Research on Design and Analysis of Experiments

- **Surrogates** (Athey, Chetty, Imbens, Kang, 2016, update coming shortly)
- **Heterogeneous treatment effects** (Wager & Athey (JASA, 2018); Athey, Tibshirani, and Wager (AOS, 2019); Friedberg, Athey, Tibshirani, and Wager (2018)
- **Offline policy estimation** (Athey and Wager, 2017; Zhou, Athey, and Wager 2018)
- **Improving estimation model used in contextual bandit algorithms** (Dimakopoulou, Zhou, Athey and Imbens, AAAI, 2018)
- **Survey** Athey & Imbens, The Econometrics of Randomized Experiments (Handbook of Experimental Economics)

TODAY:

- **Designing experiments with staggered rollouts** (Xiong, Athey, Bayati, Imbens 2019)
- **Testing hypotheses using adaptively collected data** (Hadad, Hirschberg, Zhan, Wager, Athey, 2019)

Recent work on causal inference with panel data

- **Matrix factorization in Consumer Choice settings**
  - Donnelly, Ruiz, Blei, and Athey (2019), Ruiz, Athey and Blei (AOAS, 2019), Athey, Blei, Donnelly, Ruiz, and Schmidt (AEA P&P, 2018)
  - Augment traditional mixed/nested logit models with matrix factorization for random coefficients
  - Tune for causal effects, test with held-out experiments

- **Matrix Completion for Panel Data Models**
  - Athey, Bayati, Doudchenko, Imbens, Khosravi, 2017
  - When evaluating staggered rollouts, use matrix completion to construct counterfactual outcomes

- **Synthetic Difference-in-Differences**
  - Arkhangelsky, Athey, Hirschberg, Imbens, Wager, 2018
  - Builds on ideas of double robustness in causal inference
  - Combine outcome models (e.g. matrix completion) with weighting that focuses on similar units and time periods

- **This paper:** *Design* experiments anticipating future hypothesis testing
  - Take the matrix completion approach to outcome modeling for tractability

Optimal Experimental Design for Staggered Rollouts

Ruoxuan Xiong, Susan Athey, Mohsen Bayati, Guido Imbens
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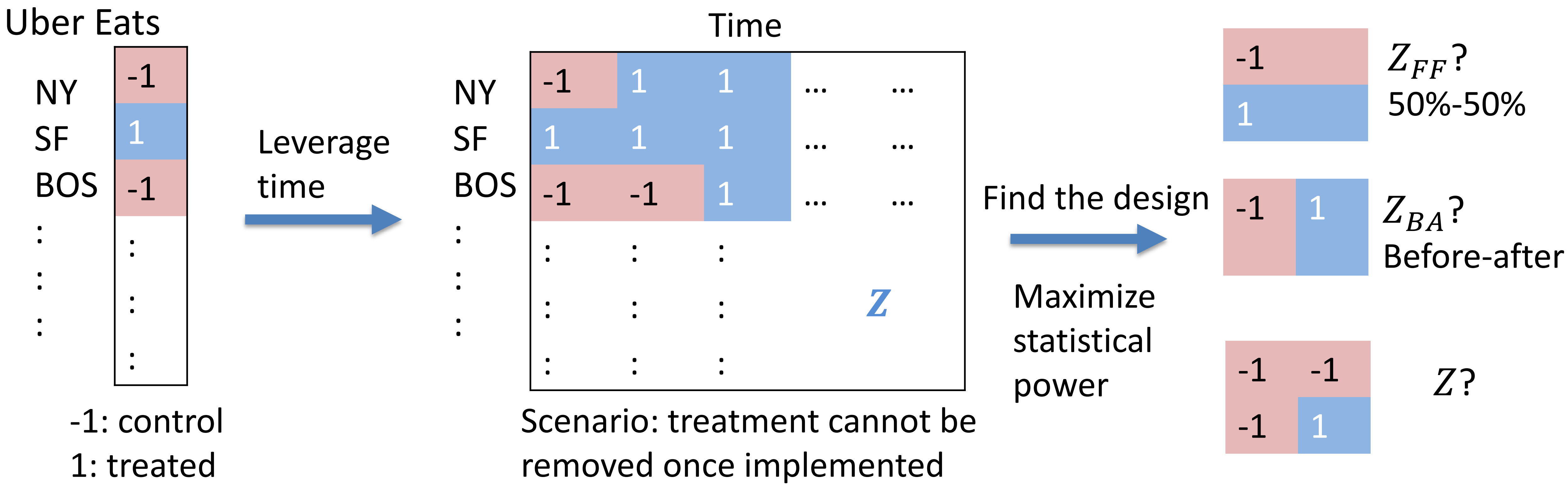
Statistical Power: A Fundamental Challenge

