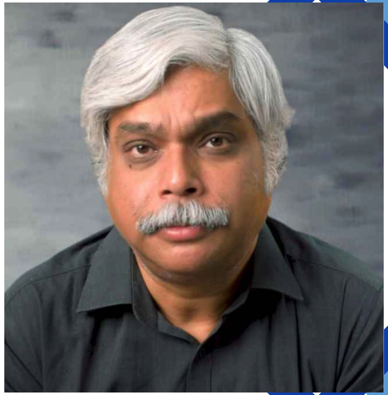
Linear Regression

Meet Your Speaker



Dr. Abhinanda Sarkar

Academic Director at Great Learning

- Alumnus - Indian Statistical Institute, Stanford University
- Faculty - MIT, Indian Institute of Management, Indian Institute of Science
- Experienced in applying probabilistic models, statistical analysis and machine learning to diverse areas
- Certified Master Black Belt in Lean Six Sigma and Design for Six Sigma in GE

Learning Objectives

By the end of this session, you should be able to:

- Relate correlation and simple linear regression in the context of understanding linear relationships.
- Explore simple linear regression models to capture the linear relationship between a pair of attributes.
- Build multiple linear regression to model relationships between two or more input attributes and the output, to predict business outcomes.
- Evaluate linear regression models and identify the levers to improve their performance.
- Discover the applications of linear regression to solve a variety of business problems.

Agenda

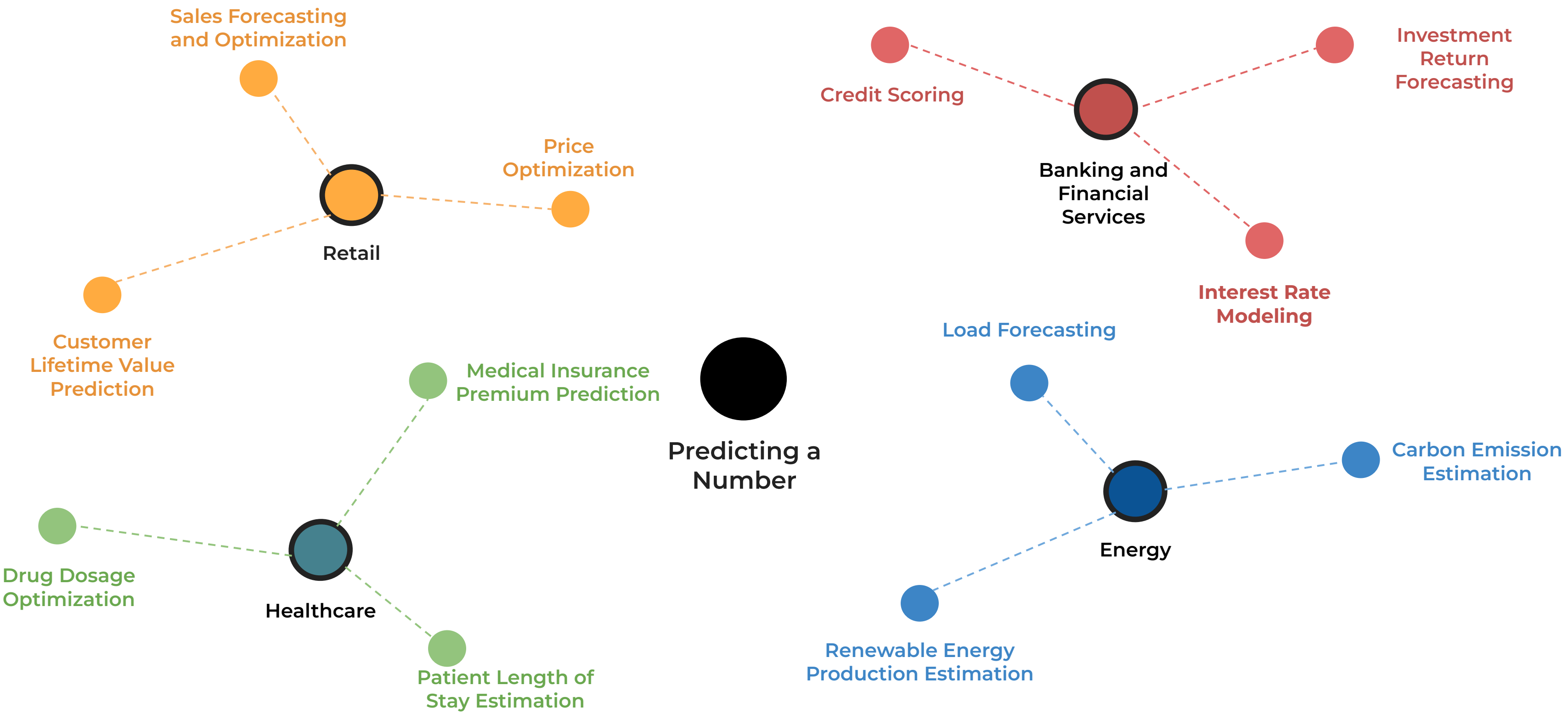
In this session, we'll discuss:

- Business Problem and Solution Space
- Correlation and Linear Relationships
- Simple Linear Regression
- Multiple Linear Regression
- Categorical Variables in Regression
- Evaluation Metrics for Regression

