7

1. Which of the following is true about the curve $x^2 - xy + y^2 = 3$ at the point $(2,\ 1)$?

(A) $\frac{dy}{dx}$ exists at $(2,\ 1)$, but there is no tangent line at that point.

(B) $\frac{dy}{dx}$ exists at $(2,\ 1)$, and the tangent line at that point is horizontal.

(C) $\frac{dy}{dx}$ exists at $(2,\ 1)$, and the tangent line at that point is neither horizontal nor vertical.

(D) $\frac{dy}{dx}$ does not exists at $(2,\ 1)$, and the tangent line at that point is vertical.

(E) $\frac{dy}{dx}$ does not exists at $(2,\ 1)$, and the tangent line at that point is horizontal.

2. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1,\ -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

3. Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\frac{1}{2}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

4. If $f(x) = \arccos\left(x^2\right)$, then f′(x)=

(A) $\frac{1}{\sqrt{1-x^4}}$

(B) $\frac{-2x}{\sqrt{1-x^4}}$

(C) $\frac{2x}{\sqrt{1-x^4}}$

(D) $\frac{-4x^3}{\sqrt{1-x^4}}$

(E) $\frac{4x^3}{\sqrt{1-x^4}}$

5. If $y = \sin^{-1}(5x)$, then $\frac{dy}{dx}$ =

7

(A) $\frac{1}{1+25x^2}$

(B) $\frac{5}{1+25x^2}$

(C) $\frac{-5}{\sqrt{1-25x^2}}$

(D) $\frac{1}{\sqrt{1-25x^2}}$

(E) $\frac{5}{\sqrt{1-25x^2}}$

6. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

(A) $-\frac{3x+y}{y^2}$

(B) $-\frac{3x+y}{x+y}$

(C) $\frac{1-3x-y}{x+y}$

(D) $-\frac{3x}{1+y}$

(E) $-\frac{3x}{x+y}$

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1-x^2}}$

(B) $\frac{xy}{\sqrt{1-x^2}}$

(C) $\frac{y}{1+x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1+x^2}$

8.

x	$f(x)$	$g(x)$	$f'(x)$
–4	0	–9	5
–2	4	–7	4
0	6	–4	2
2	7	–3	1
4	10	–2	3

The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$

(B) $-\frac{1}{4}$

(C) $-\frac{3}{100}$

(D) $\frac{1}{4}$

(E) $\frac{1}{3}$

7

9. If $\lim_{h \to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a?

(A) $\frac{\sqrt{2}}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{3}$

(D) $\frac{1}{2}$

(E) 2

10. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$

(B) $\frac{1}{4}$

(C) $\frac{7}{4}$

(D) 4

(E) 13

11. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and f′(5)=−1/2 , then $g'(-2) =$

(A) 2

(B) 1/2

(C) 1/5

(D) $-\frac{1}{5}$

(E) -2

12. Let f be the function defined by $f(x) = x^3 + x^2 + x$. Let $g(x) = f^{-1}(x)$, where g(3) = 1. What is the value of $g'(3)$?

(A) 1/39

(B) 1/34

(C) 1/6

(D) 1/3

(E) 39

13. The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

7

(A) $g(8) = \frac{1}{3}$ and $g'(8) = -\frac{1}{9}$

(B) $g(8) = \frac{1}{3}$ and $g'(8) = \frac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\frac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \frac{1}{9}$

14.

x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(3)$?

(A) $\frac{1}{13}$

(B) $\frac{1}{4}$

(C) 1

(D) 4

(E) 13

15.

x	$f(x)$	$f'(x)$
0	49	0
1	2	–8
2	–1	–80

The table above gives selected values for a differentiable and decreasing function f and its derivative. If f^{1} is the inverse function of f, what is the value of $(f^{1})'(2)$?

