

3

1. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f . Which of the following statements must be true?

(A) $f(0) = 2$
(B) $f(x) \neq 2$ for all $x \geq 0$
(C) $f(2)$ is undefined.
(D) $\lim_{x \rightarrow 2} f(x) = \infty$
(E) $\lim_{x \rightarrow \infty} f(x) = 2$

2. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) $-\frac{4}{\pi}$
(B) -1
(C) 0
(D) $\frac{\sqrt{2}}{2}$
(E) $\frac{4}{\pi}$

3.

What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

(A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66


4.

x	0	2	4	6	8	10
$f(x)$	5	7	8	0	-15	-20

Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \leq x \leq 10$?

(A) -6
(B) $-\frac{5}{2}$
(C) -2
(D) $-\frac{2}{5}$
(E) $\frac{2}{5}$

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5. If f is the function given by $f(x) = 3x^2 - x^3$, then the average rate of change of f on the closed interval $[1, 5]$ is
- (A) -21
(B) -13
(C) -12
(D) -9
6.  The function f is given by $f(x) = \int_1^x \sqrt{t^3 + 2} dt$. What is the average rate of change of f over the interval $[0, 3]$?
- (A) 1.324
(B) 1.497
(C) 1.696
(D) 2.266
(E) 2.694
7. For time $t \geq 1$, the position of a particle moving along the x -axis is given by $p(t) = \sqrt{t} - 2$. At what time t in the interval $1 \leq t \leq 16$ is the instantaneous velocity of the particle equal to the average velocity of the particle over the interval $1 \leq t \leq 16$?
- (A) 1
(B) $\frac{121}{25}$
(C) $\frac{25}{4}$
(D) 25
8. For values of h very close to 0, which of the following functions best approximates $f(x) = \frac{\tan(x+h) - \tan x}{h}$?
- (A) $\sin x$
(B) $\frac{\sin x}{x}$
(C) $\frac{\tan x}{x}$
(D) $\sec x$
(E) $\sec^2 x$
9. $\lim_{h \rightarrow 0} \frac{\ln(4+h) - \ln(4)}{h}$ is
- (A) 0
(B) $\frac{1}{4}$
(C) 1
(D) e
(E) nonexistent

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10. If $f(x) = 3x^2 + 2x$, then $f'(x) =$

- (A) $\lim_{h \rightarrow 0} \frac{(3x^2 + 2x + h) - (3x^2 + 2x)}{h}$
- (B) $\lim_{x \rightarrow 0} \frac{(3x^2 + 2x + h) - (3x^2 + 2x)}{h}$
- (C) $\lim_{h \rightarrow 0} \frac{(3(x+h)^2 + 2(x+h)) - (3x^2 + 2x)}{h}$
- (D) $\lim_{x \rightarrow 0} \frac{(3(x+h)^2 + 2(x+h)) - (3x^2 + 2x)}{h}$

11. if $f(x) = (x^2 + 1)^3$, what is $\lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x + 1}$?

- (A) -24
- (B) -8
- (C) 0
- (D) 12

12. $\lim_{x \rightarrow e} \frac{(x^{20} - 3x) - (e^{20} - 3e)}{x - e}$ is

- (A) 0
- (B) $20e^{19} - 3$
- (C) $e^{20} - 3e$
- (D) nonexistent

13. Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

- (A) -5
- (B) $-\frac{15}{4}$
- (C) -1
- (D) 6
- (E) 17

3

14.

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t , in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

- (A) 37
- (B) 49
- (C) 58
- (D) 65
- (E) 92

15. For which of the following does $\lim_{x \rightarrow \infty} f(x) = 0$?

I. $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III. $f(x) = \frac{x^{99}}{e^x}$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

16. Which of the following limits are equal to -1 ?

I. $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

II. $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{3 - x}$

III. $\lim_{x \rightarrow \infty} \frac{1 - x}{1 + x}$

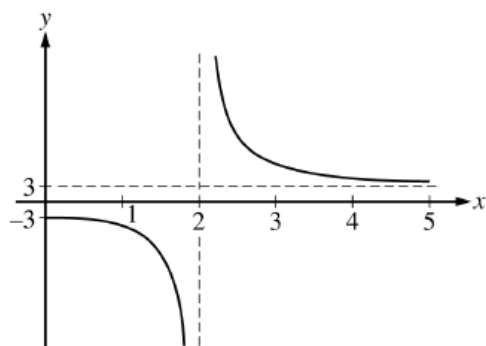
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- (A) I only
- (B) I and III only
- (C) II and III only
- (D) I, II, and III only

17. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

- (A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.
- (B) $f(3) = 15$
- (C) f attains a maximum on the open interval $(2, 4)$.
- (D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.
- (E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

18.



