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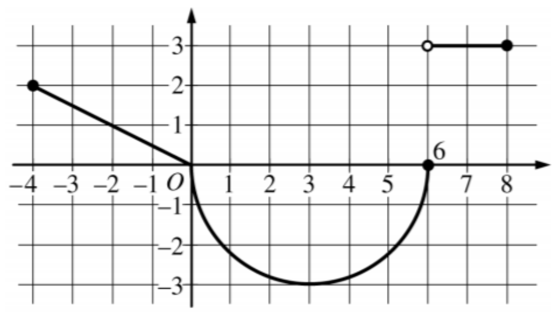
1. **CALCULUS AB**

SECTION II, Part B

Time - 1 hour

Number of questions - 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The function g is defined on the closed interval $[-4,\ 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4,\ 3]$ at which f attains its maximum value. Justify your answer.

(c) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim_{x \to -2} \frac{f(x)+7}{e^{3x+6}-1}$.

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2.

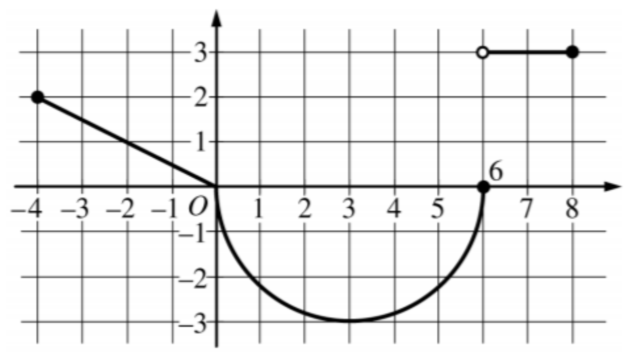
CALCULUS BC

SECTION II, Part B

Time - 1 hour

Number of questions - 4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t)\, dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.

(c) For each of $\lim_{x \to 0^-} g'(x)$ and $\lim_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim_{x \to -2} \frac{f(x)+7}{e^{3x+6}-1}$.

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3. *Let f be the function given by* $f(x) = 2xe^{2x}$.

(a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

(b) Find the absolute minimum value of *f*. Justify that your answer is an absolute minimum.

(c) What is the range of f?

(d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

Let *f* be the function given by $f(x) = 2xe^{2x}$.

4. Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

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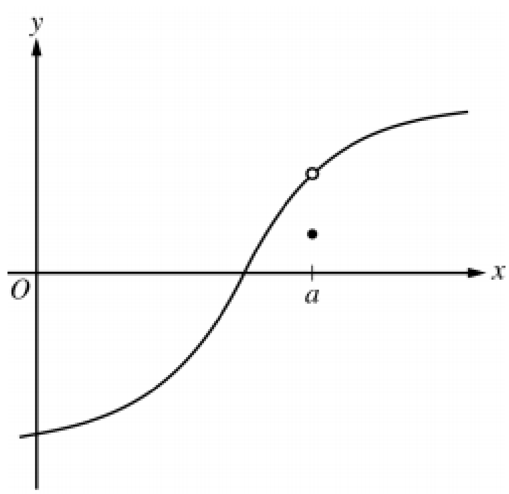

Graph of f

The graph of $y = f(x)$ is shown above. Which of the following is true?

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(A) $\lim\limits_{h \to 0} \frac{f(a+h)-f(a)}{h}$ exists.

(B) $\lim\limits_{x \to a^+} f(x) \neq \lim\limits_{x \to a^-} f(x)$

(C) $\lim\limits_{x \to a} f(x) \neq f(a)$

(D) $\lim\limits_{x \to a} f(x)$ does not exist.

6. If $\lim\limits_{x \to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$

II. f is differentiable at $x = 3$

III. $f(3) = 7$

(A) None

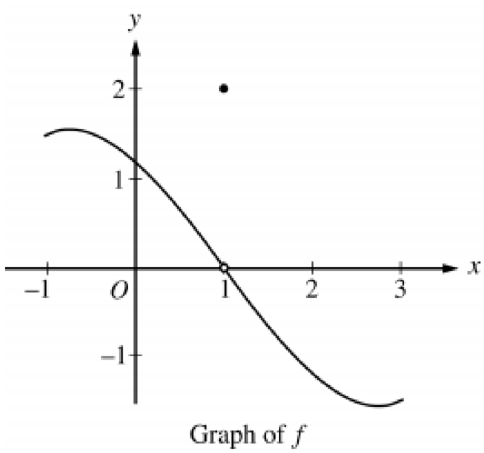
(B) II only

(C) III only

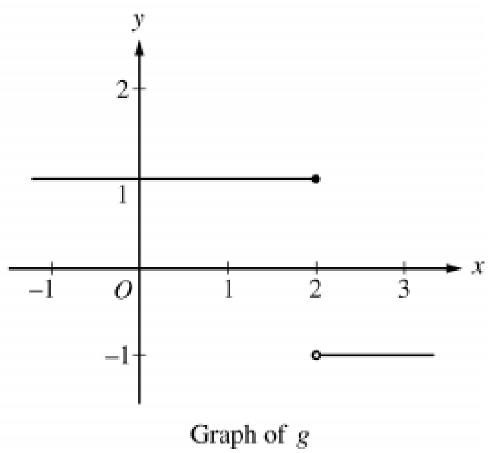
(D) I and III only

(E) I, II, and III

7.



Graph of f



Graph of g

The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim\limits_{x \to 1} f(x) = 0$

(B) $\lim\limits_{x \to 2} g(x)$ does not exist.

(C) $\lim\limits_{x \to 1} (f(x)g(x+1))$ does not exist.

(D) $\lim\limits_{x \to 1} (f(x+1)g(x))$ exists.

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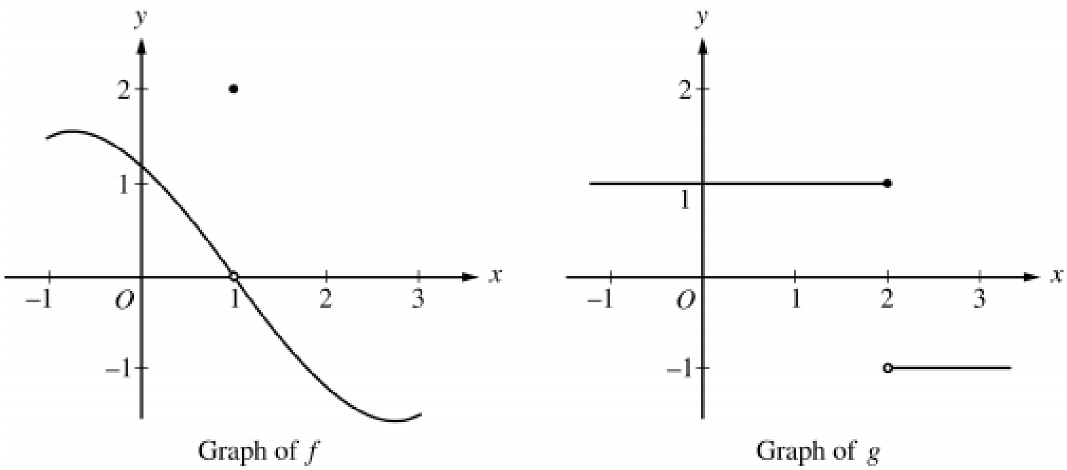
8.



The graphs of the functions *f* and *g* are shown in the figures above. Which of the following statements is false?

(A) $\lim\limits_{x\to1} f(x) = 0$

(B) $\lim\limits_{x\to2} g(x)$ does not exist.

(C) $\lim\limits_{x\to1} (f(x)g(x+1))$ does not exist.

(D) $\lim\limits_{x\to1} (f(x+1)g(x))$ exists.

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9. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

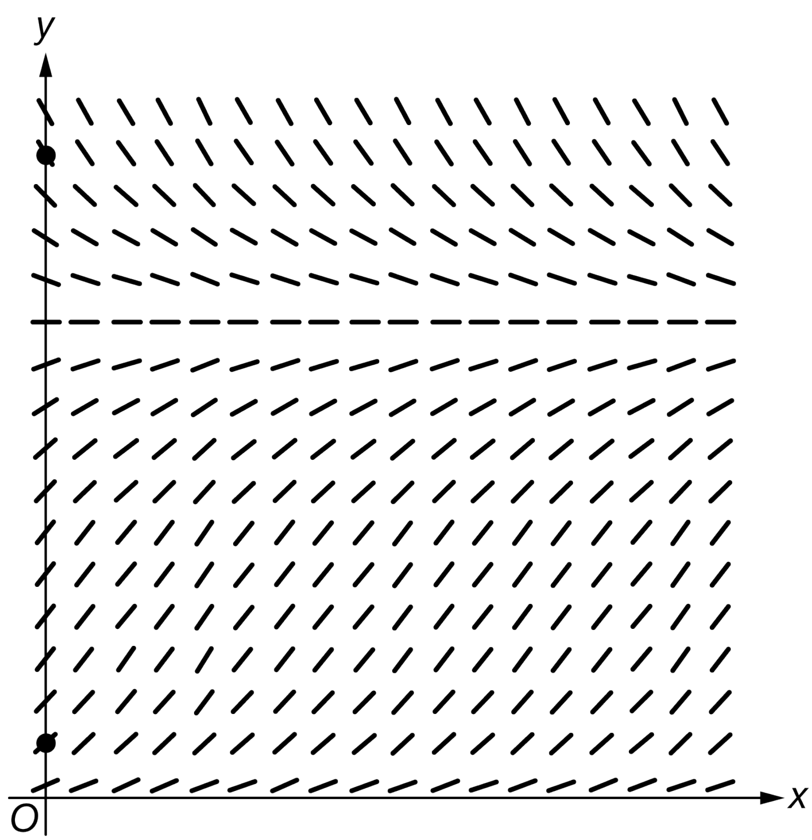
Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Consider the differential equation $\frac{dy}{dx} = \frac{-y}{4}\ln\left(\frac{y}{50}\right)$.

(a) A portion of the slope field for the differential equation is given below. Sketch the solution curves through the two indicated points for $x \geq 0$.



(b) Find $\lim\limits_{y \to 50} \frac{dy}{dx}$.

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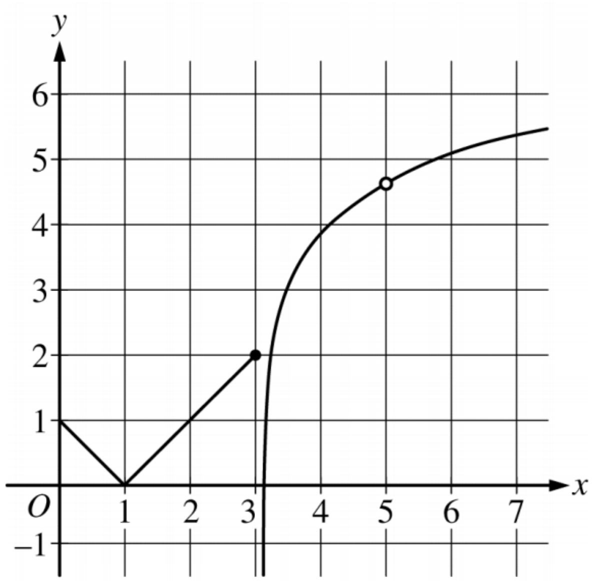
(c) Let f be the particular solution to the differential equation with initial condition $f(0) = 20$. Find $f'(0)$, and explain why f is increasing for all x.

(d) For $0 < y < 40$, at what value of y does $\frac{dy}{dx}$ attain its maximum value? Justify your answer.

10.



Graph of f

The graph of a function f is shown above. Which of the following limits does not exist?

(A) $\lim_{x \to 1^-} f(x)$

(B) $\lim_{x \to 1} f(x)$

(C) $\lim_{x \to 3^-} f(x)$

(D) $\lim_{x \to 3} f(x)$

(E) $\lim_{x \to 5} f(x)$

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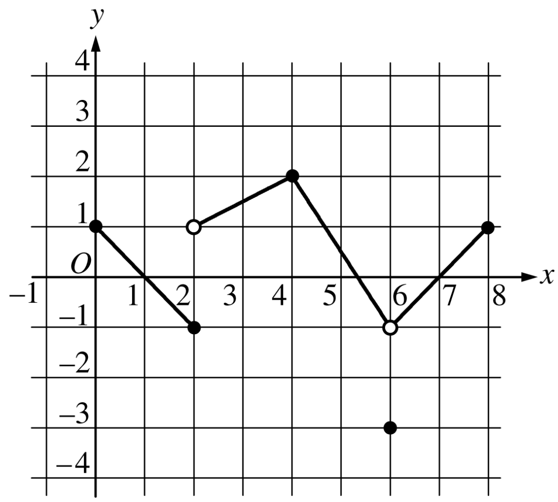


The figure above shows the graph of the function *f*. Which of the following statements are true?

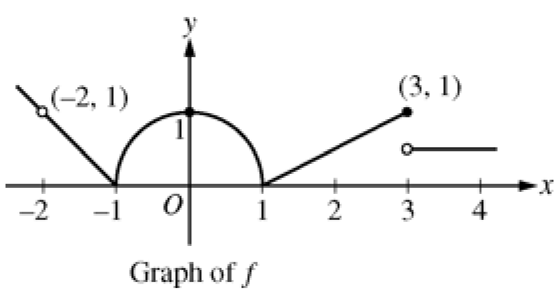
I. $\lim\limits_{x \to 2^-} f(x) = f(2)$

II. $\lim\limits_{x \to 6^-} f(x) = \lim\limits_{x \to 6^+} f(x)$

III. $\lim\limits_{x \to 6} f(x) = f(6)$

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

12.



