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PACE

Modelling, simulation & full-state control of a rotary inverted pendulum

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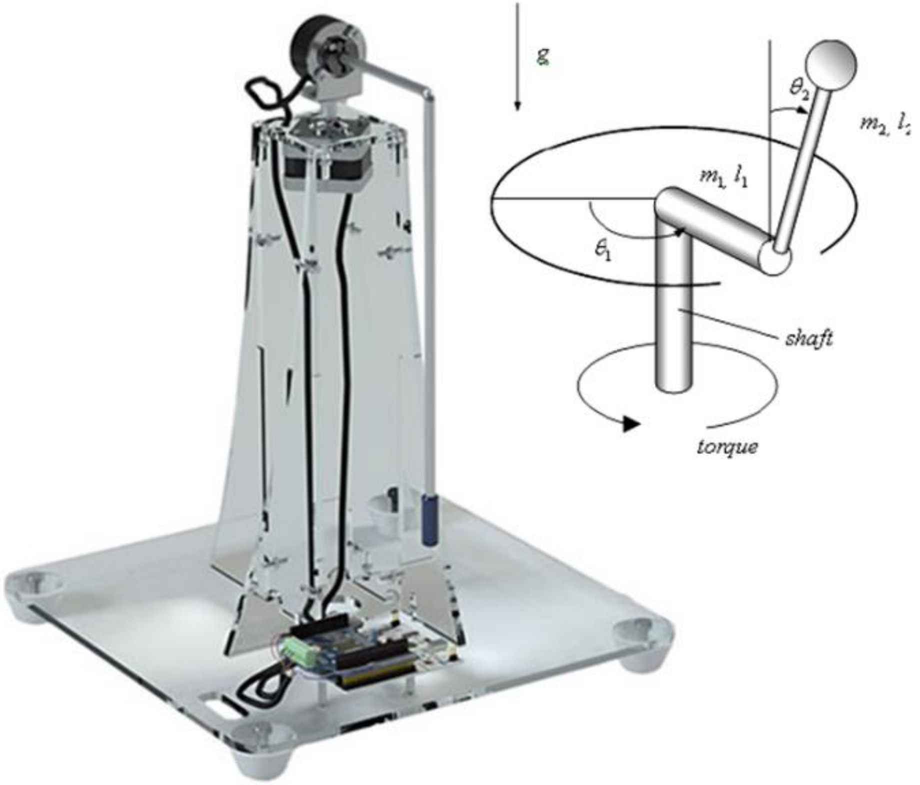
Scope

We want to elegantly control this system.

At the minute, the control software “spins up” the pendulum in quite an aggressive way, and the stabilised payload sort of drifts over time.

Challenge: Can we do better by using state-space model-based control?

Goals:
- Improve spin up using trajectory optimisation
- Stabilise beam completely without motor angle drift



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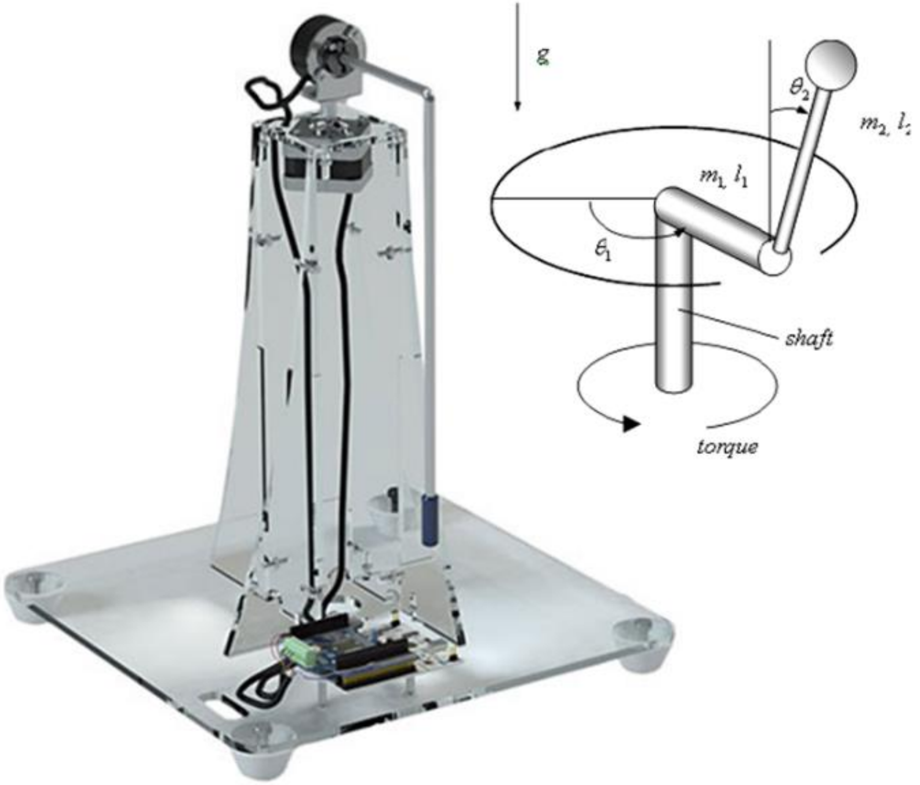
Modelling

Scope

These slides outline the steps required to derive the non-linear equations of motion (aka the dynamic model) of a rotary pendulum with only one actuator.



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Challenging topics involved:

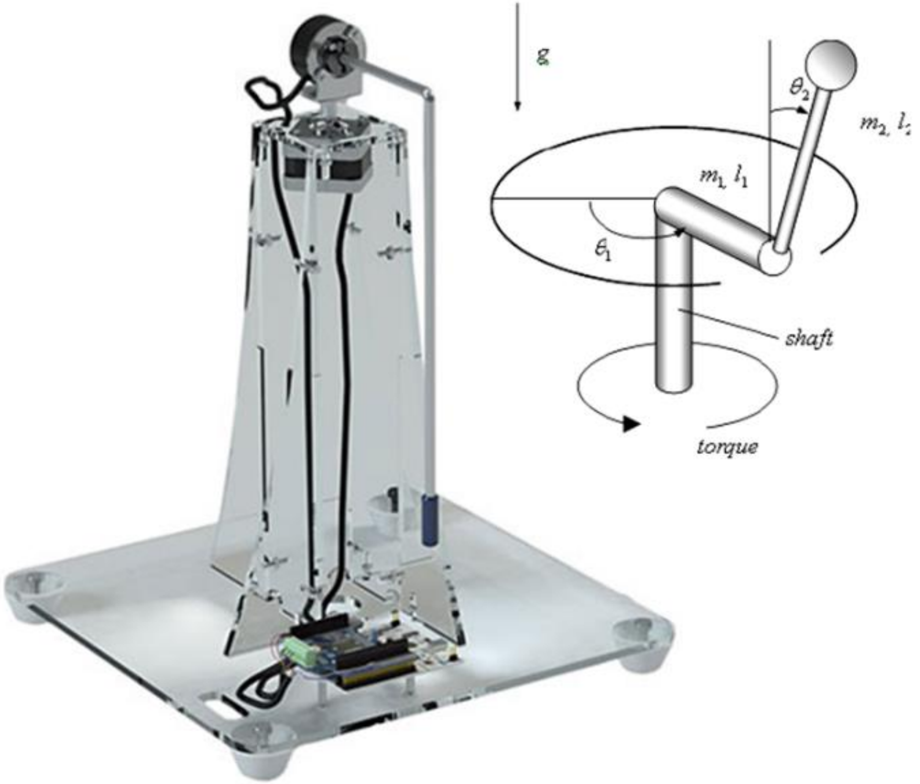
- Vector calculus
- Rigid Body Kinematics & Dynamics
  - Lagrangian Mechanics
- Simulation of Ordinary Differential Equations (ODEs)

What is a dynamic model?

When I say dynamic model, I usually mean an equation (or set of equations) that describe the system's behaviour.

Control engineers generally like this equation to be in the following form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u$$



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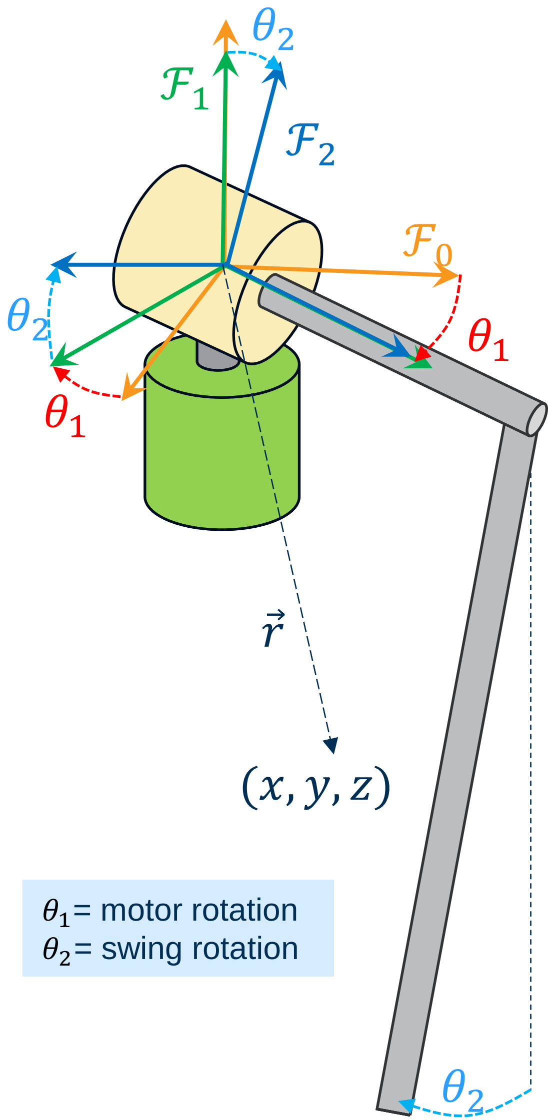
Important Kinematics

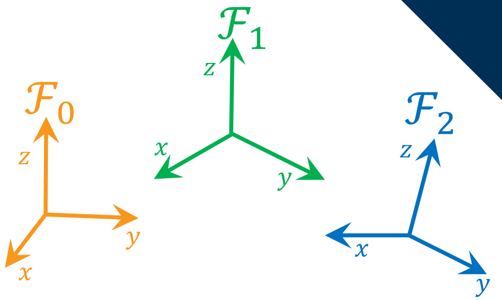


There are 3 frames of reference: $\mathcal{F}_0$, $\mathcal{F}_1$ and $\mathcal{F}_2$.

