

1)

1) t_n - true label
 x_n - feature vector
 $r_n > 0$ importance of n^{th} example

$$E(\omega) = \frac{1}{2} \sum_{n=1}^N r_n (t_n - \omega^T \phi(x_n))^2$$

$$= \frac{1}{2} \sum_{n=1}^N (t_n r_n^{\frac{1}{2}} - \omega^T \phi(x_n) r_n^{\frac{1}{2}})^2$$

Let $t'_n = t_n r_n^{\frac{1}{2}}$ for $n=1$ to N

and since $\Phi = \begin{bmatrix} \phi_0(x_1) & \phi_1(x_1) & \dots & \phi_{M+1}(x_1) \\ \phi_0(x_2) & \phi_1(x_2) & \dots & \phi_{M+1}(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_0(x_N) & \phi_1(x_N) & \dots & \phi_{M+1}(x_N) \end{bmatrix}$

Let $\Phi' = \begin{bmatrix} \phi_0(x_1) r_1^{\frac{1}{2}} & \phi_1(x_1) r_1^{\frac{1}{2}} & \dots & \phi_{M+1}(x_1) r_1^{\frac{1}{2}} \\ \phi_0(x_2) r_2^{\frac{1}{2}} & \phi_1(x_2) r_2^{\frac{1}{2}} & \dots & \phi_{M+1}(x_2) r_2^{\frac{1}{2}} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_0(x_N) r_N^{\frac{1}{2}} & \phi_1(x_N) r_N^{\frac{1}{2}} & \dots & \phi_{M+1}(x_N) r_N^{\frac{1}{2}} \end{bmatrix}$

$$\therefore E(\omega) = \frac{1}{2} \sum_{n=1}^N r_n (t'_n - \omega^T \phi'(x_n))^2$$

After taking gradient of log likelihood

$$= \sum_{n=1}^N \{ t'_n - \omega^T \Phi(x_n) \} \Phi(x_n)^T$$

and setting it to 0

$$0 = \sum_{n=1}^N t'_n \Phi(x_n)^T - \omega^T \left(\sum_{n=1}^N \Phi(x_n) \Phi(x_n)^T \right)$$

$$\omega_{ML} = [\Phi^T \Phi]^{-1} \Phi^T t'$$

here t' is the whole vector of size $= N$

2)

Instances: 506

Attributes: 14

att_1
att_2
att_3
att_4
att_5
att_6
att_7
att_8
att_9
att_10
att_11
att_12
att_13
class

Test mode:split 60.0% train, remainder test

For different values of λ

λ =

a) 0.00001

Linear Regression Model

class =

-0.1084 * att_1 +
0.0458 * att_2 +
2.7187 * att_4 +
-17.376 * att_5 +
3.8016 * att_6 +
-1.4927 * att_8 +
0.2996 * att_9 +
-0.0118 * att_10 +
-0.9465 * att_11 +
0.0093 * att_12 +
-0.5226 * att_13 +
36.3411

Time taken to build model: 0.04 second

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3689
Root mean squared error	4.5651
Relative absolute error	51.6839 %
Root relative squared error	50.8522 %
Total Number of Instances	202

b) 0.0001

Linear Regression Model

class =

```
-0.1084 * att_1 +
0.0458 * att_2 +
2.7187 * att_4 +
-17.376 * att_5 +
3.8016 * att_6 +
-1.4927 * att_8 +
0.2996 * att_9 +
-0.0118 * att_10 +
-0.9465 * att_11 +
0.0093 * att_12 +
-0.5226 * att_13 +
36.3411
```

Time taken to build model: 0.03 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3689
Root mean squared error	4.5651
Relative absolute error	51.6839 %
Root relative squared error	50.8522 %
Total Number of Instances	202

c) 0.001

Linear Regression Model

class =

```
-0.1084 * att_1 +
0.0458 * att_2 +
2.7187 * att_4 +
-17.3759 * att_5 +
```

$$\begin{aligned}
& 3.8016 * \text{att_6} + \\
& -1.4927 * \text{att_8} + \\
& 0.2996 * \text{att_9} + \\
& -0.0118 * \text{att_10} + \\
& -0.9465 * \text{att_11} + \\
& 0.0093 * \text{att_12} + \\
& -0.5226 * \text{att_13} + \\
& 36.3408
\end{aligned}$$