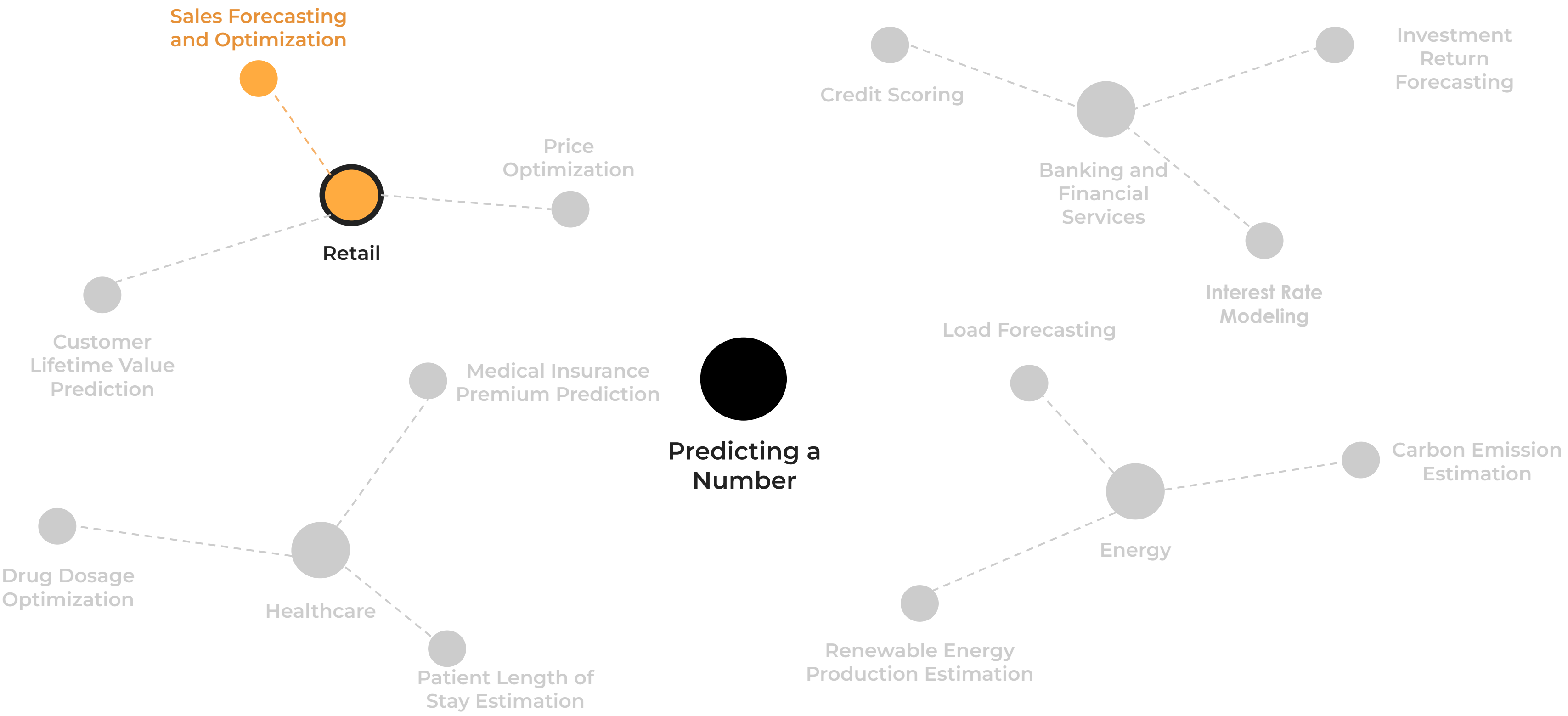
Common Business Questions

- How can we forecast sales based on historical sales data and marketing expenditure?
- How do we determine medical insurance premiums for customers based on attributes like blood pressure, blood sugar level, and smoking habits?
- How do we determine the credit card limit to be assigned to customers based on their past spending behavior, demographic information, etc?
- How can we predict future power load requirements to ensure reliable grid operation and prevent outages?

Problem Space



Problem Space



Problem Statement

- Consider an online retailer of mobiles and tablets
- Crucial to stay ahead of market trends and consumer preferences to maximize sales
- Need to effectively manage inventory and marketing efforts to attract and retain customers

Objectives

- **Accurately forecast sales to make informed decisions**
- **Identify the key levers that can influence sales**

Sales Forecasting and Optimization

Current State

Gap / Key Questions

Desired State

Unable to estimate the sales of a particular gadget for the next six months

Difficulty in allocating funds for marketing as we cannot identify factors driving the sales of a particular gadget

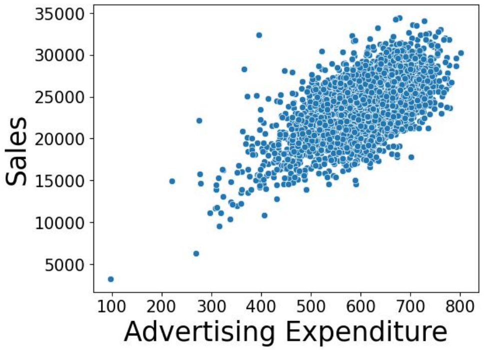
How do we predict the number of units of iPhones that will sell in the next quarter?

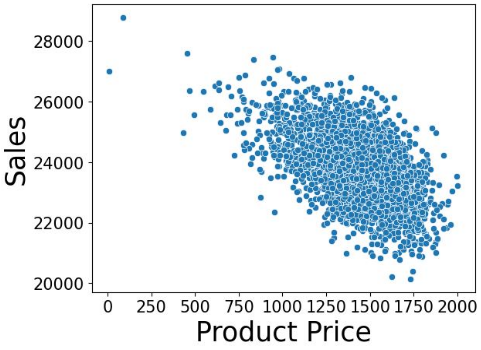
What are the factors that affect the sales of iPhones?

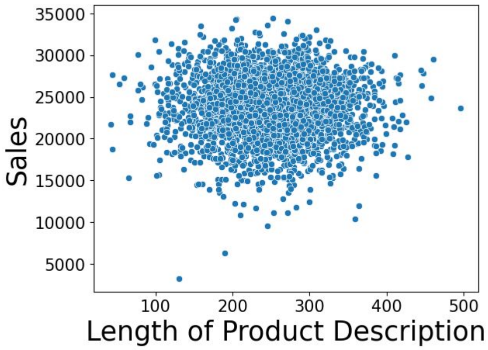
Developed a sales forecasting mechanism to estimate revenue for the next six months

One unit increase in marketing spending will result in 20 units increase in iPhone sales

Visualizing Relationships







What happens to Sales as Advertising Expenditure increases?

Advertising Expenditure ↑ ✕ **Sales** ↑

Positive relationship

What happens to Sales as Product Price increases?

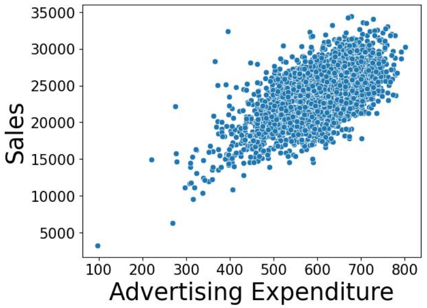
Product Price ↑ ✕ **Sales** ↓

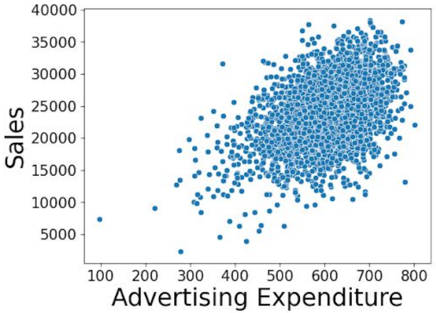
Negative relationship

No relationship

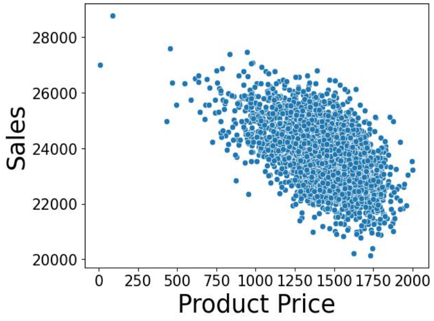
Visualizing Relationships

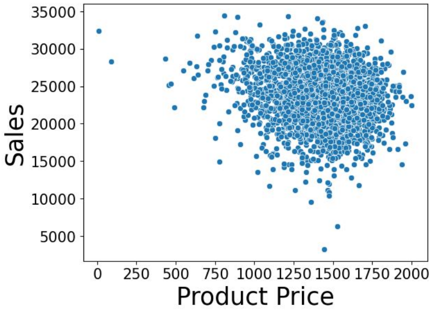
(I)