- Randomized controlled **experiments** are the gold standard to draw inference about whether a product or an intervention is **effective**.
- **Fundamental challenge**: Low power due to small sample size:
  - Avoid contamination and network effects
  - Relevant units are market, product category, school, hospital…



Related Literature

- Related work: optimal allocation of clusters for a multi-period design
  - Hussey and Hughes 2007, Hemming et al. 2015, Li et al. 2018

- Contribution of this paper
  1. Study the optimal multi-period design under a general class of linear models
  2. Allow to use historical control data to find a better design

Problem Formulation: Optimization

- Model, for all $(i, t) \in \{1, \ldots, N\} \times \{1, \ldots, T\}$

$$\boldsymbol{Y_{it}} = \alpha_i + \beta_t + u_i^T v_t + \boldsymbol{\tau z_{it}} + \varepsilon_{it}$$

- Optimization problem: given $\hat{\tau}$, solve $\boldsymbol{Z} = [\boldsymbol{z_{it}}]$

$$\min_{z_{it}} \quad Var(\hat{\tau})$$ Maximize statistical power

$$s.t. \quad z_{it} \leq z_{it+1}$$ Treatment cannot be removed once implemented

$$z_{it} \in \{\text{-}1, 1\}$$ -1: control, 1: treated

- Integer programming

- Estimation: Best Linear Unbiased Estimator (BLUE) to estimate $\boldsymbol{\tau}$
  - Explicit formula for $Var(\hat{\tau})$ in terms of $\boldsymbol{Z}$

Observed : Y_{it}, z_{it}
Unknown : $\alpha_i, \beta_t, \boldsymbol{\tau}, \varepsilon_{it}, u_i, v_t$
α_i, β_t : unit, time fixed effects
u_i : latent covariates
z_{it}: treatment status
$\boldsymbol{\tau}$: treatment effect

Time β_t

α_i

NY	-1	1	1	…	…
SF	1	1	1	…	…
BOS	-1	-1	1	…	…
:	:	:	:		
:	:	:	:	$\boldsymbol{Z}$	
:	:	:	:		

Main Result

- N units, T periods, and $Y_{it} = \alpha_i + \beta_t + u_i^T v_t + \tau z_{it} + \varepsilon_{it}$ with $z_{it} \in \{-1,1\}$

Theorem. If u_i takes finitely many values, then any design that has **staggered rollouts** with $\frac{2t-1}{2T}$ 100% treated for each **strata** at time t is an **optimal** solution to $\min\limits_{\substack{[z_{it}]\in[-1,1]^{N\times T} \\ z_{it}\leq z_{it+1}}} Var(\hat{\tau})$

- Some intuition why?

-1	-1
-1	-1
1	1
1	1

Only time effect β_t
Z_{FF} : 50%-50%

-1	1
-1	1
-1	1
-1	1

Only unit effect α_i
Z_{BA} :Before-after

-1	-1
-1	1
-1	1
1	1

Both effects: $\frac{2t-1}{2T}$

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Strata: units with the same covariate values

$$\begin{array}{rl} u_i\text{=1} & \left\{\begin{array}{cc} -1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & 1 \end{array}\right. \\ u_i\text{=-1} & \left\{\begin{array}{cc} -1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & 1 \end{array}\right. \end{array}$$