Inertial (globally fixed) frame

Encoder-fixed frame

Pendulum-fixed frame



- Let's define some position vector $\vec{r}$. We can express this as a numerical (x, y, z) vector, but these values depend on the frame of reference that we choose to express the vector in.
- Let's define: "$\vec{r}$ expressed in frame $\mathcal{F}_0$" as $\boldsymbol{r}^{\mathcal{F}_0} \in \mathbb{R}^3$.
- For our pendulum system, it can be shown that:

$$\boldsymbol{r}^{\mathcal{F}_1} = \mathbf{R}_{10}(\theta_1)\boldsymbol{r}^{\mathcal{F}_0}$$
$$\boldsymbol{r}^{\mathcal{F}_2} = \mathbf{R}_{21}(\theta_2)\boldsymbol{r}^{\mathcal{F}_1}$$

$$\boldsymbol{r}^{\mathcal{F}_2} = \underbrace{\mathbf{R}_{21}\mathbf{R}_{10}}\boldsymbol{r}^{\mathcal{F}_0}$$

$$\mathbf{R}_{20}(\theta_1, \theta_2) = \begin{bmatrix} \cos\theta_1 \cos\theta_2 & -\sin\theta_1 \cos\theta_2 & \sin\theta_2 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ -\cos\theta_1 \sin\theta_2 & \sin\theta_1 \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

This is known as a $z \rightarrow y'$ (yaw-pitch) transformation

Rotation matrices R

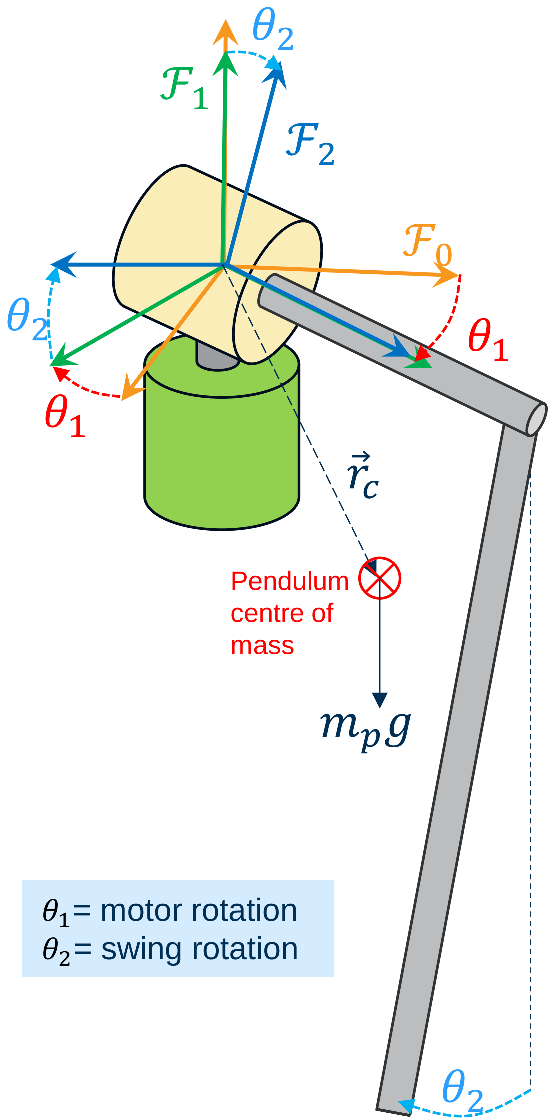
$$\mathbf{R}_{10} = \mathbf{R}_z(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_{21} = \mathbf{R}_y(\theta_2) = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

θ_1 = motor rotation
θ_2 = swing rotation

$\mathbf{R}_{20}$ describes the orientation of the pendulum body with respect to the inertial frame of reference.

Calculating the energy of the pendulum



θ_1= motor rotation
θ_2= swing rotation

Gravitational Potential Energy

$$V = m_p g \left. \boldsymbol{r}_c \right|_z$$

Vertical position of the centre of mass, expressed in the inertial frame.

$\boldsymbol{r}_c = \boldsymbol{r}_c^{\mathcal{F}_0} = \mathbf{R}_{20}(\theta_1, \theta_2)^\top \boldsymbol{r}_c^{\mathcal{F}_2}$

Rotational Kinetic energy

$$T_{rot_p} = \frac{1}{2} \boldsymbol{\omega}_p^\top \boldsymbol{I}_p \boldsymbol{\omega}_p$$

Pendulum inertia tensor, calculated in the next slides

Angular velocity of pendulum, expressed in the inertial frame, calculated in the next slides

Linear Kinetic energy

$$T_{lin_p} = \frac{1}{2} m_p \dot{\boldsymbol{r}}_c^\top \dot{\boldsymbol{r}}_c$$

Pendulum Mass

Linear velocity of pendulum centre of mass, expressed in the inertial frame. This can be shown to be:

$\dot{\boldsymbol{r}}_c = \dot{\boldsymbol{r}}_c^{\mathcal{F}_0} = \dot{\mathbf{R}}_{20}(\theta_1, \theta_2)^\top \boldsymbol{r}_c^{\mathcal{F}_2}$

We can now define the systems ‘Lagrangian’: $L = T_{rot_p} + T_{lin_p} - V$

Aside: Calculating angular velocity vector ω_p

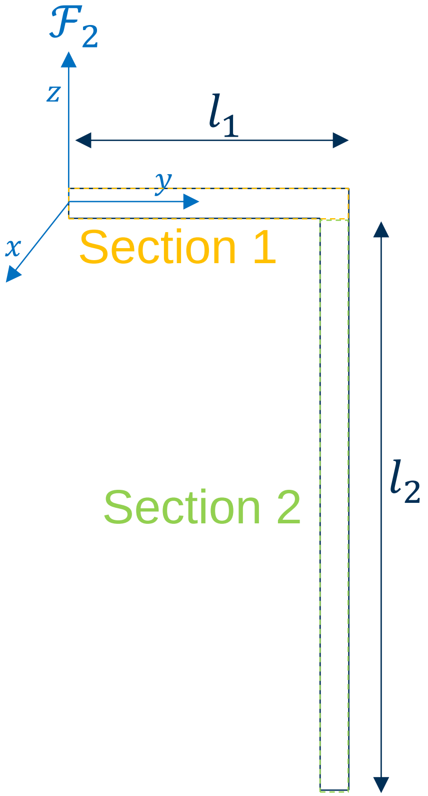
$$\dot{\mathbf{R}}_{20} = \frac{d}{dt}\mathbf{R}_{20}$$

It can be shown that: $\boldsymbol{\Omega}_{20} = \dot{\mathbf{R}}_{20}\,\mathbf{R}_{20}^{\top} = \begin{bmatrix} 0 & -\omega_{p_z} & \omega_{p_y} \\ \omega_{p_z} & 0 & -\omega_{p_x} \\ -\omega_{p_y} & \omega_{p_x} & 0 \end{bmatrix}$

Therefore we can calculate the body angular velocity vector directly as:

$$\boldsymbol{\omega}_p = \begin{bmatrix} \boldsymbol{\Omega}_{20}[3,2] \\ \boldsymbol{\Omega}_{20}[1,3] \\ \boldsymbol{\Omega}_{20}[2,1] \end{bmatrix}$$

Aside: Approximating the pendulum Inertia Tensor I_p



$\boldsymbol{I_p}$ can be thought of as the 'rotational mass' of an object. This depends on the axis of rotation, which makes $\boldsymbol{I_p}$ a 3X3 matrix.

$$\boldsymbol{I}_p^{\mathcal{F}_2} = \boldsymbol{I}_1^{\mathcal{F}_2} + \boldsymbol{I}_2^{\mathcal{F}_2}$$

Pendulum inertia tensor resolved in the moving (or 'body') frame $\mathcal{F}_2$

Section 1

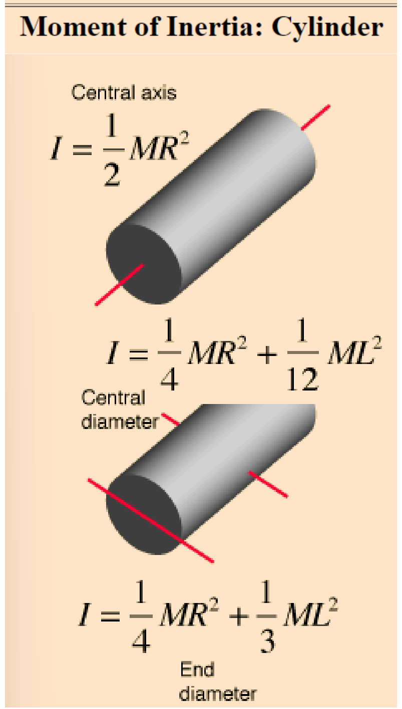
$$\boldsymbol{I}_1^{\mathcal{F}_2} = \begin{bmatrix} \frac{1}{4}m_1r^2 + \frac{1}{3}m_1l_1^2 & 0 & 0 \\ 0 & \frac{1}{2}m_1r^2 & 0 \\ 0 & 0 & \frac{1}{4}m_1r^2 + \frac{1}{3}ml_1^2 \end{bmatrix}$$

Section 2

$$\boldsymbol{I}_2^{\mathcal{F}_2} = \begin{bmatrix} \frac{1}{4}m_2r^2 + \frac{1}{12}m_2l_2^2 & 0 & 0 \\ 0 & \frac{1}{4}m_2r^2 + \frac{1}{12}m_2l_2^2 & 0 \\ 0 & 0 & \frac{1}{2}m_2r^2 \end{bmatrix} + \underbrace{m_2\left(\begin{bmatrix} 0 \\ l_1 \\ \frac{-l_2}{2} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} 0 \\ l_1 \\ \frac{-l_2}{2} \end{bmatrix} \boldsymbol{E}_3 - \begin{bmatrix} 0 \\ l_1 \\ \frac{-l_2}{2} \end{bmatrix} \begin{bmatrix} 0 \\ l_1 \\ \frac{-l_2}{2} \end{bmatrix}^{\mathsf{T}}\right)}_{\text{From the parallel axis theorem}}$$

Assumption: solid cylinder

(can improve this using measurements)



Finding the state-space 'Equations of Motion'