The graph of a function *f* is shown above. For which of the following values of *c* does $\lim\limits_{x \to c} f(x) = 1$?

(A) 0 only
(B) 0 and 3 only
(C) -2 and 0 only
(D) -2 and 3 only
(E) -2 , 0, and 3

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13. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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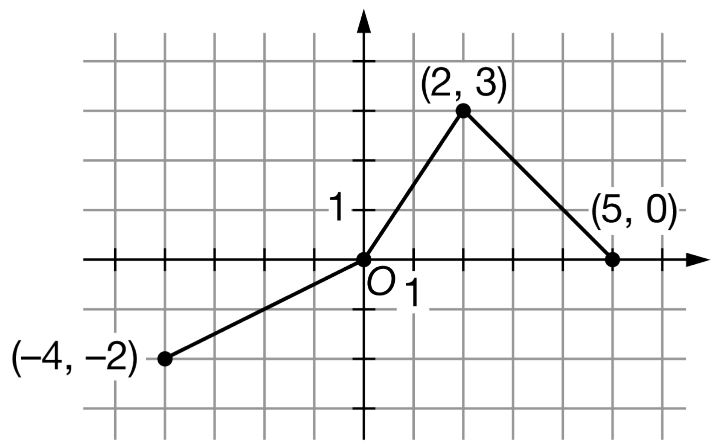
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of g'

The graph of the function g', the derivative of a function g, on the interval $-4 \leq x \leq 5$ consists of three line segments, as shown in the figure above. It is known that $g(-4) = 7$.

(a) Find the x-coordinate of each critical point of g on the open interval $-4 < x < 5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify your answers.

(c) Find the absolute maximum value of g on the interval $-4 \leq x \leq 5$. Justify your answer.

(d) Is $x = 2$ the location of a point of inflection for the graph of g ? Give a reason for your answer.

(e) Is $x = 0$ the location of a point of inflection for the graph of g ? Give a reason for your answer.

(f) Find $\lim\limits_{x \to -2} \frac{g'(x)+1}{x^2-4}$. Give a reason for your answer.

(g) Let h be the function defined by $h(x) = x \cdot g(x)$. It is known that $g(2) = 6$. Find the value of $h'(2)$, or explain why it does not exist. Show the computations that lead to your answer.

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14. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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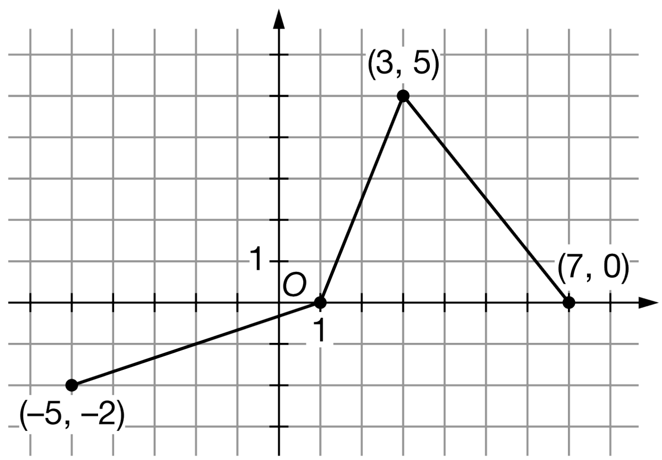
Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of h'

The graph of the function h', the derivative of a function h, on the interval $-5 \leq x \leq 7$ consists of three line segments, as shown in the figure above. It is known that $h(-5) = 6$.

(a) Find the x-coordinate of each critical point of h on the open interval $-5 < x < 7$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify your answers.

(c) Find the absolute maximum value of h on the interval $-5 \leq x \leq 7$. Justify your answer.

(d) Is $x = 3$ the location of a point of inflection for the graph of h ? Give a reason for your answer.

(e) Is $x = 1$ the location of a point of inflection for the graph of h ? Give a reason for your answer.

(f) Find $\lim\limits_{x \to -2} \frac{h'(x)+1}{x^2-4}$. Give a reason for your answer.

(g) Let g be the function defined by $g(x) = x \cdot h(x)$. It is known that $h(3) = 5$. Find the value of $g'(3)$, or explain why it does not exist. Show the computations that lead to your answer.

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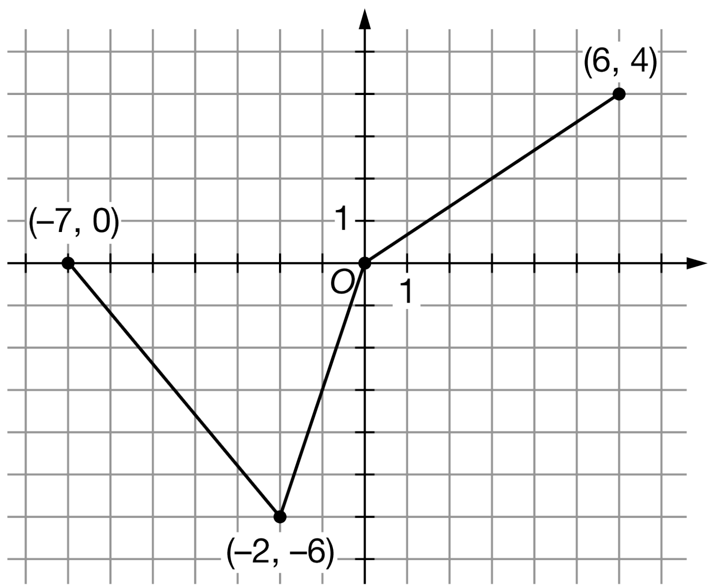

Graph of h'

The graph of the function h', the derivative of a function h, on the interval $-7 \leq x \leq 6$ consists of three line segments, as shown in the figure above. It is known that $h(-7) = 2$.

(a) Find the x-coordinate of each critical point of h on the open interval $-7 < x < 6$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify your answers.

(c) Find the absolute maximum value of h on the interval $-7 \leq x \leq 6$. Justify your answer.

(d) Is $x = -2$ the location of a point of inflection for the graph of h ? Give a reason for your answer.

(e) Is $x = 0$ the location of a point of inflection for the graph of h ? Give a reason for your answer.

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(f) Find $\lim_{x \to 3} \frac{2h'(x)-4}{x^2-9}$. Give a reason for your answer.

(g) Let g be the function defined by $g(x) = x \cdot h(x)$. It is known that $h(-2) = -13$. Find the value of $g'(-2)$, or explain why it does not exist. Show the computations that lead to your answer.

16. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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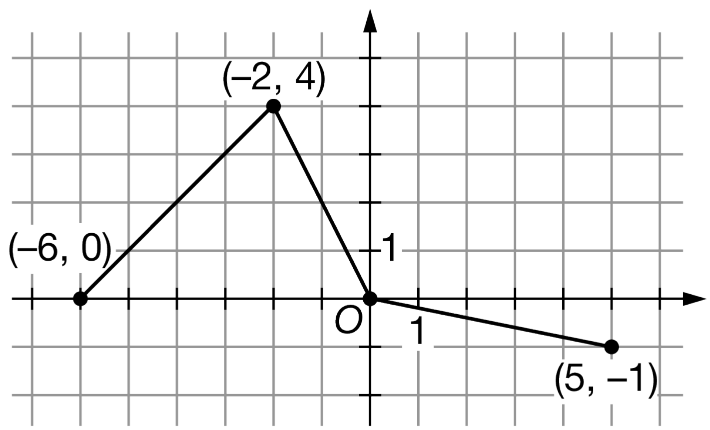

Graph of g'

The graph of the function g', the derivative of a function g, on the interval $-6 \leq x \leq 5$ consists of three line segments, as shown in the figure above. It is known that $g(-6) = -1$.

(a) Find the x-coordinate of each critical point of g on the open interval $-6 < x < 5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify your answers.

(c) Find the absolute minimum value of g on the interval $-6 \leq x \leq 5$. Justify your answer.

(d) Is $x = -2$ the location of a point of inflection for the graph of g ? Give a reason for your answer.

(e) Is $x = 0$ the location of a point of inflection for the graph of g ? Give a reason for your answer.

(f) Find $\lim\limits_{x \to -1} \frac{g'(x)-2}{x^2-1}$. Give a reason for your answer.

(g) Let h be the function defined by $h(x) = 3x \cdot g(x)$. It is known that $g(-2) = 7$. Find the value of $h'(-2)$, or explain why it does not exist. Show the computations that lead to your answer.

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17. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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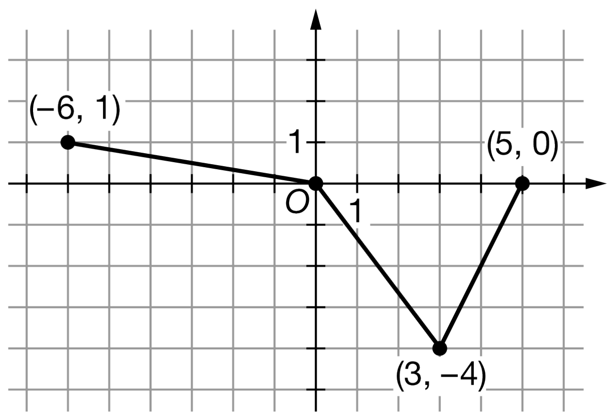
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of h'

The graph of the function h', the derivative of a function h, on the interval $-6 \leq x \leq 5$ consists of three line segments, as shown in the figure above. It is known that $h(-6) = 5$.

(a) Find the x-coordinate of each critical point of h on the open interval $-6 < x < 5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify your answers.

(c) Find the absolute minimum value of h on the interval $-6 \leq x \leq 5$. Justify your answer.

(d) Is $x = 3$ the location of a point of inflection for the graph of h ? Give a reason for your answer.

(e) Is $x = 0$ the location of a point of inflection for the graph of h ? Give a reason for your answer.

(f) Find $\lim\limits_{x \to -3} \frac{2h'(x)-1}{x^2-9}$. Give a reason for your answer.

(g) Let g be the function defined by $g(x) = 2x \cdot h(x)$. It is known that $h(3) = 2$. Find the value of $g'(3)$, or explain why it does not exist. Show the computations that lead to your answer.

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18. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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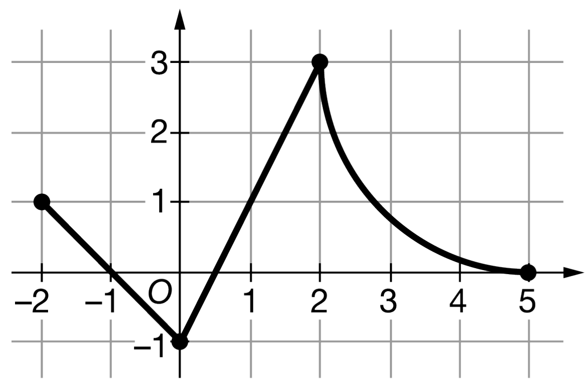
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f

The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $(3, 3-\sqrt{5})$ is on the graph of f.

a) If $\int_{-6}^{5} f(x)dx = 7$, find the value of $\int_{-6}^{-2} f(x)dx$. Show the work that leads to your answer.

b) Evaluate $\int_{3}^{5} \left(2f'(x)+4\right)dx$.

c) The function g is given by $g(x) = \int_{-2}^{x} f(t)dt$. Find the absolute maximum value of g on the interval $-2 \leq x \leq 5$. Justify your answer.

d) Find $\lim\limits_{x\to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.

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19. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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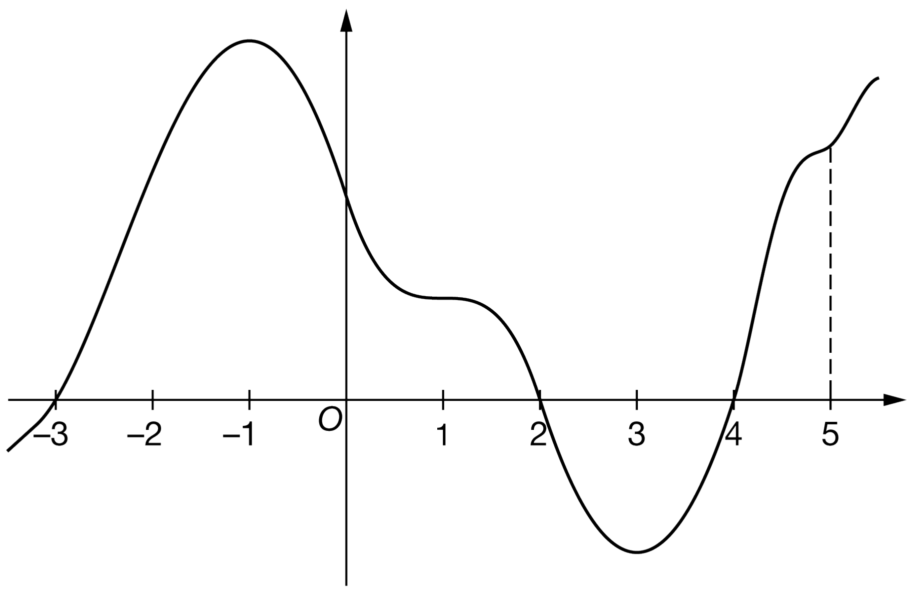

Graph of g

The graph of the differentiable function g with domain $-3.5 \leq x \leq 5.5$ is shown in the figure above. The areas of the regions bounded by the x-axis and the graph of g on the intervals $[-3, 2]$, $[2, 4]$, and $[4, 5]$ are 55, 12, and 10, respectively. The graph of g has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. Let h be the function defined by $h(x) = \int_4^x g(t)\,dt$ for $-3.5 \leq x \leq 5.5$.

