

3

1. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f . Which of the following statements must be true?

- (A) $f(0) = 2$
(B) $f(x) \neq 2$ for all $x \geq 0$
(C) $f(2)$ is undefined.
(D) $\lim_{x \rightarrow 2} f(x) = \infty$
(E) $\lim_{x \rightarrow \infty} f(x) = 2$

2. What is the average rate of change of $y = \cos(2x)$ on the interval $[0, \frac{\pi}{2}]$?

- (A) $-\frac{4}{\pi}$
(B) -1
(C) 0
(D) $\frac{\sqrt{2}}{2}$
(E) $\frac{4}{\pi}$

3.

What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval $[0, 3]$?

- (A) 8.5
(B) 8.7
(C) 22
(D) 33
(E) 66

4.

x	0	2	4	6	8	10
$f(x)$	5	7	8	0	-15	-20

Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \leq x \leq 10$?

- (A) -6
(B) $-\frac{5}{2}$
(C) -2
(D) $-\frac{2}{5}$
(E) $\frac{2}{5}$

5. If f is the function given by $f(x) = 3x^2 - x^3$, then the average rate of change of f on the closed interval $[1, 5]$ is
- (A) -21
(B) -13
(C) -12
(D) -9
6. The function f is given by $f(x) = \int_1^x \sqrt{t^3 + 2} dt$. What is the average rate of change of f over the interval $[0, 3]$?
- (A) 1.324
(B) 1.497
(C) 1.696
(D) 2.266
(E) 2.694
7. For time $t \geq 1$, the position of a particle moving along the x -axis is given by $p(t) = \sqrt{t} - 2$. At what time t in the interval $1 \leq t \leq 16$ is the instantaneous velocity of the particle equal to the average velocity of the particle over the interval $1 \leq t \leq 16$
- (A) 1
(B) $\frac{121}{25}$
(C) $\frac{25}{4}$
(D) 25
8. For values of h very close to 0, which of the following functions best approximates $f(x) = \frac{\tan(x+h)-\tan x}{h}$?
- (A) $\sin x$
(B) $\frac{\sin x}{x}$
(C) $\frac{\tan x}{x}$
(D) $\sec x$
(E) $\sec^2 x$
9. $\lim_{h \rightarrow 0} \frac{\ln(4+h)-\ln(4)}{h}$ is
- (A) 0
(B) $\frac{1}{4}$
(C) 1
(D) e
(E) nonexistent

10. If $f(x) = 3x^2 + 2x$, then $f'(x) =$

- (A) $\lim_{h \rightarrow 0} \frac{(3x^2+2x+h)-(3x^2+2x)}{h}$
- (B) $\lim_{x \rightarrow 0} \frac{(3x^2+2x+h)-(3x^2+2x)}{h}$
- (C) $\lim_{h \rightarrow 0} \frac{(3(x+h)^2+2(x+h))-(3x^2+2x)}{h}$
- (D) $\lim_{x \rightarrow 0} \frac{(3(x+h)^2+2(x+h))-(3x^2+2x)}{h}$

11. if $f(x) = (x^2 + 1)^3$, what is $\lim_{x \rightarrow -1} \frac{f(x)-f(-1)}{x+1}$?

- (A) -24
(B) -8
(C) 0
(D) 12

12. $\lim_{x \rightarrow e} \frac{(x^{20}-3x)-(e^{20}-3e)}{x-e}$ is

- (A) 0
(B) $20e^{19} - 3$
(C) $e^{20} - 3e$
(D) nonexistent

13. Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

- (A) -5
(B) $-\frac{15}{4}$
(C) -1
(D) 6
(E) 17

14.

t (hours)	0	1	2	3	4	5	6
$s(t)$ (miles)	0	25	55	92	150	210	275

The table above gives the distance $s(t)$, in miles, that a car has traveled at various times t , in hours, during a 6-hour trip. The graph of the function s is increasing and concave up. Based on the information, which of the following could be the velocity of the car, in miles per hour, at time $t = 3$?