(II)





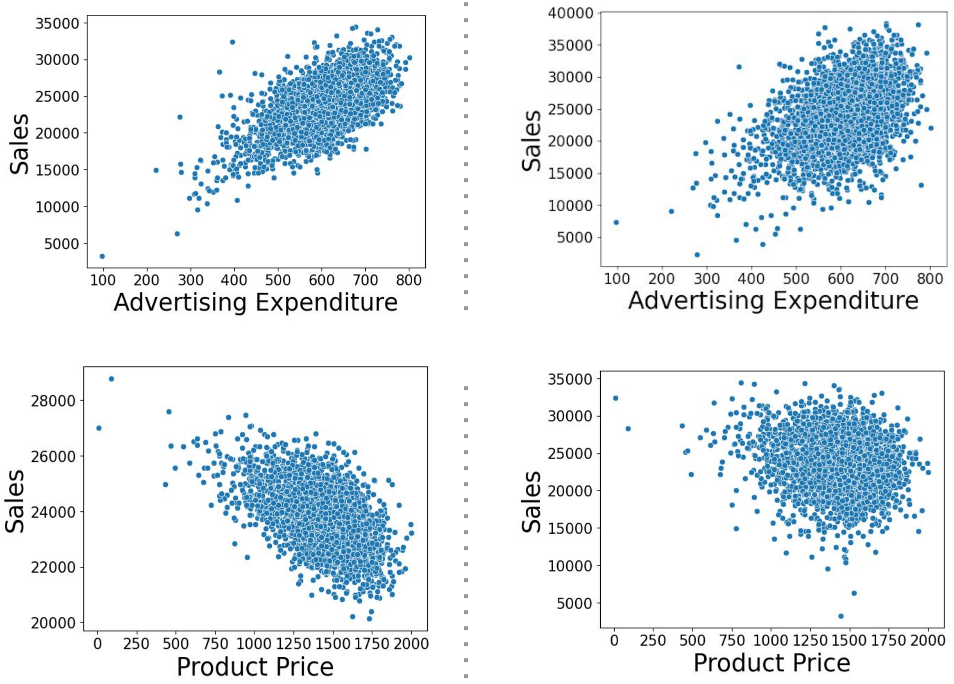
(I) In both the cases, we observe a **positive relationship** between sales and advertising expenditure

What is the **difference**?

(II) In both the cases, we observe a **negative relationship** between the sales and product price

Visualizing Relationships

(I)
(II)



Strong

Weak

The cases on the left - in both (I) and (II) - exhibit a **stronger relationship (positive** or **negative)** than the ones on the right

Correlation

- We have seen how to **visually identify relationships** between a pair of variables from two aspects - **direction** and **strength**
- But we need a **quantitative measure** of the relationship

Correlation is a **statistical measure** that describes the **strength and direction** of a **relationship** between two variables.

- Indicates the **degree** to which two variables tend to **change together**
- Quantifies both the **direction** and **strength** of the relationship

Correlation

Correlation typically **ranges between -1 and 1.**

-1	0	+1
Perfect negative correlation	No correlation	Perfect positive correlation
One variable decreases as the other increases	Variables are independent of each other	Both variables increase together

Pearson's Correlation Coefficient

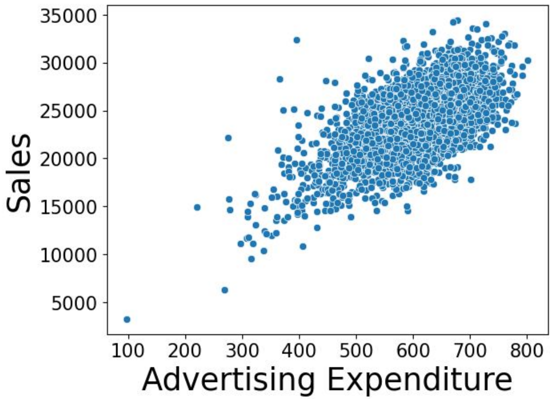
- One of the most commonly used measures of correlation.

A statistical measure that quantifies the **strength** and **direction** of the **linear relationship** between **two continuous variables.**

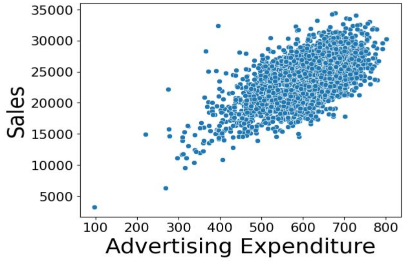
$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

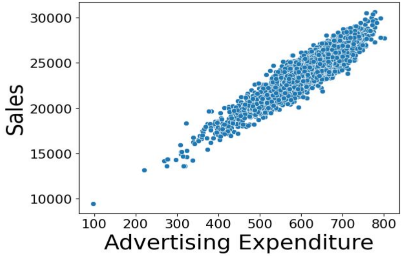
Correlation vs. Causation

- We observed advertising expenditure exhibits a strong positive correlation with sales.
- As advertising expenditure increased, sales increased.
- Does it mean advertising expenditure **causes** an increase in sales?
- **Not necessarily true!**
- There might be **other factors** at play.

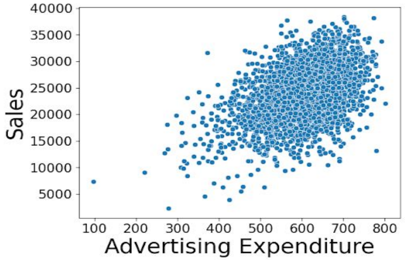


Correlation vs. Causation





Economic Zone 1



Economic Zone 2

- Let's split the data with respect to **another factor** - **economic zone.**
- Economic Zone 1 has a **booming economy** - sales will be higher here even if we don't spend as much on marketing.
- Economic Zone 2 has a **stagnant economy** - sales might have been higher due to data collected in a festive season.

