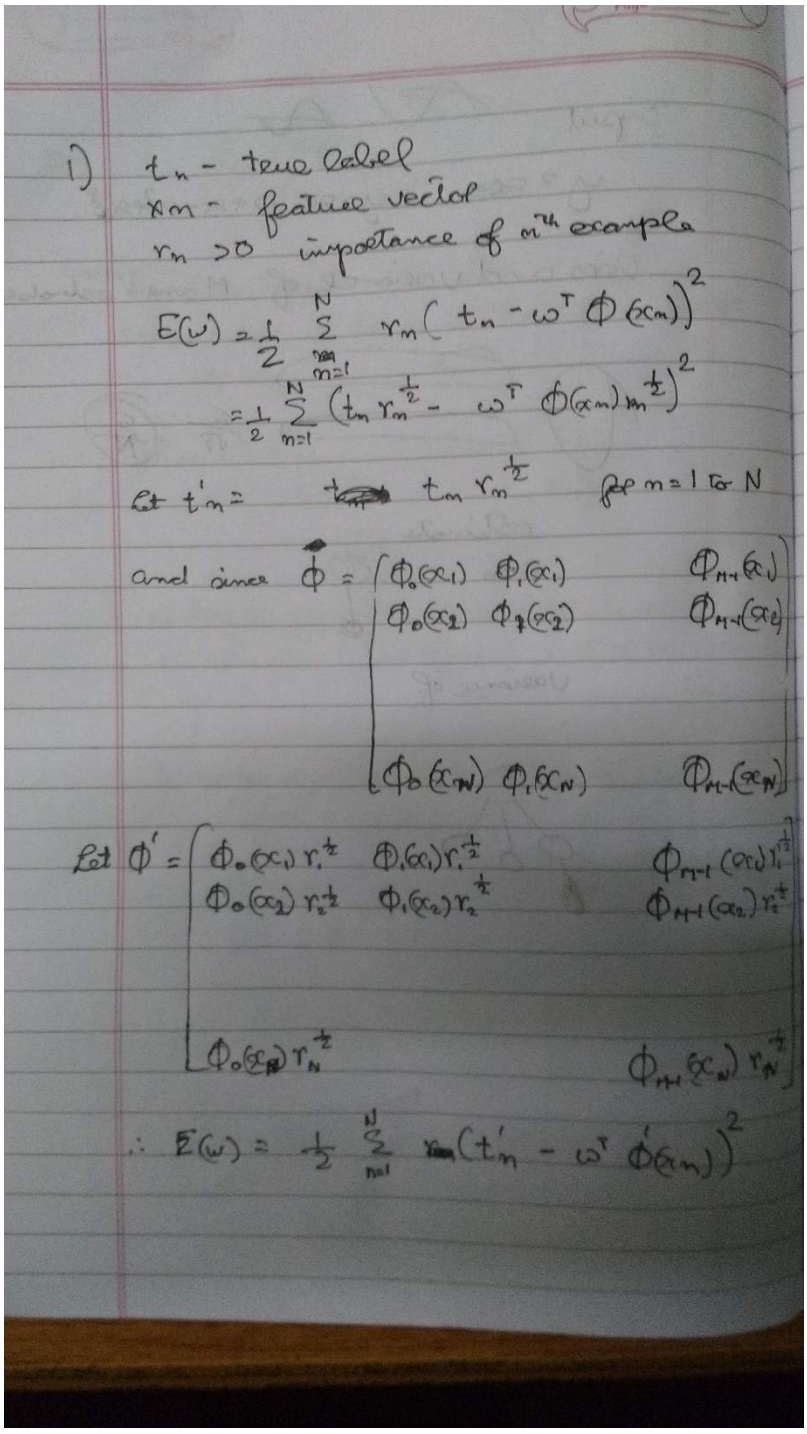
Machine learning assignment 1 Roll no - 12CS10046

