Using Markov Chains to Approximate the Cardinality of the Power Set

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Definitions

- A ***Markov chain*** is a sequence of random variables (of states) where the current state only depends on the previous state, and not the states before that
- A ***FPAUS (Fully Polynomial Almost Uniform Sampler)*** is a way a randomized algorithm, where given a set, it outputs an element from the power set of the set according to a uniform distribution
- A ***FPRAS (Fully Polynomial Random Approximation Scheme)*** is a randomized algorithm where given a set, it outputs and approximation for the cardinality of the power set.

Idea: Part 1

- We create a FPAUS by modeling a **Markov Chain**. Consider a Markov Chain over all possible power sets of a set A. Given that we are at state X_0 in the power set A,
  - With probability 1/2, stay at state X_0
  - With equal probability, choose an element a \in A. If it exists in X_0, remove it. If it does not exist in X_0, add it.
- Note that this Markov Chain is both **irreducible** and **aperiodic**, which means it is ergodic. Thus, we can conclude that this Markov Chain is rapidly mixing, and will come at a uniform distribution quickly.

Idea: Part 2

Now that we have a FPAUS, we create an FPRAS using that.

Note that the power set of the A can be expressed as (look at board).

The problem is “reducible” when considering subsets.

We can approximate each of the ratios with our FPAUS. Have an indicator random variable, run the FPAUS, and use the sample mean as the estimator.

Done.