Correlation vs. Causation

Correlation ≠ Causation

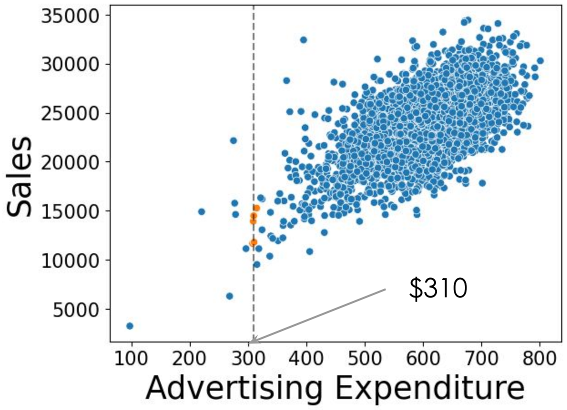
Variable 1 and **Variable 2** are **highly correlated**

≠

Variable 1 causes a change in **Variable 2**

The Need for Regression

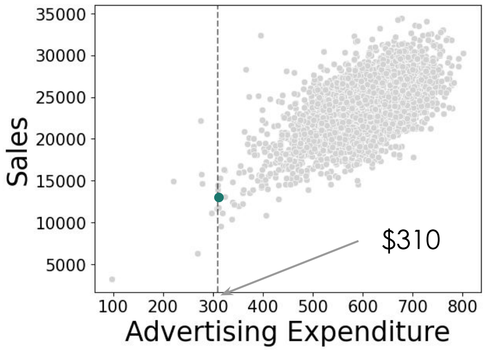
- We observed advertising expenditure exhibits a strong positive correlation with sales.
- Let's say we now decide to spend $310 for the marketing campaign of the latest iPhone.
- **How much sales should we expect?**
- **We don't know!**
- **Historically**, we've had **different sales** for **similar marketing spending.**



Correlation measures the strength and direction of the **relationship**, but **doesn't** provide a way to **predict** the output given an input.

The Need for Regression

- It is important for us to be able to determine the output (sales).
- It is also important to identify the lever(s) that drive the output (sales).
- Hence, the need for a mathematical model.



Input value
($310 marketing spend)

→

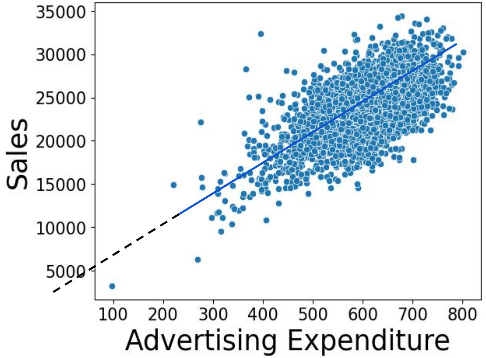
Mathematical model

→

Output value
($13040.50 sales)

Simple Linear Regression

- The **simplest** mathematical model is **linear** - a **straight line.**



Linear Regression is a **statistical model** which **estimates** the **linear relationship** between a **response** and one or more **explanatory variables.**

- Simple Linear Regression - **one explanatory** and **one response variable.**

- Assumes that there is a linear relationship between the explanatory (independent) variable and the response (dependent) variable.

advertising expenditure

sales

Simple Linear Regression

- The equation of line is represented by:

sales

$$y = b_0 + b_1 x$$

advertising expenditure

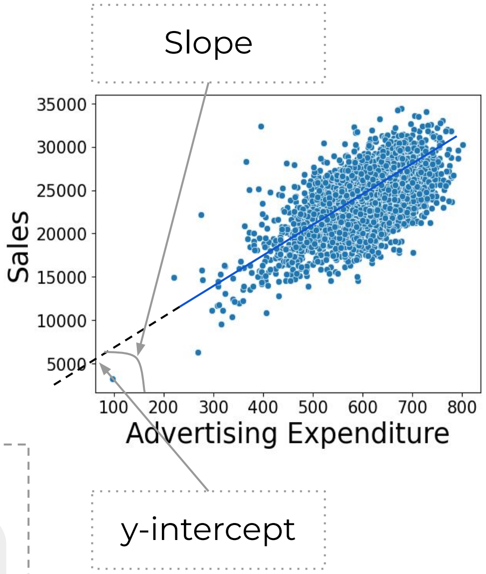
y-intercept (b_0)

The average value of the response when the explanatory variable is zero

Slope (b_1)

The average change in the response for one-unit increase in the explanatory variable

Coefficients of Simple Linear Regression



Coefficient Interpretation

- Consider the following model for our context:

sales = 1.01 + 2.45 * advertising expenditure

- For a **unit increase** in advertising expenditure, the sales will increase by **2.45 units.**

This **interpretation** is **valid ONLY IF** the **assumptions** of linear regression hold **true.**

Coefficient Interpretation

- Consider the following model for our context:

sales = 1.01 + 2.45 * advertising expenditure

- If we have zero marketing expenditure:

sales = 1.01 + 2.45 * 0 = 1.01

- Makes **business sense** — we can have **organic sales.**

What if the business context changes?

Coefficient Interpretation

- Consider the case of predicting the price of a house using the following model:

house price = 291.07 + 105.45 * square footage

- For a **unit increase** in square footage, the price of the house increases by **105.45 units.**

This **interpretation** is **valid ONLY IF** the **assumptions** of linear regression hold **true.**

Coefficient Interpretation

- Consider the case of predicting the price of a house using the following model:

house price = 291.07 + 105.45 * square footage

- In the case of zero square footage:

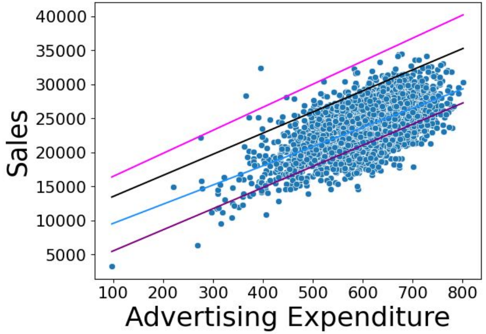
house price = 291.07 + 105.45 * 0 = 291.07

- **Doesn’t make business sense!**

y-intercept doesn’t always make business sense.

Best-Fit Line

- We observed one line that described the relationship between sales and advertising expenditure.
- But we can draw multiple lines!

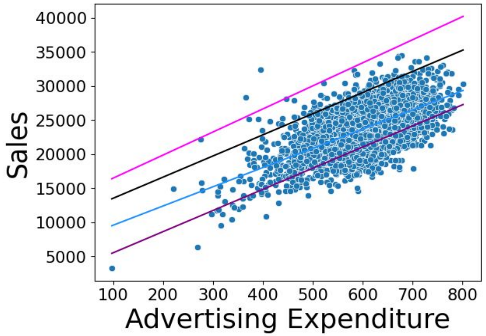


Which line do we choose?