1)



i) t_n - true label

x_n - feature vector

$r_n > 0$ importance of n^{th} example

$$E(w) = \frac{1}{2}\sum_{n=1}^{N} r_n\left(t_n - w^T \phi(x_n)\right)^2$$

$$= \frac{1}{2}\sum_{n=1}^{N}\left(t_n r_n^{\frac{1}{2}} - w^T \phi(x_n) r_n^{\frac{1}{2}}\right)^2$$

Let $t'_n = t_n r_n^{\frac{1}{2}}$ for $n = 1$ to N

and since $\phi = \begin{bmatrix} \phi_0(x_1) & \phi_1(x_1) & \cdots & \phi_{M-1}(x_1) \\ \phi_0(x_2) & \phi_1(x_2) & \cdots & \phi_{M-1}(x_2) \\ & & & \\ \phi_0(x_N) & \phi_1(x_N) & \cdots & \phi_{M-1}(x_N) \end{bmatrix}$

Let $\phi' = \begin{bmatrix} \phi_0(x_1) r_1^{\frac{1}{2}} & \phi_1(x_1) r_1^{\frac{1}{2}} & \cdots & \phi_{M-1}(x_1) r_1^{\frac{1}{2}} \\ \phi_0(x_2) r_2^{\frac{1}{2}} & \phi_1(x_2) r_2^{\frac{1}{2}} & \cdots & \phi_{M-1}(x_2) r_2^{\frac{1}{2}} \\ & & & \\ \phi_0(x_N) r_N^{\frac{1}{2}} & & & \phi_{M-1}(x_N) r_N^{\frac{1}{2}} \end{bmatrix}$

$$\therefore E(w) = \frac{1}{2}\sum_{n=1}^{N} r_n\left(t'_n - w^T \phi'(x_n)\right)^2$$

After taking gradient of log likelihood

$$= \sum_{n=1}^{N} \{ t'_n - w^T \phi'(x_n) \} \phi'(x_n)^T$$

and setting it to 0

$$0 = \sum_{n=1}^{N} t'_n \phi'(x_n)^T - w^T \left(\sum_{n=1}^{N} \phi'(x_n) \phi'(x_n)^T \right)$$

$$w_{ML} = = \left[\phi'^T \phi' \right]^{-1} \phi'^T t'$$

here t' is the whole vector of size $= N$

2)

Instances: 506
Attributes: 14
att_1
att_2
att_3
att_4
att_5
att_6
att_7
att_8
att_9
att_10
att_11
att_12
att_13
class
Test mode:split 60.0% train, remainder test

For different values of λ
λ =

a) 0.00001

Linear Regression Model

class =

-0.1084 * att_1 +
0.0458 * att_2 +
2.7187 * att_4 +
-17.376 * att_5 +
3.8016 * att_6 +
-1.4927 * att_8 +
0.2996 * att_9 +
-0.0118 * att_10 +
-0.9465 * att_11 +
0.0093 * att_12 +
-0.5226 * att_13 +
36.3411

Time taken to build model: 0.04 second

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient 0.8612
Mean absolute error 3.3689
Root mean squared error 4.5651
Relative absolute error 51.6839 %
Root relative squared error 50.8522 %
Total Number of Instances 202

b) 0.0001

Linear Regression Model

class =

-0.1084 * att_1 +
0.0458 * att_2 +
2.7187 * att_4 +
-17.376 * att_5 +
3.8016 * att_6 +
-1.4927 * att_8 +
0.2996 * att_9 +
-0.0118 * att_10 +
-0.9465 * att_11 +
0.0093 * att_12 +
-0.5226 * att_13 +
36.3411

Time taken to build model: 0.03 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient 0.8612
Mean absolute error 3.3689
Root mean squared error 4.5651
Relative absolute error 51.6839 %
Root relative squared error 50.8522 %
Total Number of Instances 202

c) 0.001

Linear Regression Model

class =

-0.1084 * att_1 +
0.0458 * att_2 +
2.7187 * att_4 +
-17.3759 * att_5 +

3.8016 * att_6 +
-1.4927 * att_8 +
0.2996 * att_9 +
-0.0118 * att_10 +
-0.9465 * att_11 +
0.0093 * att_12 +
-0.5226 * att_13 +
36.3408