Time taken to build model: 0.03 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3689
Root mean squared error	4.5651
Relative absolute error	51.6838 %
Root relative squared error	50.8522 %
Total Number of Instances	202

d) 0.01

Linear Regression Model

class =

$$\begin{aligned}
& -0.1084 * \text{att_1} + \\
& 0.0458 * \text{att_2} + \\
& 2.7188 * \text{att_4} + \\
& -17.3743 * \text{att_5} + \\
& 3.8017 * \text{att_6} + \\
& -1.4925 * \text{att_8} + \\
& 0.2995 * \text{att_9} + \\
& -0.0118 * \text{att_10} + \\
& -0.9465 * \text{att_11} + \\
& 0.0093 * \text{att_12} + \\
& -0.5225 * \text{att_13} + \\
& 36.3376
\end{aligned}$$

Time taken to build model: 0.02 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3689

Root mean squared error	4.565
Relative absolute error	51.6831 %
Root relative squared error	50.8517 %
Total Number of Instances	202

e) 0.1

Linear Regression Model

class =

$$\begin{aligned}
 &-0.1083 * \text{att_1} + \\
 &0.0458 * \text{att_2} + \\
 &2.7195 * \text{att_4} + \\
 &-17.3593 * \text{att_5} + \\
 &3.8029 * \text{att_6} + \\
 &-1.4911 * \text{att_8} + \\
 &0.2989 * \text{att_9} + \\
 &-0.0117 * \text{att_10} + \\
 &-0.9463 * \text{att_11} + \\
 &0.0093 * \text{att_12} + \\
 &-0.5224 * \text{att_13} + \\
 &36.3057
 \end{aligned}$$

Time taken to build model: 0.09 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8612
Mean absolute error	3.3684
Root mean squared error	4.5646
Relative absolute error	51.6755 %
Root relative squared error	50.8468 %
Total Number of Instances	202

f) 1

Linear Regression Model

class =

$$\begin{aligned}
 &-0.1074 * \text{att_1} + \\
 &0.0453 * \text{att_2} + \\
 &2.7262 * \text{att_4} + \\
 &-17.2099 * \text{att_5} + \\
 &3.8142 * \text{att_6} + \\
 &-1.4767 * \text{att_8} +
 \end{aligned}$$

0.2928 * att_9 +
-0.0115 * att_10 +
-0.9439 * att_11 +
0.0093 * att_12 +
-0.5212 * att_13 +
35.9922

Time taken to build model: 0.02 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8615
Mean absolute error	3.3635
Root mean squared error	4.5605
Relative absolute error	51.6015 %
Root relative squared error	50.8017 %
Total Number of Instances	202

g) 10

Linear Regression Model

class =

-0.0998 * att_1 +
0.0413 * att_2 +
2.7775 * att_4 +
-15.8401 * att_5 +
3.9043 * att_6 +
-1.3486 * att_8 +
0.2428 * att_9 +
-0.0094 * att_10 +
-0.9214 * att_11 +
0.0093 * att_12 +
-0.5098 * att_13 +
33.2887

Time taken to build model: 0.03 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8634
Mean absolute error	3.3299
Root mean squared error	4.5403
Relative absolute error	51.0848 %
Root relative squared error	50.576 %

Total Number of Instances 202

h) 100

Linear Regression Model

class =

```
-0.075 * att_1 +  
0.0272 * att_2 +  
2.8313 * att_4 +  
-9.2248 * att_5 +  
3.9836 * att_6 +  
-0.7395 * att_8 +  
0.0822 * att_9 +  
-0.0043 * att_10 +  
-0.7827 * att_11 +  
0.0086 * att_12 +  
-0.4259 * att_13 +  
22.9394
```

Time taken to build model: 0.04 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.86
Mean absolute error	3.4177
Root mean squared error	4.746
Relative absolute error	52.4325 %
Root relative squared error	52.8679 %
Total Number of Instances	202

i) 1000

Linear Regression Model

class =

```
-0.0546 * att_1 +  
0.016 * att_2 +  
-0.071 * att_3 +  
1.7399 * att_4 +  
-3.5029 * att_5 +  
2.1139 * att_6 +  
-0.0103 * att_7 +  
-0.087 * att_8 +
```


-0.0028 * att_10 +
-0.4174 * att_11 +
0.0051 * att_12 +
-0.1924 * att_13 +
22.4032

Time taken to build model: 0.02 seconds

=== Evaluation on test split ===

=== Summary ===

Correlation coefficient	0.8086
Mean absolute error	4.5596
Root mean squared error	6.389
Relative absolute error	69.951 %
Root relative squared error	71.1692 %
Total Number of Instances	202