(A) -80

(B) $-\frac{1}{8}$

(C) $-\frac{1}{80}$

(D) $\frac{1}{80}$

(E) $\frac{1}{8}$

16. If $f(x) = \sin^{-1}x$, then $f'\left(\frac{\sqrt{3}}{2}\right) =$

7

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{4}{7}$

(D) 2

17. If $y = \arctan(\cos x)$, then $\frac{dy}{dx}$ =

(A) $\frac{-\sin x}{1+\cos^2 x}$

(B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$

(C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\frac{1}{(\arccos x)^2+1}$

(E) $\frac{1}{1+\cos^2 x}$

18. $\frac{d}{dx}\left(\tan^{-1}x + 2\sqrt{x}\right) =$

(A) $-\frac{1}{\sin^2 x} + \frac{1}{\sqrt{x}}$

(B) $\frac{1}{\sqrt{1-x^2}} - 4\sqrt[3]{x}$

(C) $\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x}}$

(D) $\frac{1}{1+x^2} - 4\sqrt[3]{x}$

(E) $\frac{1}{1+x^2} + \frac{1}{\sqrt{x}}$

19. If f(x) = sin x + 2x + 1 and *g* is the inverse function of *f,* what is the value of *g′(1)* ?

(A) $\frac{1}{3}$

(B) 1

(C) 3

(D) $\frac{1}{2+\cos 1}$

(E) 2 + cos 1

20. If tan*(xy)*=*x*, then $\frac{dy}{dx} =$

(A) $\frac{1-y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$

(B) $\frac{\sec^2(xy)-y}{x}$

(C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$

(E) $\frac{\cos^2(xy)-y}{x}$

21. If $\ln(2x+y) = x+1$, then $\frac{dy}{dx} =$

(A) -2

(B) $2x+y-2$

(C) $2x+y$

(D) $4x+2y-2$

(E) $y-\frac{y}{x}$

22. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$

(B) $\frac{1}{54}$

(C) $\frac{1}{27}$

(D) $\frac{1}{6}$

(E) 6

23. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

24. In the *xy*-plane, what is the slope of the line tangent to the graph of $x^2+xy+y^2=7$ at the point $(2,1)$?

(A) $-\frac{4}{3}$

(B) $-\frac{5}{4}$

(C) -1

(D) $-\frac{4}{5}$

(E) $-\frac{3}{4}$

25. Suppose $\ln x - \ln y = y-4$, where *y* is a differentiable function of *x* and $y=4$ when $x=4$. What is the value of $\frac{dy}{dx}$ when $x=4$?

7

(A) 0
(B) $\frac{1}{5}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$
(E) $\frac{17}{5}$

26. If $x + 2xy - y^2 = 2$, then at the point (1,1), $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$
(B) $\frac{1}{2}$
(C) 0
(D) $-\frac{3}{2}$
(E) nonexistent

27. If $(x + 2y) \cdot \frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$
(B) 0
(C) 2
(D) $\frac{10}{3}$
(E) Undefined

28. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3,2)?

(A) 0
(B) 4/9
(C) 7/9
(D) 6/7
(E) 5/3

29. If $x^2y - 3x = y^3 - 3$, then at the point $(-1, 2)$, $\frac{dy}{dx} =$

(A) $-\frac{7}{11}$
(B) $-\frac{7}{13}$
(C) $-\frac{1}{2}$
(D) $-\frac{3}{14}$
(E) 7

30. What is the slope of the line tangent to the curve $y + 2 = \frac{x^2}{2} - 2\sin y$ at the point(2,0) ?

(A) -2
(B) 0
(C) $\frac{1}{2}$
(D) $\frac{2}{3}$
(E) 2

31. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at (2, 1) is

(A) $-\frac{3}{2}$
(B) -1
(C) $-\frac{5}{14}$
(D) $-\frac{3}{14}$
(E) 0

32. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2+y}{x+2y^2}$
(B) $-\frac{x^2+y}{x+y^2}$
(C) $-\frac{x^2+y}{x+2y}$
(D) $-\frac{x^2+y}{2y^2}$
(E) $\frac{-x^2}{1+2y^2}$