- (A) 37
- (B) 49
- (C) 58
- (D) 65
- (E) 92

15. For which of the following does $\lim_{x \rightarrow \infty} f(x) = 0$?

I. $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III. $f(x) = \frac{x^{99}}{e^x}$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I and III only

16. Which of the following limits are equal to -1 ?

I. $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$

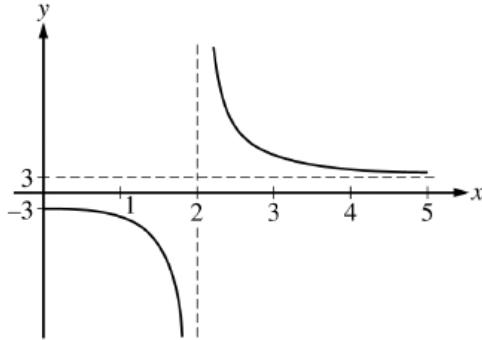
II. $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{3-x}$

III. $\lim_{x \rightarrow \infty} \frac{1-x}{1+x}$

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- (A) I only
 (B) I and III only
 (C) II and III only
 (D) I, II, and III only
17. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?
 (A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.
 (B) $f(3) = 15$
 (C) f attains a maximum on the open interval $(2, 4)$.
 (D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.
 (E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

18.



The function f is given by $f(x) = \frac{ax^2+12}{x^2+b}$. The figure above shows a portion of the graph of f . Which of the following could be the values of the constants a and b ?

- (A) $a = -3, b = 2$
 (B) $a = 2, b = -3$
 (C) $a = 2, b = -2$
 (D) $a = 3, b = -4$
 (E) $a = 3, b = 4$
19. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?
 (A) $y = \frac{\sin(5x)}{x}$
 (B) $y = 5x$
 (C) $y = \frac{1}{x-5}$
 (D) $y = \frac{5x}{1-x}$
 (E) $y = \frac{20x^2-x}{1+4x^2}$
20. What is $\lim_{x \rightarrow \infty} \frac{x^2-4}{2+x-4x^2}$?

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- (A) -2
 (B) $-\frac{1}{4}$
 (C) $\frac{1}{2}$
 (D) 1
 (E) The limit does not exist.
21. $\lim_{x \rightarrow \infty} \frac{x^3}{e^{3x}}$ is
 (A) 0
 (B) $\frac{2}{9}$
 (C) $\frac{2}{3}$
 (D) 1
 (E) infinite
22. Let f be a function that is continuous on the closed interval $[1,3]$ with $f(1) = 10$ and $f(3) = 18$. Which of the following statements must be true?
 (A) $10 \leq f(2) \leq 18$
 (B) f is increasing on the interval $[1, 3]$.
 (C) $f(x) = 17$ has at least one solution in the interval $[1, 3]$.
 (D) $f'(x) = 8$ has at least one solution in the interval $(1, 3)$.
 (E) $\int_1^3 f(x) dx > 20$
23. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?
 (A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.
 (B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$
 (C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$
 (D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$
 (E) For all h in the open interval $(0,1)$, $\lim_{x \rightarrow h} g(x) = g(h)$
- 24.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k=$

3

- (A) 0
 (B) $\frac{1}{2}$
 (C) 1
 (D) 2
 (E) 3

25.

x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

The table above shows selected values of a continuous function f . For $0 \leq x \leq 13$, what is the fewest possible number of times $f(x) = 4$?

- (A) one
 (B) two
 (C) three
 (D) four
26. Let f be a function such that $f(1) = -2$ and $f(5) = 7$. Which of the following conditions ensures that $f(c) = 0$ for some value c in the open interval $(1, 5)$?
- (A) $\int_1^5 f(x)dx$ exists.
 (B) f is increasing on the closed interval $[1, 5]$.
 (C) f is continuous on the closed interval $[1, 5]$.
 (D) f is defined for all values of x in the closed interval $[1, 5]$.
27. Let f be a function that is differentiable on the open interval $(1, 10)$. If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?
- I. f has at least 2 zeros.
 II. The graph of f has at least one horizontal tangent.
 III. For some c , $2 < c < 5$, $f(c) = 3$.
- (A) None
 (B) I only
 (C) I and II only
 (D) I and III only
 (E) I, II, and III
28. If a function f is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