Best-Fit Line

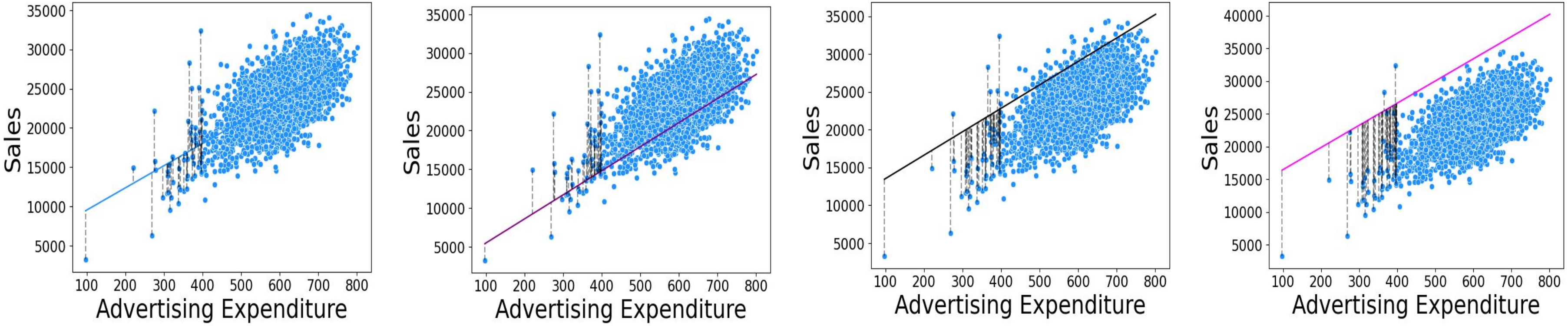
- We first need to understand the **difference** between these **lines.**

- We have actual data points (actual sales) and predicted data points (model's predicted sales).



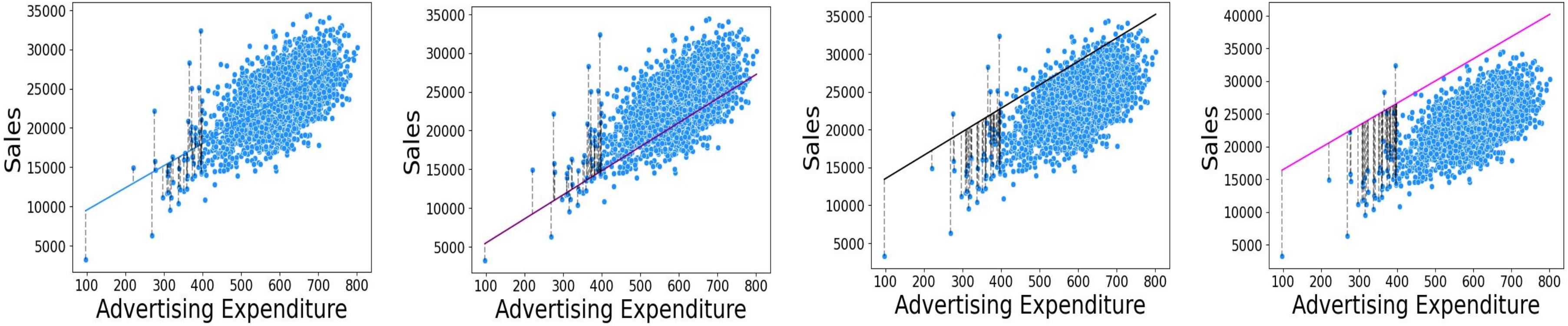
Prediction Error = Actual Value - Predicted Value

Best-Fit Line



- There are multiple data points to consider.
- Take the aggregate of the errors across the data points.

Best-Fit Line



- The **line** with the **least aggregate error** across all data points is the one **we want.**

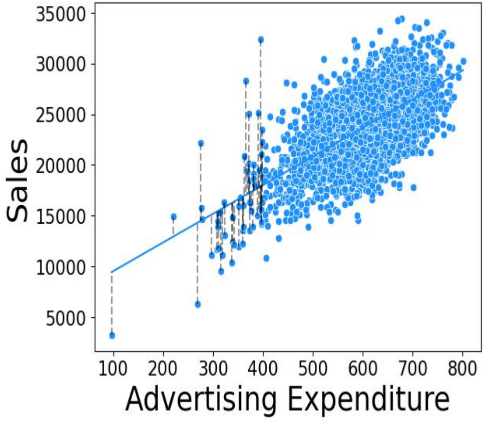
This is called the **best-fit line.**

Best-Fit Line Computation

- How to find the error?

$$Error = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)$$

Actual Value — **Predicted Value**



- **Difference** between actual and predicted values can be **positive** or **negative**
- Direct addition will give a false picture of low overall error

Best-Fit Line Computation

- Take **absolute values**

$$Error = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i|$$

$$\Downarrow$$

$$Error = \frac{1}{n}\sum_{i=1}^{n} |y_i - (b_0 + b_1 x_i)|$$

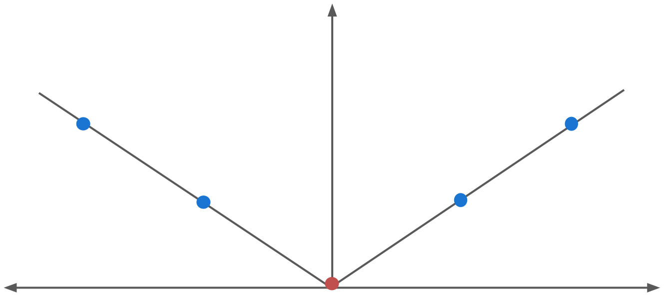
How to minimize the error?

Best-Fit Line Computation

- Need to find the **values** of the **coefficients** (b_0 and b_1) that yield the **minimum error**
- Use **differentiation**

Differentiate the **error** with respect to the **coefficients** (b_0 and b_1)

- Differentiating absolute values is **mathematically inconvenient**



Differentiable

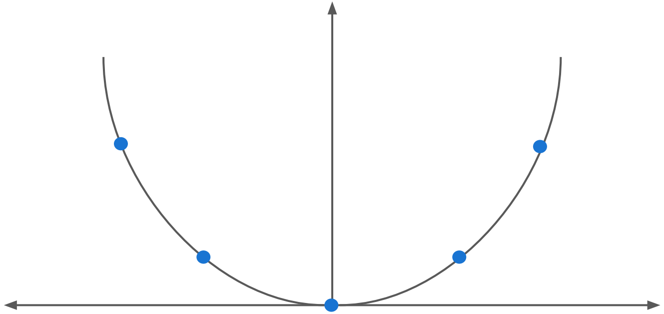
Not differentiable

Best-Fit Line Computation

- Use **squared values** instead

$$Error = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- **Accommodates** both **positive** and **negative** errors
- **Mathematically convenient** - differentiable



- **Differentiable**

Best-Fit Line Computation

- Use **squared values** instead

$$Error = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$\Downarrow$$

$$Error = \frac{1}{n}\sum_{i=1}^{n}(y_i - (b_0 + b_1 x_i))^2$$

This is known as the **Method of Least Squares**

Multiple Linear Regression

- We have checked the relationship between sales and advertising expenditure
- What if there is another variable which can be used to predict the sales?