33. If $3x^2 + 2xy + y^2 = 2$, then the value of dy/dx at x = 1 is

(A) -2
(B) 0
(C) 2
(D) 4
(E) not defined

34. If $\sin x = e^y$, $0 < x < \pi$, what is dy/dx in terms of x?

(A) -tan x
(B) -cot x
(C) cot x
(D) tan x
(E) csc x

35. If $y=\ln(x^2+y^2)$, then the value of $\frac{dy}{dx}$ at the point (1, 0) is

7

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) undefined

36. If $x^2 + xy + y^3 = 0$, then, in terms of *x* and *y*, $\frac{dy}{dx}$=

(A) $-\frac{2x+y}{x+3y^2}$
(B) $-\frac{x+3y^2}{2x+y}$
(C) $\frac{-2x}{1+3y^2}$
(D) $\frac{-2x}{x+3y^2}$
(E) $-\frac{2x+y}{x+3y^2-1}$

37. The slope of the line tangent to the graph of ln*(xy)*=*x* at the point where *x*=1 is

(A) 0
(B) 1
(C) e
(D) e^2
(E) 1-e

38. If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\frac{1}{1+x^2}$
(B) $\frac{2x}{1+x^2}$
(C) $2x\left(1 + x^2\right)$
(D) $2x\left(e^{1+x^2}\right)$
(E) $2x \ln\left(1 + x^2\right)$

39. If $y = xy + x^2 + 1$, then when *x*=-1, $\frac{dy}{dx}$ is

(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) -1
(D) -2
(E) nonexistent

40. If x^2+*xy*=10, then when $x = 2$, $\frac{dy}{dx} =$

7

(A) $-\frac{7}{2}$

(B) -2

(C) $\frac{2}{7}$

(D) $\frac{3}{2}$

(E) $\frac{7}{2}$

41. If $x^2+y^2=25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4,3) ?

(A) $-\frac{25}{27}$

(B) $-\frac{7}{27}$

(C) $\frac{7}{27}$

(D) $\frac{3}{4}$

(E) $\frac{25}{27}$

42. If $x^2 + xy - 3y = 3$, then at the point (2, 1), $\frac{dy}{dx} =$

(A) 5

(B) 4

(C) $\frac{7}{3}$

(D) 2

43. If $x^2 + xy - 3y = 3$, then at the point (2, 1), $\frac{dy}{dx} =$

(A) 5

(B) 4

(C) $\frac{7}{3}$

(D) 2

44. if $e^{xy} - y^2 = e - 4$, then at $x = 1/2$ and $y = 2$, $dy/dx =$

(A) e/4

(B) e/2

(C) 4e/(8-e)

(D) 4e/(4-e)

(E) (8-4e)/e

45. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$

7

(A) 0

(B) $\frac{x}{2}$

(C) $\frac{2x}{3y^2}$

(D) $2x - 3y^2$

(E) $\frac{2x}{1+3y^2}$

46. If cos (xy) = y-1, then the value of dy/dx when $x = \pi/2$ and y = 1 is

(A) $\frac{-2}{(2-\pi)}$

(B) $\frac{-2}{(2+\pi)}$

(C) 0

(D) $\frac{2}{(2-\pi)}$

(E) $\frac{2}{(2+\pi)}$

47. Let $y = f(x)$ be a twice-differentiable function such that $f(1) = 2$ and $\frac{dy}{dx} = y^3 + 3$. What is the value of $\frac{d^2y}{dx^2}$ at $x = 1$?