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- (A) The graph of f has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f'(-1) = 0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x = 3$.
(E) The graph of f intersects both axes.
29. Let f be a function such that $f(1) = -2$ and $f(5) = 7$. Which of the following conditions ensures that $f(c) = 0$ for some value c in the open interval f of c , equals 0?
(A) $\int_1^5 f(x)dx$ exists
(B) f is increasing on the closed interval $[1, 5]$.
(C) f is continuous on the closed interval $[1, 5]$.
(D) f is defined for all values of x in the closed interval $[1, 5]$
30. Let f be a continuous function on the closed interval $[-3, 6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that
(A) $f(0)=0$
(B) $f'(c) = \frac{4}{9}$ for at least one c between -3 and 6
(C) $-1 \leq f(x) \leq 3$ for all x between -3 and 6
(D) $f(c)=1$ for at least one c between -3 and 6
(E) $f(c)=0$ for at least one c between -1 and 3
31. A polynomial $p(x)$ has a relative maximum at $(-2, 4)$, a relative minimum at $(1, 1)$, a relative maximum at $(5, 7)$ and no other critical points. How many zeros does $p(x)$ have?
(A) One
(B) Two
(C) Three
(D) Four
(E) Five
32. Let g be a function such that $g(-1) = 0$ and $g(2) = 5$. Which of the following conditions guarantees that there is an x , $-1 < x < 2$, for which $g(x) = 3$?
(A) g is defined for all x in $(-1, 2)$.
(B) g is continuous for all x in $[-1, 2]$.
(C) g is increasing on $[-1, 2]$.
(D) There exists an x in $(-1, 2)$ such that $g'(x) = 5$.
(E) $\int_{-1}^2 g(x)dx = 3$

33. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?
- (A) $\lim_{x \rightarrow 0^+} f(x) = \infty$ and $\lim_{x \rightarrow 2} f(x) = \infty$
(B) $\lim_{x \rightarrow 0^+} f(x) = 2$ and $\lim_{x \rightarrow \infty} f(x) = 0$
(C) $\lim_{x \rightarrow 0^+} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = 2$
(D) $\lim_{x \rightarrow 2} f(x) = \infty$ and $\lim_{x \rightarrow \infty} f(x) = 2$
34. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?
- (A) $y = \frac{1}{x^2+1}$
(B) $y = \frac{1}{x^3+1}$
(C) $y = \frac{1}{e^x-1}$
(D) $y = \frac{1}{e^x+1}$
35. What are the equations of the horizontal asymptotes of the graph of $y = \frac{2x}{\sqrt{x^2-1}}$?
- (A) $y = 0$ only
(B) $y = 1$ only
(C) $y = 2$ only
(D) $y = -2$ only and $y = 2$ only
(E) $y = -1$ only and $y = 1$ only
36. Let f be the function given by $f(x) = \frac{(x-4)(2x-3)}{(x-1)^2}$. If the line $y = b$ is a horizontal asymptote to the graph of f , then $b =$
- (A) 0
(B) 1
(C) 2
(D) 3
(E) 4
37. The function f is given by $f(x) = \sin\left(\frac{x+1}{x^2}\right)$. Which of the following statements are true?
- I. The graph of f has a horizontal asymptote at $y=0$.
II. The graph of f has a horizontal asymptote at $y=1$.
III. The graph of f has a vertical asymptote at $x=0$.

3

- (A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only
38. Let f be the function defined by $f(x) = \frac{(3x+8)(5-4x)}{(2x+1)^2}$. Which of the following is a horizontal asymptote to the graph of f ?
(A) $y = -6$
(B) $y = -3$
(C) $y = -\frac{1}{2}$
(D) $y = 0$
(E) $y = \frac{3}{2}$
39. The graph of which of the following equations has $y = 1$ as an asymptote?
(A) $y = \ln x$
(B) $y = \sin x$
(C) $y = \frac{x}{x+1}$
(D) $y = \frac{x^2}{x-1}$
(E) $y = e^{-x}$
40. The function f is not differentiable at $x = 5$. Which of the following statements must be true?
(A) f is not continuous at $x = 5$.
(B) $\lim_{x \rightarrow 5} f(x)$ does not exist.
(C) $\lim_{x \rightarrow 5} \frac{f(x)-f(5)}{x-5}$ does not exist.
(D) $\int_0^5 f(x) dx$ does not exist.

41.

x	0	1	2	3	4	5
$f(x)$	1	-5	-4	2	-10	-15

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[0, 5]$?

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- (A) Zero, because $f(x)$ is not equal to 0 for any of the values in the table.
- (B) One, because f is continuous on the interval $[0, 5]$ and $f(0) > 0 > f(5)$.
- (C) Two, because the values for $f(x)$ in the table change from positive to negative twice.
- (D) Three, because f is continuous on the interval $[0, 5]$ and $f(0) > 0 > f(1)$, $f(1) < 0 < f(3)$, and $f(3) > 0 > f(5)$.