(a) Find the x-coordinate of each critical point of h on the interval $-3.5 < x < 5.5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify each of your answers.

(c) Find the x-coordinate of each point of inflection for the graph of h on the interval $-3.5 < x < 5.5$. Justify your answer.

(d) Find the value of $h(5)$.

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(e) Find the value of $h(-3)$. Show the computations that lead to your answer.

(f) Must there exist a value of r, for $-3 < r < 2$, such that $h'(r)$ is equal to the average rate of change of h over the interval $-3 \leq x \leq 2$? Justify your answer.

(g) Find $\lim\limits_{x \to 5} \frac{h(x)}{2x}$. Show the computations that lead to your answer.

(h) The function k is defined by $k(x) = x \cdot h(x)$. It is known that $g(5) = 15$. Find $k'(5)$. Show the computations that lead to your answer.

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20. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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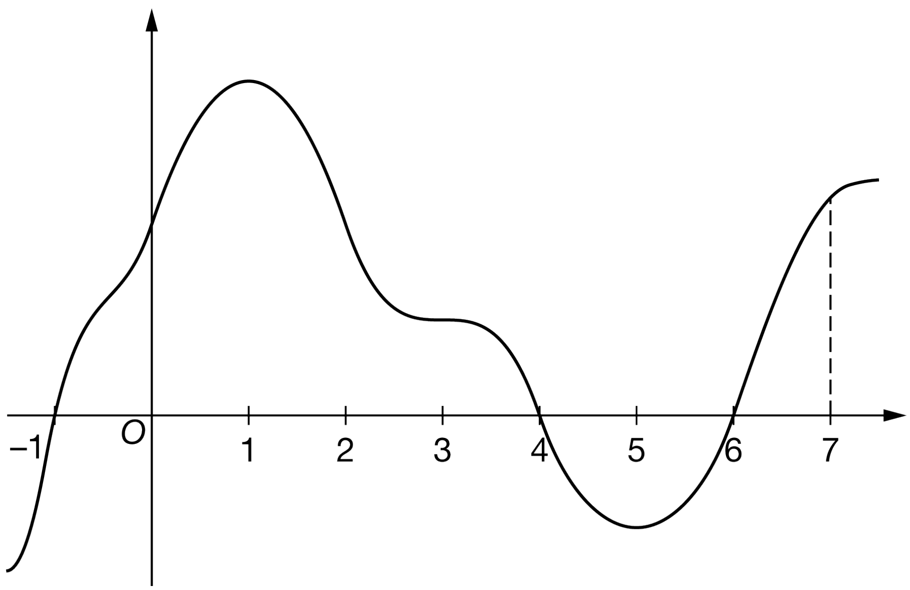

Graph of f

The graph of the differentiable function f with domain $-1.5 \le x \le 7.5$ is shown in the figure above. The areas of the regions bounded by the x-axis and the graph of f on the intervals $[-1, 4]$, $[4, 6]$, and $[6, 7]$ are 56, 10, and 8, respectively. The graph of f has horizontal tangents at $x = 1$, $x = 3$, and $x = 5$. Let h be the function defined by $h(x) = \int_6^x f(t)dt$ for $-1.5 \le x \le 7.5$.

(a) Find the x-coordinate of each critical point of h on the interval $-1.5 < x < 7.5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify each of your answers.

(c) Find the x-coordinate of each point of inflection for the graph of h on the interval $-1.5 < x < 7.5$. Justify your answer.

(d) Find the value of $h(7)$.

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(e) Find the value of $h(-1)$. Show the computations that lead to your answer.

(f) Must there exist a value of d, for $-1 < d < 4$, such that $h'(d)$ is equal to the average rate of change of h over the interval $-1 \leq x \leq 4$? Justify your answer.

(g) Find $\lim_{x \to 7} \frac{h(x)}{4x}$. Show the computations that lead to your answer.

(h) The function q is defined by $q(x) = 3x \cdot h(x)$. It is known that $f(7) = 14$. Find $q'(7)$. Show the computations that lead to your answer.

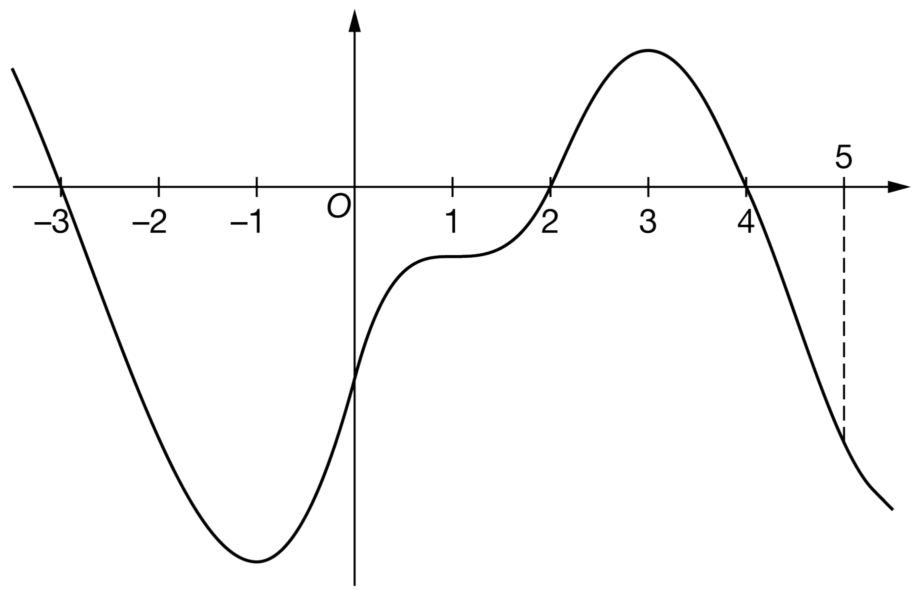
1.1

21. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of h

The graph of the differentiable function h with domain $-3.5 \leq x \leq 5.5$ is shown in the figure above. The areas of the regions bounded by the x-axis and the graph of h on the intervals $[-3, 2]$, $[2, 4]$, and $[4, 5]$ are 58, 11, and 8, respectively. The graph of h has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. Let g be the function defined by $g(x) = \int_4^x h(t)\,dt$ for $-3.5 \leq x \leq 5.5$.

(a) Find the x-coordinate of each critical point of g on the interval $-3.5 < x < 5.5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify each of your answers.

(c) Find the x-coordinate of each point of inflection for the graph of g on the interval $-3.5 < x < 5.5$. Justify your answer.

(d) Find the value of $g(5)$.

1.1

(e) Find the value of $g(-3)$. Show the computations that lead to your answer.

(f) Must there exist a value of b, for $-3 < b < 2$, such that $g'(b)$ is equal to the average rate of change of g over the interval $-3 \leq x \leq 2$? Justify your answer.

(g) Find $\lim\limits_{x \to 5} \frac{g(x)}{3x}$. Show the computations that lead to your answer.

(h) The function a is defined by $a(x) = x^2 \cdot g(x)$. It is known that $h(5) = -16$. Find $a'(5)$. Show the computations that lead to your answer.

1.1

22. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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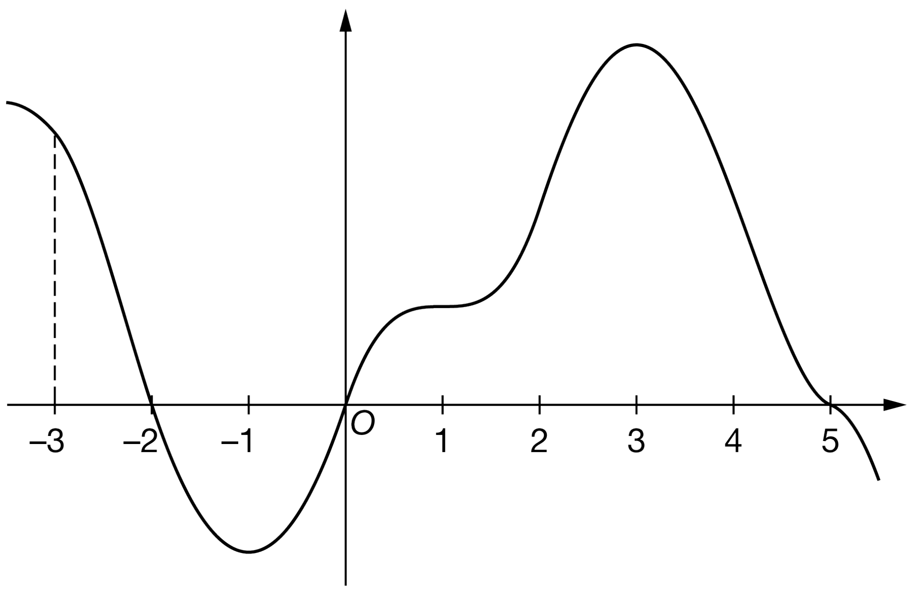
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of g

The graph of the differentiable function g with domain $-3.5 \leq x \leq 5.5$ is shown in the figure above. The areas of the regions bounded by the x-axis and the graph of g on the intervals $[-3,-2]$, $[-2,0]$, and $[0,5]$ are 10, 13, and 59, respectively. The graph of g has horizontal tangents at $x=-1$, $x=1$, and $x=3$. Let h be the function defined by $h(x)=\int_0^x g(t)dt$ for $-3.5 \leq x \leq 5.5$.

(a) Find the x-coordinate of each critical point of h on the interval $-3.5 < x < 5.5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify each of your answers.

(c) Find the x-coordinate of each point of inflection for the graph of h on the interval $-3.5 < x < 5.5$. Justify your answer.

(d) Find the value of $h(5)$.

1.1

(e) Find the value of $h(-3)$. Show the computations that lead to your answer.

(f) Must there exist a value of p, for $-3 < p < 2$, such that $h'(p)$ is equal to the average rate of change of h over the interval $-3 \leq x \leq 2$? Justify your answer.

(g) Find $\lim_{x \to -3} \frac{h(x)}{2x}$. Show the computations that lead to your answer.

(h) The function f is defined by $f(x) = x \cdot h(x)$. It is known that $g(-3) = 18$. Find $f'(-3)$. Show the computations that lead to your answer.

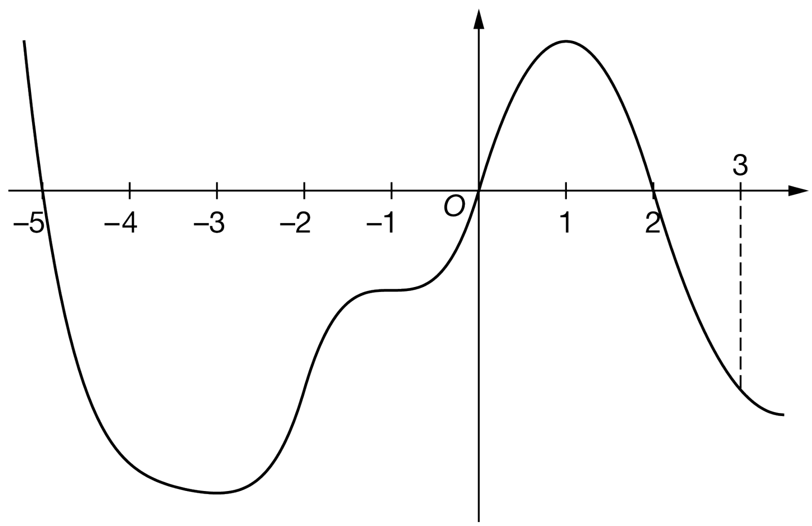
1.1

23. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f

The graph of the differentiable function f with domain $-5.5 \leq x \leq 3.5$ is shown in the figure above. The areas of the regions bounded by the x-axis and the graph of f on the intervals $[-5, 0]$, $[0, 2]$, and $[2, 3]$ are 57, 12, and 7, respectively. The graph of f has horizontal tangents at $x = -3$, $x = -1$, and $x = 1$. Let g be the function defined by $g(x) = \int_2^x f(t)\,dt$ for $-5.5 \leq x \leq 3.5$.

(a) Find the x-coordinate of each critical point of g on the interval $-5.5 < x < 3.5$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify each of your answers.

(c) Find the x-coordinate of each point of inflection for the graph of g on the interval $-5.5 < x < 3.5$. Justify your answer.

(d) Find the value of $g(3)$.

(e) Find the value of $g(-5)$. Show the computations that lead to your answer.

1.1

(f) Must there exist a value of k, for $-5 < k < 0$, such that $g'(k)$ is equal to the average rate of change of g over the interval $-5 \le x \le 0$? Justify your answer.

(g) Find $\lim_{x\to 3} \frac{5g(x)}{2x}$. Show the computations that lead to your answer.

(h) The function r is defined by $r(x) = x^2 \cdot g(x)$. It is known that $f(3) = -12$. Find $r'(3)$. Show the computations that lead to your answer.