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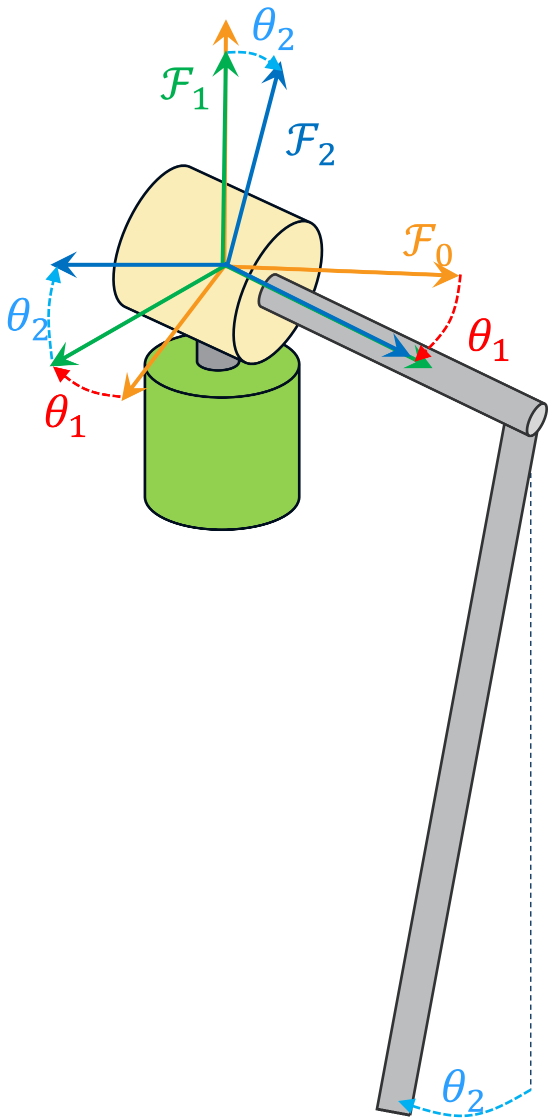


Let's define our state coordinates, $\boldsymbol{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$.

$\rightarrow$ $\boldsymbol{q}$ completely describes the position of our pendulum

Euler-Lagrange Equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial L}{\partial \boldsymbol{q}} = \boxed{\boldsymbol{F}}$$

The 'external force' vector. If we ignore friction damping, then $\boldsymbol{F} = \begin{bmatrix} u \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, where u is the torque applied by the motor.

This equation basically creates the "F=ma" of the system: the equation that describes the behaviour of the pendulum. Also known as a "Dynamic Model".

By substituting L into this and expanding / rearranging, the result ends up looking something like this:

"Mass Matrix" $\boxed{\boldsymbol{M}(\boldsymbol{q})}\ddot{\boldsymbol{q}} + \boxed{\boldsymbol{N}(\boldsymbol{q}, \dot{\boldsymbol{q}})} = \boldsymbol{F}$

Vector describing gravity & Coriolis effects

$\equiv$

Often it is nice to express this in the <u>*equivalent*</u> 'first order ODE form', by defining a new set of variables

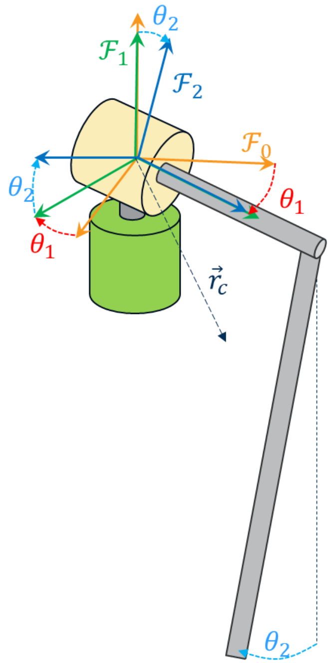
$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{M}^{-1}(\boldsymbol{F} - \boldsymbol{N}) \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{\boldsymbol{q}} \\ -\boldsymbol{M}^{-1}\boldsymbol{N} \end{bmatrix}}_{\boldsymbol{f}(\boldsymbol{x})} + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{x})} u$$

$$\boxed{\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u}$$

This model form is a "<u>Control affine</u>" system, meaning the input u appears linearly

Simulating the free-response system (u=0)

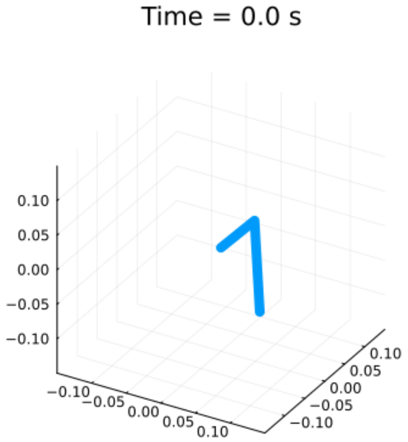


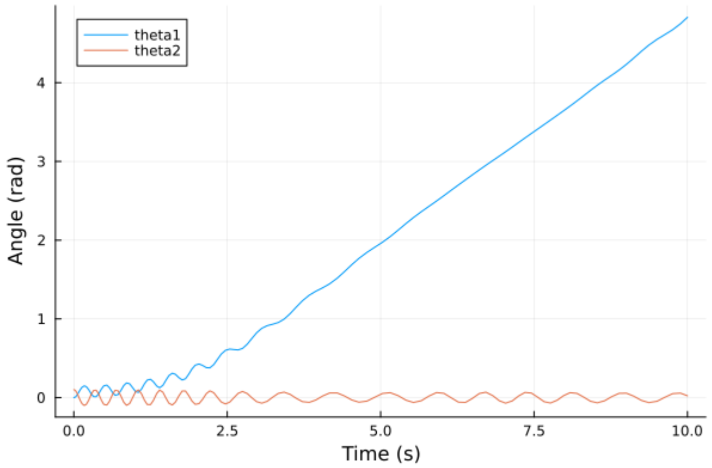
Given a starting state $\boldsymbol{x}(\boldsymbol{t}=\mathbf{0})=\boldsymbol{x}_0$, the model equations $\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x})$ can be 'solved' (aka 'simulated') using a variety of techniques (aka 'ODE solvers').

- This can be done in Python, Matlab, C++, Julia and more.
  - ➢ I use ODE solvers in Julia's DifferentialEquations.jl toolbox, this is just my personal preference.

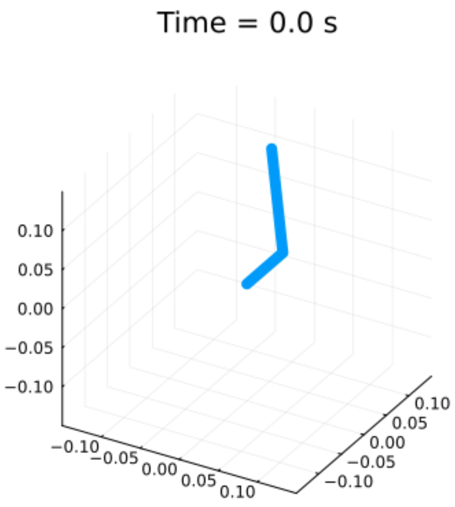
You can then create an animation of the model simulation.

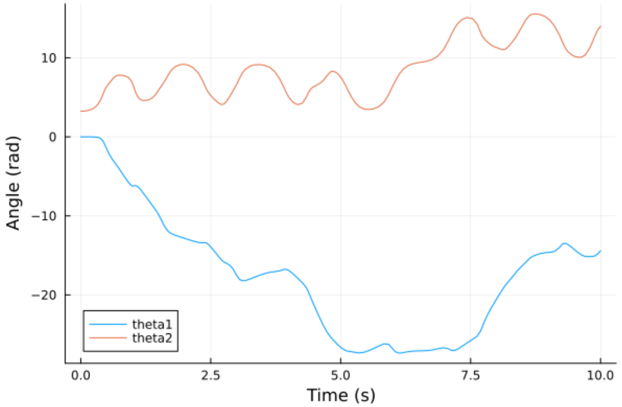
Example 1: $\boldsymbol{x}_0=\begin{bmatrix}\theta_{1_0}\\ \theta_{2_0}\\ \dot{\theta}_{1_0}\\ \dot{\theta}_{2_0}\end{bmatrix}=\begin{bmatrix}0\\ 0.1\\ 0\\ 0\end{bmatrix},\quad u=0$





Example 2: $\boldsymbol{x}_0=\begin{bmatrix}0\\ \pi+0.1\\ 0\\ 0\end{bmatrix},\quad u=0$



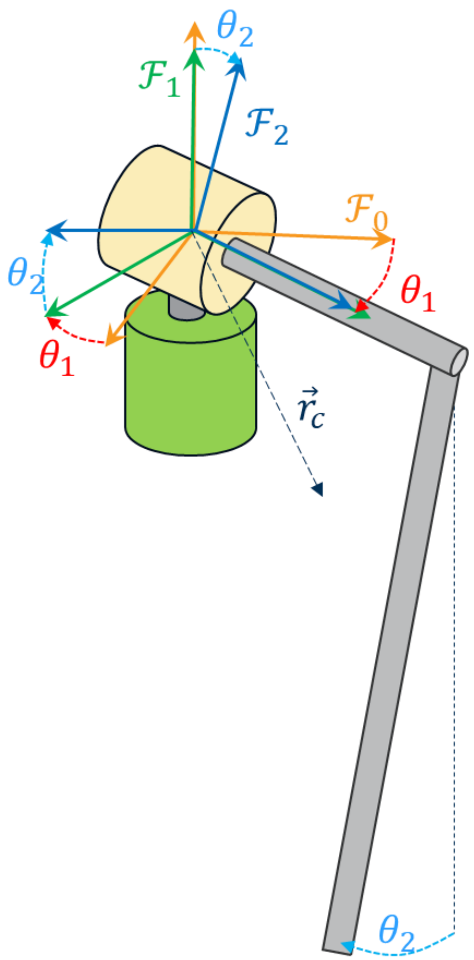


Very chaotic!
(no damping is modelled)

Adding damping to the model

In reality, there will be *viscous damping* at the joints.