Uber Eats

u_i: quality of restaurants

u_i=1: good

u_i=-1: bad

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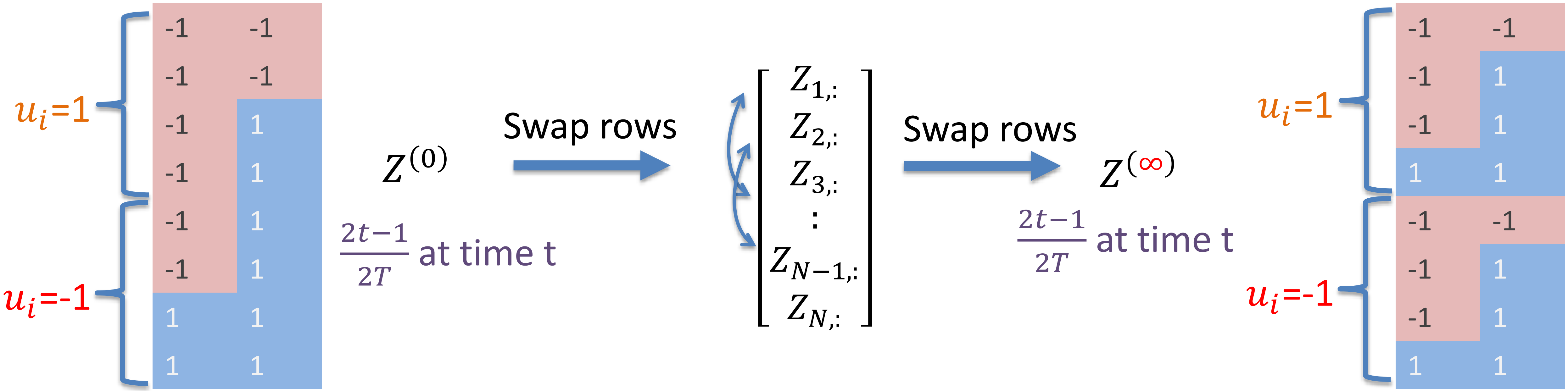
Remark 1: This result holds regardless of whether u_i is known or latent

Remark 2: For general u_i use a clustering algorithm (e.g., K-means) to stratify

Next: A data-driven heuristic to stratify, when u_i's are latent

A Local Search Heuristic to Stratify



Step 1. Preprocess historical data: split historical data into several matrices

Step 2. Minimax decision rule: synthetic experiment τ on each matrix with estimator $\hat{\tau}^{(j)}$

- Find the Z to minimize $\max_j |\hat{\tau}^{(j)} - \tau|$. **(Robust measure)**

Step 3. Local Search Z (simulated annealing)

Empirical Application

- Purchases from a large grocery store
  - 17,880,248 transactions between May 2005 and May 2007
  - Aggregated weekly net expenditure by household
    - 63,375 households over 97 weeks
    - 85.4% entries to be 0
- Hypothetical experiments:
  - Promotions, coupons, ads, etc.
  - …

Empirical Application: Comparison

- Design matrices

$$\begin{matrix} -1 & -1 \\ -1 & -1 \\ 1 & 1 \\ 1 & 1 \end{matrix}$$

Z_{FF}

50% control 50% treated at every time period

$$\begin{matrix} -1 & 1 \\ -1 & 1 \\ -1 & 1 \\ -1 & 1 \end{matrix}$$

Z_{BA}

First half all control second half all treated

$$\begin{matrix} -1 & -1 \\ -1 & -1 \\ -1 & 1 \\ -1 & 1 \end{matrix}$$

Z_{FF+BA}

First half all control second half half treated

$$\begin{matrix} -1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & 1 \end{matrix}$$