Input 1
($310 advertising expenditure)

+

Input 2
(3.88% discount percentage)

Mathematical model

Output value
($13341.70 sales)

Multiple Linear Regression

- Multiple Linear Regression - **two or more explanatory** and **one response variable**
- Extension of Simple Linear Regression

Input 1
($310 advertising expenditure)

+

Input 2
(3.88% discount percentage)

→ Multiple Linear Regression → Output value
($13341.70 sales)

Multiple Linear Regression

- Multiple Linear Regression equation - two explanatory variables

sales

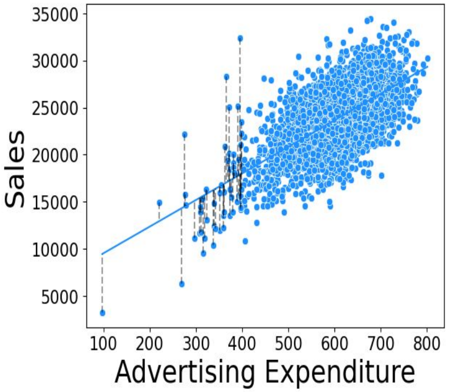
advertising expenditure

discount percentage

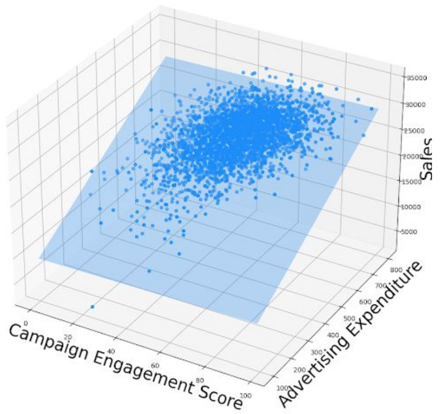
$$y = b_0 + b_1x_1 + b_2x_2$$

Coefficients of Multiple Linear Regression

Multiple Linear Regression



$$y = b_0 + b_1 x_1$$



$$y = b_0 + b_1x_1 + b_2x_2$$

- For **one explanatory variable**, the **equation** was that of a **line**
- For **two explanatory variables**, the **equation** will be that of a **plane**

Coefficient Interpretation

- Consider the following model for our context

sales = 1.01 + 2.45 * advertising expenditure + 7.88 * discount percentage

- For a **unit increase** in advertising expenditure, the sales will increase by **2.45 units**, provided all other variables are held constant

- For a **unit increase** in discount percentage, the sales will increase by **7.88 units**, provided all other variables are held constant

These **interpretations** are **valid ONLY IF** the **assumptions** of linear regression hold **true**

Multiple Linear Regression

- Multiple Linear Regression equation - more than two explanatory variables

sales | **advertising expenditure** | **discount percentage** | **product price**

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3$$

Coefficients of Multiple Linear Regression

Categorical Variables in Regression

- So far we've worked with **numerical variables**
- But real-world data often contains **categorical variables**
- Consider the following case

numerical

advertising expenditure

categorical

popularity

sales

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4$$

discount percentage

numerical

product price

numerical

Categorical Variables in Regression

- Categorical variables are not numbers - even if they might be represented by numbers
- Can’t be utilized directly in a linear regression model
- Need to be converted into a numerical format

Encoding

Label Encoding

Used when the categories have an inherent sense of order

One-hot Encoding

Used when the categories have no inherent sense of order

Label Encoding

- Assigns a unique integer to each category
- Order of the integers represents the order of the categories

Popularity		Popularity
Very Low		1
Low	Label Encoding →	2
Moderate		3
High		4
Very High		5

One-hot Encoding

- A new column is created for each category
- If the data point contains the category, corresponding column has value 1
- If the data point doesn't contain the category, corresponding column has value 0

Region
East
South
West
East
North

One-hot Encoding

Region _North	Region _East	Region _West	Region _South
0	1	0	0
0	0	0	1
0	0	1	0
0	1	0	0
1	0	0	0

Evaluation Metrics

- We have seen multiple models so far
- We don’t know **‘how well’** these models are **performing**
- Need to **evaluate** the models to gauge if they’re performing ‘well’
- **Model performance** is measured using **metrics**
- **Quantify** how well the model predictions align with the actual values

Evaluation Metrics

Mean Absolute Error

Evaluation Metrics

- An intuitive metric

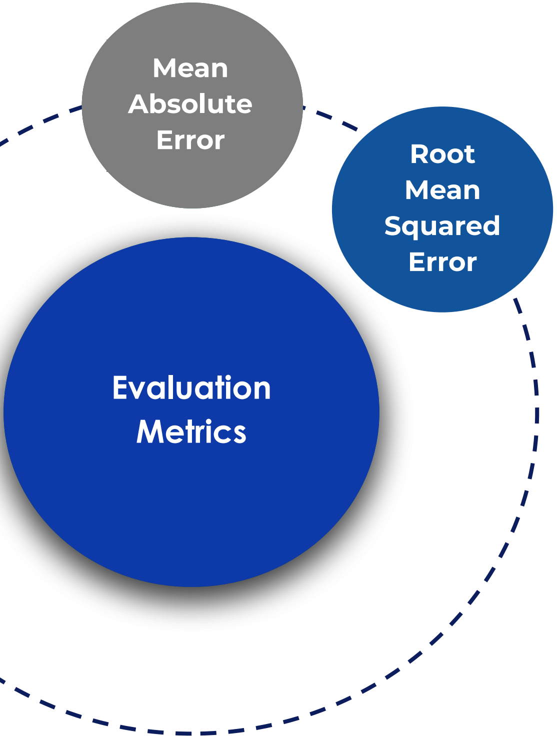
$$MAE = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i|$$

- Gives an idea of how much the model predictions deviate from the actual observations
- Relative to the range of the response

Evaluation Metrics

- Problem with considering the absolute value of errors is it doesn't penalize larger errors
- Needed to ensure that the model learns to do better when encountering edge cases

Evaluation Metrics

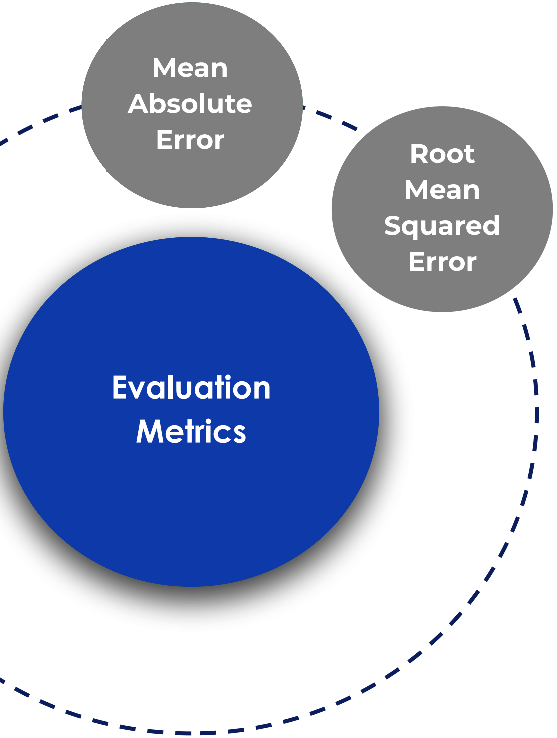


- Problem with considering the absolute value of errors is it doesn't penalize larger errors
- Needed to ensure that the model learns to do better when encountering edge cases