- This is a friction torque proportional to the joint velocity:



$$\tau_{fr_i} \approx -\boxed{d_{fr_i}}\dot{\theta}_i$$

A damping constant of proportionality for joint i

In vector form:

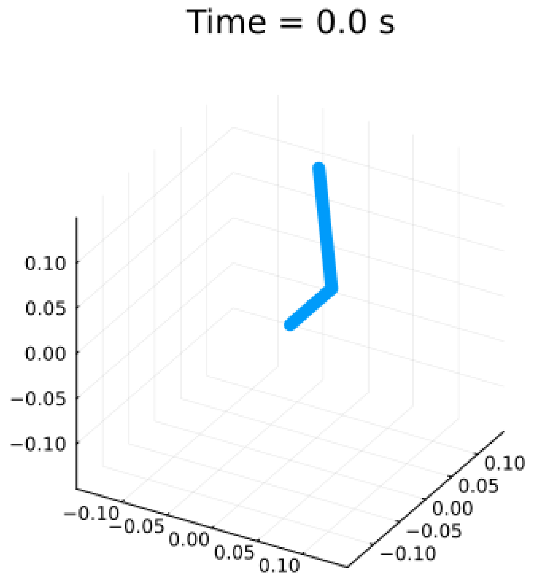
$$\boldsymbol{\tau}_{fr} \approx -\boxed{\boldsymbol{D}}\dot{\boldsymbol{\theta}}$$

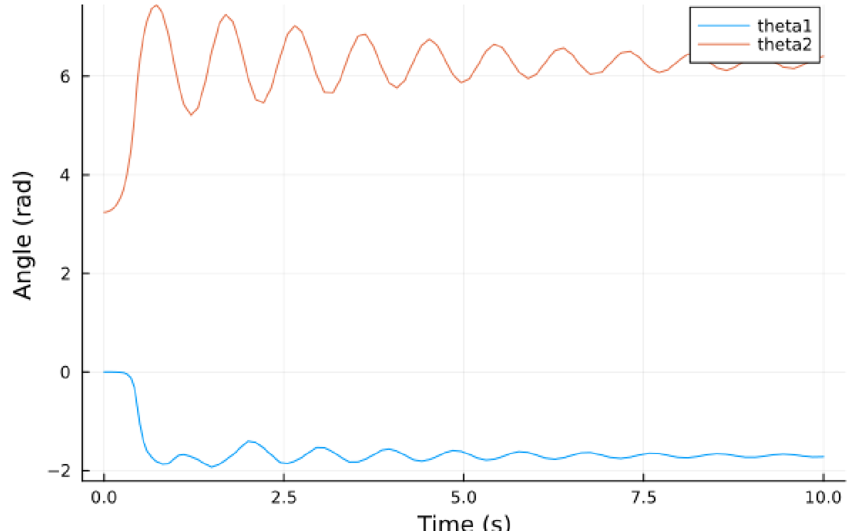
A diagonal Damping matrix

The model now becomes:

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{N}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{D}\dot{\boldsymbol{\theta}} = \boldsymbol{F}$$

$$\dot{\boldsymbol{x}} = \underbrace{\begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{M}^{-1}(\boldsymbol{F} - \boldsymbol{N} + \boldsymbol{D}\dot{\boldsymbol{\theta}}) \end{bmatrix}}_{f(x,u)}$$





Stepper motor stabilization control

Scope

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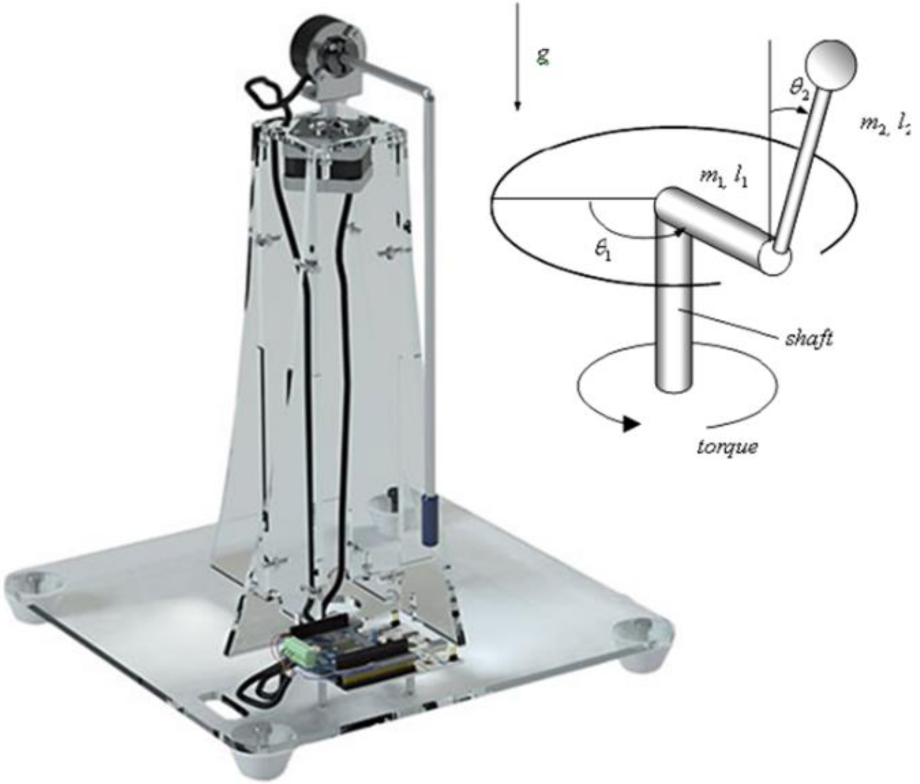
We want to explore techniques for stabilising the pendulum in its unstable position.

- Can we control both angles, θ_1 and θ_2?

- **Key point**: We cannot command the torque of the stepper motor, but we can command the acceleration…
- We need to do a bit of reformulation of the equations of motion to reflect this.



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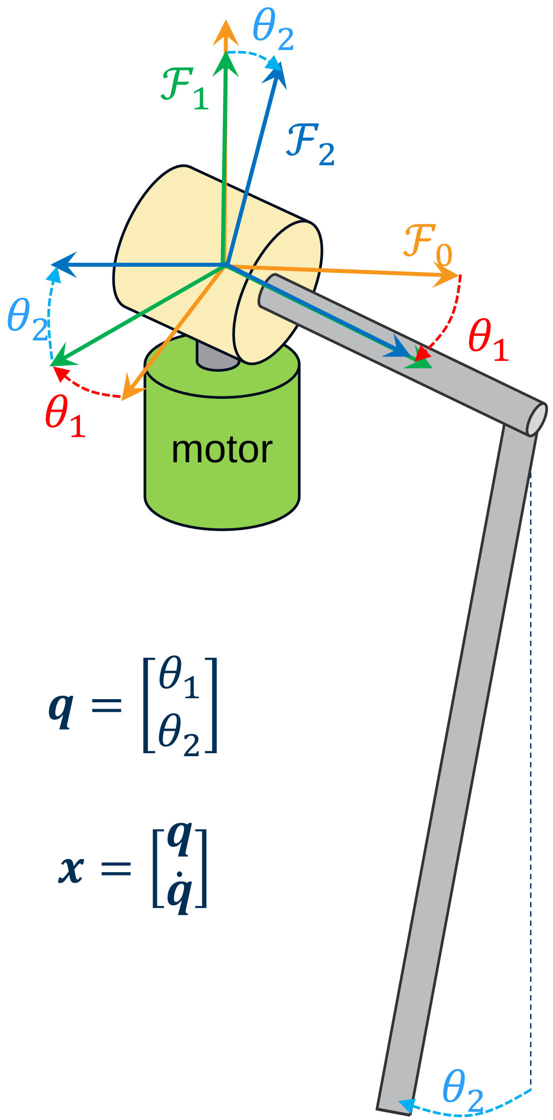
What if our control input is $u = \ddot{\theta}_1$, instead of the motor torque?



$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

Let's assume we can precisely control the acceleration of the stepper motor $\ddot{\theta}_1$. In terms of physics, this means the motor imparts a torque on the system that ensures $\ddot{\theta}_1$ follows a desired acceleration, $u(\mathrm{t})$.

Ideally, we want our system dynamic model of the following form:

$$\boxed{\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u}$$

where u is now acceleration and not torque.

How can we build the equation above?

- ➢ Using Lagrange Multiplier approach: see next slide.

What if our control input is $u = \ddot{\theta}_1$, instead of the motor torque?

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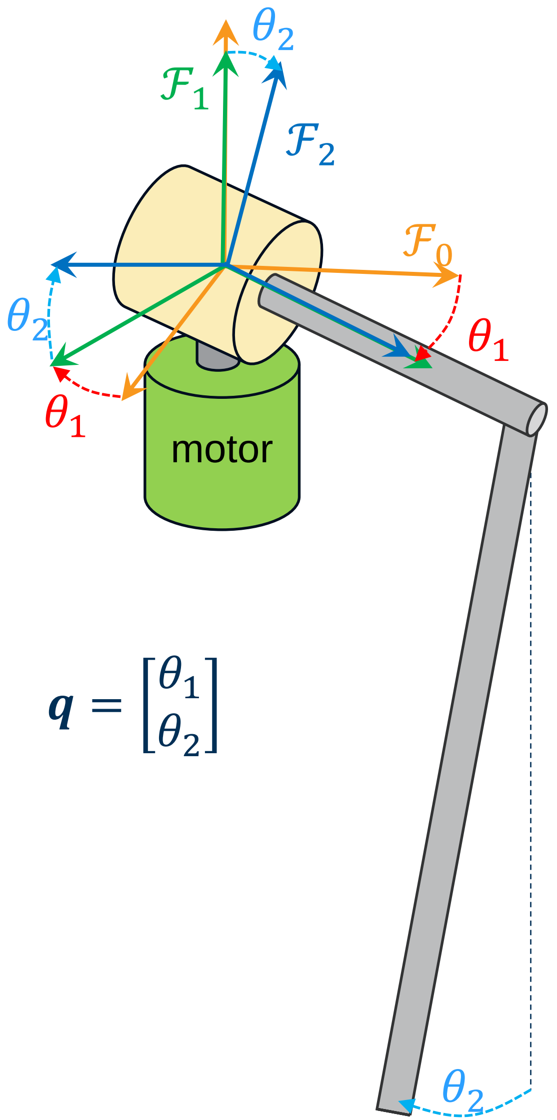


$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

-> Lets express $u = \ddot{\theta}_1 = \underbrace{[1 \;\; 0]}_{\Upsilon} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \Upsilon \ddot{q}$. We will call this our constraint.