42.

x	-2	-1	0	1	2
$g(x)$	-4	-1	4	11	20

The table above gives values of a function g at selected values of x . Which of the following statements, if true, would be individually sufficient to conclude that there exists a number c in the interval $[-2, 2]$ such that $g(c) = 6$?

- I. g is defined for all x in the interval $[-2, 2]$.
- II. g is increasing on the interval $[-2, 2]$.
- III. g is continuous on the interval $[-2, 2]$.

- (A) II only
- (B) III only
- (C) I and III only
- (D) I, II, and III

43.

x	-2	-1	0	1	2	3
$f(x)$	-2	5	2	-4	-1	3

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[-2, 3]$?

- (A) Zero
- (B) One
- (C) Two
- (D) Three

44.

x	2	3	4
$g(x)$	-2	1	-4

Selected values of the continuous function g are given in the table above. Which of the following statements must be true?

- (A) The minimum value of g on the interval $[2, 4]$ is -4 .
- (B) $g'(x) = -1$ has at least one solution in the interval $[2, 4]$.
- (C) $g'(x) = 0$ has at least two solutions in the interval $[2, 4]$.
- (D) $g(x) = 0$ has at least two solutions in the interval $[2, 4]$.

45.

x	0	1	3	7
$g(x)$	24	35	42	68

The table above gives values of a function g at selected values of x . Which of the following statements, if true, would be sufficient to conclude that there exists a number c in the interval $[0, 7]$ such that $g(c) = 50$?

- I. g is defined for all x in the interval $[0, 7]$.
- II. g is increasing for all x in the interval $[0, 7]$.
- III. g is continuous for all x in the interval $[0, 7]$.

- (A) II only
- (B) III only
- (C) I and III only
- (D) I, II, and III

46. Let f be the function given by $f(x) = \frac{9+2xe^{-\frac{x}{4}}}{\cos(\frac{x}{2})}$. The Intermediate Value Theorem applied to f on the closed interval $[24, 28]$ guarantees a solution in $[24, 28]$ to which of the following equations?

- (A) $f(x) = 0$
- (B) $f(x) = 9.090$
- (C) $f(x) = 12.235$
- (D) $f(x) = 76.999$

47. The function f is continuous on the closed interval $[a, b]$, where a and b are constants and $a < b$. If $f(a) < 0 < f(b)$, which of the following statements must be true?

- (A) There is at least one value of x in the interval (a, b) such that $f(x) = 0$.
- (B) There is at least one value of x in the interval (a, b) such that $f'(x) = \frac{f(b)-f(a)}{b-a}$.
- (C) $f'(x) > 0$ for all x in the interval (a, b) .
- (D) $f(a) < f(x) < f(b)$ for all x in the interval (a, b) .

48. $f(x) = \frac{x^4+x^3+x^2+x+1}{380-\ln\left(\frac{x^2+1}{2}\right)}$

Let f be the function defined above. The Intermediate Value Theorem applied to f on the closed interval $[10, 12]$ guarantees a solution in $[10, 12]$ to which of the following equations?

- (A) $f(x) = 0$
- (B) $f(x) = 27.372$
- (C) $f(x) = 42.421$
- (D) $f(x) = 67.205$

49. $\lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

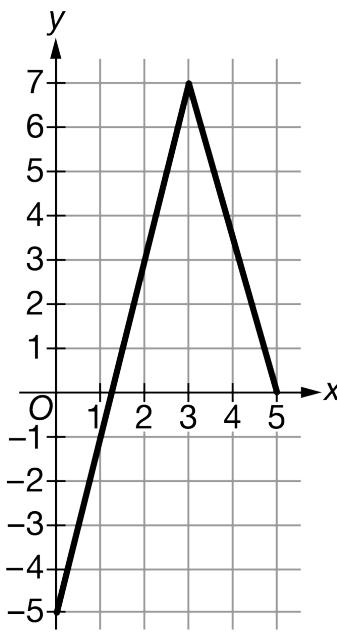
- (A) -5
(B) -2
(C) 1
(D) 3
(E) nonexistent
50. $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4+1}}{4x^2+3}$
(A) 1/3
(B) 3/4
(C) 3/2
(D) 9/4
(E) infinite
51. $\lim_{x \rightarrow \infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$
(A) 4
(B) 1
(C) 1/4
(D) 0
(E) -1
52. For which of the following pairs of functions f and g is $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$ infinite?
(A) $f(x) = x^2 + 2x$ and $g(x) = x^2 + \ln x$
(B) $f(x) = 3x^3$ and $g(x) = x^4$
(C) $f(x) = 3^x$ and $g(x) = x^3$
(D) $f(x) = 3e^x + x^3$ and $g(x) = 2e^x + x^2$
(E) $f(x) = \ln(3x)$ and $g(x) = \ln(2x)$
53. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$, then $a+c=$
(A) -5
(B) -1
(C) 0
(D) 1
(E) 5
54. $\lim_{x \rightarrow -\infty} \frac{3+2^x}{4-5^x}$ is