1.1

24. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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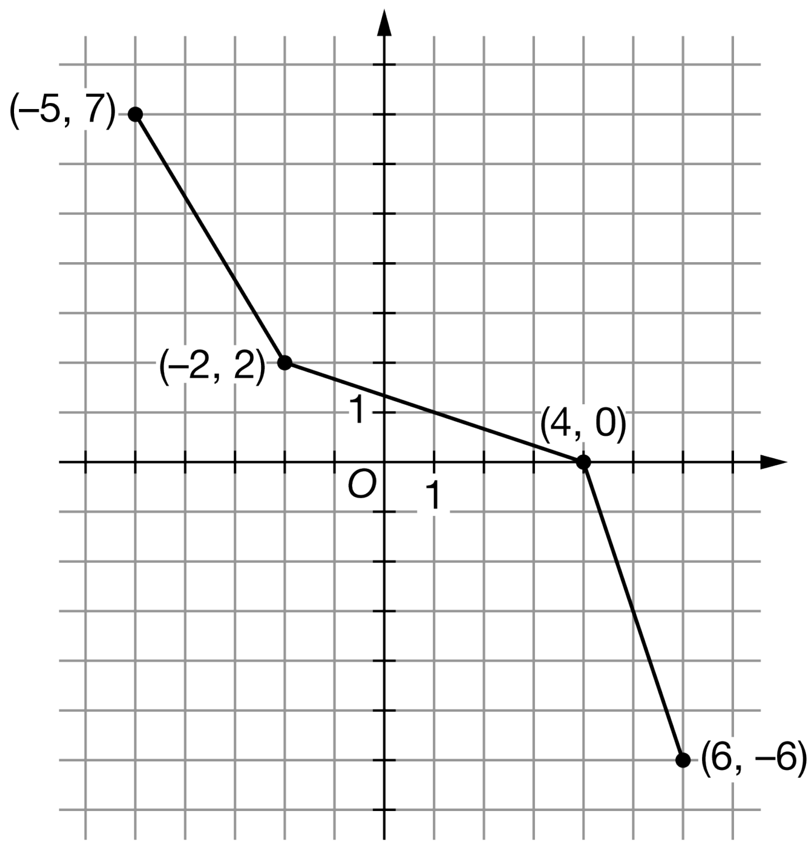
Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f

The graph of the function f on the interval $-5 \leq x \leq 6$ consists of three line segments, as shown in the figure above. Let h be the function defined by $h(x) = \int_{-2}^{x} f(t)\,dt$ for $-5 \leq x \leq 6$.

(a) For the function h, is $x = 4$ the location of a relative minimum, a relative maximum, or neither? Give a reason for your answer.

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(b) Find the absolute maximum value of h on the interval $-5 \leq x \leq 6$. Justify your answer.

(c) On what open intervals contained in $-5 < x < 6$ is the graph of h both increasing and concave down? Give a reason for your answer.

(d) Find the value of $h''(-2)$, or explain why it does not exist.

(e) Find $\lim_{x \to -2} \frac{h(x)}{x-2}$. Show the computations that lead to your answer.

(f) Let k be the function defined by $k(x) = h(x^2)$. Find the value of $k'\left(\sqrt{5}\right)$ or explain why it does not exist. Show the computations that lead to your answer. (Note: $\sqrt{5}$ can be keyboarded as $\mathtt{sqrt}(5)$)

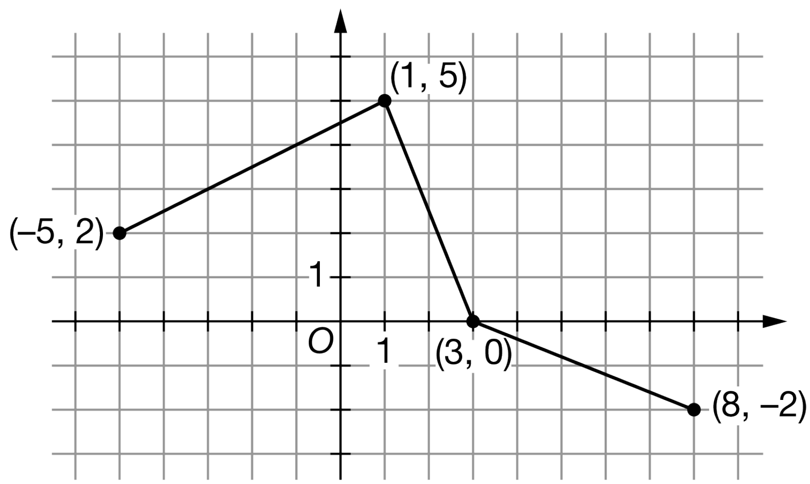
1.1

25. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of g

The graph of the function g on the interval $-5 \le x \le 8$ consists of three line segments, as shown in the figure above. Let m be the function defined by $m(x) = \int_1^x g(t)\,dt$ for $-5 \le x \le 8$.

(a) For the function m, is $x = 3$ the location of a relative minimum, a relative maximum, or neither? Give a reason for your answer.

(b) Find the absolute maximum value of m on the interval $-5 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $-5 < x < 8$ is the graph of m both increasing and concave down? Give a reason for your answer.

(d) Find the value of $m''(1)$, or explain why it does not exist.

(e) Find $\lim\limits_{x \to 1} \frac{m(x)}{x+1}$. Show the computations that lead to your answer.

(f) Let k be the function defined by $k(x) = m(x^2)$. Find the value of $k'(1)$ or explain why it does not exist.

1.1

Show the computations that lead to your answer.

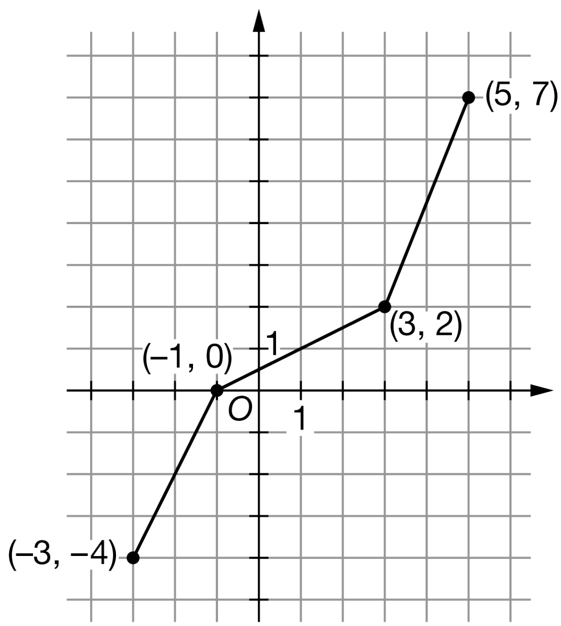
1.1

26. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of g

The graph of the function g on the interval $-3 \leq x \leq 5$ consists of three line segments, as shown in the figure above. Let k be the function defined by $k(x) = \int_3^x g(t)\,dt$ for $-3 \leq x \leq 5$.

(a) For the function k, is $x = -1$ the location of a relative minimum, a relative maximum, or neither? Give a reason for your answer.

(b) Find the absolute minimum value of k on the interval $-3 \leq x \leq 5$. Justify your answer.

(c) On what open intervals contained in $-3 < x < 5$ is the graph of k both increasing and concave up? Give a reason for your answer.

(d) Find the value of $k''(-1)$, or explain why it does not exist.

1.1

(e) Find $\lim_{x \to 3} \frac{k(x)}{x+3}$. Show the computations that lead to your answer.

(f) Let h be the function defined by $h(x) = k(x^2)$ Find the value of $h'\left(\sqrt{3}\right)$ or explain why it does not exist. Show the computations that lead to your answer. (Note: $\sqrt{3}$ can be keyboarded as $\mathtt{sqrt}(3)$)

1.1

27. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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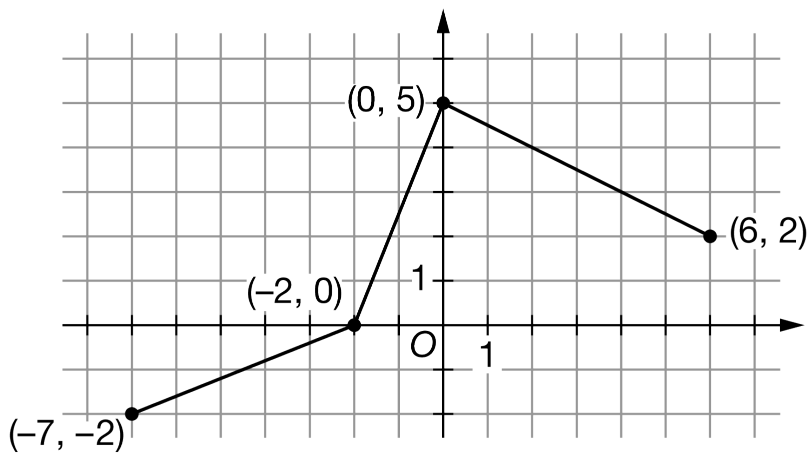
Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of h

The graph of the function h on the interval $-7 \leq x \leq 6$ consists of three line segments, as shown in the figure above. Let r be the function defined by $r(x) = \int_0^x h(t)dt$ for $-7 \leq x \leq 6$.

(a) For the function r, is $x = -2$ the location of a relative minimum, a relative maximum, or neither? Give a reason for your answer.

(b) Find the absolute minimum value of r on the interval $-7 \leq x \leq 6$. Justify your answer.

(c) On what open intervals contained in $-7 < x < 6$ is the graph of r both increasing and concave down? Give a reason for your answer.

(d) Find the value of $r''(-2)$, or explain why it does not exist.

(e) Find $\lim_{x \to 0} \frac{r(x)}{x-4}$.Show the computations that lead to your answer.

(f) Let v be the function defined by $v(x) = r\left(x^2\right)$. Find the value of $v'\left(\sqrt{3}\right)$ or explain why it does not exist.

1.1

Show the computations that lead to your answer. (Note: $\sqrt{3}$ can be keyboarded as $\mathtt{sqrt}(3)$)

1.1

28. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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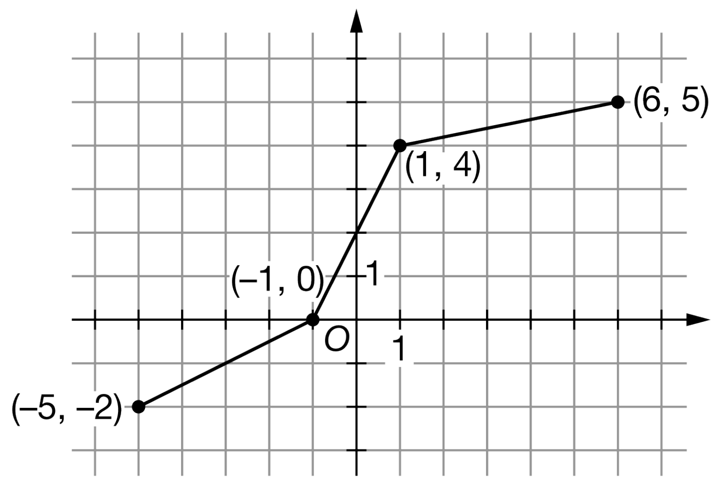

Graph of h

The graph of the function h on the interval $-5 \leq x \leq 6$ consists of three line segments, as shown in the figure above. Let p be the function defined by $p(x) = \int_{1}^{x} h(t)dt$ for $-5 \leq x \leq 6$.

(a) For the function p, is $x = -1$ the location of a relative minimum, a relative maximum, or neither? Give a reason for your answer.

(b) Find the absolute minimum value of p on the interval $-5 \leq x \leq 6$. Justify your answer.

(c) On what open intervals contained in $-5 < x < 6$ is the graph of p both decreasing and concave up? Give a reason for your answer.

(d) Find the value of $p''(1)$, or explain why it does not exist.

(e) Find $\lim_{x \to 1} \frac{p(x)}{x+4}$. Show the computations that lead to your answer.

(f) Let k be the function defined by $k(x) = p\left(x^2\right)$. Find the value of $k'\left(\sqrt{2}\right)$ or explain why it does not exist.

1.1

Show the computations that lead to your answer. (Note: $\sqrt{2}$ can be keyboarded as $\mathtt{sqrt(2)}$)

1.1

29. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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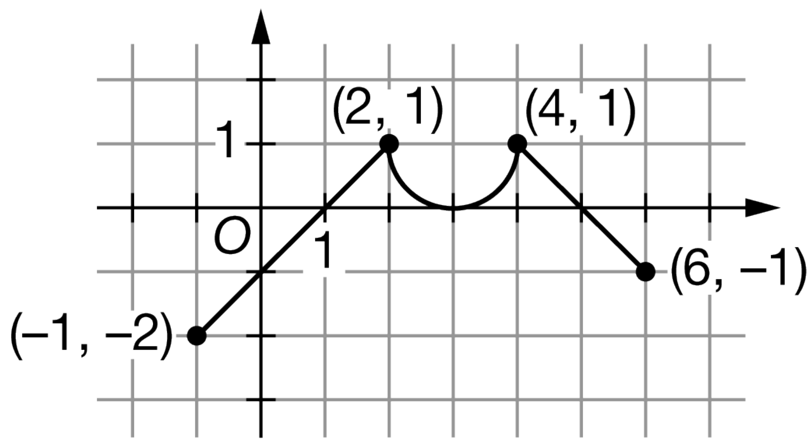

Graph of h

The continuous function h has domain $-1 \leq x \leq 6$. The graph of h, consisting of two line segments and a semicircle, is shown in the figure above. The graph of h has a horizontal tangent at $x = 3$. Let g be the function defined by $g(x) = \int_2^x h(t)dt$ for $-1 \leq x \leq 6$.

(a) Find the x-coordinate of each critical point of g on the interval $-1 \leq x \leq 6$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify each of your answers.

(c) For $-1 \leq x \leq 6$, on what open intervals is g decreasing and concave up? Give a reason for your answer.

(d) Find the value of $g(4)$. Show the computations that lead to your answer.

(e) Find the value of $g(-1)$. Show the computations that lead to your answer.

(f) Find the value of $g''(0)$, or explain why it does not exist.

1.1

(g) Must there exist a value of d, for $-1 < d < 3$, such that $g'(d)$ is equal to the average rate of change of g over the interval $-1 \leq x \leq 3$? Justify your answer.

(h) Find $\lim_{x\to4} \frac{g'(x)}{3x}$. Show the computations that lead to your answer.

(i) The function a is defined by $a(x) = g(2x)$. Find $a'(0)$. Show the computations that lead to your answer.