3) Done

4)

4 Let us assume N input values X
 $X = (x_1, x_2, \dots, x_m)^T$ and their corresponding
 target values. $t = (t_1, t_2, \dots, t_m)^T$

Assuming Gaussian distribution of t
 we have the following

$$p(t|x, \omega, \beta) = N(t | \phi(x)^T \omega, \beta^{-1})$$

where $\phi = (\phi_0, \phi_1, \dots, \phi_{m+1})^T$ and in its
 simplest form $\phi_i(x) = x^i$

So because of independence assumption

$$\begin{aligned} p(t|X, \omega) &= \prod_{i=1}^N p(t_i | x_i, \omega) \\ &= \prod_{i=1}^N \frac{\beta}{\sqrt{2\pi}} \exp\left\{-\frac{\beta}{2} (t_i - \phi(x)^T \omega)^2\right\} \\ &= \frac{1}{(2\pi/\beta)^{N/2}} \exp\left\{-\frac{1}{2/\beta} |t - \phi(X)^T \omega|^2\right\} \\ &= N(t | \phi(X)^T \omega, \beta^{-1} I) \end{aligned}$$

where $\phi(X)$ is given by.

$$\begin{pmatrix} \Phi_0(x_1) & \Phi_1(x_1) & & & \Phi_{M-1}(x_1) \\ \Phi_0(x_2) & \Phi_1(x_2) & & & \Phi_{M-1}(x_2) \\ & & & & \\ & & & & \\ \Phi_0(x_n) & & & & \Phi_{M-1}(x_n) \end{pmatrix}$$

Let us also assume a Gaussian as prior distribution for ω

$$P(\omega|\alpha) = N(\omega|0, \alpha^{-1}I) = \frac{\alpha^{M/2}}{2\pi} \exp\left(-\frac{\alpha}{2} \omega^T \omega\right)$$

Using Bayes theorem, the posterior distribution and the likelihood function and applying simple procedure we get

$$P(\omega|t, X, \alpha) \propto P(t|X, \omega) P(\omega|\alpha)$$

$$\propto \exp\left\{-\frac{\beta}{2} (t - \Phi(X)^T \omega)^T (t - \Phi(X)^T \omega)\right\}$$

$$\cdot \exp\left\{-\frac{\alpha}{2} \omega^T \omega\right\}$$

$$\propto \exp\left\{-\frac{1}{2} (\omega - \bar{\omega})^T C^{-1} (\omega - \bar{\omega})\right\}$$

where $\bar{\omega} = \beta (\beta \Phi(X) \Phi(X)^T + \alpha I)^{-1} \Phi(X)^T t$
and covariance matrix $C = (\beta \Phi(X) \Phi(X)^T + \alpha I)^{-1}$

$$\text{So } P(\omega|t, X, \alpha) \sim N(\bar{\omega} = \beta C \Phi(X)^T t, C)$$

5)

classmate
Date _____
Page _____

Q5
$$\sum_{d=1}^D \frac{(M+d-1)!}{(M-1)!d!}$$

~~confuse~~ number of terms will be configuration of terms will be like ~~sig~~
 $x_1^{i_1} x_2^{i_2} \dots x_m^{i_m}$

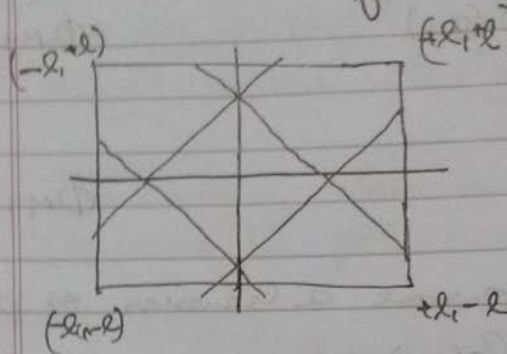
where $i_1, i_2, \dots, i_m \leq D$
 and $i_1, i_2, \dots, i_m \geq 0$

~~that~~
 number of possible permutation of i
 $= \frac{M+D-1}{M-1} C_{M-1}^{D+M-1}$

\therefore number of different polynomials

$$\sum_{d=1}^D \frac{(M+d-1)!}{(M-1)!d!} = \sum_{d=1}^D \frac{(M+d-1)!}{(M-1)!d!}$$