- (A) $-2/5$
(B) 0
(C) $3/4$
(D) non existent
55. $\lim_{x \rightarrow \infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is
(A) -3
(B) -2
(C) 2
(D) 3
(E) nonexistent
56.
$$g(x) = \begin{cases} x^2 + 1 & \text{for } x < 3 \\ 13 - x & \text{for } x \geq 3 \end{cases}$$
Let g be the function given above. What is the average rate of change of g on the closed interval $[1, 5]$?
(A) $\frac{1}{2}$
(B) $\frac{3}{2}$
(C) 5
(D) Undefined
57. $\lim_{x \rightarrow \infty} \frac{x+7}{\sqrt{2x^2+49}}$ is
(A) 0
(B) $\frac{1}{2}$
(C) $\frac{1}{\sqrt{2}}$
(D) 1
58. $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2+27}}{\sqrt{x^2+27x+3}}$ is
(A) 2
(B) 3
(C) 4
(D) infinite
59. If f is a function such that $\lim_{x \rightarrow \infty} f(x) = 0$, which of the following could be an expression for $f(x)$?

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- (A) $\frac{x^2}{x^2 + 4}$
(B) $\frac{\sin x}{x}$
(C) $\cos x$
(D) e^x

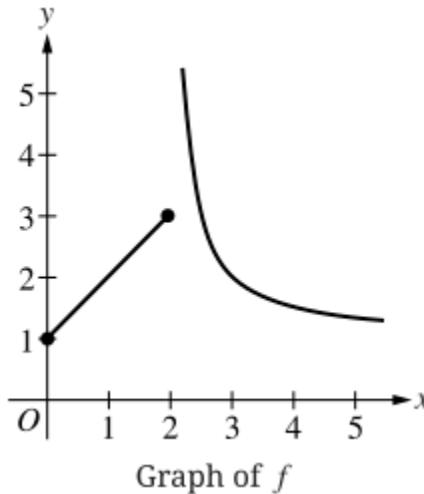
60.

Graph of f

The graph of a function f is shown above. If g is the function defined by $g(x) = \frac{x^2+1}{f(x)}$, what is the value of $g'(2)$?

- (A) $-\frac{8}{9}$
(B) $\frac{1}{9}$
(C) 1
(D) $\frac{32}{9}$
61. The function g is defined for all positive x , and $0 < g(x) < x$. Of the following, $\lim_{x \rightarrow \infty} \frac{x}{g(x)}$ could not be
(A) 0
(B) 1
(C) 3
(D) ∞

62.



- I. $\lim_{x \rightarrow 2^-} f(x)$ exists.
 II. $\lim_{x \rightarrow 2^+} f(x)$ exists.
 III. $\lim_{x \rightarrow \infty} f(x) = \infty$
- (A) I only
 (B) I and II only
 (C) I and III only
 (D) II and III only
63. Let f be the function given by $f(x) = \frac{e^{2x}}{1-e^{2x}}$. Which of the following describes all asymptotes of the graph of f ?
 (A) No vertical asymptotes and two horizontal asymptotes
 (B) One vertical asymptote and no horizontal asymptotes
 (C) One vertical asymptote and two horizontal asymptotes
 (D) Two vertical asymptotes and two horizontal asymptotes
64. Let f be the function defined by $f(x) = e^{2x}$. The average rate of change of f over the interval $[1, b]$ is 20, where $b > 1$. Which of the following is an equation that could be used to find the value of b ?
 (A) $f(b) = 20$
 (B) $f(b) - f(1) = 20$
 (C) $\frac{f(b)-f(1)}{b-1} = 20$
 (D) $\frac{f(b)+f(1)}{2} = 20$

65.

x	-3	-1	1	3
$g(x)$	12	0	1	-4

Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-3, 3]$?