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

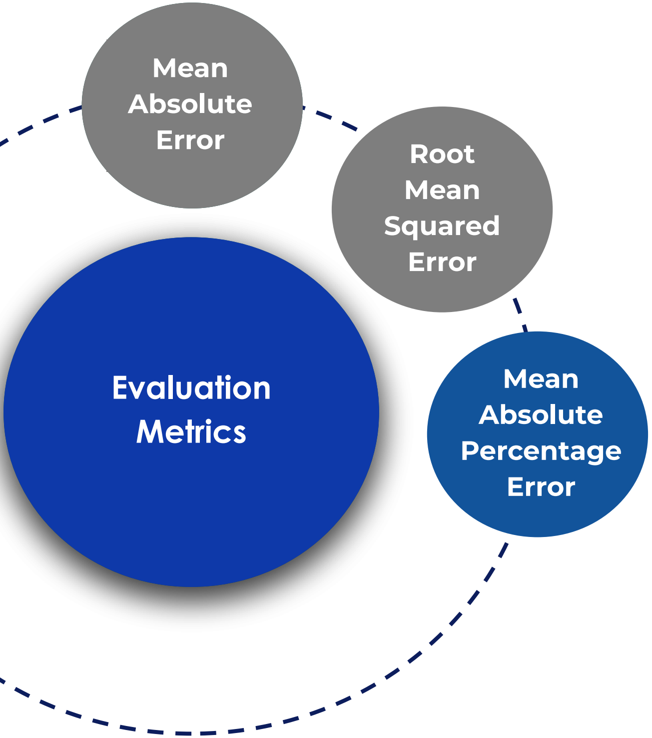
- Relative to the range of the response

Evaluation Metrics



- MAE and RMSE are relative to the scale of the response
- Cannot compare models across different data and scale of response value

Evaluation Metrics

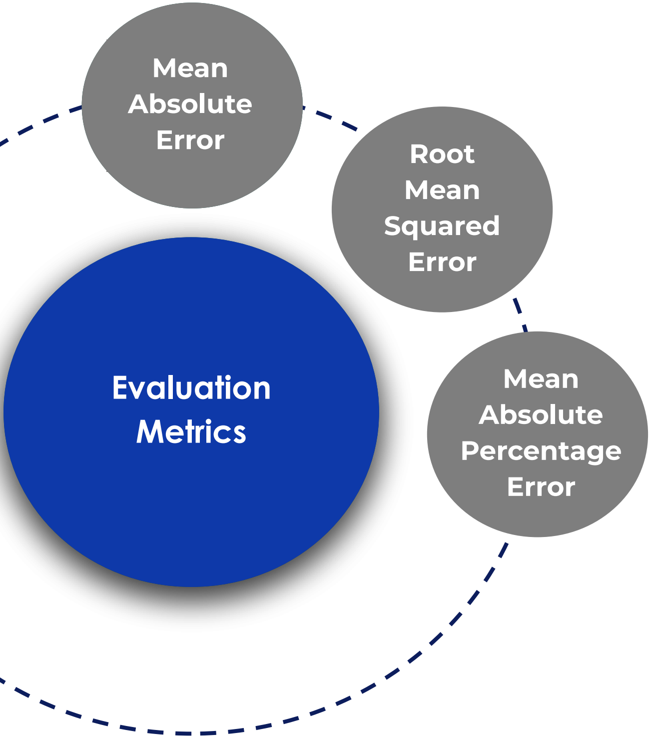


- MAE and RMSE are relative to the scale of the response
- Cannot compare models across different data and scale of response value

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| * 100$$

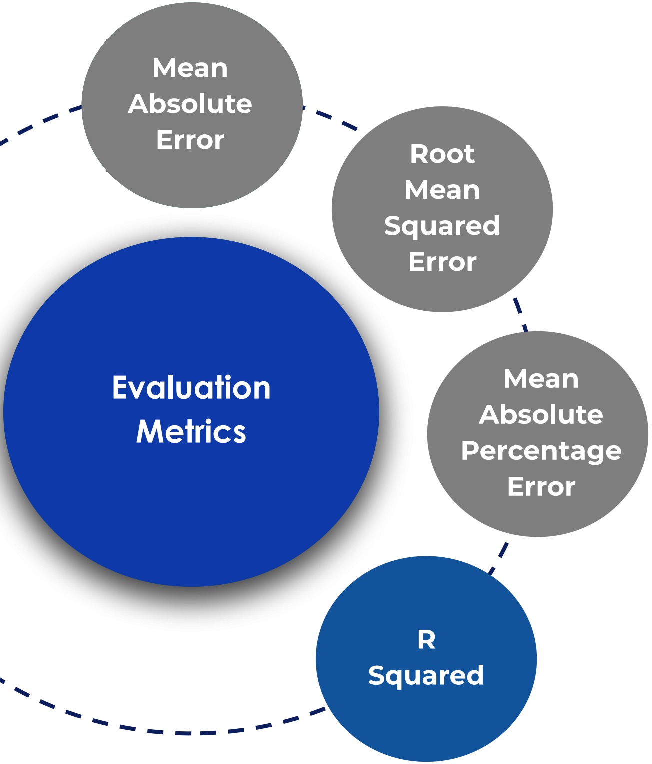
- Indifferent of the range of the response
- Needs to be adjusted when the actual value of the response is zero

Evaluation Metrics



- Previous metrics do not clearly quantify how well the model explains the variability in the data