(A) 12

(B) 66

(C) 132

(D) 165

48. If $e^x - y = xy^3 + e^2 - 18$, what is the value of *dy/dx* at the point (2, 2) ?

(A) e^2 - 32

(B) $\frac{e^2-9}{24}$

(C) $\frac{e^2-8}{25}$

(D) $\frac{e^2}{13}$

49. Given that $3x - \tan y = 4$, what is $\frac{dy}{dx}$ in terms of y ?

(A) $\frac{dy}{dx} = 3\sin^2 y$

(B) $\frac{dy}{dx} = 3\cos^2 y$

(C) $\frac{dy}{dx} = 3\cos y \cot y$

(D) $\frac{dy}{dx} = \frac{3}{1+9y^2}$

50. Given that 3x − tan y = 4, what is $\frac{dy}{dx}$ in terms of y?

7

(A) $\frac{dy}{dx} = 3\sin^2 y$

(B) $\frac{dy}{dx} = 3\cos^2 y$

(C) $\frac{dy}{dx} = 3\cos y \cot y$

(D) $\frac{dy}{dx} = \frac{3}{1+9y^2}$

51. Let *f* be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all *x* and the point (0,1) is on the graph of *f*, what is the value of $g'(1)$?

(A) $\frac{1}{2+e}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 3

(E) $2 + e$

52. The function *h* is given by $h(x) = x^5 + 3x - 2$ and $h(1) = 2$. If h^{-1} is the inverse of *h*, what is the value of $\left(h^{-1}\right)'(2)$?

(A) $\frac{1}{83}$

(B) $\frac{1}{8}$

(C) $\frac{1}{2}$

(D) 1

(E) 8

53. What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?

(A) 2

(B) $\frac{1}{2}$

(C) 0

(D) $-\frac{1}{2}$

(E) -2

54. An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

(A) $x - 2y = 0$

(B) $x - y = 0$

(C) $x = 0$

(D) $y = 0$

(E) $\pi x - 2y = 0$

55. The function $y = g(x)$ is differentiable and increasing for all real numbers. On what intervals is the function $y = g\left(x^3 - 6x^2\right)$ increasing?

(A) $(-\infty, 0]$ and $[4, \infty)$ only

(B) $[0, 4]$ only

(C) $[2, \infty)$ only

(D) $[6, \infty)$ only

(E) $(-\infty, \infty)$

56. The points $(-1,\ -1)$ and $(1,\ -5)$ are on graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

(A) $(1,\ -5)$ is a local maximum of *f*.

(B) $(1,\ -5)$ is a point of inflection of the graph of *f*.

(C) $(-1,\ -1)$ is a local maximum of *f*.

(D) $(-1,\ -1)$ is a local minimum of *f*.

(E) $(-1,\ -1)$ is a point of inflection of the graph of *f*.

57. For which of the following increasing functions f does $\left(f^{-1}\right)'(20) = \frac{1}{5}$?

(A) $f(x) = x + 5$

(B) $f(x) = x^3 + 5x + 20$

(C) $f(x) = x^5 + 5x + 14$

(D) $f(x) = e^x + 5x + 19$

58. For which of the following decreasing functions f does $\left(f^{-1}\right)'(10) = -\frac{1}{8}$?

(A) $f(x) = -5x + 15$

(B) $f(x) = -2x^3 - 2x + 14$

(C) $f(x) = -x^5 - 4x + 15$

(D) $f(x) = e^{-2x} - x + 9$

59. A decreasing function g satisfies $g(4) = 6$ and $g'(4) = -2$. Which of the following statements about the inverse of g must be true?

(A) $\left(g^{-1}\right)'(6) = 4$

(B) $\left(g^{-1}\right)'(-2) = 4$

(C) $\left(g^{-1}\right)'(6) = -2$

(D) $\left(g^{-1}\right)'(6) = -\frac{1}{2}$

60. An increasing function f satisfies $f(10) = 5$ and $f'(10) = 8$. Which of the following statements about the inverse of f must be true?