The function f is given by $f(x) = \frac{ax^2+12}{x^2+b}$. The figure above shows a portion of the graph of f . Which of the following could be the values of the constants a and b ?

- (A) $a = -3$, $b = 2$
- (B) $a = 2$, $b = -3$
- (C) $a = 2$, $b = -2$
- (D) $a = 3$, $b = -4$
- (E) $a = 3$, $b = 4$

19. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

- (A) $y = \frac{\sin(5x)}{x}$
- (B) $y = 5x$
- (C) $y = \frac{1}{x-5}$
- (D) $y = \frac{5x}{1-x}$
- (E) $y = \frac{20x^2-x}{1+4x^2}$

20. What is $\lim_{x \rightarrow \infty} \frac{x^2-4}{2+x-4x^2}$?

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- (A) -2
- (B) $-\frac{1}{4}$
- (C) $\frac{1}{2}$
- (D) 1
- (E) The limit does not exist.

21. $\lim_{x \rightarrow \infty} \frac{x^3}{e^{3x}}$ is

- (A) 0
- (B) $\frac{2}{9}$
- (C) $\frac{2}{3}$
- (D) 1
- (E) infinite

22. Let f be a function that is continuous on the closed interval $[1,3]$ with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?

- (A) $10 \leq f(2) \leq 18$
- (B) f is increasing on the interval $[1, 3]$.
- (C) $f(x) = 17$ has at least one solution in the interval $[1, 3]$.
- (D) $f'(x) = 8$ has at least one solution in the interval $(1, 3)$.
- (E) $\int_1^3 f(x) dx > 20$

23. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

- (A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.
- (B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$
- (C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$
- (D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$
- (E) For all h in the open interval $(0,1)$, $\lim_{x \rightarrow h} g(x) = g(h)$

24.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

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- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2
- (E) 3

25.

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

The table above shows selected values of a continuous function f . For $0 \leq x \leq 13$, what is the fewest possible number of times $f(x) = 4$?

- (A) one
 - (B) two
 - (C) three
 - (D) four
26. Let f be a function such that $f(1) = -2$ and $f(5) = 7$. Which of the following conditions ensures that $f(c) = 0$ for some value c in the open interval $(1, 5)$?
- (A) $\int_1^5 f(x) dx$ exists.
 - (B) f is increasing on the closed interval $[1, 5]$.
 - (C) f is continuous on the closed interval $[1, 5]$.
 - (D) f is defined for all values of x in the closed interval $[1, 5]$.
27. Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?
- I. f has at least 2 zeros.
 - II. The graph of f has at least one horizontal tangent.
 - III. For some c , $2 < c < 5$, $f(c) = 3$.
- (A) None
 - (B) I only
 - (C) I and II only
 - (D) I and III only
 - (E) I, II, and III
28. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

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- (A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.
- (B) $f'(-1) = 0$
- (C) The graph of f has a horizontal asymptote.
- (D) The graph of f has a horizontal tangent line at $x = 3$.
- (E) The graph of f intersects both axes.
29. Let f be a function such that $f(1) = -2$ and $f(5) = 7$. Which of the following conditions ensures that $f(c) = 0$ for some value c in the open interval f of c , equals 0?
- (A) $\int_1^5 f(x)dx$ exists
- (B) f is increasing on the closed interval $[1, 5]$.
- (C) f is continuous on the closed interval $[1, 5]$.
- (D) f is defined for all values of x in the closed interval $[1, 5]$
30. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that
- (A) $f(0) = 0$
- (B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6
- (C) $-1 \leq f(x) \leq 3$ for all x between -3 and 6
- (D) $f(c) = 1$ for at least one c between -3 and 6
- (E) $f(c) = 0$ for at least one c between -1 and 3
31. A polynomial $p(x)$ has a relative maximum at $(-2, 4)$, a relative minimum at $(1, 1)$, a relative maximum at $(5, 7)$ and no other critical points. How many zeros does $p(x)$ have?
- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) Five
32. Let g be a function such that $g(-1) = 0$ and $g(2) = 5$. Which of the following conditions guarantees that there is an x , $-1 < x < 2$, for which $g(x) = 3$?
- (A) g is defined for all x in $(-1, 2)$.
- (B) g is continuous for all x in $[-1, 2]$.
- (C) g is increasing on $[-1, 2]$.
- (D) There exists an x in $(-1, 2)$ such that $g'(x) = 5$.
- (E) $\int_{-1}^2 g(x)dx = 3$