1.1

30. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

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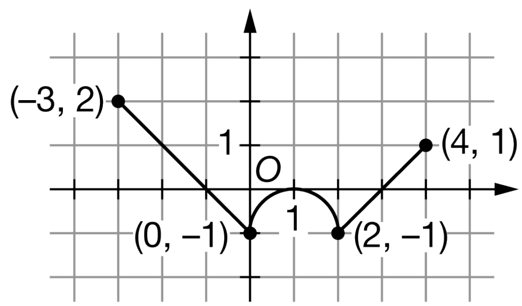

Graph of f

The continuous function f has domain $-3 \leq x \leq 4$. The graph of f, consisting of two line segments and a semicircle, is shown in the figure above. The graph of f has a horizontal tangent at $x = 1$. Let g be the function defined by $g(x) = \int_0^x f(t)dt$ for $-3 \leq x \leq 4$.

(a) Find the x-coordinate of each critical point of g on the interval $-3 < x < 4$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g. Justify each of your answers.

(c) For $-3 < x < 4$, on what open intervals is g decreasing and concave up? Give a reason for your answer.

(d) Find the value of $g(2)$. Show the computations that lead to your answer.

(e) Find the value of $g(-3)$. Show the computations that lead to your answer.

(f) Find the value of $g''(-2)$, or explain why it does not exist.

(g) Must there exist a value of b, for $-3 < b < 1$, such that $g'(b)$ is equal to the average rate of change of g over the interval $-3 \leq x \leq 1$? Justify your answer.

(h) Find $\lim_{x \to 2} \frac{g'(x)}{4x}$. Show the computations that lead to your answer.

1.1

(i) The function z is defined by $z(x) = g\left(x^2\right)$. Find $z'(-2)$. Show the computations that lead to your answer.

1.1

31. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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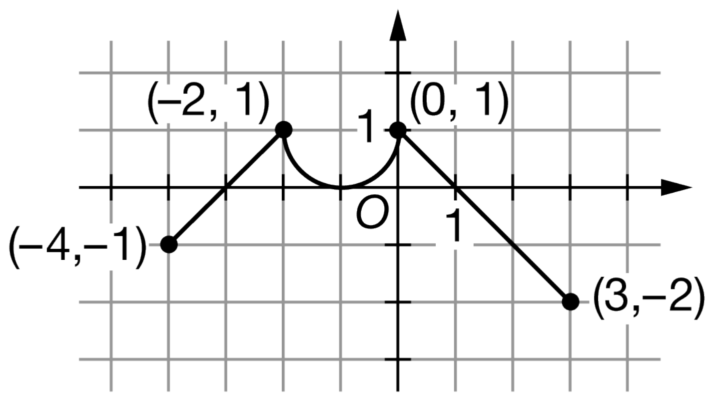

Graph of f

The continuous function f has domain $-4 \leq x \leq 3$. The graph of f, consisting of two line segments and a semicircle, is shown in the figure above. The graph of f has a horizontal tangent at $x = -1$. Let h be the function defined by $h(x) = \int_0^x f(t)dt$ for $-4 \leq x \leq 3$.

(a) Find the x-coordinate of each critical point of h on the interval $-4 < x < 3$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify each of your answers.

(c) For $-4 < x < 3$, on what open intervals is h increasing and concave down? Give a reason for your answer.

(d) Find the value of $h(-2)$. Show the computations that lead to your answer.

(e) Find the value of $h(3)$. Show the computations that lead to your answer.

(f) Find the value of $h''(2)$, or explain why it does not exist.

(g) Must there exist a value of k, for $-4 < k < 0$, such that $h'(k)$ is equal to the average rate of change of h over the interval $-4 \leq x \leq 0$? Justify your answer.

1.1

(h) Find $\lim_{x \to -2} \frac{h'(x)}{3x}$. Show the computations that lead to your answer.

(i) The function r is defined by $r(x) = h(-3x)$. Find $r'(-1)$. Show the computations that lead to your answer.

1.1

32. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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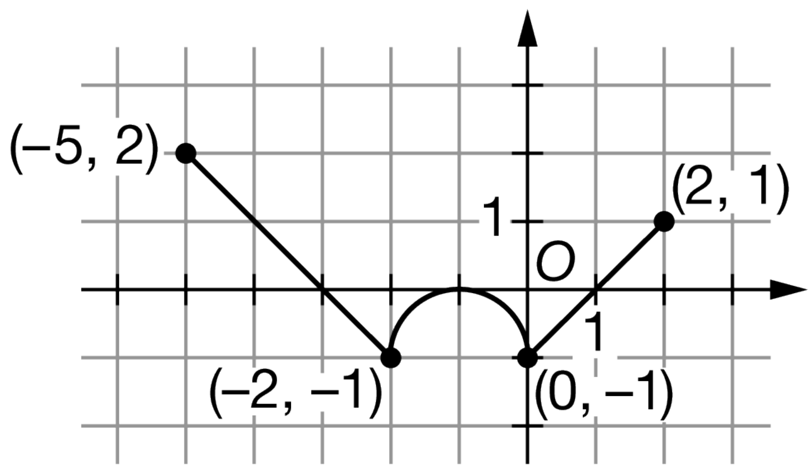


Graph of g

The continuous function g has domain $-5 \le x \le 2$. The graph of g, consisting of two line segments and a semicircle, is shown in the figure above. The graph of g has a horizontal tangent at $x = -1$. Let h be the function defined by $h(x) = \int_{-2}^{x} g(t)\,dt$ for $-5 \le x \le 2$.

(a) Find the x-coordinate of each critical point of h on the interval $-5 < x < 2$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify each of your answers.

(c) For $-5 < x < 2$, on what open intervals is h increasing and concave up? Give a reason for your answer.

(d) Find the value of $h(0)$. Show the computations that lead to your answer.

(e) Find the value of $h(-5)$. Show the computations that lead to your answer.

(f) Find the value of $h''(-3)$, or explain why it does not exist.

1.1

(g) Must there exist a value of p, for $-3 < p < 1$, such that $h'(p)$ is equal to the average rate of change of h over the interval $-3 \leq x \leq 1$? Justify your answer.

(h) Find $\lim\limits_{x \to -2} \frac{h'(x)}{4x}$. Show the computations that lead to your answer.

(i) The function f is defined by $f(x) = h(-2x)$. Find $f'(1)$. Show the computations that lead to your answer.

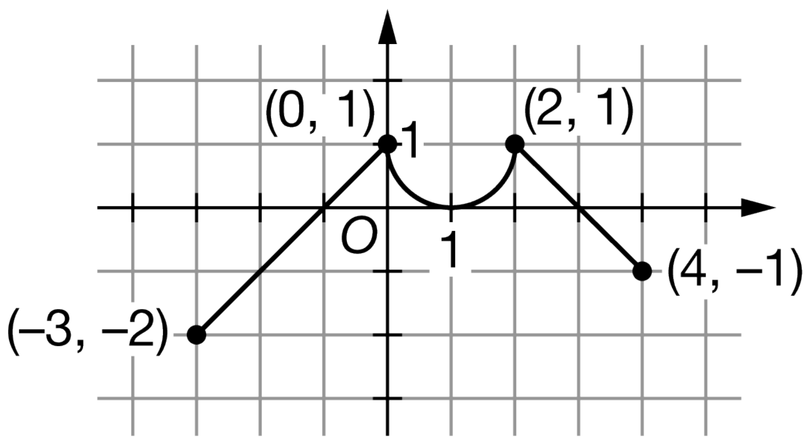
1.1

33. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of g

The continuous function g has domain $-3 \leq x \leq 4$. The graph of g, consisting of two line segments and a semicircle, is shown in the figure above. The graph of g has a horizontal tangent at $x = 1$. Let h be the function defined by $h(x) = \int_0^x g(t)\,dt$ for $-3 \leq x \leq 4$.

(a) Find the x-coordinate of each critical point of h on the interval $-3 < x < 4$.

(b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for h. Justify each of your answers.

(c) For $-3 < x < 4$, on what open intervals is h decreasing and concave down? Give a reason for your answer.

(d) Find the value of $h(2)$. Show the computations that lead to your answer.

(e) Find the value of $h(-3)$. Show the computations that lead to your answer.

(f) Find the value of $h''(-1)$, or explain why it does not exist.

1.1

(g) Must there exist a value of r, for $-3 < r < 1$, such that $h'(r)$ is equal to the average rate of change of h over the interval $-3 \le x \le 1$? Justify your answer.

(h) Find $\lim_{x\to 2} \frac{h'(x)}{5x}$. Show the computations that lead to your answer.

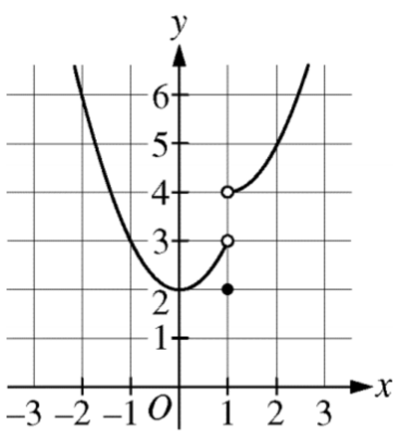
(i) The function k is defined by $k(x) = h(x^2)$. Find $k'(-2)$. Show the computations that lead to your answer.

34.

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) ln2
(B) ln8
(C) ln16
(D) 4
(E) nonexistent

35.



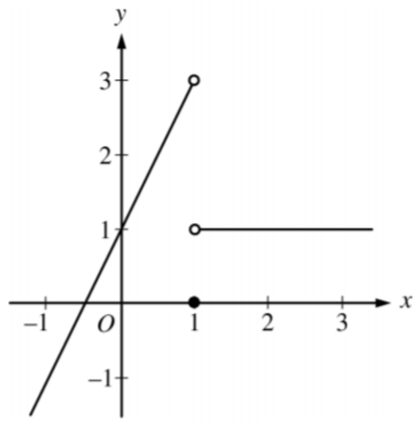
Graph of f

The graph of the function f is shown in the figure above. The value of $\lim_{x\to 0} f(1-x^2)$ is

(A) 1
(B) 2
(C) 3
(D) 4
(E) nonexistent

1.1

36.

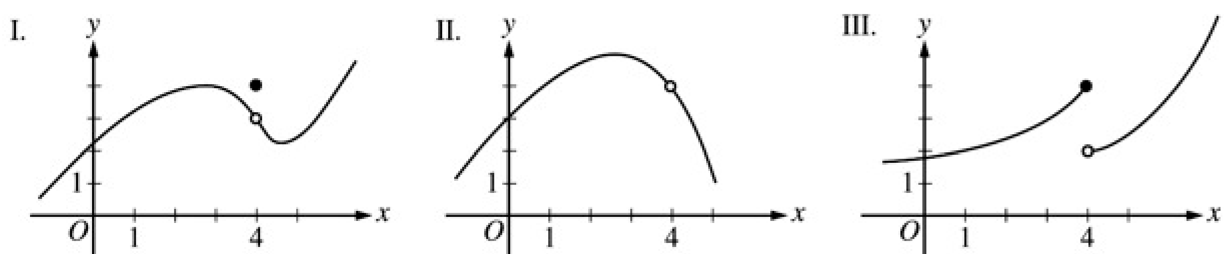


Graph of f

The graph of $y = f(x)$ is shown above. What is $\lim\limits_{x \to 1} f(x)$?

(A) 0
(B) 1
(C) 3
(D) The limit does not exist.

37. For which of the following does $\lim\limits_{x \to 4} f(x)$ exist?

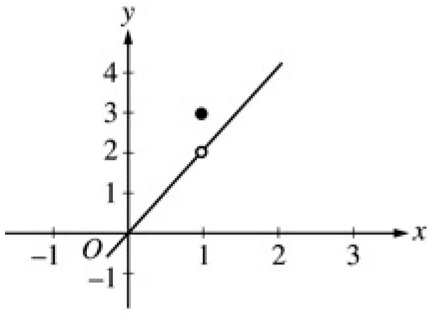


Graph of f Graph of f Graph of f

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

1.1

38.



Graph of f

The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1} \sin(f(x))$ is

(A) 0.909

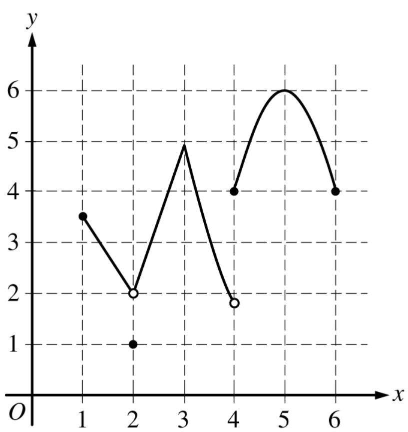
(B) 0.841

(C) 0.141

(D) -0.416

(E) nonexistent

39.



Graph of f

The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

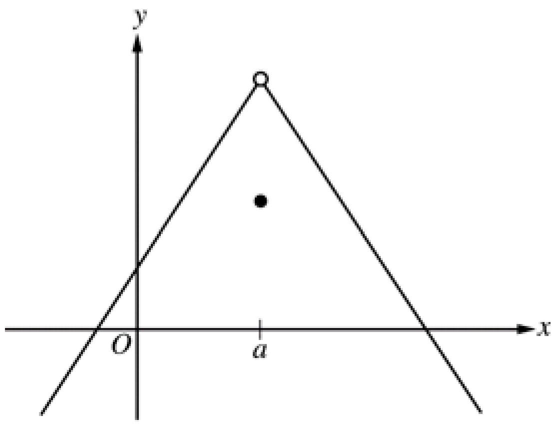
(C) $\lim_{x \to 4} f(x)$ exists.