7

(A) $\left(f^{-1}\right)'(5) = 10$

(B) $\left(f^{-1}\right)'(8) = 10$

(C) $\left(f^{-1}\right)'(5) = 8$

(D) $\left(f^{-1}\right)'(5) = \frac{1}{8}$

61. Let $f$and g be inverse functions that are differentiable for all x. If $f(3) = -2$ and $g'(-2) = -4$, which of the following statements must be false?

I. $f'(0) = \frac{1}{4}$
II. $f'(3) = -\frac{1}{4}$
III. $f'(5) = -\frac{1}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

62. Let f and g be inverse functions that are differentiable for all x. If $f(-5) = 7$ and $g'(7) = 3$, which of the following statements must be false?

I. $f'(3) = -\frac{1}{3}$
II. $f'(-5) = \frac{1}{3}$
III. $f'(7) = \frac{1}{3}$

(A) I only

(B) II only

(C) III only

(D) I and III only

63. Which of the following does not require the use of the chain rule to find $\frac{dy}{dx}$?

(A) $y = \sin^{-1}(3x^2 - 4)$

(B) $3x^6 - 4y^2 = 2xy^5 + 7$

(C) $y = 3x^2 - \sqrt{x} + \frac{2}{x}$

(D) $\cos(x + y) + 2^y - x = 0$

64. Which of the following does not require the use of the chain rule to find $\frac{dy}{dx}$?

(A) $y = \cos^{-1}(10x^5 - x^2)$

(B) $4x^{10} + 6y^3 = x^2y^5 - 2$

(C) $y = 10\sqrt{x} - \frac{4}{x^3} + x^5$

(D) $\sin(2x - y) + e^{2y} + \frac{x}{6} = 0$

65. Which of the following could be used to find the slope of the line tangent to the curve $\sin^{-1}(2x^2+y^2)=\frac{2}{x}+y^2$?

(A) $\frac{1}{\sqrt{1-(2x^2+y^2)^2}}=\frac{-2}{x^2}+2y\frac{dy}{dx}$

(B) $\frac{4x+2y}{\sqrt{1-(2x^2+y^2)^2}}=\frac{-2}{x^2}+2y$

(C) $\frac{4x+2y\frac{dy}{dx}}{\sqrt{1-(2x^2+y^2)^2}}=\frac{-2}{x^2}+2y\frac{dy}{dx}$

(D) $\frac{4x+2y\frac{dy}{dx}}{\sqrt{1-(2x^2+y^2)}}=\frac{-2}{x^2}+2y\frac{dy}{dx}$

66.

x	4	8
$f(x)$	11	6
$f'(x)$	-4	-3

The table above gives selected values for a differentiable and decreasing function f and its derivative. Let g be the decreasing function given by $g(x)=f(4x)-f(2x)$, where $g(2)=f(8)-f(4)=-5$. Which of the following describes a correct process for finding $(g^{-1})'(-5)$?

(A) $(g^{-1})'(-5)=\frac{1}{g'(g^{-1}(-5))}=\frac{1}{g'(2)}$ and $g'(2)=4f'(8)-2f'(4)$

(B) $(g^{-1})'(-5)=\frac{1}{g'(g^{-1}(-5))}=\frac{1}{g'(2)}$ and $g'(2)=f'(8)-f'(4)$

(C) $(g^{-1})'(-5)=g'(g^{-1}(-5))=g'(2)$ and $g'(2)=f'(8)-f'(4)$

(D) $(g^{-1})'(-5)=g'(g^{-1}(-5))=g'(2)$ and $g'(2)=4f'(8)-2f'(4)$

67. Which of the following could be used to find the slope of the line tangent to the curve $\tan^{-1}(x-2y+2)=x^2-3y+\tan^{-1}(2)-1$?