6 Consider the following diagram of Lasso



Now since L is very large

the Lasso occupies a part in the space. Thus in 2 dimension probability that one of the dim is zero is $\frac{1}{2}$

Thus for M dimensions the prob that of the dimensions are 0 is

$$\frac{M!}{(M-k)! k!} \left(\frac{1}{M}\right)^k \left(1 - \frac{1}{M}\right)^{M-k}$$

$$\text{or } \binom{M}{k} \left(\frac{1}{M}\right)^k \left(1 - \frac{1}{M}\right)^{M-k}$$

7)

Instances: 63

Attributes: 10

x^1

x^2

x^3

x^4

x^5

x^6

x^7

x^8

x^9

class

Test mode: evaluate on training data

For a =

a) 0.1

$\lambda =$

i) 0.001

Linear Regression Model

class =

$0.4953 * x^1 +$

$-0.0001 * x^9 +$

-0.0207

Time taken to build model: 0.03 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.9222
Mean absolute error	0.236
Root mean squared error	0.2809
Relative absolute error	36.0293 %
Root relative squared error	38.6637 %
Total Number of Instances	63

ii) 0.01

Linear Regression Model

class =

$$0.4951 * x^1 +$$

$$-0.0001 * x^9 +$$

$$-0.0207$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.9222
Mean absolute error	0.236
Root mean squared error	0.2809
Relative absolute error	36.0281 %
Root relative squared error	38.6637 %
Total Number of Instances	63

iii) 0.1

Linear Regression Model

class =

$$0.0896 * x^3 +$$

$$-0.0001 * x^9 +$$

$$-0.0226$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.7091
Mean absolute error	0.441
Root mean squared error	0.5123
Relative absolute error	67.316 %
Root relative squared error	70.5198 %
Total Number of Instances	63

iv) 1

Linear Regression Model

class =

$$0.0011 * x^7 +$$

$$-0.0001 * x^9 +$$

$$-0.0107$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.4225
Mean absolute error	0.5827
Root mean squared error	0.6703
Relative absolute error	88.9447 %
Root relative squared error	92.2722 %
Total Number of Instances	63

v) 10

Linear Regression Model

class =

$$0.3873 * x^1 +$$

$$-0.001 * x^5 +$$

$$-0.0002 * x^7 +$$

$$0 * x^9 +$$

$$-0.0147$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.9315
Mean absolute error	0.2571
Root mean squared error	0.3142
Relative absolute error	39.2476 %
Root relative squared error	43.2522 %
Total Number of Instances	63

vi) 100

Linear Regression Model

class =

$$0.1199 * x^1 +$$

$$-0.0087$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.7842
Mean absolute error	0.5022
Root mean squared error	0.5717
Relative absolute error	76.6562 %
Root relative squared error	78.7014 %
Total Number of Instances	63

vii) 1000

Linear Regression Model

class =

$$0.0183 * x^1 + \\ -0.0129$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.7842
Mean absolute error	0.6263
Root mean squared error	0.7006
Relative absolute error	95.6043 %
Root relative squared error	96.4519 %
Total Number of Instances	63

b) 0.5

$\lambda =$

i) 0.001

Linear Regression Model

class =

$$0.5151 * x^1 + \\ -0.0001 * x^9 + \\ -0.0995$$

Time taken to build model: 0.06 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.8016
Mean absolute error	0.4265
Root mean squared error	0.5302
Relative absolute error	55.5615 %
Root relative squared error	59.7835 %
Total Number of Instances	63

ii) 0.01
Linear Regression Model

class =

$$0.5149 * x^1 +$$

$$-0.0001 * x^9 +$$

$$-0.0995$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
 === Summary ===

Correlation coefficient	0.8016
Mean absolute error	0.4265
Root mean squared error	0.5302
Relative absolute error	55.5624 %
Root relative squared error	59.7835 %
Total Number of Instances	63

iii) 0.1
Linear Regression Model

class =

$$-0.0022 * x^6 +$$

$$0.0002 * x^8 +$$

$$0 * x^9 +$$

$$-0.0434$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
 === Summary ===

Correlation coefficient	0.24
Mean absolute error	0.7399
Root mean squared error	0.861
Relative absolute error	96.399 %