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

1.1

40.



Graph of f

The graph of the function f is shown above. Which of the following statements must be false?

(A) $f(a)$ exists.

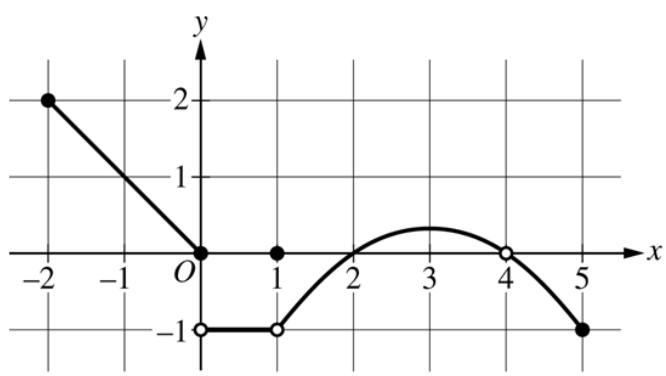
(B) $f(x)$ is defined for $0 < x < a$.

(C) f is not continuous at $x = a$.

(D) $\lim\limits_{x \to a} f(x)$ exists.

(E) $\lim\limits_{x \to a} f'(x)$ exists.

41.



Graph of f

The graph of the function f is shown above. For what values of a does $\lim\limits_{x \to a} f(x) = 0$?

(A) 2 only

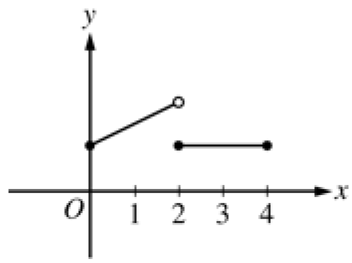
(B) 2 and 4

(C) 0 and 2 only

(D) 0, 1, and 2

1.1

42.



Graph of f

The figure above shows the graph of a function f with domain $0 \leq x \leq 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, and III

43.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	–0.1054	–0.0101	–0.001	0.001	0.0099	0.0953

The function f is continuous and increasing for $x \geq -1$. The table above gives values of f at selected values of x. Of the following, which is the best approximation for $\lim_{x \to 0} e^{-2f(x)}$?

(A) -2
(B) 0
(C) 1
(D) The limit does not exist

44. If f is a continuous function such that $f(2) = 6$, which of the following statements must be true?

1.1

(A) $\lim_{x \to 1} f(2x) = 3$

(B) $\lim_{x \to 2} f(2x) = 12$

(C) $\lim_{x \to 2} \frac{f(x)-f(2)}{x-2} = 6$

(D) $\lim_{x \to 2} f\left(x^2\right) = 36$

(E) $\lim_{x \to 2} (f(x))^2 = 36$

45.

$\lim_{x \to -5} f(x) = 4$	$\lim_{x \to 5} f(x) = 2$	$\lim_{x \to 5} g(x) = 5$

The table above gives selected limits of the functions *f* and *g*. What is $\lim_{x \to 5} (f(-x) + 3g(x))$?

(A) 19

(B) 17

(C) 13

(D) 9

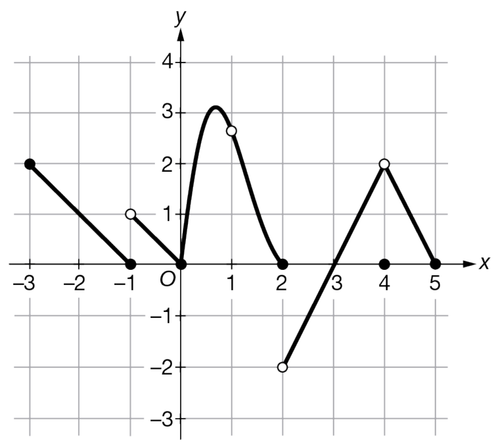
1.1

46. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

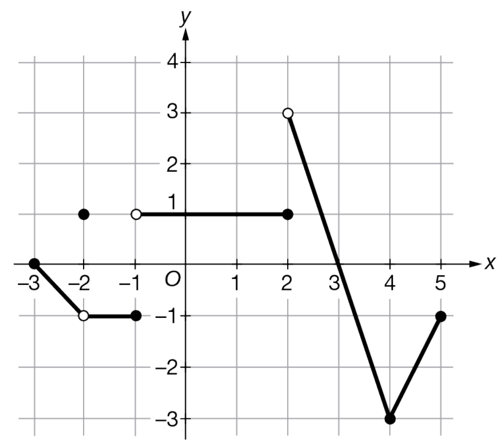
Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



Graph of f



Graph of g

The graphs of the functions f and g are shown above on the interval $-3 \leq x \leq 5$. The graphs consist of line segments, except where $f(x) = \frac{5\sin(\pi x)}{2-2x^3}$ on the interval $0 \leq x \leq 2$.

(a) Write a difference quotient that best approximates the instantaneous rate of change of g at $x = 4.5$.

(b) Let $h(x) = g(f(x))$. Find $\lim\limits_{x\to 4} h(x)$. Use correct limit notation in your answer.

(c) Let $k(x) = f(x) + g(x)$. Consider $x = -1$, $x = 2$, and $x = 4$. At which one of these values does k have a jump discontinuity? Justify your answer for the jump discontinuity using correct limit notation.

(d) Values of $f(x)$ at selected values of x are given in the table below. What type of discontinuity does f have at $x = 1$? Based on the values in the table, what is a reasonable estimate for $\lim\limits_{x\to 1} f(x)$? Give a reason for your answer.

x	0.995	0.996	0.997	0.998	0.999	1	1.001	1.002	1.003	1.004	1.005
$f(x)$	2.631	2.628	2.626	2.623	2.621	Undefined	2.615	2.613	2.610	2.607	2.605

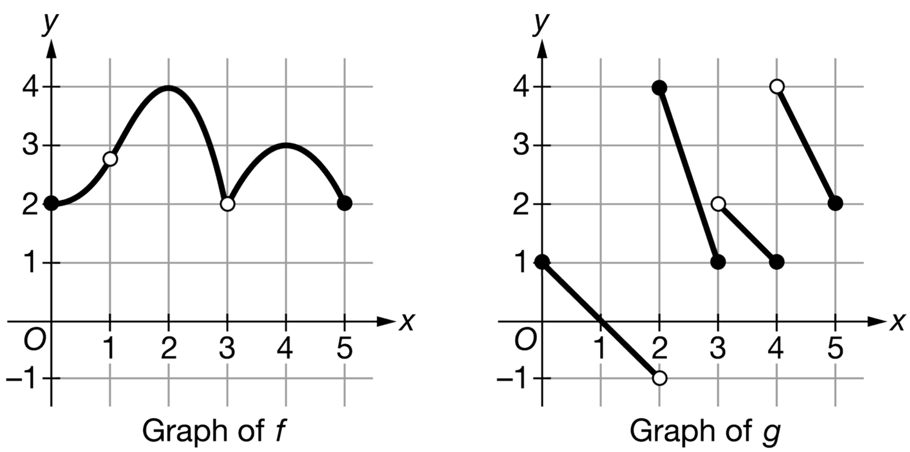
1.1

47. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



The graphs of the functions f and g are shown above on the interval $0 \leq x \leq 5$.

(a) Write a difference quotient that best approximates the instantaneous rate of change of g at $x = 2.5$.

(b) Let h be the function defined by $h(x) = g(f(x))$. Find the value of $\lim_{x \to 3} h(x)$ or explain why the limit does not exist. Use correct limit notation in your answer.

(c) Let k be the function defined by $k(x) = (4 - f(x))g(x)$. Consider $x = 2$ and $x = 4$. Determine whether k is continuous at each of these values. Justify your answers using correct limit notation.

(d) Values of $f(x)$ at selected values of x are given in the table below. What type of discontinuity does f have at $x = 1$? Based on the values in the table, what is a reasonable estimate for $\lim_{x \to 1} f(x)$? Give a reason for your answer.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	2.634	2.770	2.784	Undefined	2.787	2.801	2.946

1.1

48. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

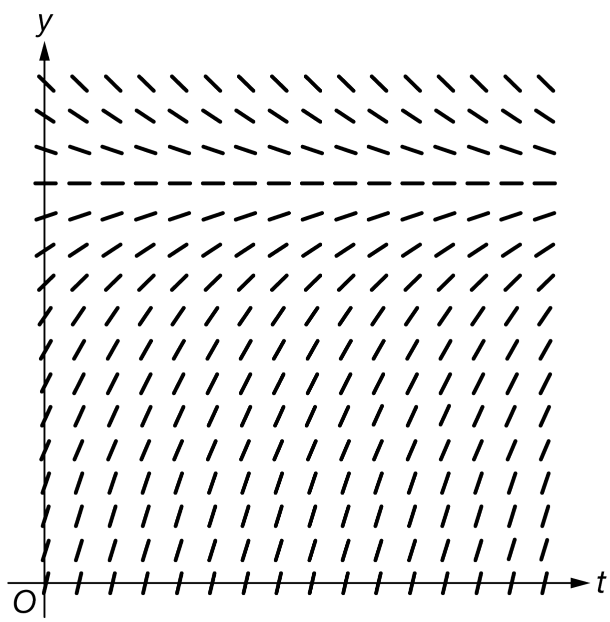
Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

A medication is administered to a patient. The amount, in milligrams, of the medication in the patient at time t hours is modeled by a function $y = A(t)$ that satisfies the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$. At time $t = 0$ hours, there are 0 milligrams of the medication in the patient.

(a) A portion of the slope field for the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$ is given below. Sketch the solution curve through the point $(0, 0)$.



(b) Using correct units, interpret the statement $\lim_{t \to \infty} A(t) = 12$ in the context of this problem.

(c) Use separation of variables to find $y = A(t)$, the particular solution to the differential equation $\frac{dy}{dt} = \frac{12-y}{3}$ with initial condition $A(0) = 0$.

1.1

(d) A different procedure is used to administer the medication to a second patient. The amount, in milligrams, of the medication in the second patient at time t hours is modeled by a function $y = B(t)$ that satisfies the differential equation $\frac{dy}{dt} = 3 - \frac{y}{t+2}$. At time $t = 1$ hour, there are 2.5 milligrams of the medication in the second patient. Is the rate of change of the amount of medication in the second patient increasing or decreasing at time $t = 1$? Give a reason for your answer.

49. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sin x}{x} =$

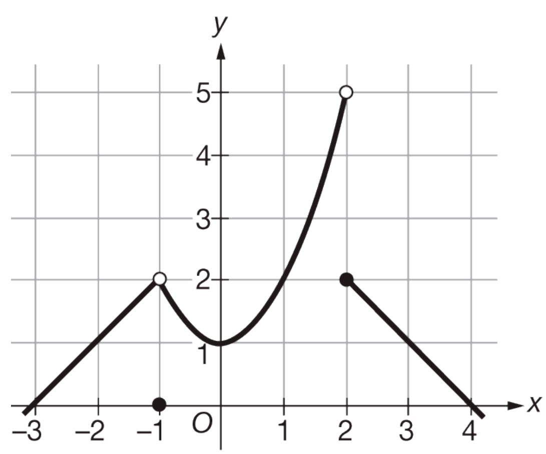
(A) $-\frac{4}{\pi^2}$

(B) 0

(C) $\frac{2}{\pi}$

(D) 1

50.



Graph of f

The graph of the function f is shown above. What is $\lim\limits_{x \to -1} f(f(x))$?

(A) 1

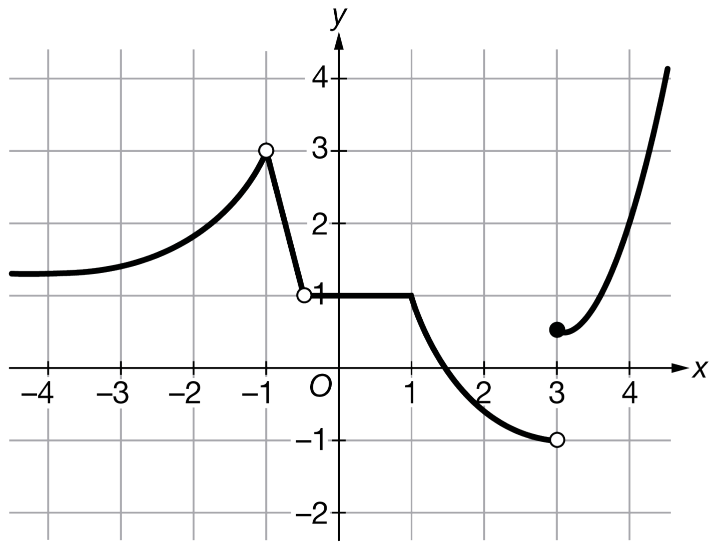
(B) 2

(C) 5

(D) nonexistent

1.1

51.



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to -1} f(f(x))$?

(A) 3

(B) $\frac{1}{2}$

(C) -1

(D) nonexistent

52. $$f(x) = \begin{cases} 5x - 3 & \text{for } x < 2 \\ 9 & \text{for } x = 2 \\ 4x + 3 & \text{for } x > 2 \end{cases}$$

Let f be the piecewise function defined above. The value of $\lim_{x \to 2^+} f(x)$ is

(A) 7

(B) 9

(C) 11

(D) nonexistent

53. $$f(x) = \begin{cases} x^2 - 4x + 4 & \text{for } x < 2 \\ 3 & \text{for } x = 2 \\ x^2 - 4x + 8 & \text{for } x > 2 \end{cases}$$

Let f be the piecewise function defined above. The value of $\lim_{x \to 2^+} f(x)$ is

1.1

(A) 0
(B) 3
(C) 4
(D) nonexistent

54.