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33. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?
- (A) $\lim_{x \rightarrow 0^+} f(x) = \infty$ and $\lim_{x \rightarrow 2} f(x) = \infty$
- (B) $\lim_{x \rightarrow 0^+} f(x) = 2$ and $\lim_{x \rightarrow \infty} f(x) = 0$
- (C) $\lim_{x \rightarrow 0^+} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = 2$
- (D) $\lim_{x \rightarrow 2} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = 2$
34. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?
- (A) $y = \frac{1}{x^2+1}$
- (B) $y = \frac{1}{x^3+1}$
- (C) $y = \frac{1}{e^x-1}$
- (D) $y = \frac{1}{e^x+1}$
35. What are the equations of the horizontal asymptotes of the graph of $y = \frac{2x}{\sqrt{x^2-1}}$?
- (A) $y = 0$ only
- (B) $y = 1$ only
- (C) $y = 2$ only
- (D) $y = -2$ only and $y = 2$ only
- (E) $y = -1$ only and $y = 1$ only
36. Let f be the function given by $f(x) = \frac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f , then $b =$
- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4
37. The function f is given by $f(x) = \sin\left(\frac{x+1}{x^2}\right)$. Which of the following statements are true?
- I. The graph of f has a horizontal asymptote at $y=0$.
- II. The graph of f has a horizontal asymptote at $y=1$.
- III. The graph of f has a vertical asymptote at $x=0$.

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- (A) I only
- (B) II only
- (C) III only
- (D) I and III only
- (E) II and III only

38. Let f be the function defined by $f(x) = \frac{(3x+8)(5-4x)}{(2x+1)^2}$. Which of the following is a horizontal asymptote to the graph of f ?

- (A) $y = -6$
- (B) $y = -3$
- (C) $y = -\frac{1}{2}$
- (D) $y = 0$
- (E) $y = \frac{3}{2}$

39. The graph of which of the following equations has $y = 1$ as an asymptote?

- (A) $y = \ln x$
- (B) $y = \sin x$
- (C) $y = \frac{x}{x+1}$
- (D) $y = \frac{x^2}{x-1}$
- (E) $y = e^{-x}$

40. The function f is not differentiable at $x = 5$. Which of the following statements must be true?

- (A) f is not continuous at $x = 5$.
- (B) $\lim_{x \rightarrow 5} f(x)$ does not exist.
- (C) $\lim_{x \rightarrow 5} \frac{f(x) - f(5)}{x - 5}$ does not exist.
- (D) $\int_0^5 f(x) dx$ does not exist.

41.

x	0	1	2	3	4	5
$f(x)$	1	-5	-4	2	-10	-15

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[0, 5]$?

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- (A) Zero, because $f(x)$ is not equal to 0 for any of the values in the table.
- (B) One, because f is continuous on the interval $[0, 5]$ and $f(0) > 0 > f(5)$.
- (C) Two, because the values for $f(x)$ in the table change from positive to negative twice.
- (D) Three, because f is continuous on the interval $[0, 5]$ and $f(0) > 0 > f(1)$, $f(1) < 0 < f(3)$, and $f(3) > 0 > f(5)$.

42.

x	-2	-1	0	1	2
$g(x)$	-4	-1	4	11	20

The table above gives values of a function g at selected values of x . Which of the following statements, if true, would be individually sufficient to conclude that there exists a number c in the interval $[-2, 2]$ such that $g(c) = 6$?

- I. g is defined for all x in the interval $[-2, 2]$.
- II. g is increasing on the interval $[-2, 2]$.
- III. g is continuous on the interval $[-2, 2]$.