- (A) $\frac{3-(-3)}{(-4)-12}$
 (B) $\frac{(-4)-12}{3-(-3)}$
 (C) $\frac{12+(-4)}{2}$
 (D) $\frac{12+0+1+(-4)}{4}$

66. Let f be the function defined by $f(x) = \sec x + \csc x$. Which of the following expressions is the average rate of change of f over the interval $[\frac{\pi}{4}, \frac{3\pi}{8}]$?

- (A) $\frac{f(\frac{3\pi}{8})+f(\frac{\pi}{4})}{2}$
 (B) $\frac{f(\frac{3\pi}{8})+f(\frac{\pi}{4})}{\frac{3\pi}{8}+\frac{\pi}{4}}$
 (C) $\frac{f(\frac{3\pi}{8})-f(\frac{\pi}{4})}{\frac{3\pi}{8}-\frac{\pi}{4}}$
 (D) $f(\frac{3\pi}{8}) - f(\frac{\pi}{4})$

67. Let f be the function defined by $f(x) = 2 \sin x + \cos x$. The average rate of change of f over the interval $[0, b]$ is 0.05, where $b > 0$. Which of the following is an equation that could be used to find the value of b ?

- (A) $f(b) = 0.05$
 (B) $f(b) - f(0) = 0.05$
 (C) $\frac{f(b)-f(0)}{b-0} = 0.05$
 (D) $\frac{f(b)+f(0)}{2} = 0.05$

68.

x	-2	-1	0	1	2
$g(x)$	-3	2	1	0	5

Selected values of a function g are shown in the table above. What is the average rate of change of g over the interval $[-2, 2]$?

- (A) $\frac{2-(-2)}{5-(-3)}$
 (B) $\frac{5-(-3)}{2-(-2)}$
 (C) $\frac{5+(-3)}{2}$
 (D) $\frac{-3+2+1+0+5}{5}$

69.

x	0	2	6	12	20
$f(x)$	-8	-2	0	2	18

The function f is differentiable and has values as shown in the table above. Of the following, on which interval is the average rate of change of f greatest?

- (A) $[0, 2]$
 (B) $[2, 6]$
 (C) $[6, 12]$
 (D) $[12, 20]$
70. Let f be the function defined by $f(x) = 2x^3 - x$. Which of the following expressions is the average rate of change of f on the interval $[1, 3]$?
- (A) $\frac{f(3)+f(1)}{2}$
 (B) $\frac{f(3)+f(1)}{3+1}$
 (C) $\frac{f(3)-f(1)}{3-1}$
 (D) $f(3) - f(1)$
71. Let f be the function defined by $f(x) = \frac{1-5x-2x^2}{3x^2+7}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = -\frac{2}{3}$
 (B) $y = \frac{1}{3}$
 (C) $y = \frac{2}{3}$
 (D) There is no horizontal asymptote to the graph of f .
72. Let f be the function defined by $f(x) = \frac{2^x+5}{e^x+1}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = 0$
 (B) $y = \frac{2}{e}$
 (C) $y = 1$
 (D) There is no horizontal asymptote to the graph of f .
73. Let f be the function given by $f(x) = \frac{1+e^x \sin x}{e^{x-1}-1}$. What are all horizontal asymptotes of the graph of f ?
- (A) $y = -1$ only
 (B) $y = 1$ only
 (C) $y = -1$ and $y = 1$
 (D) The graph of f has no horizontal asymptotes.

74. Let f be the function defined by $f(x) = \frac{3^x+2}{e^{2x}+1}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = 0$
(B) $y = \frac{3}{e^2}$
(C) $y = 1$
(D) There is no horizontal asymptote to the graph of f .
75. Let f be the function defined by $f(x) = \frac{2-x+3x^2+5x^3-7x^4}{x^4-2x^3-5x^2+2x-3}$ for $x > 0$. Which of the following is a horizontal asymptote to the graph of f ?
- (A) $y = -7$
(B) $y = 2$
(C) $y = 7$
(D) There is no horizontal asymptote to the graph of f .
76. Let f be the function given by $f(x) = \frac{(\cos x)e^{2x}-1}{e^{2x-1}+2}$. What are all horizontal asymptotes to the graph of f ?
- (A) $y = -\frac{1}{2}$ only
(B) $y = e$ only
(C) $y = -\frac{1}{2}$ and $y = e$
(D) The graph of f has no horizontal asymptotes.
77. $\lim_{x \rightarrow \infty} \frac{10-6x^2}{5+3e^x}$ is
- (A) -2
(B) 0
(C) 2
(D) nonexistent