Problems on *statistics and data analysis (MVComp2)*

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Problem sheet 7

To be handed in during the exercise group on 20.June.2016

1. *combining measurements* *(10 points)*

If you combine independent measurements by multiplying their likelihoods, $\mathcal{L} = \mathcal{L}_1\mathcal{L}_2$, why would that imply that the Fisher-matrices add, $F_{\mu\nu} = F^{(1)}_{\mu\nu} + F^{(2)}_{\mu\nu}$? Why do the errors decrease if one has more data?

2. *invariance of statistical errors* *(10 points)*

Why is the magnitude of statistical errors of model parameters independent if you fit to a linear combination of data points instead to the data points directly? Would the errors be identical if you choose a different system of units?

3. *generalised fit of a polynomial* *(10 points)*

Imagine that you've got n data points y_i at positions x_i, each with a Gaussian error σ_i. The data points originate from a polynomial model of the form $y(x) = \sum_{\alpha=0}^{m} a_\alpha x^\alpha$, where obviously $n \gg m$. Please

(a) formulate a χ^2-functional for the fitting problem,

(b) derive a linear system of equations by using the minimum conditions $\partial_\alpha \chi^2 = 0$,

(c) write this system in a matrix-vector-notation,

(d) formulate the coefficients as *moments including a weighting* with the errors σ_i,

(e) invert the linear system for getting the model parameters a_α,

(f) solve the special case $m = 0$: What's the solution for a_0?

(g) What happens if $n = m$?

4. *posterior distribution* *(10 points)*

An experiment for measuring two parameters is characterised by a 2×2 Fisher-matrix $F_{\mu\nu}$.

(a) Please formulate the Gaussian posterior distribution $p(x_\mu, x_\nu)\mathrm{d}x_\mu\mathrm{d}x_\nu$ in terms of the Fisher-matrix, if no prior is assumed.

(b) Determine the correlation coefficient $r_{\mu\nu}$ for the posterior Gaussian distribution.

(c) Carry out a marginalisation of $p(x_\mu, x_\nu)\mathrm{d}x_\mu\mathrm{d}x_\nu$ over x_ν deriving the variance σ_m^2, and

(d) a conditionalisation with the condition $x_y \equiv 0$ determining σ_c^2, and give expressions for σ_m^2 and σ_c^2 in terms of the Fisher-matrix entries.

(e) In what way would you interpret the eigenvalues of the Fisher-matrix?

5. *estimating* *(10 points)*

In physics one often encounters order of magnitude-estimates involving a product of many numbers, for instance in the *Drake equation* describing the number of alien civilisations in the Milky Way. Why do estimates have smaller relative statistical errors if there are more factors in the relation (each with a similar uncertainty)?

6. *python exercise* *(10 points)*

Write a python script which generates artifical data following the model $y(x) = ax + b$ including Gaussian noise with variance σ^2.

(a) Fit the model to the artifical data a large number of times and derive the distributions $p(a)\mathrm{d}a$ and $p(b)\mathrm{d}b$ of the model parameters,

(b) as well as their correlection coefficient r_{ab}. Plot as well the cloud of points (a, b) and see if its orientation corresponds with the correlation coefficient.

(c) What happens to the distributions $p(a)\mathrm{d}a$ and $p(b)\mathrm{d}b$ if you decrease the noise σ^2?

(d) Illustrate that the width of the distributions for a and b changes if you change the x-range: Why is that the case?