Dynamics equations

$$M\ddot{q} + N + \Upsilon^\top \tau_1 = 0 \quad (1)$$

rearrange

$$\ddot{q} = M^{-1}(-N - \Upsilon^\top \tau_1) = 0$$

Constraint equations

$$\Upsilon\, \ddot{q} - u(t) = 0$$

Sub into

$$\Upsilon\, M^{-1}(-N - \Upsilon^\top \tau_1) - u(t) = 0$$

rearrange

$$\tau_1 = -(\Upsilon M^{-1} \Upsilon^\top)^{-1}(\Upsilon M^{-1} N + u)$$

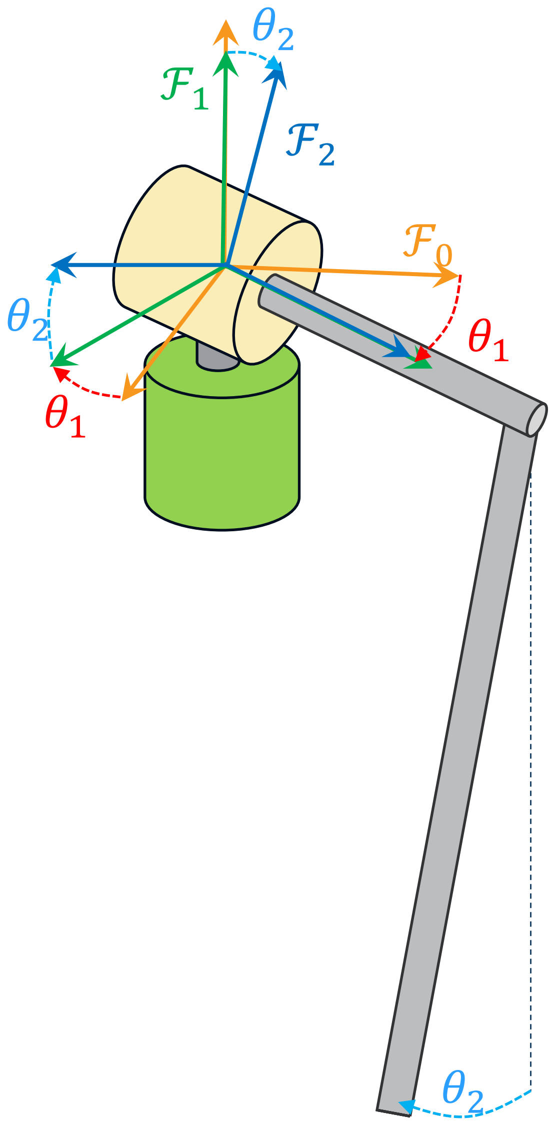
This can be thought of as the motor torque which achieves the constraint.

Sub into (1)

$$M\ddot{q} + N - \Upsilon^\top (\Upsilon M^{-1} \Upsilon^\top)^{-1} (\Upsilon M^{-1} N + u) = 0$$

$$M\ddot{q} + N_a = B_a u \quad \begin{cases} N_a = N - \Upsilon^\top (\Upsilon M^{-1} \Upsilon^\top)^{-1} \Upsilon M^{-1} N \\ B_a = \Upsilon^\top (\Upsilon M^{-1} \Upsilon^\top)^{-1} \end{cases}$$

What if our control input is $u = \ddot{\theta}_1$, instead of the motor torque?



$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{N}_a = \boldsymbol{B}_a u \quad \begin{cases} \boldsymbol{N}_a = \boldsymbol{N} - \boldsymbol{\Upsilon}^\top(\boldsymbol{\Upsilon}\boldsymbol{M}^{-1}\boldsymbol{\Upsilon}^\top)^{-1}\boldsymbol{\Upsilon}\boldsymbol{M}^{-1}\boldsymbol{N} \\ \boldsymbol{B}_a = \boldsymbol{\Upsilon}^\top(\boldsymbol{\Upsilon}\boldsymbol{M}^{-1}\boldsymbol{\Upsilon}^\top)^{-1} \end{cases}$$

First order form

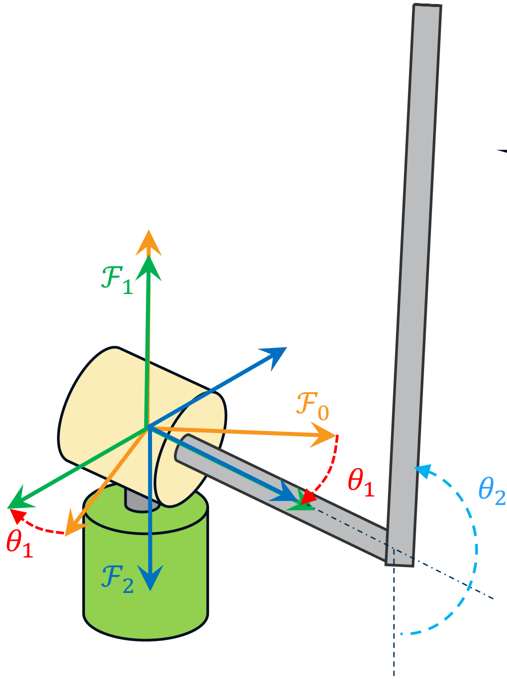
$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{\boldsymbol{q}} \\ -\boldsymbol{M}^{-1}\boldsymbol{N}_a \end{bmatrix}}_{\boldsymbol{f}(\boldsymbol{x})} + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{B}_a \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{x})} u$$

$$\boxed{\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u}$$

Stabilisation feedback control

Challenge: What should $u(t)$ be if we want to *stabilise* the pendulum, once we have spun it up?

One common approach is to linearise the dynamics about the unstable equilibrium and then apply some classical linear control techniques, for example LQR.



$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u$$

Linearise about unstable equilibrium $\boldsymbol{x}_e$

$$\Delta\dot{\boldsymbol{x}} \approx \underbrace{\left.\frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}_e}}_{A} \Delta\boldsymbol{x} + \underbrace{\left.\frac{\partial \boldsymbol{g}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}_e}}_{B} u$$

Linearised dynamics

$$\dot{\boldsymbol{x}} \approx A\Delta\boldsymbol{x} + Bu$$

(but only when close to the equilibrium!)

Define equilibrium:

$$\boldsymbol{x}_e = \begin{bmatrix}\theta_1\\ \theta_2\\ \dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}_{equil} = \begin{bmatrix}\theta_{1\,equil}\\ k\pi\\ 0\\ 0\end{bmatrix}, where\ k = any\ integer$$

-> lets choose $\theta_{1\,equil} = 0\ rad$

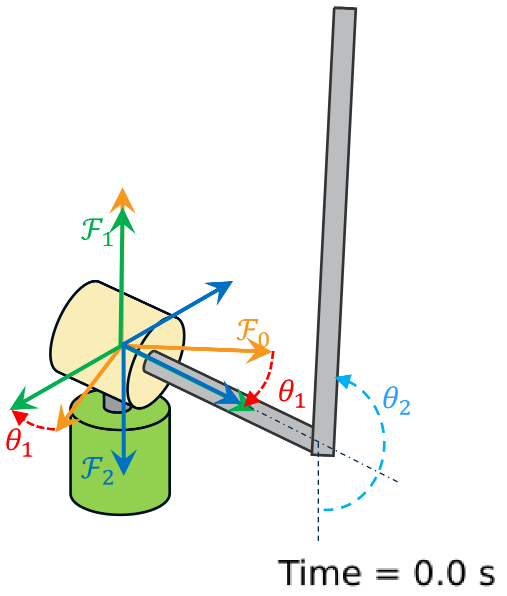
$$\Delta\boldsymbol{x} \coloneqq \boldsymbol{x} - \boldsymbol{x}_e$$
$$\Delta\dot{\boldsymbol{x}} = \dot{\boldsymbol{x}} - \cancel{\dot{\boldsymbol{x}}_e} = \dot{\boldsymbol{x}}$$

If we have done everything right, our linearized system matrices should look something like this:

```
StateSpace{Continuous, Float64}
A =
 0.0   0.0                1.0  0.0
 0.0   0.0                0.0  1.0
 0.0   0.0                0.0  0.0
 0.0  57.13998570144878   0.0  1.205924321549014
B =
 0.0
 0.0
 1.0
 0.8370424320555033
```

Stabilisation feedback control with LQR

Challenge: What should $u(t)$ be if we want to *stabilise* the pendulum, once we have spun it up?

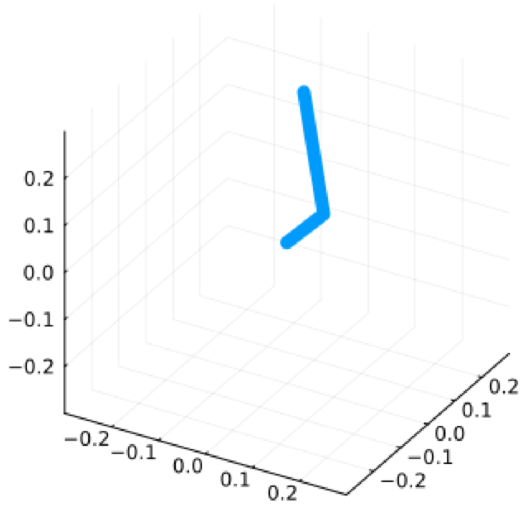


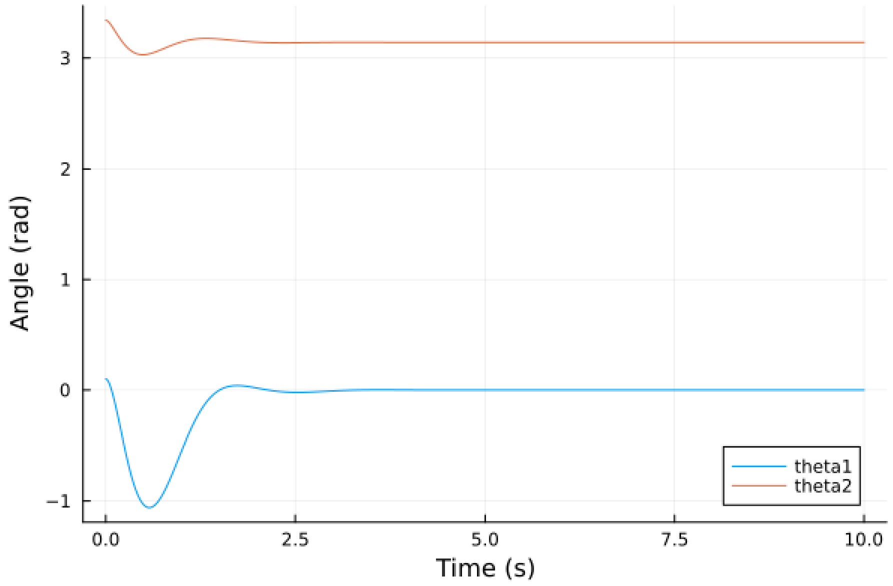
$$\dot{\boldsymbol{x}} = \boldsymbol{A}\Delta\boldsymbol{x} + \boldsymbol{B}u$$