Time taken to build model: 0.03 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3689
Root mean squared error	4.5651
Relative absolute error	51.6838 %
Root relative squared error	50.8522 %
Total Number of Instances	202

d) 0.01

Linear Regression Model

class =

-0.1084 * att_1 +
0.0458 * att_2 +
2.7188 * att_4 +
-17.3743 * att_5 +
3.8017 * att_6 +
-1.4925 * att_8 +
0.2995 * att_9 +
-0.0118 * att_10 +
-0.9465 * att_11 +
0.0093 * att_12 +
-0.5225 * att_13 +
36.3376

Time taken to build model: 0.02 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3689

Root mean squared error	4.565
Relative absolute error	51.6831 %
Root relative squared error	50.8517 %
Total Number of Instances	202

e) 0.1

Linear Regression Model

class =

-0.1083 * att_1 +
0.0458 * att_2 +
2.7195 * att_4 +
-17.3593 * att_5 +
3.8029 * att_6 +
-1.4911 * att_8 +
0.2989 * att_9 +
-0.0117 * att_10 +
-0.9463 * att_11 +
0.0093 * att_12 +
-0.5224 * att_13 +
36.3057

Time taken to build model: 0.09 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3684
Root mean squared error	4.5646
Relative absolute error	51.6755 %
Root relative squared error	50.8468 %
Total Number of Instances	202

f) 1

Linear Regression Model

class =

-0.1074 * att_1 +
0.0453 * att_2 +
2.7262 * att_4 +
-17.2099 * att_5 +
3.8142 * att_6 +
-1.4767 * att_8 +

```
     0.2928 * att_9 +
    -0.0115 * att_10 +
    -0.9439 * att_11 +
     0.0093 * att_12 +
    -0.5212 * att_13 +
    35.9922

Time taken to build model: 0.02 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient                  0.8615
Mean absolute error                      3.3635
Root mean squared error                  4.5605
Relative absolute error                 51.6015 %
Root relative squared error             50.8017 %
Total Number of Instances              202
```

g) 10

```
Linear Regression Model

class =

    -0.0998 * att_1 +
     0.0413 * att_2 +
     2.7775 * att_4 +
   -15.8401 * att_5 +
     3.9043 * att_6 +
    -1.3486 * att_8 +
     0.2428 * att_9 +
    -0.0094 * att_10 +
    -0.9214 * att_11 +
     0.0093 * att_12 +
    -0.5098 * att_13 +
    33.2887

Time taken to build model: 0.03 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient                  0.8634
Mean absolute error                      3.3299
Root mean squared error                  4.5403
Relative absolute error                 51.0848 %
Root relative squared error             50.576  %
```

Total Number of Instances 202

h) 100

Linear Regression Model

class =

-0.075 * att_1 +
0.0272 * att_2 +
2.8313 * att_4 +
-9.2248 * att_5 +
3.9836 * att_6 +
-0.7395 * att_8 +
0.0822 * att_9 +
-0.0043 * att_10 +
-0.7827 * att_11 +
0.0086 * att_12 +
-0.4259 * att_13 +
22.9394

Time taken to build model: 0.04 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient	0.86
Mean absolute error	3.4177
Root mean squared error	4.746
Relative absolute error	52.4325 %
Root relative squared error	52.8679 %
Total Number of Instances	202

i) 1000

Linear Regression Model

class =

-0.0546 * att_1 +
0.016 * att_2 +
-0.071 * att_3 +
1.7399 * att_4 +
-3.5029 * att_5 +
2.1139 * att_6 +
-0.0103 * att_7 +
-0.087 * att_8 +

-0.0028 * att_10 +
-0.4174 * att_11 +
0.0051 * att_12 +
-0.1924 * att_13 +
22.4032

Time taken to build model: 0.02 seconds

=== Evaluation on test split ===
=== Summary ===

Correlation coefficient	0.8086
Mean absolute error	4.5596
Root mean squared error	6.389
Relative absolute error	69.951 %
Root relative squared error	71.1692 %
Total Number of Instances	202