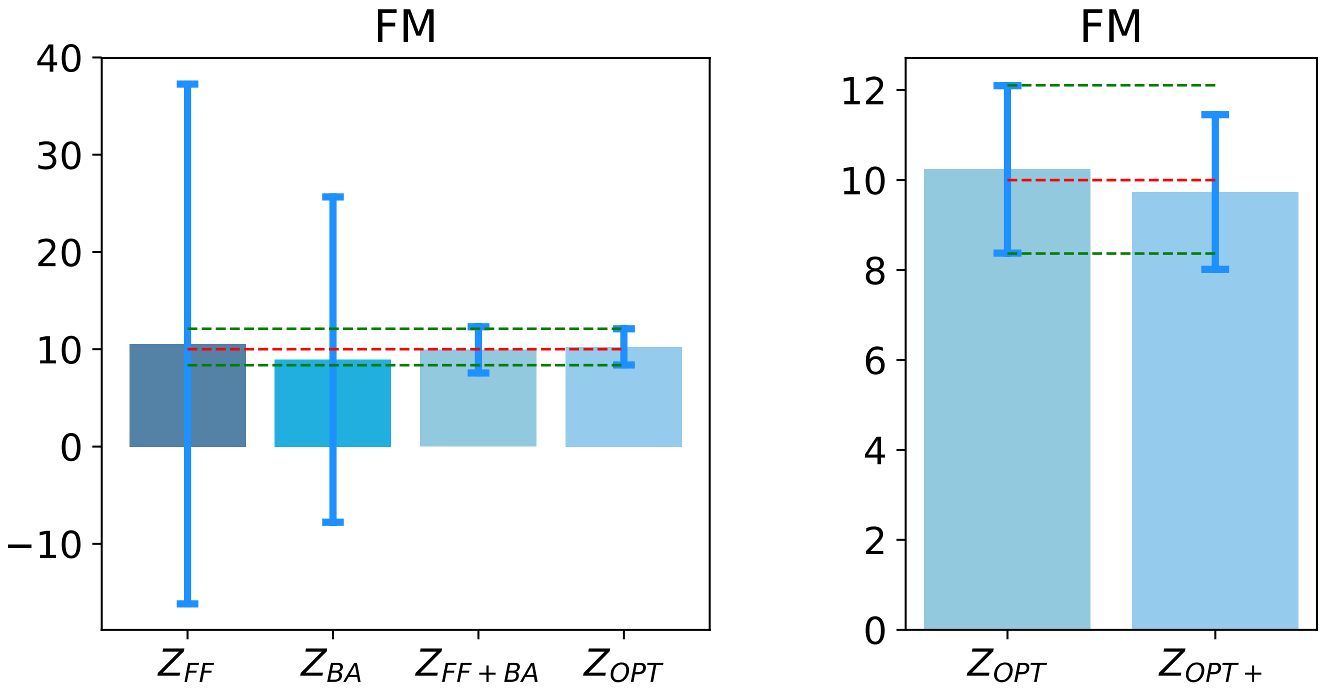
Z_{OPT}

Staggered rollouts

% treated: $\frac{2t-1}{2T}$

Results

- Z_{OPT} outperforms benchmark matrices $Z_{FF}, Z_{BA}, Z_{FF+BA}$ for all estimation methods
- Factor model has RMSE about half of linear fixed effect model
- Using historical data, local search heuristic with minimax decision rule provides a better matrix Z_{OPT+} than Z_{OPT} for all estimation methods



RMSE: 26.73 16.76 2.37 **1.88** 1.88 **1.74**

Red dash line: true value
Bar height: average estimated value
Error bar: variance

Conclusion

- **Optimal** multi-period design
  - Linear treated portion in time with **stratification** (randomize the treatment allocation within each strata, the units with the same covariates' value)

- A data driven data-driven heuristic to stratify
  - Based on local search with minimax decision rule using historical control data

- **Extension:** Treatment effect varies over time
  - Optimal design starts with lower % treated, increases faster, in order to learn time-dependent treatment effects

- Empirics: synthetic experiments on several real world data sets
  - Various domains, including retail and health care
  - Superior performance of the analytical solution and our algorithm compared with the benchmark designs

Confidence Intervals for Policy Evaluation in Adaptive Experiments

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November, 2019

The promise

Multi-armed bandit experiments can be a substantially more efficient optimization method than traditional experiments. [1]

[1] Scott, S. L. (2015). Multi-armed bandit experiments in the online service economy. Applied Stochastic Models in Business and Industry, 31(1), 37-45.