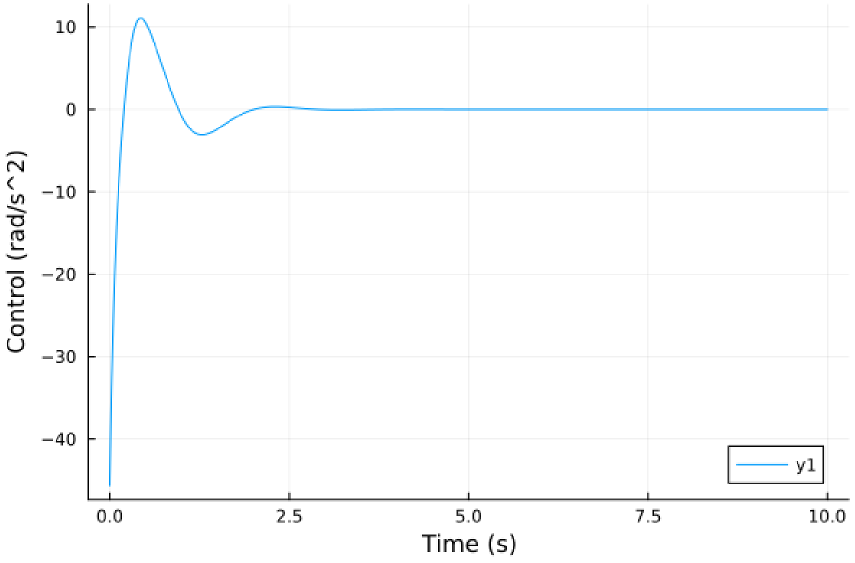
Lets choose: $u = \boldsymbol{K}\Delta\boldsymbol{x}$

- Where $\boldsymbol{K}$ are some carefully calculated gains.
  - ➢ LQR is where these gains are calculated in a particular way as to minimise a cost function.

LQR simulation







Stabilisation feedback control with LQR

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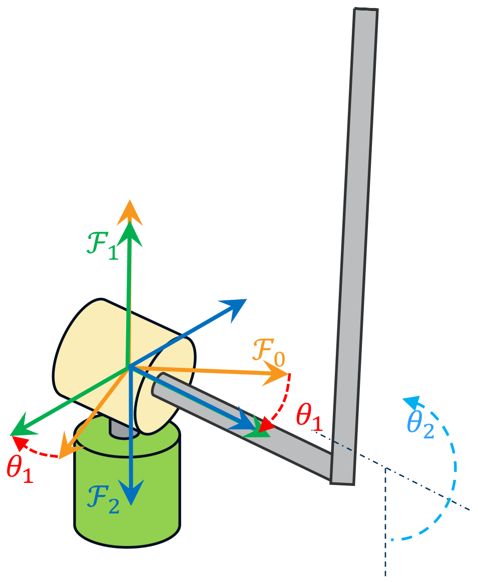
Challenge: What should $u(t)$ be if we want to *stabilise* the pendulum, once we have spun it up?



Linearised dynamic model

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\Delta\boldsymbol{x} + \boldsymbol{B}u$$

Where $u = \ddot{\theta}_1$ is our control input $\quad \boldsymbol{x} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$ = pendulum 'state'

A PD control law looks like: $\boxed{u = \boldsymbol{K}\Delta\boldsymbol{x},}$ where $\boldsymbol{K}$ is a vector of constant gains, which can be calculated from the dynamic model (e.g. by using LQR):

$$u = \ddot{\theta}_1 = \boldsymbol{K}\Delta\boldsymbol{x} = \underbrace{\begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}}_{\boldsymbol{K}} \underbrace{\begin{bmatrix} \theta_1 - \theta_{1_{equil}} \\ \theta_2 - \theta_{2_{equil}} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}_{\Delta\boldsymbol{x}}$$

Try to calculate directly from encoder readings using finite difference approach. Should be okay I think (as long as timestep is fast)

- $\theta_{1_{equil}}$ is our chosen θ_1 angle that we want to stay at. This can be anything, but I suggest we keep it at 0.
- $\theta_{2_{equil}}$ is our chosen θ_2 angle that we want to stay at. If we set $\theta_2 = 0$ as the downward vertical, then $\theta_{2_{equil}} = (2\text{k} - 1)\pi$ rad, where k =any integer.
  - You may want to compute θ_2 as modulo$(\theta_2 + \pi, 2\pi) - \pi$ to keep it within +/- π radians
- Note that I have not written out $\dot{\theta}_{1_{equil}}$ or $\dot{\theta}_{2_{equil}}$ in $\Delta\boldsymbol{x}$ because these are both 0 at the equilibrium.

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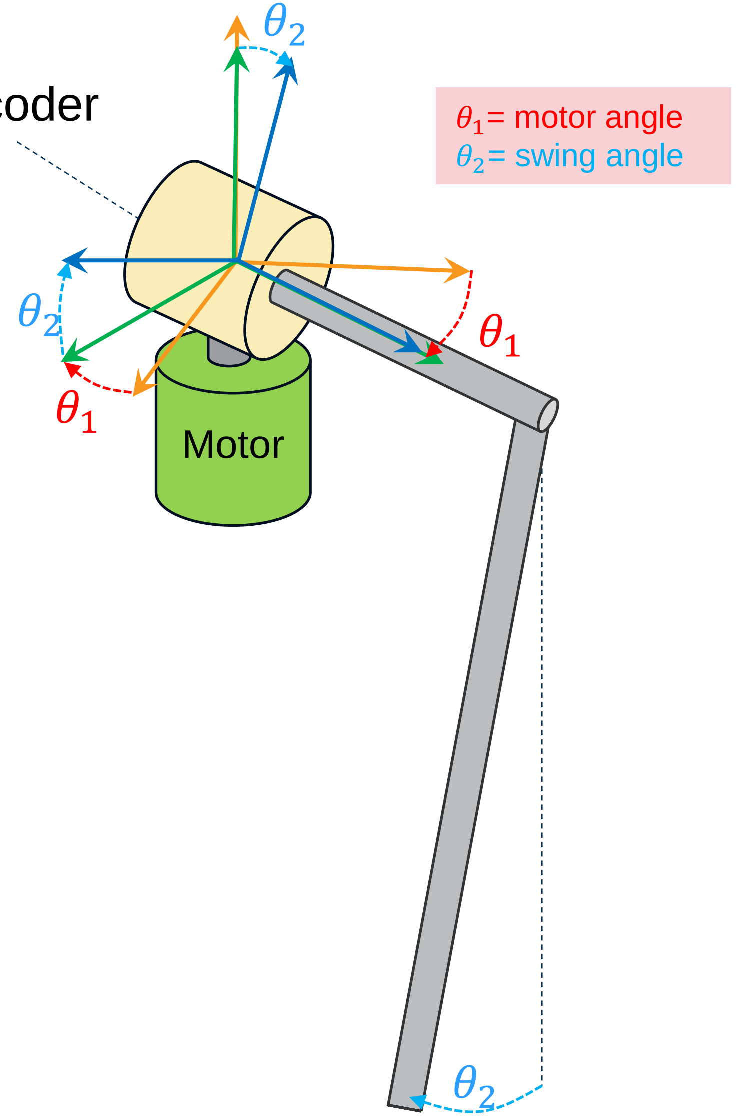

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END OF SLIDES

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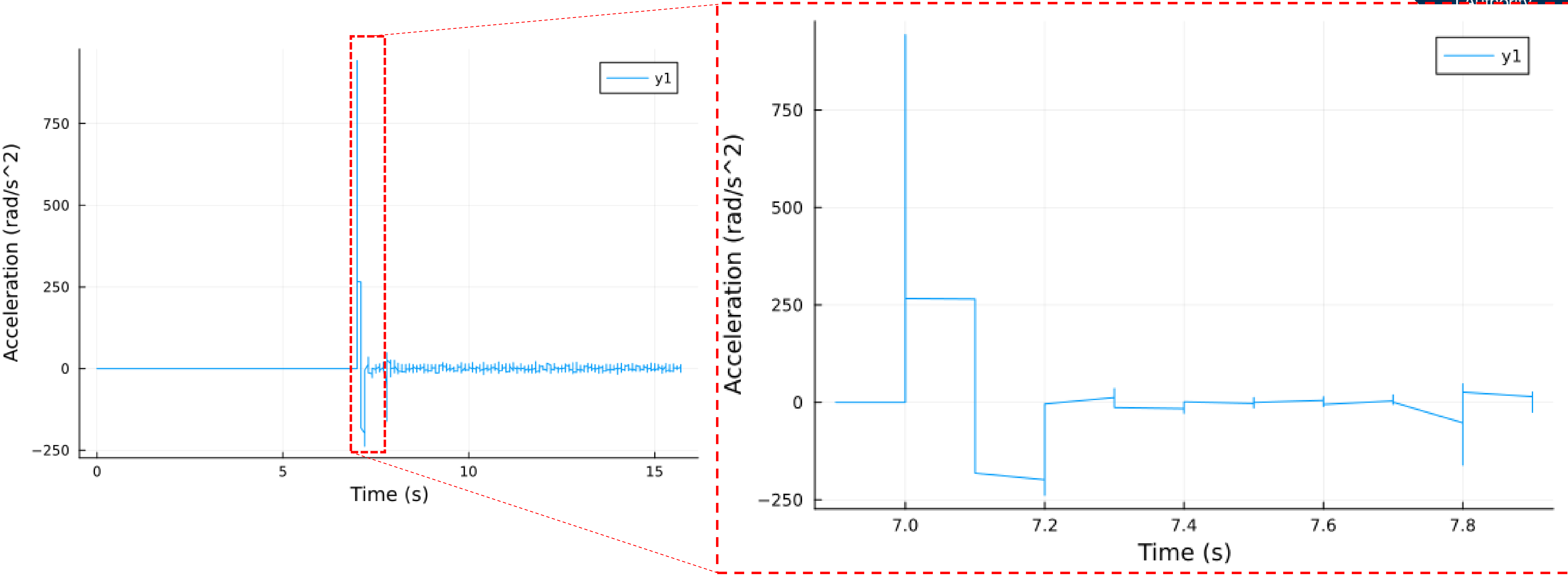
WIP images / future plans

Important things to double check:

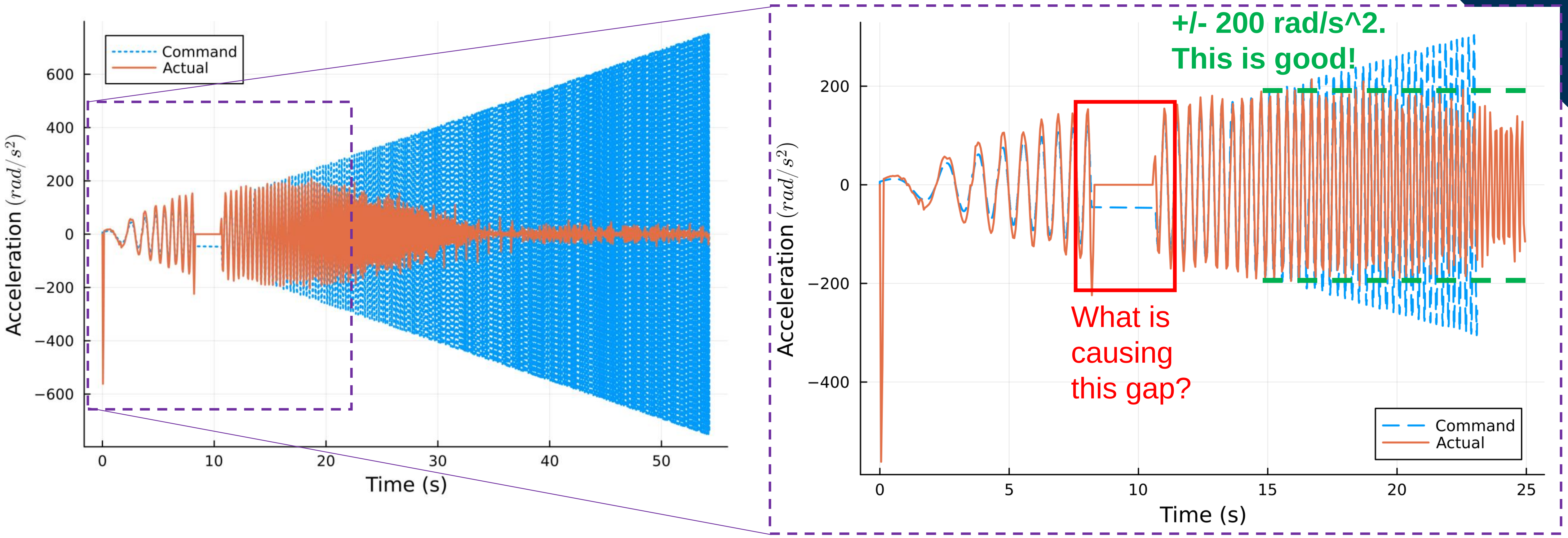
```
#define MAX_SPEED 2000
#define MIN_SPEED 800
#define MAX_ACCEL 6000
#define MAX_DECEL 6000

#define MAX_SPEED_MODE_2 2000
#define MIN_SPEED_MODE_2 1000
#define MAX_SPEED_MODE_1 2000
#define MIN_SPEED_MODE_1 800
#define MAX_SPEED_MODE_3 2000
#define MIN_SPEED_MODE_3 600
#define MAX_SPEED_MODE_4 2000
#define MIN_SPEED_MODE_4 400
#define MAX_SPEED_MODE_5 2000
#define MIN_SPEED_MODE_5 800
```

Important things to double check:



- Are we logging at 10Hz? or is the command signal 10 Hz.
- **Either way, we need faster please!**

Chirp test results



“Actual” values are calculated from encoder position using Savitzky Golay filter

Swing-up

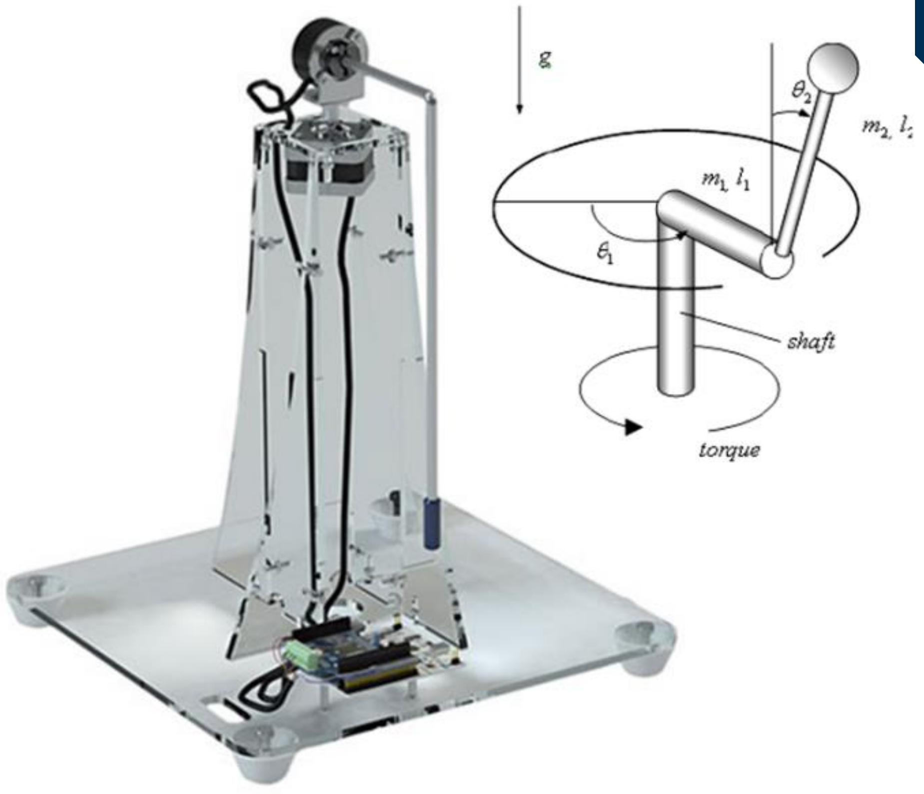
Trajectory Optimisation

We want to ‘swing up’ the pendulum.

How can we do this intelligently?

There are many approaches, for example

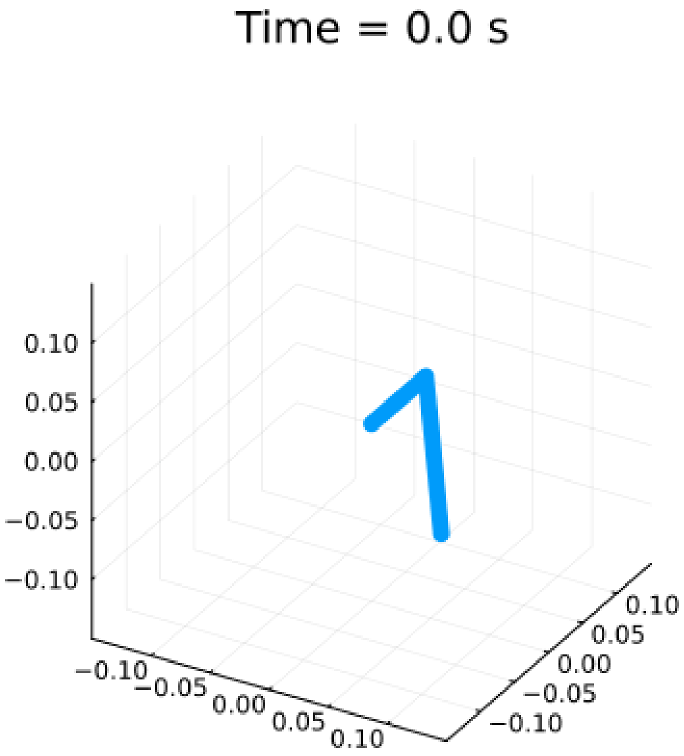
- **Energy Shaping**
- **Trajectory optimisation**
- **Neural Networks**

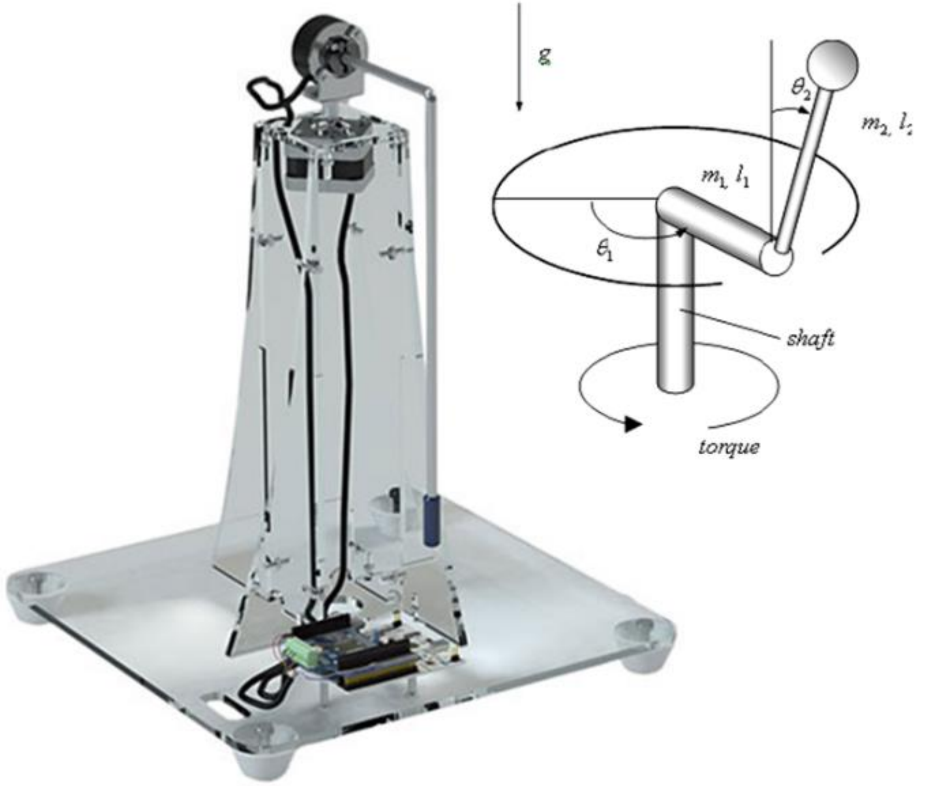


Trajectory Optimisation

Can we determine a trajectory for the motor that swings up the pendulum, while considering:

- Motor Torque / Velocity / acceleration limits
- Spatial constraints





How?