$f(2)=3$	$\lim\limits_{x\to 2} f(x)=4$
$g(2)=-6$	$\lim\limits_{x\to 2} g(x)=-6$
$h(2)=-3$	$\lim\limits_{x\to 2} h(x)=2$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim\limits_{x\to 2}(h(x)(5f(x)+g(x)))$?

(A) -27
(B) -20
(C) 28
(D) 34

55.

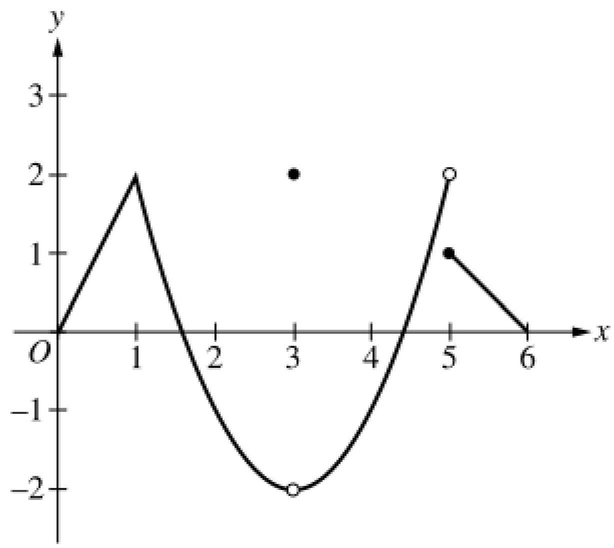
$f(3)=3$	$\lim\limits_{x\to 3} f(x)=2$
$g(3)=8$	$\lim\limits_{x\to 3} g(x)=8$
$h(3)=4$	$\lim\limits_{x\to 3} h(x)=2$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim\limits_{x\to 3}(h(x)(2f(x)+3g(x)))$?

(A) 120
(B) 104
(C) 56
(D) 32

1.1

56.



Graph of f

The graph of the function f is shown. For which of the following values of a does $\lim_{x \to a} f(x)$ not exist?

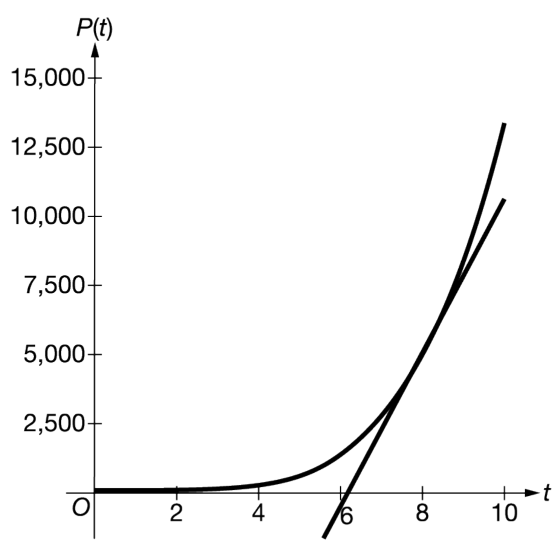
(A) 1 only

(B) 5 only

(C) 3 and 5 only

(D) 1, 3, and 5

57.



The size of a population of bacteria is modeled by the function P, where $P(t)$ gives the number of bacteria and t gives the number of hours after midnight for $0 \leq t \leq 10$. The graph of the function P and the line tangent to P at $t = 8$ are shown above. Which of the following gives the best estimate for the instantaneous rate of change of P at $t = 8$?

1.1

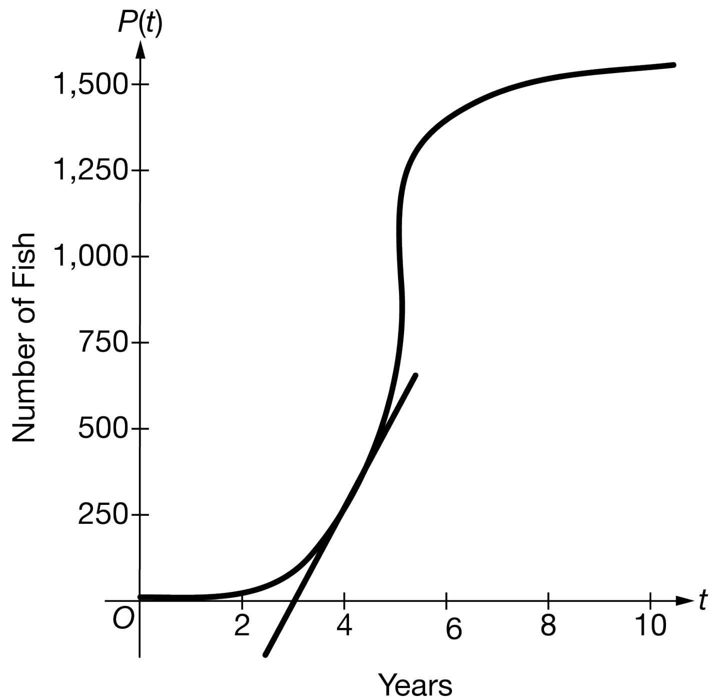
(A) $P(8)$

(B) The slope of the line joining $(0, P(0))$ and $(8, P(8))$

(C) The slope of the line joining $(0, P(0))$ and $(10, P(10))$

(D) The slope of the line joining $(7.9, P(7.9))$ and $(8.1, P(8.1))$

58.



The size of a population of fish in a pond is modeled by the function P, where $P(t)$ gives the number of fish and t gives the number of years after the first year of introduction of the fish to the pond for $0 \leq t \leq 10$. The graph of the function P and the line tangent to P at $t = 4$ are shown above. Which of the following gives the best estimate for the instantaneous rate of change of P at $t = 4$?

(A) $P(4)$

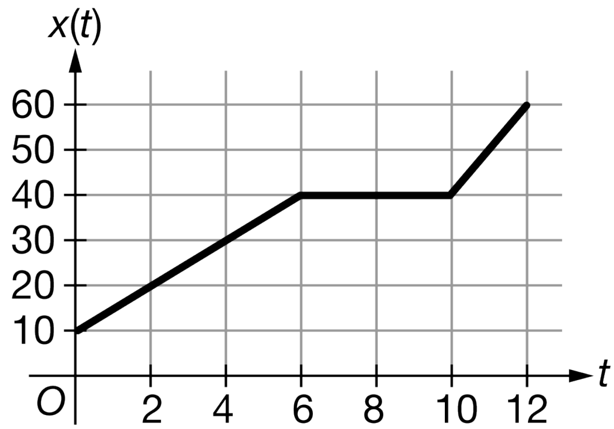
(B) The slope of the line joining $(0, P(0))$ and $(4, P(4))$

(C) The slope of the line joining $(0, P(0))$ and $(8, P(8))$

(D) The slope of the line joining $(3.9, P(3.9))$ and $(4.1, P(4.1))$

1.1

59.



A particle is moving on the x-axis and the position of the particle at time t is given by $x(t)$, whose graph is shown above. Which of the following is the best estimate for the speed of the particle at time $t = 4$?

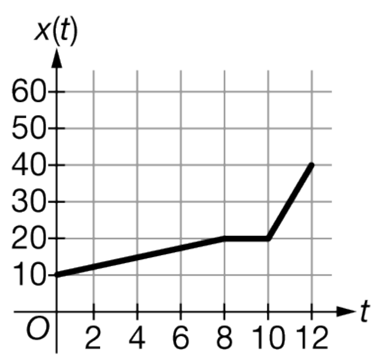
(A) 0

(B) 5

(C) $\frac{15}{2}$

(D) 10

60.



A particle is moving on the x-axis, and the position of the particle at time t is given by $x(t)$, whose graph is shown above. Which of the following is the best estimate for the speed of the particle at time $t = 6$?

(A) 0

(B) $\frac{5}{4}$

(C) $\frac{35}{12}$

(D) $\frac{10}{3}$

61. A car is driven on a straight road, and the distance traveled by the car after time $t = 0$ is given by a quadratic function s, where $s(t)$ is measured in feet and t is measured in seconds for $0 \leq t \leq 12$. Of the following, which gives the best estimate of the velocity of the car, in feet per second, at time $t = 6$ seconds?

1.1

(A) $s(6)$

(B) $\frac{s(6)}{6}$

(C) $\frac{s(8)-s(4)}{8-4}$

(D) $\frac{s(7)-s(5)}{7-5}$

62. An automobile is driven on a straight road, and the distance traveled by the automobile after time $t=0$ is given by a quadratic function s, where $s(t)$ is measured in feet and t is measured in seconds for $0 \leq t \leq 12$. Of the following, which gives the best estimate of the velocity of the automobile, in feet per second, at time $t=8$ seconds?

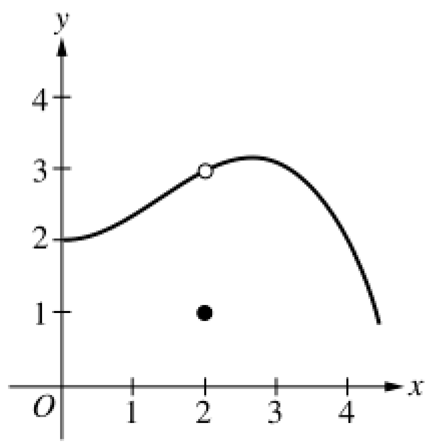
(A) $s(8)$

(B) $\frac{s(8)}{8}$

(C) $\frac{s(12)-s(2)}{12-2}$

(D) $\frac{s(9)-s(7)}{9-7}$

63.



Graph of f

The graph of the function f is shown. What is $\lim_{x \to 2} f(x)$?

(A) 0

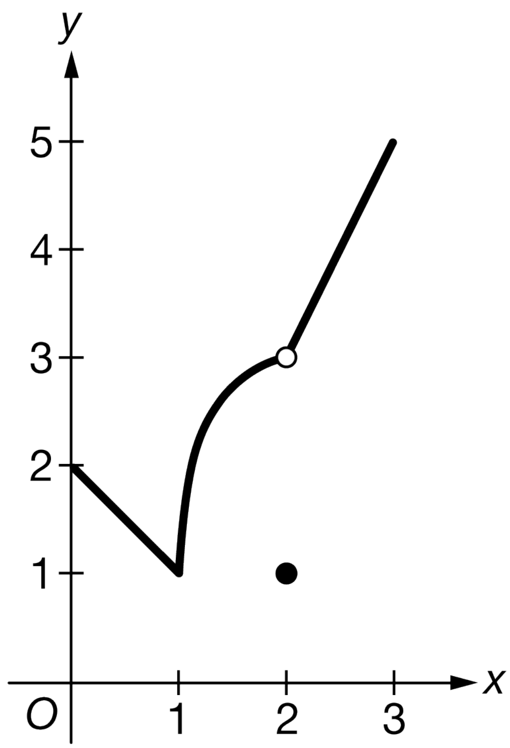
(B) 1

(C) 3

(D) The limit does not exist.

1.1

64.



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2} f(x)$?

(A) 1

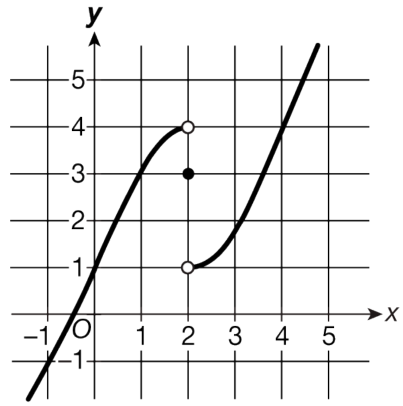
(B) 2

(C) 3

(D) The limit does not exist.

1.1

65.



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2^+} f(x)$?

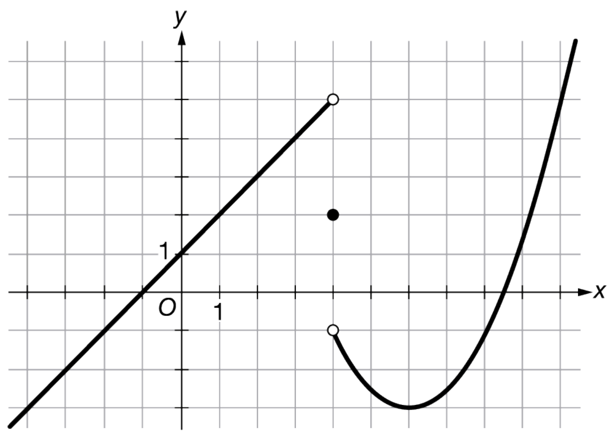
(A) 1

(B) 3

(C) 4

(D) nonexistent

66.



Graph of h

The graph of the function h is shown above. What is $\lim_{x \to 4} h(x)$?

(A) -1

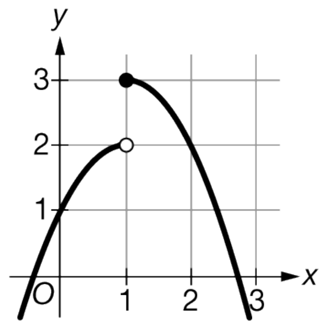
(B) 2

(C) 5

(D) nonexistent

1.1

67.



Graph of f

The graph of a function f is shown above. Which of the following statements is true?