- (A) II only
- (B) III only
- (C) I and III only
- (D) I, II, and III

43.

x	-2	-1	0	1	2	3
$f(x)$	-2	5	2	-4	-1	3

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[-2, 3]$?

- (A) Zero
- (B) One
- (C) Two
- (D) Three

44.

x	2	3	4
$g(x)$	-2	1	-4

Selected values of the continuous function g are given in the table above. Which of the following statements must be true?

- (A) The minimum value of g on the interval $[2, 4]$ is -4 .
- (B) $g'(x) = -1$ has at least one solution in the interval $[2, 4]$.
- (C) $g'(x) = 0$ has at least two solutions in the interval $[2, 4]$.
- (D) $g(x) = 0$ has at least two solutions in the interval $[2, 4]$.

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45.

x	0	1	3	7
$g(x)$	24	35	42	68

The table above gives values of a function g at selected values of x . Which of the following statements, if true, would be sufficient to conclude that there exists a number c in the interval $[0, 7]$ such that $g(c) = 50$?

- I. g is defined for all x in the interval $[0, 7]$.
- II. g is increasing for all x in the interval $[0, 7]$.
- III. g is continuous for all x in the interval $[0, 7]$.


- (A) II only
- (B) III only
- (C) I and III only
- (D) I, II, and III

46.  Let f be the function given by $f(x) = \frac{9+2xe^{-\frac{x}{4}}}{\cos(\frac{x}{2})}$. The Intermediate Value Theorem applied to f on the closed interval $[24, 28]$ guarantees a solution in $[24, 28]$ to which of the following equations?

- (A) $f(x) = 0$
- (B) $f(x) = 9.090$
- (C) $f(x) = 12.235$
- (D) $f(x) = 76.999$

47. The function f is continuous on the closed interval $[a, b]$, where a and b are constants and $a < b$. If $f(a) < 0 < f(b)$, which of the following statements must be true?

- (A) There is at least one value of x in the interval (a, b) such that $f(x) = 0$.
- (B) There is at least one value of x in the interval (a, b) such that $f'(x) = \frac{f(b)-f(a)}{b-a}$.
- (C) $f'(x) > 0$ for all x in the interval (a, b) .
- (D) $f(a) < f(x) < f(b)$ for all x in the interval (a, b) .

48.  $f(x) = \frac{x^4+x^3+x^2+x+1}{380-\ln\left(\frac{x^2+1}{2}\right)}$

Let f be the function defined above. The Intermediate Value Theorem applied to f on the closed interval $[10, 12]$ guarantees a solution in $[10, 12]$ to which of the following equations?

- (A) $f(x) = 0$
- (B) $f(x) = 27.372$
- (C) $f(x) = 42.421$
- (D) $f(x) = 67.205$

49. $\lim_{n \rightarrow \infty} \frac{3n^3-5n}{n^3-2n^2+1}$ is

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- (A) -5
- (B) -2
- (C) 1
- (D) 3
- (E) nonexistent

50. $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4+1}}{4x^2+3}$

- (A) 1/3
- (B) 3/4
- (C) 3/2
- (D) 9/4
- (E) infinite

51. $\lim_{x \rightarrow \infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$

- (A) 4
- (B) 1
- (C) 1/4
- (D) 0
- (E) -1

52. For which of the following pairs of functions f and g is $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$ infinite?

- (A) $f(x) = x^2 + 2x$ and $g(x) = x^2 + \ln x$
- (B) $f(x) = 3x^3$ and $g(x) = x^4$
- (C) $f(x) = 3^x$ and $g(x) = x^3$
- (D) $f(x) = 3e^x + x^3$ and $g(x) = 2e^x + x^2$
- (E) $f(x) = \ln(3x)$ and $g(x) = \ln(2x)$

53. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$, then $a+c=$

- (A) -5
- (B) -1
- (C) 0
- (D) 1
- (E) 5

54. $\lim_{x \rightarrow -\infty} \frac{3+2^x}{4-5^x}$ is

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- (A) $-2/5$
- (B) 0
- (C) $3/4$
- (D) non existent

55. $\lim_{x \rightarrow \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is

- (A) -3
- (B) -2
- (C) 2
- (D) 3
- (E) nonexistent

56. $g(x) = \begin{cases} x^2 + 1 & \text{for } x < 3 \\ 13 - x & \text{for } x \geq 3 \end{cases}$

Let g be the function given above. What is the average rate of change of g on the closed interval $[1, 5]$?