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TBC. Need time to write-up

Flexible payload trajectory optimization

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Tip position error

$e_p(\mathbf{q},t) = \|\mathbf{p}_{tip,d}(t) - \mathbf{p}_{tip}(\mathbf{q})\|$

Desired position

Actual position

Tip orientation error

$e_a(\mathbf{q},t) = 1 - (\hat{\mathbf{w}}_{tip,d}(t)^\top \hat{\mathbf{w}}_{tip}(\mathbf{q}))^2$

Desired pose (quaternion)

Actual pose (quaternion)

Total payload Kinetic Energy throughout motion

$$\min\ q(t_f) + \int_{t_i}^{t_f} T(q,\dot{q})dt$$

$$\min_{\mathbf{x},\dot{\mathbf{x}},\mathbf{u}} \underbrace{\alpha \boxed{e_p(\mathbf{q}(t_f))} + \beta \boxed{e_o(\mathbf{q}(t_f))}}_{h(\mathbf{q}(t_f))} + \int_{t_i}^{t_f} \boxed{T(\mathbf{q},\dot{\mathbf{q}})} dt$$

$$\begin{aligned} s.t.\quad & \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}, && \forall t \in [t_i, t_f] \\ & \mathbf{x}(t) \in \mathbf{x}_{safe}, && \forall t \in [t_i, t_f] \\ & \|\mathbf{u}\|_2 \leq \mathbf{J}_t(\mathbf{x})^\top \mathbf{F}_{t,max} && \forall t \in [t_i, t_f] \\ & \dot{\mathbf{q}}(t_i) = \dot{\mathbf{q}}(t_f) = \ddot{\mathbf{q}}(t_f) = \mathbf{0} \end{aligned}$$

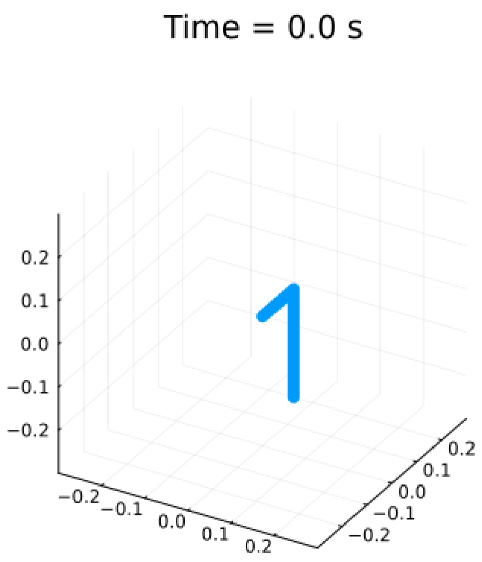
Model dynamics

Spatial constraints

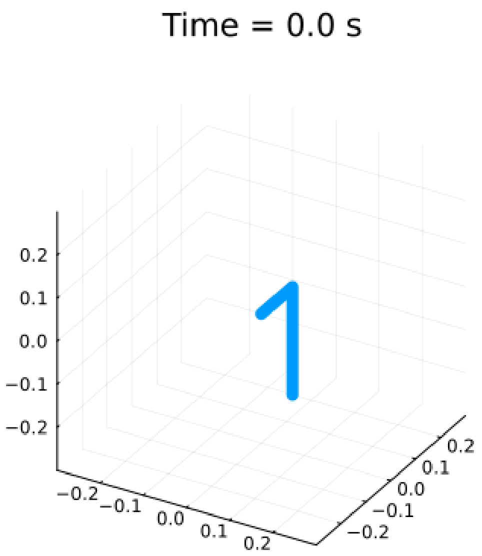
Tool force limits

Start / end conditions

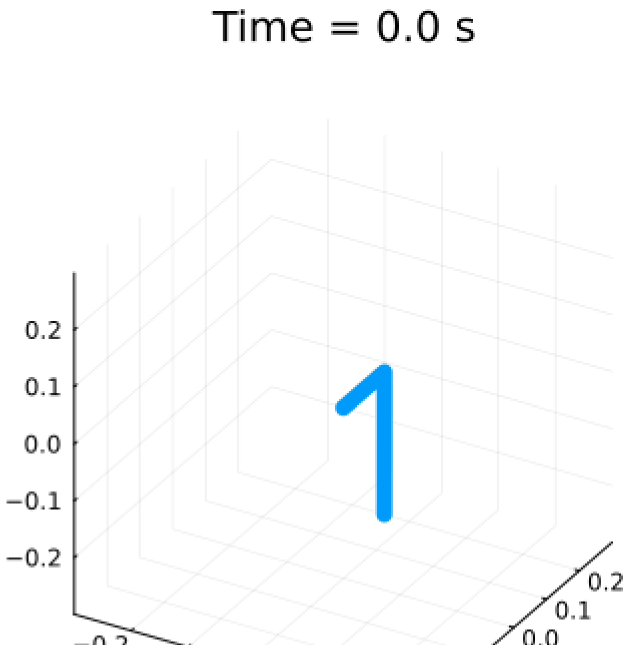
Real life



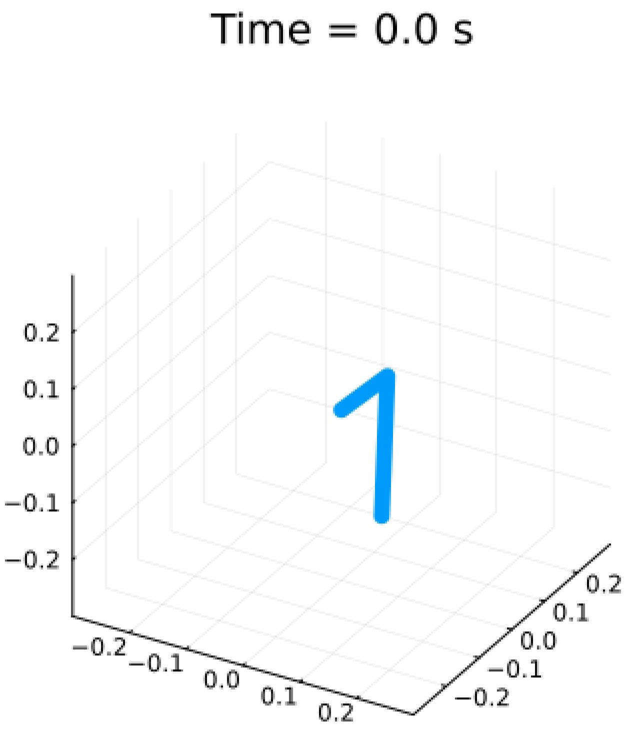
Simulated



Optimised

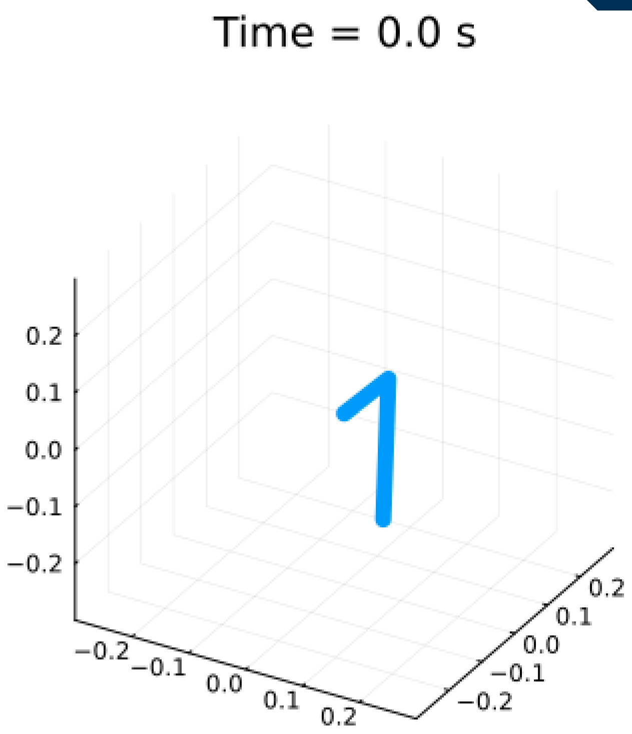


Real life



Optimised

Simulated



System Identification

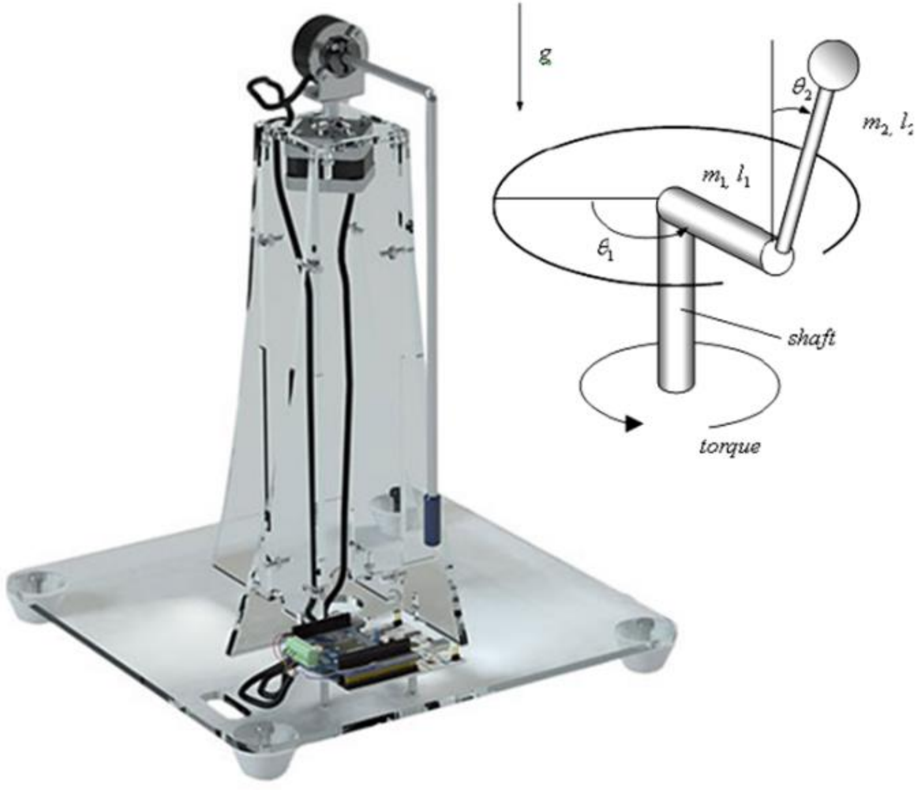
Why System Identification?

Our analytical model may be poor due to making bad estimates of:

$$m_1, m_2, d_{fr_2}, \boldsymbol{r}_c^{\mathcal{F}_2}$$

Can we improve our model parameters using observed data? YES

$$\boldsymbol{p} = m_1, \\ m_2, d_{fr_2}, \boldsymbol{r}_c^{\mathcal{F}_2}$$



Next steps

- Improve model parameters & verify

- Implement on real set-up