3) Done

4)

4 Let us assume N input values $X = (x_1, x_2, \ldots x_N)^T$ and their corresponding target values. $t = (t_1, t_2 \cdots t_N)^T$

Assuming Gaussian distribution of t we have the following

$$p(t|x, \omega, \beta) = \mathcal{N}(t|\phi(x)^T \omega, \beta^{-1})$$

where $\phi = (\phi_0, \phi_1 \ldots \phi_{M-1})^T$ and in its simplest form $\phi_i(x) = x^i$

So because of independence assumption

$$p(t|X, \omega) = \prod_{i=1}^{N} p(t_i|x_i, \omega)$$

$$= \prod_{i=1}^{N} \sqrt{\frac{\beta}{2\pi}} \exp\left\{-\frac{\beta}{2}(t_i - \phi(x)^T \omega)^2\right\}$$

$$= \frac{1}{(2\pi/\beta)^{N/2}} \exp\left\{-\frac{1}{2/\beta}|t - \phi(x)^T \omega|^2\right\}$$

$$= \mathcal{N}(t|\phi(X)^T \omega, \beta^{-1} I)$$

where $\phi(X)$ is as given eq.

$$\begin{bmatrix} \phi_0(x_1) & \phi_1(x_{21}) & & \phi_{M-1}(x_1) \\ \phi_0(x_2) & \phi_1(x_2) & & \phi_{M-1}(x_2) \\ & & & \\ \phi_0(x_n) & & & \phi_{M-1}(x_n) \end{bmatrix}$$

Let us also assume a Gaussian as prior distribution for ω

$$p(\omega|\alpha) = \mathcal{N}(\omega|0, \alpha^{-1}I) = \left(\frac{\alpha}{2\pi}\right)^{M/2} \exp\left(-\frac{\alpha}{2}\omega^T\omega\right)$$

Using Bayes theorem, the posterior distribution and the likelihood function and applying simple procedure we get

$$p(\omega|t, X, \alpha) \propto p(t|X, \omega)\, p(\omega|\alpha)$$

$$\propto \exp\left\{-\frac{\beta}{2}(t - \Phi(X)^T\omega)^T(t - \Phi(X)^T\omega)\right\} \cdot \exp\left\{-\frac{\alpha}{2}\omega^T\omega\right\}$$