(A) $\frac{1}{1+(x-2y+2)^2}=2x-3\frac{dy}{dx}$

(B) $\frac{-1}{1+(x-2y+2)^2}=2x-3$

(C) $\frac{1-2\frac{dy}{dx}}{1+(x-2y+2)^2}=2x-3\frac{dy}{dx}$

(D) $\frac{1-2\frac{dy}{dx}}{1+(x-2y+2)}=2x-3\frac{dy}{dx}$

68.

x	3	6
$f(x)$	4	5
$f'(x)$	$\frac{1}{3}$	$\frac{3}{4}$

The table above gives selected values for a differentiable and increasing function f and its derivative. Let g be the increasing function given by $g(x) = f(x) + f(2x)$, where $g(3) = f(3) + f(6) = 9$. Which of the following describes a correct process for finding $(g^{-1})'(9)$?

(A) $(g^{-1})'(9) = \frac{1}{g'(g^{-1}(9))} = \frac{1}{g'(3)}$ and $g'(3) = f'(3) + 2f'(6)$

(B) $(g^{-1})'(9) = \frac{1}{g'(g^{-1}(9))} = \frac{1}{g'(3)}$ and $g'(3) = f'(3) + f'(6)$

(C) $(g^{-1})'(9) = g'(g^{-1}(9)) = g'(3)$ and $g'(3) = f'(3) + f'(6)$

(D) $(g^{-1})'(9) = g'(g^{-1}(9)) = g'(3)$ and $g'(3) = f'(3) + 2f'(6)$

69.

x	$f(x)$	$f'(x)$
1	2	3
2	3	4
3	5	2

The function f is increasing and differentiable. Selected values of f and its derivative f' are given in the table above. What is the value of $\left(f^{-1}\right)'(3)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

70. $\frac{d}{dx}\left(x^5 - 5^x\right) =$

(A) $\frac{x^6}{6} - \frac{5^x}{\ln 5}$

(B) $5x^4 - 5^x$

(C) $5x^4 - x \cdot 5^{x-1}$

(D) $5x^4 - (\ln 5)\, 5^x$

71. $\frac{d}{dx}\left(\cot^{-1} x\right) =$

7

(A) $-\csc^2 x$

(B) $\sec^2 x$

(C) $-\frac{1}{1+x^2}$

(D) $\frac{1}{1+x^2}$

72. $\frac{d}{dx}\left(\tan^{-1}(3x)\right) =$

(A) $3\sec^2(3x)$

(B) $-3\csc^2(3x)$

(C) $\frac{3}{\sqrt{1-(3x)^2}}$

(D) $\frac{3}{1+(3x)^2}$

73. $\frac{d}{dx}\left(\csc^{-1}(e^x)\right) =$

(A) $-e^x \cdot \cot(e^x) \cdot \csc(e^x)$

(B) $e^x \cdot \cot\left(e^x\right) \cdot \csc^3(e^x)$

(C) $\frac{e^x}{\sqrt{1-e^{2x}}}$

(D) $\frac{-1}{e^x\sqrt{1-\left(\frac{1}{e^{2x}}\right)}}$

74. If $f(x) = \arcsin x$, then $\lim\limits_{x\to\frac{1}{2}} \frac{f(x)-f\left(\frac{1}{2}\right)}{x-\frac{1}{2}}$ is

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{2}{\sqrt{3}}$

(D) nonexistent

75. $\frac{d}{dx}\left(\sec^{-1}x\right) =$

(A) $-\sin x$

(B) $\tan x \ \sec x$

(C) $\frac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}}$

(D) $\frac{-1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}}$

76. If $f(x) = \arctan x$, then $\lim\limits_{x\to\sqrt{3}} \frac{f(x)-f\left(\sqrt{3}\right)}{x-\sqrt{3}}$ is

7

(A) 0
(B) $\frac{1}{4}$
(C) $\frac{\pi}{3}$
(D) nonexistent

77. Let f be the function defined by $f(x) = x^3 + x$. If g is the inverse function of f, what is the slope of the line tangent to the graph of g at the point $(2, 1)$?