Adaptive Experimentation

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What do we want to *optimize*?

- Standard for bandits: *Low regret* during the experiment

But for innovation, understanding, scientific discovery, also want:

- Statistical *accuracy*
- Valid *confidence intervals*
- *Power* for the hypothesis test

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These goals can be *at odds* with each other.

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Inference challenges

You are given data $\{(W_t, Y_t)\}_{t=1}^{T}$ coming from an adaptive experiment. How would you estimate the value of arm w?

[1] [Xu et al., 2013, Bowden and Trippa, 2017, Nie et al., 2018, Shin et al., 2019]

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Sample average

$$\widehat{Q}^{Sample}(w) = \frac{1}{n_w} \sum_{t:W_t=w} Y_t$$

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Sample average

$$\widehat{Q}^{Sample}(w) = \frac{1}{n_w} \sum_{t: W_t = w} Y_t$$

- In a randomized control trial: *unbiased*, *consistent*
- In a sequential experiment: possibly *biased* and *inconsistent* [1]

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Inference challenges

An *unbiased* estimator

Inverse Propensity Weighted Estimator

$$\widehat{Q}^{IPW}(w) = \frac{1}{T}\sum_{t=1}^{T}\frac{\mathbb{I}\{W_t = w\}}{e_t(w)}Y_t$$

- **Notation:** $e_t(w)$ is probability of selecting arm w at t

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  - Adaptive data collection introduces sample bias
  - Up/down weight observations to counter bias

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  - Adaptive data collection introduces sample bias
  - Up/down weight observations to counter bias
- **Problem:** High variance if $e_t(w)$ small

Inference challenges

A *lower variance* and *unbiased* estimator

Augmented Inverse Propensity Weighted Estimator

$$\widehat{Q}^{AIPW}(w) = \frac{1}{T}\sum_{t=1}^{T} \frac{\mathbb{I}\left\{W_t = w\right\}}{e_t(w)} Y_t + \left(1 - \frac{\mathbb{I}\left\{W_t = w\right\}}{e_t(w)}\right)\hat{\mu}_t$$

- **Notation:** $\hat{\mu}_t$ is any other estimator (including sample mean)
- *Additional term* helps *decrease variance*...
- ...but in bandit experiments, *variance is still too high*

(Remainder of the talk: IPW only)

Example

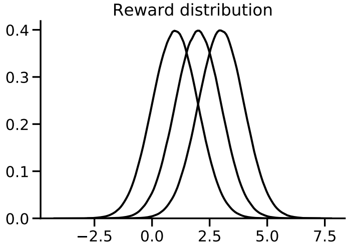
A simple example with *three treatments (arms)*.

True distribution
(unknown)

$Y_t(1) \sim \mathcal{N}(3,1)$ High reward

$Y_t(2) \sim \mathcal{N}(2,1)$

$Y_t(3) \sim \mathcal{N}(1,1)$ Low reward (Control)



Example

Let's compare

Adaptive vs *random* estimates of $Q(w)$.

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Let's compare

Adaptive vs *random* estimates of $Q(w)$.

Random

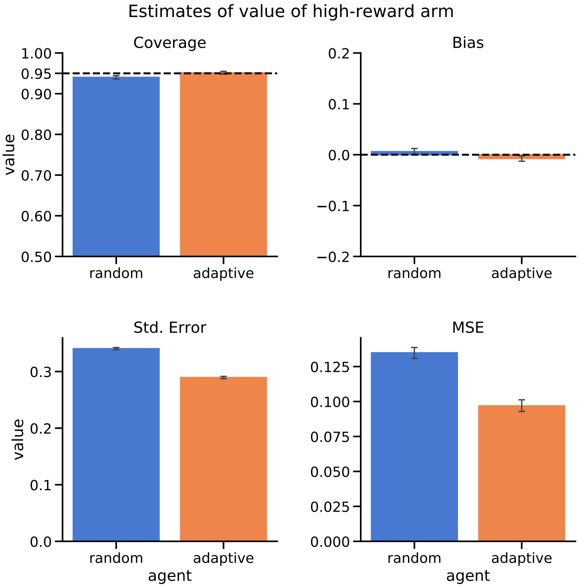
- Assign arms at random, with equal probability