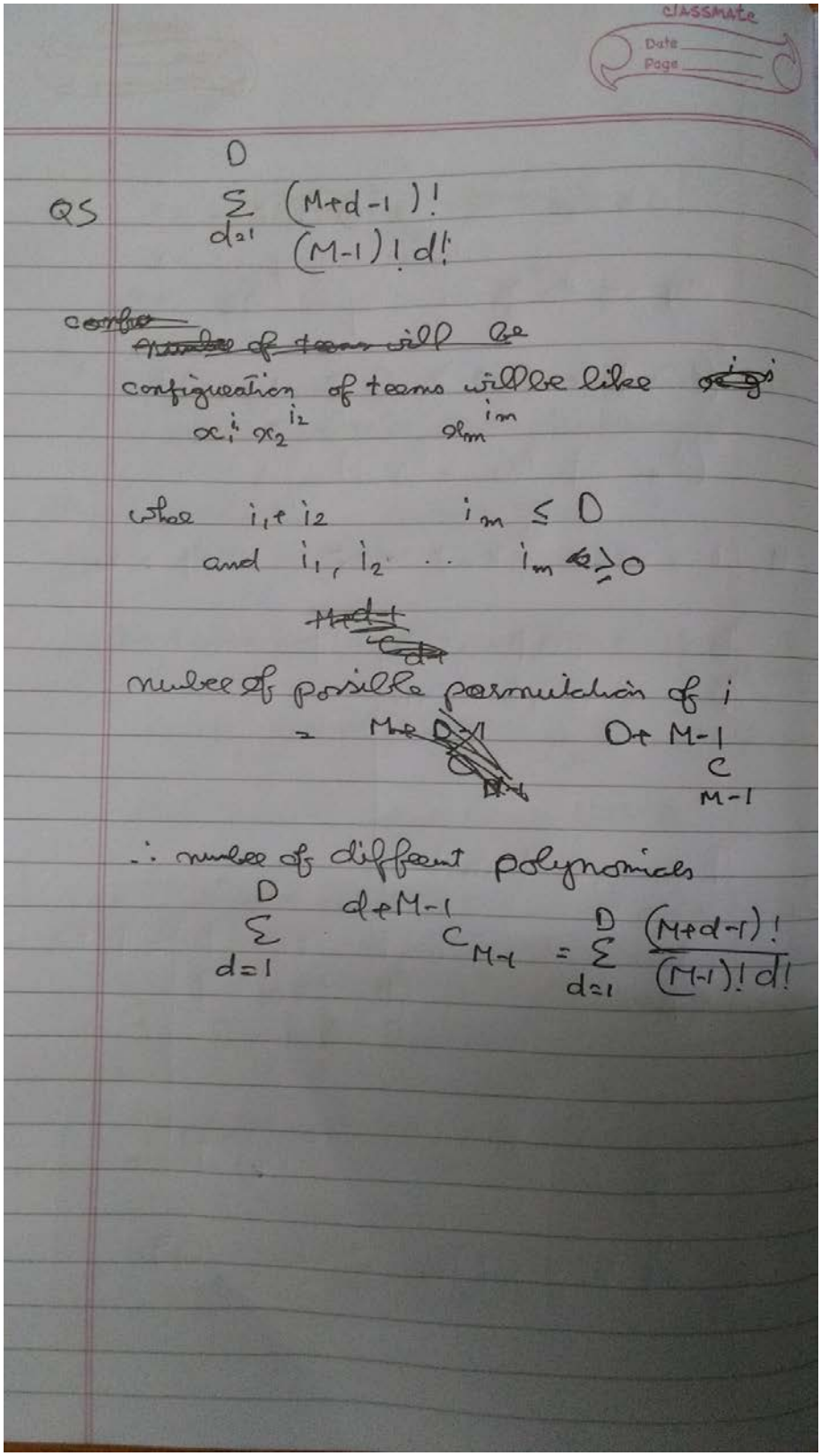
$$\propto \exp\left\{-\frac{1}{2}(\omega-\bar{\omega})^T C^{-1}(\omega-\bar{\omega})\right\}$$

where $\bar{\omega} = \beta\left(\beta\Phi(X)\Phi(X)^T + \alpha I\right)^{-1}\Phi(X)t$

and covariance matrix $C = \left(\beta\Phi(X)\Phi(X)^T + \alpha I\right)^{-1}$

So $p(\omega|t, X, \alpha) \sim \mathcal{N}(\bar{\omega} = \beta C\Phi(X)t, C)$

5)



$$\sum_{d=1}^{D} \frac{(M+d-1)!}{(M-1)!\,d!}$$

configuration of terms will be like $x_1^{i_1} x_2^{i_2} \dots x_m^{i_m}$

where $i_1 + i_2 \dots i_m \leq D$

and $i_1, i_2 \dots i_m \geq 0$

number of possible permutation of i

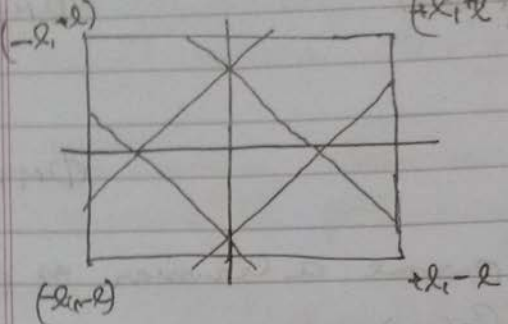
$= {}^{M+D-1}C_{M-1}$ or ${}^{D+M-1}C_{M-1}$

$\therefore$ number of different polynomials

$$\sum_{d=1}^{D} {}^{d+M-1}C_{M-1} = \sum_{d=1}^{D} \frac{(M+d-1)!}{(M-1)!\,d!}$$

6) ..