Root relative squared error	97.078 %
Total Number of Instances	63

iv) 1
Linear Regression Model

class =

$$0.7385 * x^1 +$$
$$-0.0675 * x^3 +$$
$$-0.0928$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.8424
Mean absolute error	0.3923
Root mean squared error	0.4814
Relative absolute error	51.114 %
Root relative squared error	54.2743 %
Total Number of Instances	63

v) 10
Linear Regression Model

class =

$$0.3749 * x^1 +$$
$$0 * x^9 +$$
$$-0.0932$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===
=== Summary ===

Correlation coefficient	0.7914
Mean absolute error	0.4505
Root mean squared error	0.5631
Relative absolute error	58.6898 %
Root relative squared error	63.4884 %
Total Number of Instances	63

vi) 100
Linear Regression Model

class =

$$0.1333 * x^1 + \\ -0.0901$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.7141
Mean absolute error	0.6022
Root mean squared error	0.7337
Relative absolute error	78.4577 %
Root relative squared error	82.7272 %
Total Number of Instances	63

vii) 1000

Linear Regression Model

class =

$$0.0203 * x^1 + \\ -0.0948$$

Time taken to build model: 0.04 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.7141
Mean absolute error	0.7367
Root mean squared error	0.8609
Relative absolute error	95.9842 %
Root relative squared error	97.0669 %
Total Number of Instances	63

c) 1

$\lambda =$

i) 0.001

Linear Regression Model

class =

$$0.695 * x^1 +$$

$$-0.0099 * x^5 +$$

$$-0.1009$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.593
Mean absolute error	0.796
Root mean squared error	0.9924
Relative absolute error	81.5531 %
Root relative squared error	80.517 %
Total Number of Instances	63

ii) 0.01

Linear Regression Model

class =

$$0.6945 * x^1 +$$

$$-0.0099 * x^5 +$$

$$-0.1009$$

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.593
Mean absolute error	0.796
Root mean squared error	0.9924
Relative absolute error	81.5492 %
Root relative squared error	80.517 %
Total Number of Instances	63

iii) 0.1

Linear Regression Model

class =

$$0.7037 * x^1 +$$

$$-0.0104 * x^5 +$$

$$0.0051 * x^6 +$$

$$-0.0006 * x^8 +$$

$$-0.1953$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.6118
Mean absolute error	0.7922
Root mean squared error	0.9751
Relative absolute error	81.1621 %
Root relative squared error	79.1093 %
Total Number of Instances	63

iv) 1

Linear Regression Model

class =

$$0.8336 * x^1 +$$
$$-0.096 * x^3 +$$
$$-0.0941$$

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.6052
Mean absolute error	0.7952
Root mean squared error	0.9851
Relative absolute error	81.4653 %
Root relative squared error	79.925 %
Total Number of Instances	63

v) 10

Linear Regression Model

class =

$$0.3693 * x^1 +$$
$$-0.0001 * x^9 +$$
$$-0.0971$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.5636
Mean absolute error	0.8103

Root mean squared error	1.0381
Relative absolute error	83.0203 %
Root relative squared error	84.2186 %
Total Number of Instances	63

vi) 100

Linear Regression Model

class =

0.1054 * x^1 +
-0.0848

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.4061
Mean absolute error	0.9156
Root mean squared error	1.168
Relative absolute error	93.8028 %
Root relative squared error	94.7577 %
Total Number of Instances	63

vii) 1000

Linear Regression Model

class =

+
-0.0892

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0
Mean absolute error	0.9761
Root mean squared error	1.2326
Relative absolute error	100 %
Root relative squared error	100 %
Total Number of Instances	63

d) 2

$\lambda =$

i) 0.001

Linear Regression Model

class =

$$0.9374 * x^1 + \\ -0.1023 * x^3 + \\ 0.0731$$

Time taken to build model: 0.03 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3342
Mean absolute error	1.7087
Root mean squared error	2.1675
Relative absolute error	93.4849 %
Root relative squared error	94.2496 %
Total Number of Instances	63

ii) 0.01

Linear Regression Model

class =

$$0.936 * x^1 + \\ -0.1021 * x^3 + \\ 0.0731$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3342
Mean absolute error	1.7086
Root mean squared error	2.1675
Relative absolute error	93.4841 %
Root relative squared error	94.2496 %
Total Number of Instances	63

iii) 0.1

Linear Regression Model

class =

$$0.9222 * x^1 + \\ -0.1 * x^3 + \\ 0.0734$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3342
Mean absolute error	1.7088
Root mean squared error	2.1675
Relative absolute error	93.495 %
Root relative squared error	94.2509 %
Total Number of Instances	63

