Problems on *statistics and data analysis (MVComp2)*

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Problem sheet 5

To be handed in during the exercise group on 6.June.2016

1. *Saturn rings* *(10 points)*

A survey is carried out to ask many people (secretly and independently) their guess how thick the rings of Saturn are. The answers received are such that the logarithm of the thickness is uniformly distributed (see fig.1) . . . why is this the case?

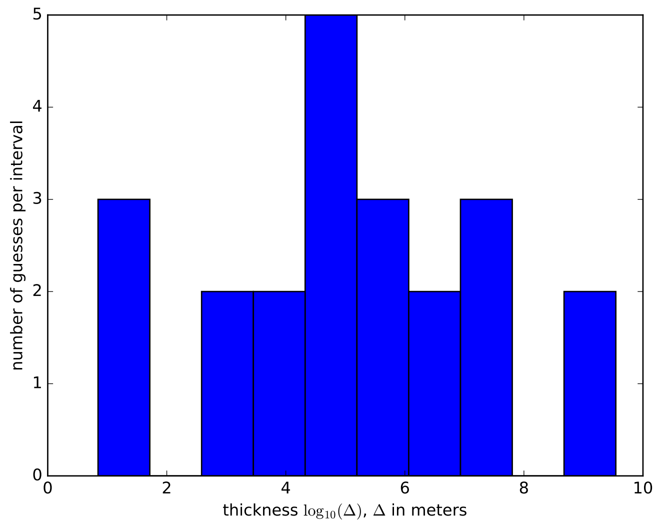

Figure 1: Distribution of the logarithm of the thickness of Saturn rings, according to the survey of ex.1

2. *Confidence intervals for n-dimensional Gaussian distributions* *(10 points)*

Imagine an n-dimensional Gaussian distribution with unit-matrix as covariance matrix: what radius contains a fraction $erf(1/\sqrt{2})$ of the total probability in $n = 2, 3, 4$ dimensions (erf is the error function)? Give an explicit value for each of these three n.

3. *Hölder's inequality* *(10 points)*

Show using Hölder's inequality that $\sqrt{\langle x^2 \rangle} \leq \langle x^p \rangle^{\frac{1}{p}}$, with integer $p > 2$. Why is this inequality always fulfilled when x follows a Gaussian distribution with variance σ^2?

4. *Cauchy distribution in 2D* *(10 points)*

Write down an explicit expression for a Cauchy distribution in 2D, with some degree of correlation between the variables x and y. Determine the normalisation of the distribution. Then show what you get when you perform a marginalisation over y.

5. *Minkowski's inequality* *(10 points)*

Starting from Minkowski's inequality,

$$\langle (x+y)^p \rangle^{\frac{1}{p}} \leq \langle x^p \rangle^{\frac{1}{p}} + \langle y^p \rangle^{\frac{1}{p}}$$

show that the equality holds if $y = \alpha x$. Then show that the Cauchy-Schwarz inequality follows from Minkowski's inequality, with an appropriate choice of p.

6. *Python exercise* *(10 points)*

Write a Python script to generate a sample of Gaussian distributed random numbers using the rejection method. Then compare the runtime performance of the rejection and the inversion method: which one is faster? For the inversion method you can use the Python script you wrote for Exercise Sheet 04.