(1)

$$y_{n+1} = y_n + \frac{1}{2}(k_1 + k_2)\Delta t \quad \text{--- (i)}$$

$$k_1 = f(y_n) = \frac{dy}{dt}$$

$$k_2 = f(y_n + k_1 \Delta t) =$$

$$= \frac{d}{dt}(y_n + k_1 \Delta t)$$

$$= \frac{d}{dt} y_n + \frac{dk_1}{dt}\Delta t$$

$$= \frac{dy_n}{dt} + \frac{d^2 y_n}{dt^2}\Delta t$$

Putting that value in eq^n (i)

$$y_{n+1} = y_n + \frac{1}{2}\left(\frac{dy_n}{dt} + \frac{dy_n}{dt} + \frac{d^2 y_n}{dt^2}\Delta t\right)\Delta t$$

$$y_{n+1} = y_n + \frac{dy_n}{dt}\Delta t + \frac{1}{2}\frac{d^2 y_n}{dt}\Delta t^2 \quad \text{--- (ii)}$$

~~This looks like bit taylor's expansion~~

Now, taylor expand $y(t_n + \Delta t)$ for $y(t_n)$ considering Δt very small.

$$y(t_n + \Delta t) = y(t_n) + \frac{dy(t_n)}{dt}\Delta t + \frac{1}{2}\frac{d^2 y(t_n)}{dt^2}\Delta t^2$$

$$+ \frac{1}{6}\frac{d^3 y}{dt^3}(\Delta t)^3 + \dots\dots \quad \text{--- (iii)}$$

Now, ~~considering~~. from (ii) and (iii)

$$y_{n+1} - y(t_n + \Delta t) = \frac{1}{6}\frac{d^3y}{dt^3}(\Delta t)^3 + \ldots\ldots$$

which shows that the truncation error per step is the third-order in the step size. Δt,