iv) 1

Linear Regression Model

class =

$$0.8069 * x^1 + \\ -0.0825 * x^3 + \\ 0.0758$$

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3331
Mean absolute error	1.7112
Root mean squared error	2.1696
Relative absolute error	93.6228 %
Root relative squared error	94.3428 %
Total Number of Instances	63

v) 10

Linear Regression Model

class =

$$0.2828 * x^1 + \\ 0.0881$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.2596
Mean absolute error	1.7492
Root mean squared error	2.2224
Relative absolute error	95.7019 %
Root relative squared error	96.6379 %
Total Number of Instances	63

vi) 100

Linear Regression Model

class =

$0.1257 * x^1 +$
0.0815

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.2596
Mean absolute error	1.7793
Root mean squared error	2.2513
Relative absolute error	97.3491 %
Root relative squared error	97.8915 %
Total Number of Instances	63

vii) 1000

Linear Regression Model

class =

+

0.0763

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0
Mean absolute error	1.8277
Root mean squared error	2.2997
Relative absolute error	100 %

Root relative squared error	100	%
Total Number of Instances	63	

e) 10

$\lambda =$

i) 0.001

class =

$$-4.6864 * x^2 +$$

$$0.4606 * x^4 +$$

$$5.7743$$

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3906
Mean absolute error	8.0141
Root mean squared error	9.7644
Relative absolute error	92.2647 %
Root relative squared error	92.0566 %
Total Number of Instances	63

ii) 0.01

Linear Regression Model

class =

$$-4.6712 * x^2 +$$

$$0.4589 * x^4 +$$

$$5.7577$$

Time taken to build model: 0.08 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3906
Mean absolute error	8.0138
Root mean squared error	9.7644
Relative absolute error	92.2619 %
Root relative squared error	92.0567 %
Total Number of Instances	63

iii) 0.1

Linear Regression Model

class =

$$\begin{aligned} &-4.5249 * x^2 + \\ &0.4425 * x^4 + \\ &5.5971 \end{aligned}$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3905
Mean absolute error	8.0115
Root mean squared error	9.7654
Relative absolute error	92.235 %
Root relative squared error	92.066 %
Total Number of Instances	63

iv) 1

Linear Regression Model

class =

$$\begin{aligned} &3.7124 * x^1 + \\ &-2.7438 * x^2 + \\ &-0.7805 * x^3 + \\ &0.0289 * x^6 + \\ &0.0005 * x^9 + \\ &4.402 \end{aligned}$$

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.483
Mean absolute error	7.8584
Root mean squared error	9.3402
Relative absolute error	90.4723 %
Root relative squared error	88.0579 %
Total Number of Instances	63

v) 10

Linear Regression Model

class =

-1.3356 * x^2 +
0.0013 * x^8 +
2.3379

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0.3696
Mean absolute error	8.1603
Root mean squared error	10.0002
Relative absolute error	93.948 %
Root relative squared error	94.2794 %
Total Number of Instances	63

vi) 100

Linear Regression Model

class =

+
-0.6515

Time taken to build model: 0.02 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0
Mean absolute error	8.6859
Root mean squared error	10.6069
Relative absolute error	100 %
Root relative squared error	100 %
Total Number of Instances	63

vii) 1000

Linear Regression Model

class =

+
-0.6515

Time taken to build model: 0.01 seconds

=== Evaluation on training set ===

=== Summary ===

Correlation coefficient	0
Mean absolute error	8.6859
Root mean squared error	10.6069
Relative absolute error	100 %
Root relative squared error	100 %
Total Number of Instances	63

8)

$$\text{Bias of mean estimator} = \left(\frac{1}{100} \sum_{i=1}^{100} m_i \right) - M$$

Where M is mean ie 5 here and m_i is mean of i^{th} subset

$$\text{Variance of mean estimator} = \frac{1}{100} \sum_{i=1}^{100} (v_i - V)^2$$

Where v_i = is variance of i^{th} subset

$$\text{Bias of variance estimator} = \frac{1}{1000} \sum_{i=1}^{1000} (x_i - X)^2 - V$$

Where x_i = i^{th} term and V is variance ie 2 and X is mean of all x_i

Variance of variance estimator = is the variance of all the 100 variances of the subset

Bias of mean estimator

-0.0542

Variance of mean estimator

0.3891

Bias of variance estimator

1.7724

Variance of variance estimator

2.8490