Adaptive

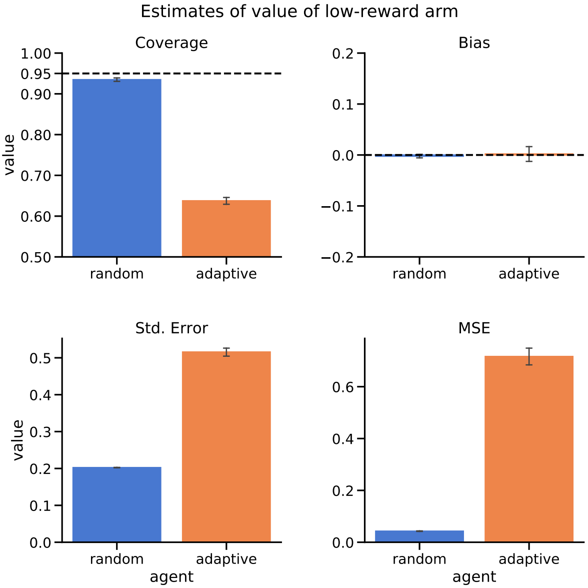
- As we learn which arms are better, increase assignment probabilities to them (Thompson Sampling)

Which data-collection algorithm
yields better *coverage*, *power*, and *mse*?

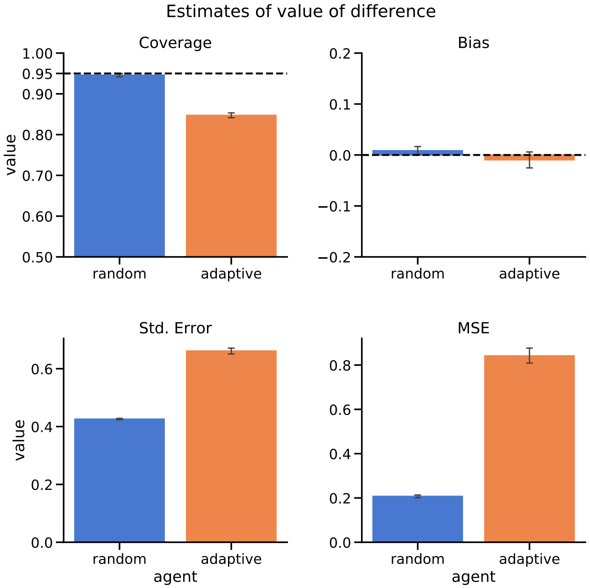
High-reward arm is estimated well



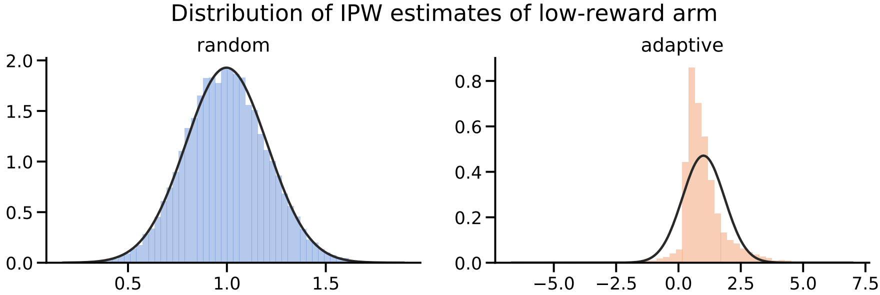
Low-reward arm is estimated poorly



Difference (High-Low) is estimated poorly too



Adaptivity hurts normality



central limit theorem may not hold

$\Longrightarrow$

Wrong confidence intervals!

Our contribution

Introduce *evaluation weights* $h_t(w)$.