- (A) $\frac{1}{2}$
- (B) $\frac{3}{2}$
- (C) 5
- (D) Undefined

57. $\lim_{x \rightarrow \infty} \frac{x+7}{\sqrt{2x^2+49}}$ is

- (A) 0
- (B) $\frac{1}{2}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) 1

58. $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2+27}}{\sqrt{x^2+27x+3}}$ is

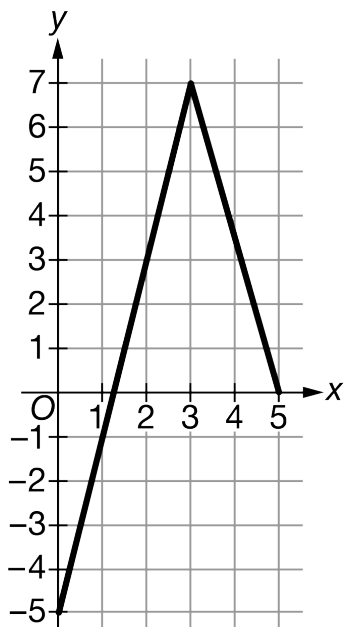
- (A) 2
- (B) 3
- (C) 4
- (D) infinite

59. If f is a function such that $\lim_{x \rightarrow \infty} f(x) = 0$, which of the following could be an expression for $f(x)$?

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- (A) $\frac{x^2}{x^2 + 4}$
(B) $\frac{\sin x}{x}$
(C) $\cos x$
(D) e^x

60.

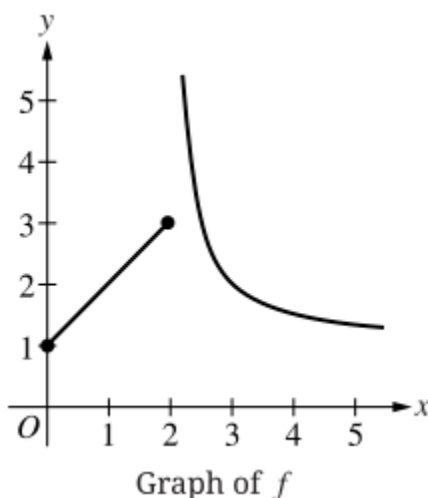
Graph of f

The graph of a function f is shown above. If g is the function defined by $g(x) = \frac{x^2+1}{f(x)}$, what is the value of $g'(2)$?

- (A) $-\frac{8}{9}$
(B) $\frac{1}{9}$
(C) 1
(D) $\frac{32}{9}$
61. The function g is defined for all positive x , and $0 < g(x) < x$. Of the following, $\lim_{x \rightarrow \infty} \frac{x}{g(x)}$ could not be
- (A) 0
(B) 1
(C) 3
(D) ∞

3

62.



The figure shows the graph of a piecewise function f with domain $[0, \infty)$. The function f has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 1$. Which of the following statements are true?

- I. $\lim_{x \rightarrow 2^-} f(x)$ exists.
- II. $\lim_{x \rightarrow 2} f(x)$ exists.
- III. $\lim_{x \rightarrow \infty} f(x) = \infty$

- (A) I only
- (B) I and II only
- (C) I and III only
- (D) II and III only

63. Let f be the function given by $f(x) = \frac{e^{2x}}{1 - e^{2x}}$. Which of the following describes all asymptotes of the graph of f ?

- (A) No vertical asymptotes and two horizontal asymptotes
- (B) One vertical asymptote and no horizontal asymptotes
- (C) One vertical asymptote and two horizontal asymptotes
- (D) Two vertical asymptotes and two horizontal asymptotes

64. Let f be the function defined by $f(x) = e^{2x}$. The average rate of change of f over the interval $[1, b]$ is 20, where $b > 1$. Which of the following is an equation that could be used to find the value of b ?

- (A) $f(b) = 20$
- (B) $f(b) - f(1) = 20$
- (C) $\frac{f(b) - f(1)}{b - 1} = 20$
- (D) $\frac{f(b) + f(1)}{2} = 20$

3

65.

x	-3	-1	1	3
$g(x)$	12	0	1	-4

Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-3, 3]$?