6 Consider the following diagram of Lasso



Now since l is very large
the lasso occupies a point in the space.
Thus in 2 dimension probability that one of the dim is zero is $\frac{1}{2}$

~~This~~ Thus for M dimensions the prob that of the dimensions are 0 is

$$\frac{M!}{(M-K)!\,K!}\left(\frac{1}{M}\right)^{K}\left(1-\frac{1}{M}\right)^{M-K}$$

or $$\binom{M}{K}\left(\frac{1}{M}\right)^{K}\left(1-\frac{1}{M}\right)^{M-K}$$

7)

Instances: 63
Attributes: 10
x^1
x^2
x^3
x^4
x^5
x^6
x^7
x^8
x^9
class
Test mode:evaluate on training data

For a =

a) 0.1

λ =

i) 0.001
Linear Regression Model

class =

0.4953 * x^1 +
-0.0001 * x^9 +
-0.0207

Time taken to build model: 0.03 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.9222
Mean absolute error	0.236
Root mean squared error	0.2809
Relative absolute error	36.0293 %
Root relative squared error	38.6637 %
Total Number of Instances	63

ii) 0.01
Linear Regression Model

class =

0.4951 * x^1 +
-0.0001 * x^9 +
-0.0207

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.9222
Mean absolute error	0.236
Root mean squared error	0.2809
Relative absolute error	36.0281 %
Root relative squared error	38.6637 %
Total Number of Instances	63

iii) 0.1
Linear Regression Model

class =

0.0896 * x^3 +
-0.0001 * x^9 +
-0.0226

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.7091
Mean absolute error	0.441
Root mean squared error	0.5123
Relative absolute error	67.316 %
Root relative squared error	70.5198 %
Total Number of Instances	63

iv) 1

Linear Regression Model

class =

0.0011 * x^7 +

-0.0001 * x^9 +
-0.0107

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.4225
Mean absolute error	0.5827
Root mean squared error	0.6703
Relative absolute error	88.9447 %
Root relative squared error	92.2722 %
Total Number of Instances	63

v) 10
Linear Regression Model

class =

0.3873 * x^1 +
-0.001 * x^5 +
-0.0002 * x^7 +
0 * x^9 +
-0.0147

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.9315
Mean absolute error	0.2571
Root mean squared error	0.3142
Relative absolute error	39.2476 %
Root relative squared error	43.2522 %
Total Number of Instances	63

vi) 100

Linear Regression Model

class =

0.1199 * x^1 +
-0.0087

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.7842
Mean absolute error	0.5022
Root mean squared error	0.5717
Relative absolute error	76.6562 %
Root relative squared error	78.7014 %
Total Number of Instances	63

vii) 1000

Linear Regression Model

class =

0.0183 * x^1 +
-0.0129

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.7842
Mean absolute error	0.6263
Root mean squared error	0.7006
Relative absolute error	95.6043 %
Root relative squared error	96.4519 %
Total Number of Instances	63

b) 0.5

λ =

i) 0.001

Linear Regression Model

class =

0.5151 * x^1 +
-0.0001 * x^9 +
-0.0995

Time taken to build model: 0.06 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.8016
Mean absolute error	0.4265
Root mean squared error	0.5302
Relative absolute error	55.5615 %
Root relative squared error	59.7835 %
Total Number of Instances	63

ii) 0.01
Linear Regression Model

class =

0.5149 * x^1 +
-0.0001 * x^9 +
-0.0995

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.8016
Mean absolute error	0.4265
Root mean squared error	0.5302
Relative absolute error	55.5624 %
Root relative squared error	59.7835 %
Total Number of Instances	63

iii) 0.1
Linear Regression Model

class =

-0.0022 * x^6 +
0.0002 * x^8 +
0 * x^9 +
-0.0434

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.24
Mean absolute error	0.7399
Root mean squared error	0.861
Relative absolute error	96.399 %