Adaptively-Weighted Augmented Inverse Propensity Score Estimator

$$\widehat{Q}_T^{AW}(w) := \sum_{t=1}^{T} \frac{h_t(w)}{\sum_{s=1}^{T} h_s(w)} \left\{ \frac{\mathbb{I}\{W_t = w\}}{e_t(w)} Y_t + \left(1 - \frac{\mathbb{I}\{W_t = w\}}{e_t(w)}\right) \hat{\mu}_t \right\}$$

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- **Intuition:** $h_t(w)$ controls *flow of information*
- **Main result:** Conditions for CLT

Assumptions

A1: Bounded moments

Rewards have bounded fourth moments.

A2: Bounded $\hat{\mu}_t$

Estimate $\hat{\mu}_t$ is uniformly bounded, depends only on past data.

Assumptions

A3: Assignments

Assignment probabilities $e_t(w)$ depend on past data and converge in a.s. to a constant.

$$e_t(w) \xrightarrow[T\to\infty]{a.s.} e_\infty$$

A4: Evaluation weights

Weights $h_t(w)$ are non-negative, depend on past data.

Main result

A5: Growth and decay conditions (Main assumption)

Unnormalized weights $h_t(w)$ and assignment probabilities $e_t(w)$ jointly satisfy

$$\frac{\sum_{t=1}^{T} \frac{h_t(w)^p}{e_t(w)^{p-1}}}{\left(\sum_{t=1}^{T} \mathbb{E}\left[\frac{h_t(w)^2}{e_t(w)}\right]\right)^{p/2}} \xrightarrow[T\to\infty]{p} \begin{cases} \infty & \text{if } p = 1 \\ 1 & \text{if } p = 2 \\ 0 & \text{if } p \in \{3,4\} \end{cases}$$

In addition, for $p \in \{2,3,4\}$ this is bounded.

Joint constraint:

- *Data-collection algorithm*: $e_t(w)$ must decay slowly
- *Evaluation weights*: $h_t(w)$ chosen appropriately

Main result: Consistency and Asymptotic Normality

Theorem

If Assumption 1-5 are satisfied, then $\widehat{Q}_T^{AW}$ is a consistent estimator of the true arm value Q, and the following statistic is asymptotically normal.

$$\frac{\widehat{Q}_T(w)^{AW} - Q(w)}{\widehat{V}^{\frac{1}{2}}} \xrightarrow[T \to \infty]{d} \mathcal{N}(0, 1)$$

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- **Notation:** $\widehat{V}_T$ is estimate of variance

Weighting schemes

Propensity-score based (`ps`)

Seeks minimum variance. When applied to IPW, gets us back approximately to simple averaging. Does not always satisfy A5.

