single-output technology, and let Y be the production set of this technology. Show that Y satisfies constant returns to scale if and only if $f(\cdot)$ is homogeneous of degree one.

5.B.3 Show that for a single-output technology, Y is convex if and only if the production function $f(\cdot)$ is concave.

5.C.9 Derive the profit function $\pi(p)$ and supply function (or correspondence) $y(p)$ for the single-output technologies whose production functions $f(z)$ are given by

(a) $f(z) = \sqrt{z_1 + z_2}$.

(b) $f(z) = \sqrt{\min\{z_1, z_2\}}$.

(c) $f(z) = (z_1^\rho + z_2^\rho)^{1/\rho}$, for $\rho \leq 1$.

5.C.10 Derive the cost function $c(w, q)$ and conditional factor demand functions (or correspondences) $z(w.q)$ for each of the following single-output constant return technologies with production functions given by

(a) $f(z) = z_1 + z_2$ (perfect substitutable inputs)

(b) $f(z) = \min\{z_1, z_2\}$ (leontief technology)

(c) $f(z) = (z_1^\rho + z_2^\rho)^{1/rho}, \rho \leq 1$ (constant elasticity of substitution technology)

5.C.11 Show that $\partial z_l(w, q)/\partial q > 0$ if and only if marginal cost at q is increasing in w_l.

5.D.1 Show that $AC(\bar{q}) = C'(\bar{q})$ at any $\bar{q}$ satisfying $AC(\bar{q}) \leq AC(q)$ for all q. Does this result depend on the differentiability of $C(\cdot)$ everywhere?

5.D.2 Depict the supply locus for a case with partially sunk costs, that is, where $C(q) = K + C_v(q)$ if $q > 0$ and $0 < C(0) < K$.

5.D.3 Suppose that a firm can produce good L from $L-1$ factor inputs ($L > 2$). Factor prices are $w \in \mathbb{R}^{L-1}$ and the price of output is p. The firm's differentiable cost function is $c(w, q)$. Assume that this function is strictly convex in q. However, although $c(w, q)$ is the cost function when all factors can be freely adjusted, factor 1 cannot be adjusted in the short run.
Suppose that the firm is initially at a point where it is producing its long-run profit-maximizing output level of good L given prices w and p, $q(w, p)$ [i.e., the level that is optimal under the long-run cost conditions described by $c(w, q)$], and that all inputs are optimally adjusted [i.e., $z_l = z_l(w, q(w, p))$ for all $l = 1, ..., L-1$, where $z_l(\cdot, \cdot)$ is the long-run input demand function]. Show that the firm's profit-maximizing output response to a marginal increase in the price of good L is larger in the long run than in the short run. [*Hint*: Define a short-run cost function $c_s(w, q|z_1)$ that gives the minimized costs of producing output level q given that input 1 is fixed at level z_1.]