Root relative squared error 97.078 %
Total Number of Instances 63

iv) 1
Linear Regression Model

class =

0.7385 * x^1 +
-0.0675 * x^3 +
-0.0928

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient 0.8424
Mean absolute error 0.3923
Root mean squared error 0.4814
Relative absolute error 51.114 %
Root relative squared error 54.2743 %
Total Number of Instances 63

v) 10
Linear Regression Model

class =

0.3749 * x^1 +
0 * x^9 +
-0.0932

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient 0.7914
Mean absolute error 0.4505
Root mean squared error 0.5631
Relative absolute error 58.6898 %
Root relative squared error 63.4884 %
Total Number of Instances 63

vi) 100
Linear Regression Model

class =

0.1333 * x^1 +
-0.0901

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.7141
Mean absolute error	0.6022
Root mean squared error	0.7337
Relative absolute error	78.4577 %
Root relative squared error	82.7272 %
Total Number of Instances	63

vii) 1000
Linear Regression Model

class =

0.0203 * x^1 +
-0.0948

Time taken to build model: 0.04 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.7141
Mean absolute error	0.7367
Root mean squared error	0.8609
Relative absolute error	95.9842 %
Root relative squared error	97.0669 %
Total Number of Instances	63

c) 1

λ =
i) 0.001
Linear Regression Model

class =

0.695 * x^1 +

-0.0099 * x^5 +
-0.1009

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.593
Mean absolute error	0.796
Root mean squared error	0.9924
Relative absolute error	81.5531 %
Root relative squared error	80.517 %
Total Number of Instances	63

ii) 0.01
Linear Regression Model

class =

0.6945 * x^1 +
-0.0099 * x^5 +
-0.1009

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.593
Mean absolute error	0.796
Root mean squared error	0.9924
Relative absolute error	81.5492 %
Root relative squared error	80.517 %
Total Number of Instances	63

iii) 0.1
Linear Regression Model

class =

0.7037 * x^1 +
-0.0104 * x^5 +
0.0051 * x^6 +
-0.0006 * x^8 +
-0.1953

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.6118
Mean absolute error	0.7922
Root mean squared error	0.9751
Relative absolute error	81.1621 %
Root relative squared error	79.1093 %
Total Number of Instances	63

iv) 1
Linear Regression Model

class =

0.8336 * x^1 +
-0.096 * x^3 +
-0.0941

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.6052
Mean absolute error	0.7952
Root mean squared error	0.9851
Relative absolute error	81.4653 %
Root relative squared error	79.925 %
Total Number of Instances	63

v) 10
Linear Regression Model

class =

0.3693 * x^1 +
-0.0001 * x^9 +
-0.0971

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.5636
Mean absolute error	0.8103

Root mean squared error 1.0381
Relative absolute error 83.0203 %
Root relative squared error 84.2186 %
Total Number of Instances 63

vi) 100
Linear Regression Model

class =

0.1054 * x^1 +
-0.0848

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient 0.4061
Mean absolute error 0.9156
Root mean squared error 1.168
Relative absolute error 93.8028 %
Root relative squared error 94.7577 %
Total Number of Instances 63

vii) 1000
Linear Regression Model

class =

+
-0.0892

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient 0
Mean absolute error 0.9761
Root mean squared error 1.2326
Relative absolute error 100 %
Root relative squared error 100 %
Total Number of Instances 63

d) 2

λ =

i) 0.001

Linear Regression Model

class =

0.9374 * x^1 +
-0.1023 * x^3 +
0.0731

Time taken to build model: 0.03 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.3342
Mean absolute error	1.7087
Root mean squared error	2.1675
Relative absolute error	93.4849 %
Root relative squared error	94.2496 %
Total Number of Instances	63

ii) 0.01

Linear Regression Model

class =

0.936 * x^1 +
-0.1021 * x^3 +
0.0731

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.3342
Mean absolute error	1.7086
Root mean squared error	2.1675
Relative absolute error	93.4841 %
Root relative squared error	94.2496 %
Total Number of Instances	63

iii) 0.1

Linear Regression Model

class =

0.9222 * x^1 +
-0.1 * x^3 +
0.0734

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.3342
Mean absolute error	1.7088
Root mean squared error	2.1675
Relative absolute error	93.495 %
Root relative squared error	94.2509 %
Total Number of Instances	63