- (A) $\frac{3-(-3)}{(-4)-12}$
(B) $\frac{(-4)-12}{3-(-3)}$
(C) $\frac{12+(-4)}{2}$
(D) $\frac{12+0+1+(-4)}{4}$

66. Let f be the function defined by $f(x) = \sec x + \csc x$. Which of the following expressions is the average rate of change of f over the interval $[\frac{\pi}{4}, \frac{3\pi}{8}]$?

- (A) $\frac{f(\frac{3\pi}{8})+f(\frac{\pi}{4})}{2}$
(B) $\frac{f(\frac{3\pi}{8})+f(\frac{\pi}{4})}{\frac{3\pi}{8}+\frac{\pi}{4}}$
(C) $\frac{f(\frac{3\pi}{8})-f(\frac{\pi}{4})}{\frac{3\pi}{8}-\frac{\pi}{4}}$
(D) $f(\frac{3\pi}{8}) - f(\frac{\pi}{4})$

67. Let f be the function defined by $f(x) = 2 \sin x + \cos x$. The average rate of change of f over the interval $[0, b]$ is 0.05, where $b > 0$. Which of the following is an equation that could be used to find the value of b ?

- (A) $f(b) = 0.05$
(B) $f(b) - f(0) = 0.05$
(C) $\frac{f(b)-f(0)}{b-0} = 0.05$
(D) $\frac{f(b)+f(0)}{2} = 0.05$

68.

x	-2	-1	0	1	2
$g(x)$	-3	2	1	0	5

Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-2, 2]$?

- (A) $\frac{2-(-2)}{5-(-3)}$
(B) $\frac{5-(-3)}{2-(-2)}$
(C) $\frac{5+(-3)}{2}$
(D) $\frac{-3+2+1+0+5}{5}$

3

69.

x	0	2	6	12	20
$f(x)$	-8	-2	0	2	18

The function f is differentiable and has values as shown in the table above. Of the following, on which interval is the average rate of change of f greatest?

- (A) $[0, 2]$
(B) $[2, 6]$
(C) $[6, 12]$
(D) $[12, 20]$
70. Let f be the function defined by $f(x) = 2x^3 - x$. Which of the following expressions is the average rate of change of f on the interval $[1, 3]$?
- (A) $\frac{f(3)+f(1)}{2}$
(B) $\frac{f(3)+f(1)}{3+1}$
(C) $\frac{f(3)-f(1)}{3-1}$
(D) $f(3) - f(1)$
71. Let f be the function defined by $f(x) = \frac{1-5x-2x^2}{3x^2+7}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = -\frac{2}{3}$
(B) $y = \frac{1}{3}$
(C) $y = \frac{2}{3}$
(D) There is no horizontal asymptote to the graph of f .
72. Let f be the function defined by $f(x) = \frac{2^x+5}{e^x+1}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = 0$
(B) $y = \frac{2}{e}$
(C) $y = 1$
(D) There is no horizontal asymptote to the graph of f .
73. Let f be the function given by $f(x) = \frac{1+e^x \sin x}{e^{x-1}-1}$. What are all horizontal asymptotes of the graph of f ?
- (A) $y = -1$ only
(B) $y = 1$ only
(C) $y = -1$ and $y = 1$
(D) The graph of f has no horizontal asymptotes.

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74. Let f be the function defined by $f(x) = \frac{3^x+2}{e^{2x}+1}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = 0$
(B) $y = \frac{3}{e^2}$
(C) $y = 1$
(D) There is no horizontal asymptote to the graph of f .
75. Let f be the function defined by $f(x) = \frac{2-x+3x^2+5x^3-7x^4}{x^4-2x^3-5x^2+2x-3}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = -7$
(B) $y = 2$
(C) $y = 7$
(D) There is no horizontal asymptote to the graph of f .
76. Let f be the function given by $f(x) = \frac{(\cos x)e^{2x}-1}{e^{2x-1}+2}$. What are all horizontal asymptotes to the graph of f ?
- (A) $y = -\frac{1}{2}$ only
(B) $y = e$ only
(C) $y = -\frac{1}{2}$ and $y = e$
(D) The graph of f has no horizontal asymptotes.
77. $\lim_{x \rightarrow \infty} \frac{10-6x^2}{5+3e^x}$ is
- (A) -2
(B) 0
(C) 2
(D) nonexistent