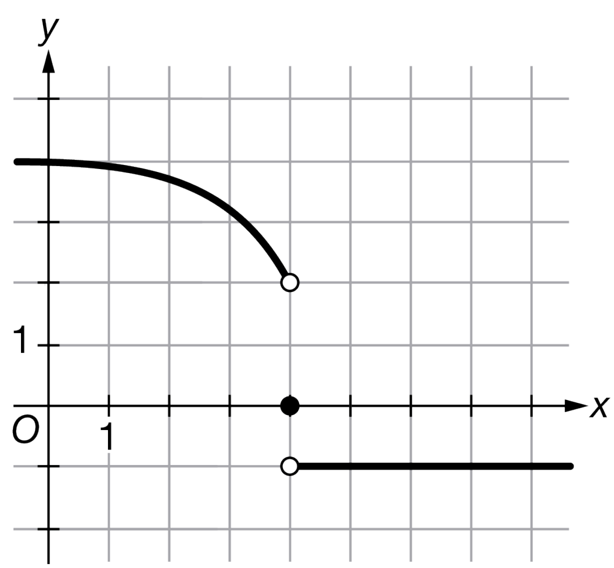
(A) $\lim_{x \to 1} f(x) = 2.5$

(B) $\lim_{x \to 1} f(x) = 3$

(C) $\lim_{x \to 1} f(x)$ does not exist because the left-hand and right-hand limits of $f(x)$ as x approaches 1 do not exist.

(D) $\lim_{x \to 1} f(x)$ does not exist because while the left-hand and right-hand limits of $f(x)$ as x approaches 1 exist, their values are not equal.

68.



Graph of h

The graph of the function h is shown above. What is $\lim_{x \to 4} h(x)$?

(A) -1

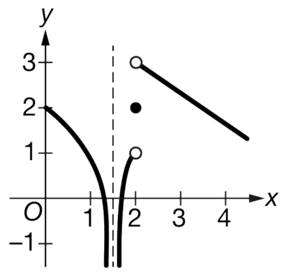
(B) 0

(C) 2

(D) nonexistent

1.1

69.

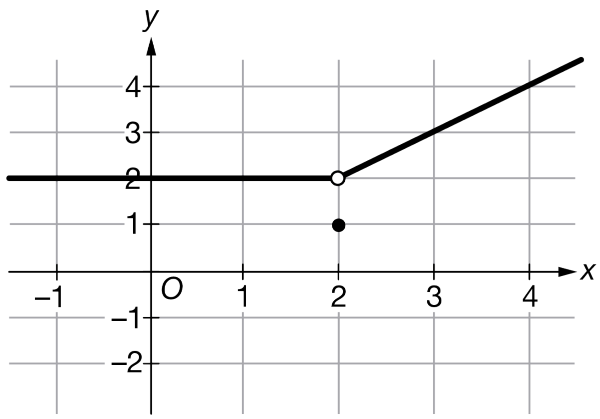


Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2^+} f(x)$?

(A) 3
(B) 2
(C) 1
(D) nonexistent

70.



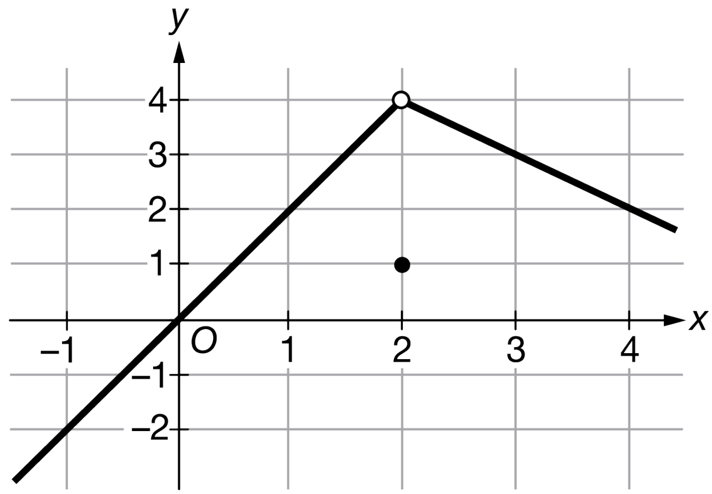
Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2} f(x)$?

(A) 0
(B) 1
(C) 2
(D) The limit does not exist.

1.1

71.



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to 2} f(x)$?

(A) $\frac{1}{2}$

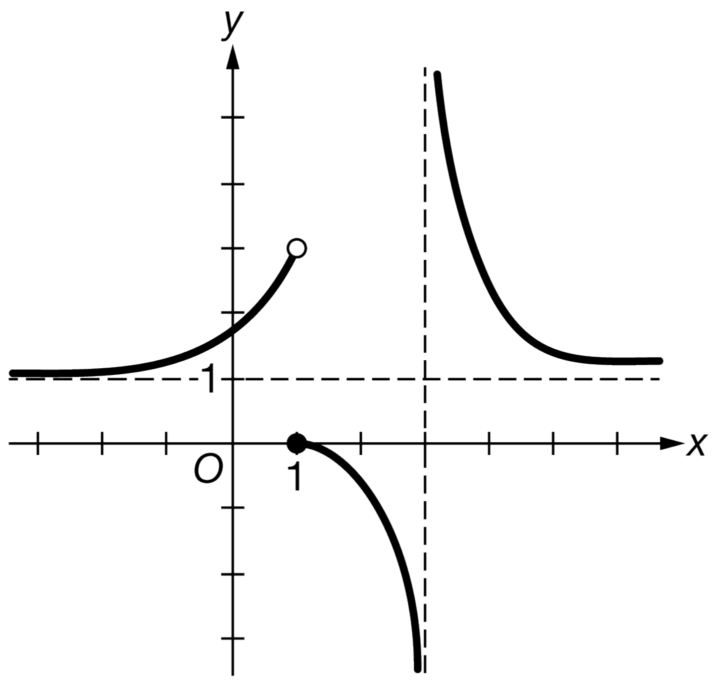
(B) 1

(C) 4

(D) The limit does not exist.

1.1

72.



Graph of f

The graph of a function f is shown above. Which of the following statements is true?

(A) $\lim_{x\to 1} f(x) = 1.5$

(B) $\lim_{x\to 1} f(x) = 0$

(C) $\lim_{x\to 1} f(x)$ does not exist because the left-hand and right-hand limits of $f(x)$ as x approaches 1 do not exist.

(D) $\lim_{x\to 1} f(x)$ does not exist because while the left-hand and right-hand limits of $f(x)$ as x approaches 1 exist, their values are not equal.

73. Of the following tables, which best reflects the values of a function g for which $\lim_{x\to 9} g(x) = 5$?

1.1

(A)

x	4.85	4.90	4.95	4.99	5.01	5.05	5.10	5.15
$g(x)$	8.700	8.800	8.900	8.980	9.020	9.100	9.200	9.300

(B)

x	8.85	8.90	8.95	8.99	9.01	9.05	9.10	9.15
$g(x)$	4.925	4.950	4.975	4.995	5.005	5.025	5.050	5.075

(C)

x	8.85	8.90	8.95	8.99	9.01	9.05	9.10	9.15
$g(x)$	4.950	4.967	4.983	4.997	10.030	10.070	10.120	10.170

(D)

x	8.85	8.90	8.95	8.99	9.01	9.05	9.10	9.15
$g(x)$	6.667	10.000	20.000	100.000	4.570	4.590	4.615	4.640

74.

x	10	10.9	10.99	10.999	11.001	11.01	11.1	12
$f(x)$	29	31.7	31.97	31.997	32.003	32.03	32.3	35

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x\to 11} f(x) = 32$

(B) $\lim_{x\to 11} f(x) = \infty$

(C) $\lim_{x\to 32} f(x) = 11$

(D) $\lim_{x\to 32} f(x) = \infty$

75.

x	1	1.9	1.99	1.999	1.9999	2.0001	2.001	2.01	2.1	3
$f(x)$	-4	-1.399	-1.040	-1.004	-1.000	6.001	6.012	6.121	7.261	25

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x\to 2} f\left(x\right) = -1$

(B) $\lim_{x\to 2} f\left(x\right) = 6$

(C) $\lim_{x\to 2^-} f\left(x\right) = -1$ and $\lim_{x\to 2^+} f\left(x\right) = 6$

(D) $\lim_{x\to 2^-} f\left(x\right) = 6$ and $\lim_{x\to 2^+} f\left(x\right) = -1$

1.1

76.

x	2	2.9	2.99	2.999	3.001	3.01	3.1	4
$f(x)$	-8	-80	-800	-8000	8000	800	80	8

The table above gives values of a function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x\to 3} f(x) = 0$

(B) $\lim_{x\to 3} f(x) = 3$

(C) $\lim_{x\to 3} f(x) = 10$

(D) $\lim_{x\to 3} f(x)$ does not exist.

77. Of the following tables, which best reflects the values of a function g for which $\lim_{x\to 7} g(x) = 6$?

(A)

x	5.85	5.90	5.95	5.99	6.01	6.05	6.10	6.15
$g(x)$	7.126	7.075	7.033	7.006	6.995	6.977	6.964	6.960

(B)

x	6.85	6.90	6.95	6.99	7.01	7.05	7.10	7.15
$g(x)$	5.620	5.837	5.961	5.998	5.999	5.964	5.863	5.709

(C)

x	6.85	6.90	6.95	6.99	7.01	7.05	7.10	7.15
$g(x)$	5.919	5.942	5.969	5.993	7.017	7.087	7.177	7.269

(D)

x	6.85	6.90	6.95	6.99	7.01	7.05	7.10	7.15
$g(x)$	1.362	5.954	10.691	14.690	6.010	6.049	6.095	6.140

78.

x	5	5.9	5.99	5.999	6.001	6.01	6.1	7
$f(x)$	2	20	200	2000	-2000	-200	-20	-2

The table above gives values of a function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim_{x\to 6} f(x) = 0$

(B) $\lim_{x\to 6} f(x) = 6$

(C) $\lim_{x\to 6} f(x) = 10$

(D) $\lim_{x\to 6} f(x)$ does not exist.

1.1

79.

x	1.9	1.99	1.999	1.9999	2.0001	2.001	2.01	2.1
$f(x)$	9.80	9.85	9.90	9.95	9.95	9.90	9.85	9.80

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim\limits_{x \to 2} f(x) = 10$

(B) $\lim\limits_{x \to 2} f(x) = \infty$

(C) $\lim\limits_{x \to 10} f(x) = 2$

(D) $\lim\limits_{x \to 10} f(x) = \infty$

80.

x	3.9	3.99	3.999	3.9999	4.0001	4.001	4.01	4.1
$f(x)$	5.8	5.85	5.9	5.95	6.999	6.99	6.9	6

The table above gives values of the function f at selected values of x. Which of the following conclusions is supported by the data in the table?

(A) $\lim\limits_{x \to 4} f(x) = 6$

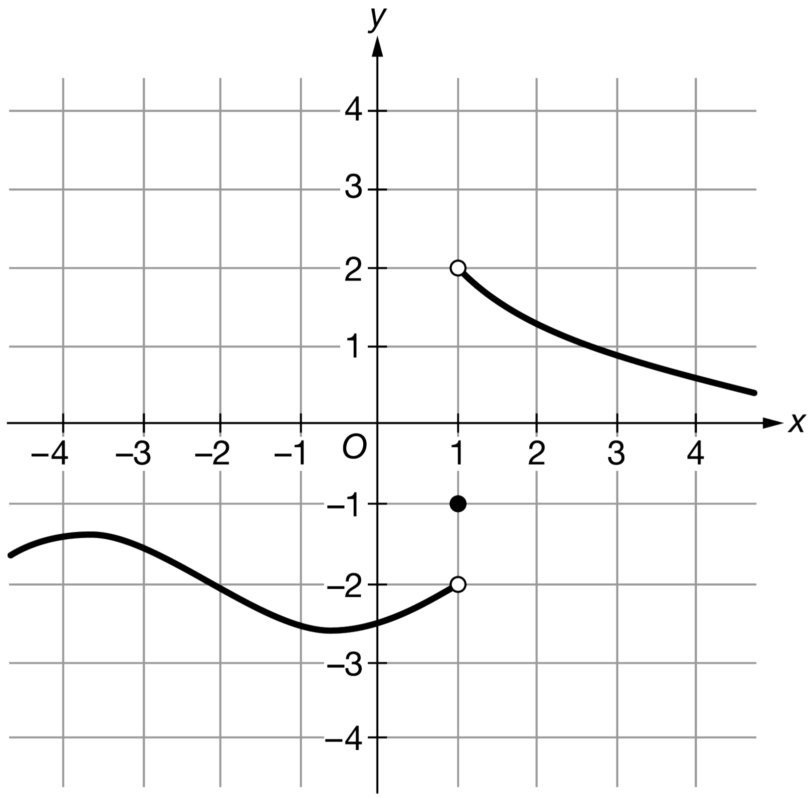
(B) $\lim\limits_{x \to 4} f(x) = 7$

(C) $\lim\limits_{x \to 4^-} f(x) = 6$ and $\lim\limits_{x \to 4^+} f(x) = 7$

(D) $\lim\limits_{x \to 4^-} f(x) = 7$ and $\lim\limits_{x \to 4^+} f(x) = 6$

1.1

81.



Graph of f

The graph of the function f is shown in the figure above. The value of $\lim_{x \to 1^+} f(x)$ is

(A) -2
(B) -1
(C) 2
(D) nonexistent

82. Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x \to 2} f(x) = 7$?

(A) The value of the function f at $x = 2$ is 7.
(B) The value of the function f at $x = 7$ is 2.
(C) As x approaches 2, the values of $f(x)$ approach 7.
(D) As x approaches 7, the values of $f(x)$ approach 2.

83. Let f be a function that is defined for all real numbers x. Of the following, which is the best interpretation of the statement $\lim_{x \to 3} f(x) = 5$?

1.1

(A) The value of the function f at $x = 3$ is 5.

(B) The value of the function f at $x = 5$ is 3.

(C) As x approaches 3, the values of $f(x)$ approach 5.

(D) As x approaches 5, the values of $f(x)$ approach 3.

84. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?