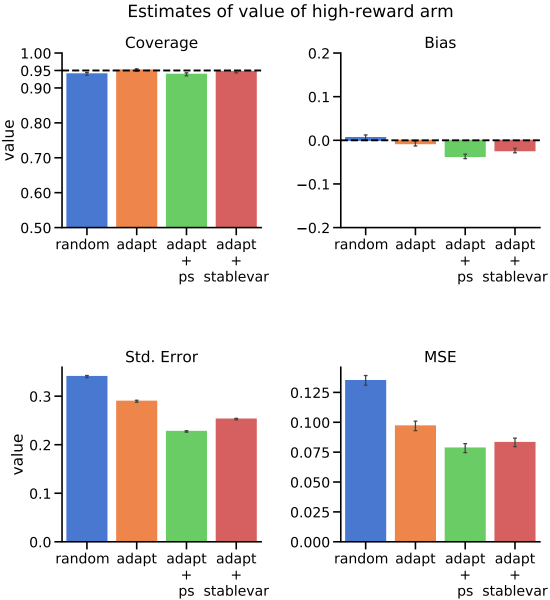
$$h_t = e_t(w)$$

Variance-stabilizing `stablevar`

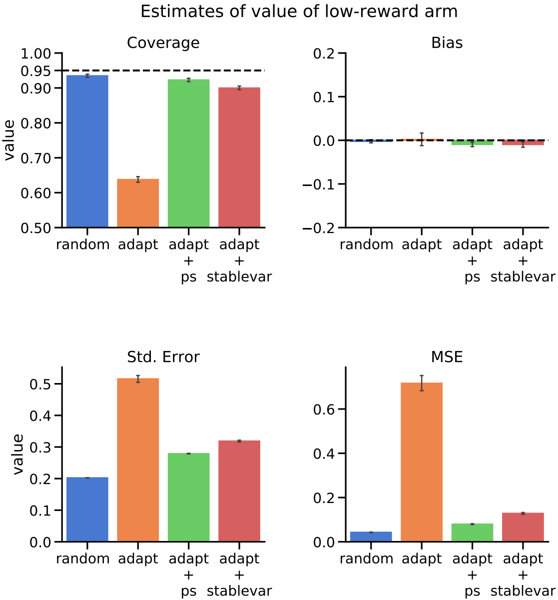
Ensures assumption A5(2) is satisfied.

$$h_t^2 = \left(1 - \sum_{s=1}^{t-1} h_s^2 e_s^{-1}\right) \mathbb{E}\left[\frac{e_t(w)}{\sum_{s=t}^{T} e_s} \,\Big|\, \mathcal{F}_{t-1}\right] e_t(w)$$

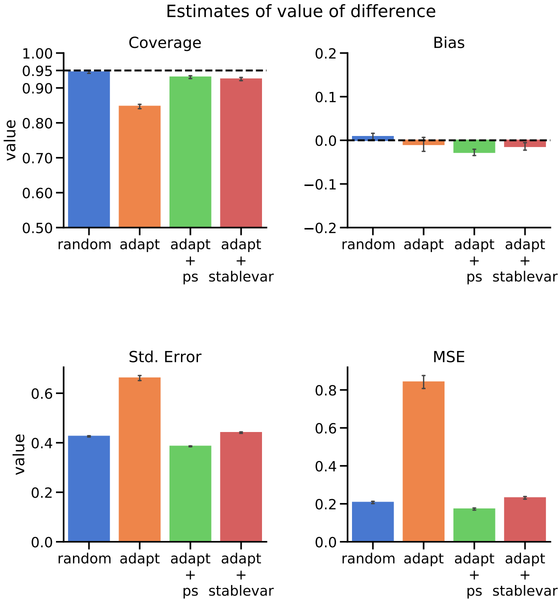
High-reward arms are (still) estimated well



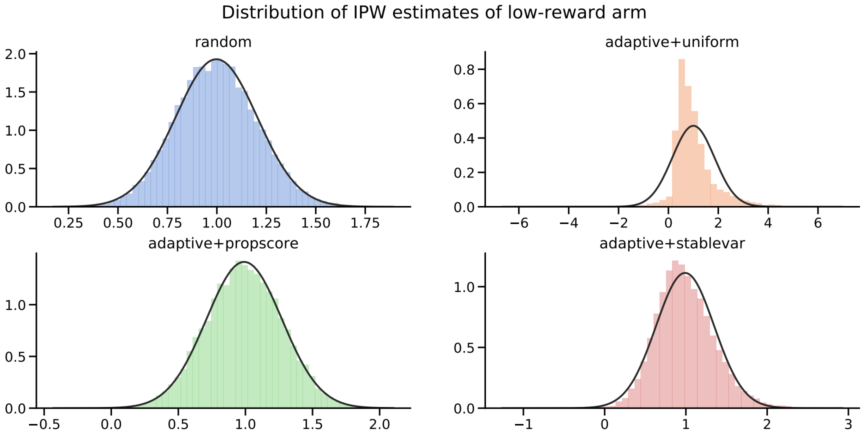
Low-reward arms are estimated well too



Difference is estimated well too



Normality is restored



Concluding thoughts

Appropriately chosen adaptive evaluation weights:

- Yield valid confidence intervals
- Produce estimates with smaller MSE
- Still allow for adaptive data collection

However: For hypothesis testing, beware of too-aggressive algorithms!