(A) $-\frac{1}{4}$
(B) $-\frac{1}{13}$
(C) $\frac{1}{13}$
(D) $\frac{1}{4}$

78. What is the slope of the line tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ at the point $\left(\frac{9}{4}, \frac{1}{4}\right)$?

(A) -3
(B) $-\frac{1}{3}$
(C) 1
(D) $\frac{4}{3}$

79. If $x^3 - 2xy + 3y^2 = 7$, then $\frac{dy}{dx} =$

(A) $\frac{3x^2+4y}{2x}$
(B) $\frac{3x^2-2y}{2x-6y}$
(C) $\frac{3x^2}{2x-6y}$
(D) $\frac{3x^2}{2-6y}$

80. If $x = \frac{1}{y+1}$, then $\frac{dy}{dx} =$

(A) $-(y+1)^2$
(B) $\frac{-1}{(y+1)^2}$
(C) $\frac{1}{(y+1)^2}$
(D) $\frac{y}{(y+1)^2}$

81. What is the slope of the line tangent to the curve $x^2y + y^2 = 21$ at the point $(2, 3)$?

7

(A) $-\frac{9}{2}$
(B) -2
(C) $-\frac{6}{5}$
(D) $\frac{9}{10}$

82. If $x\cos y = \pi - 2y$, then $\frac{dy}{dx} =$

(A) $\dfrac{\cos y}{x\sin y - 2}$
(B) $\dfrac{\cos y}{2 + x\sin y}$
(C) $\dfrac{-\cos y}{2 + x\sin y}$
(D) $\dfrac{x\sin y - \cos y}{2}$

83. If $2xy^2 - 3x^2y = 6x$, then $\frac{dy}{dx} =$

(A) $\frac{6}{4y-6x}$
(B) $\frac{2y^2-2xy-6}{3x^2}$
(C) $\frac{-2y^2+6xy+6}{4xy-3x^2}$
(D) $\frac{-2y^2+6xy+3x^2+6}{4xy}$

84. If $y = \ln(3x + 4y)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{3x+4y}$
(B) $\frac{3}{3x+4y}$
(C) $\frac{7}{3x+4y}$
(D) $\frac{3}{3x+4y-4}$

85. If $e^y - e^{y^2} = x - x^3$, then the value of $\frac{dy}{dx}$ at the point $(0, 1)$ is

(A) $-\frac{1}{e}$
(B) $\frac{e-1}{2e}$
(C) $\frac{1+2e}{e}$
(D) undefined

86. If $3x^2 + 5x^2y^2 = 2y$, then $\frac{dy}{dx} =$

7

(A) $\frac{-6x}{20xy-2}$

(B) $\frac{-10xy^2-6x}{10x^2y-2}$

(C) $\frac{2-6x-10xy^2}{10x^2y}$

(D) $\frac{6x+10xy^2+10x^2y}{2}$

87. If $y = \ln(2x^2 - 3y^2)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{2x^2-3y^2}$

(B) $\frac{4x}{2x^2-3y^2}$

(C) $\frac{4x-6y}{2x^2-3y^2}$

(D) $\frac{4x}{2x^2-3y^2+6y}$

88. If $e^{2y} - e^{(y^2-y)} = x^4 - x^2$, then the value of $\frac{dy}{dx}$ at the point $(1, 0)$ is

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 2

89.

x	$f(x)$	$g(x)$	$f'(x)$
1	2	0	$\frac{\pi}{2}$
2	π	1	$\frac{\pi}{3}$
π	4	2	$\frac{\pi}{4}$

Let f and g be differentiable functions, where g is the inverse of f. Selected values of f, g, and f' are shown in the table. What is the value of $g'(\pi)$?

(A) $\frac{\pi}{4}$

(B) $\frac{3}{\pi}$

(C) $\frac{\pi}{3}$

(D) $\frac{4}{\pi}$