iv) 1
Linear Regression Model

class =

0.8069 * x^1 +
-0.0825 * x^3 +
0.0758

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.3331
Mean absolute error	1.7112
Root mean squared error	2.1696
Relative absolute error	93.6228 %
Root relative squared error	94.3428 %
Total Number of Instances	63

v) 10
Linear Regression Model

class =

0.2828 * x^1 +
0.0881

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.2596
Mean absolute error	1.7492
Root mean squared error	2.2224
Relative absolute error	95.7019 %
Root relative squared error	96.6379 %
Total Number of Instances	63

vi) 100
Linear Regression Model

class =

0.1257 * x^1 +
0.0815

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.2596
Mean absolute error	1.7793
Root mean squared error	2.2513
Relative absolute error	97.3491 %
Root relative squared error	97.8915 %
Total Number of Instances	63

vii) 1000
Linear Regression Model

class =

+
0.0763

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0
Mean absolute error	1.8277
Root mean squared error	2.2997
Relative absolute error	100 %

Root relative squared error 100 %
Total Number of Instances 63

e) 10

λ =

i) 0.001
class =

-4.6864 * x^2 +
0.4606 * x^4 +
5.7743

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient 0.3906
Mean absolute error 8.0141
Root mean squared error 9.7644
Relative absolute error 92.2647 %
Root relative squared error 92.0566 %
Total Number of Instances 63

ii) 0.01
Linear Regression Model

class =

-4.6712 * x^2 +
0.4589 * x^4 +
5.7577

Time taken to build model: 0.08 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient 0.3906
Mean absolute error 8.0138
Root mean squared error 9.7644
Relative absolute error 92.2619 %
Root relative squared error 92.0567 %
Total Number of Instances 63

iii) 0.1

Linear Regression Model

class =

-4.5249 * x^2 +
0.4425 * x^4 +
5.5971

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.3905
Mean absolute error	8.0115
Root mean squared error	9.7654
Relative absolute error	92.235 %
Root relative squared error	92.066 %
Total Number of Instances	63

iv) 1
Linear Regression Model

class =

3.7124 * x^1 +
-2.7438 * x^2 +
-0.7805 * x^3 +
0.0289 * x^6 +
0.0005 * x^9 +
4.402

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.483
Mean absolute error	7.8584
Root mean squared error	9.3402
Relative absolute error	90.4723 %
Root relative squared error	88.0579 %
Total Number of Instances	63

v) 10
Linear Regression Model

class =

-1.3356 * x^2 +
0.0013 * x^8 +
2.3379

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.3696
Mean absolute error	8.1603
Root mean squared error	10.0002
Relative absolute error	93.948 %
Root relative squared error	94.2794 %
Total Number of Instances	63

vi) 100
Linear Regression Model

class =

+
-0.6515

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0
Mean absolute error	8.6859
Root mean squared error	10.6069
Relative absolute error	100 %
Root relative squared error	100 %
Total Number of Instances	63

vii) 1000
Linear Regression Model

class =

+
-0.6515

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0
Mean absolute error	8.6859
Root mean squared error	10.6069
Relative absolute error	100 %
Root relative squared error	100 %
Total Number of Instances	63

8)

Bias of mean estimator = $\left(\frac{1}{100} \sum_{i=1}^{100} m_i \right) - M$

Where M is mean ie 5 here and m_i is mean of i^{th} subset

Variance of mean estimator = $\frac{1}{100} \sum_{i=1}^{100} (v_i - V)^2$

Where v_i = is variance of i^{th} subset

Bias of variance estimator = $\frac{1}{1000} \sum_{i=1}^{1000} (x_i - X)^2$ - V

Where x_i = i^{th} term and V is variance ie 2 and X is mean of all x_i

Variance of variance estimator = is the variance of all the 100 variances of the subset

Bias of mean estimator
-0.0542

Variance of mean estimator
0.3891

Bias of variance estimator
1.7724

Variance of variance estimator
2.8490