Evaluation Metrics

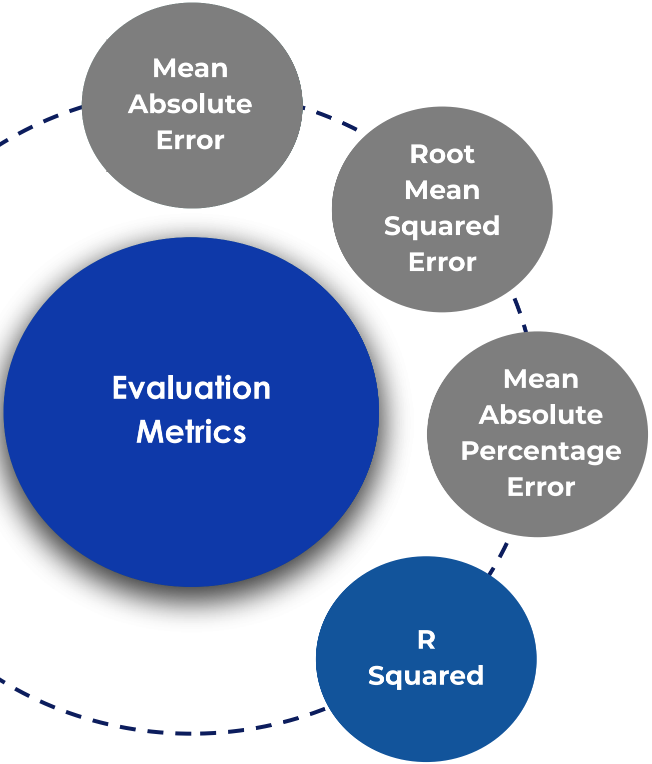


- Previous metrics do not clearly quantify how well the model explains the variability in the data

$$R^2 = 1 - \frac{\sum_{1=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{1=1}^{n}(y_i - \bar{y})^2}$$

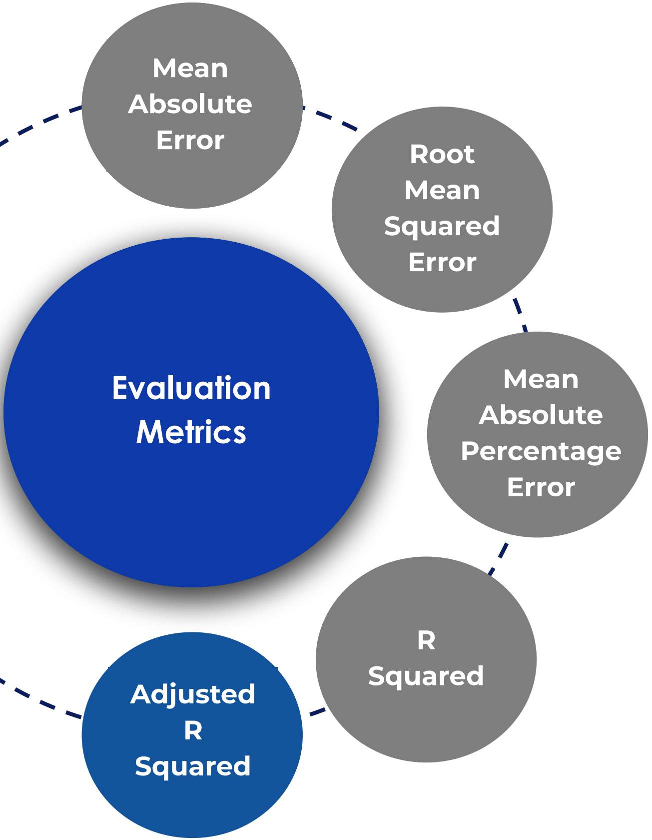
- Generally ranges between 0 and 1

Evaluation Metrics



- Tends to increase when adding more explanatory variables
- Does not account for the value addition from the added explanatory variables

Evaluation Metrics

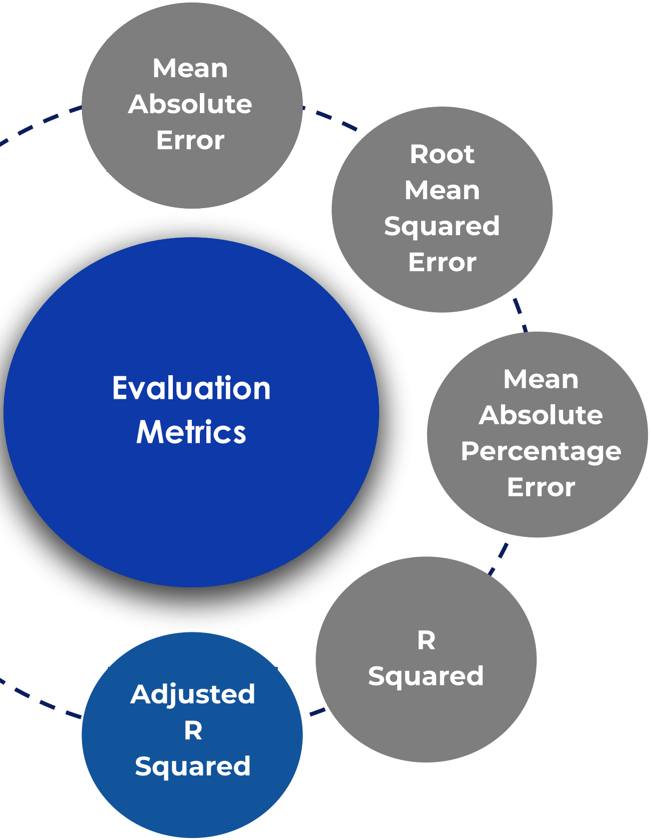


- Tends to increase when adding more explanatory variables
- Does not account for the value addition from the added explanatory variables

$$Adjusted\ R^2 = 1 - \frac{(1 - R^2) * (n - 1)}{n - k - 1}$$

- Accounts for the number of explanatory variables in the model

Evaluation Metrics



- Gives a sense of which variables actually help in prediction and which ones do not
- Provides a balance between model fit and complexity (number of explanatory variables)

$$Adjusted\ R^2 = 1 - \frac{(1 - R^2) * (n - 1)}{n - k - 1}$$

Summary

Here’s a quick recap of what we've learned:

- **Business Problem and Solution Space:** Identifies the specific problem that linear regression aims to solve and defines the scope of its application in business contexts.
- **Correlation and Linear Relationships:** Explores how correlation measures the strength and direction of linear relationships between variables, laying the foundation for understanding linear regression.
- **Simple Linear Regression:** Introduces the basic concept of simple linear regression, which models the relationship between a dependent variable and one independent variable using a straight line.

Summary

- **Multiple Linear Regression:** Expands on the concept of simple linear regression by incorporating multiple independent variables to predict a dependent variable, accommodating more complex relationships.
- **Categorical Variables in Regression:** Discusses strategies for encoding categorical variables in regression models to include qualitative data effectively in predictive analysis.
- **Evaluation Metrics for Regression:** Covers key metrics such as Mean Squared Error (MSE), R-squared, and others used to assess the accuracy and performance of regression models in predicting outcomes.

Learning Outcomes

You should now be able to:

- Explain how correlation measures the strength and direction of linear relationships, and apply this understanding to build simple linear regression models effectively.
- Gain proficiency in constructing and interpreting simple linear regression models to analyze and predict relationships between two variables.
- Develop multiple linear regression models to enable the prediction of business outcomes using multiple input variables.

Learning Outcomes

- Evaluate linear regression models using key metrics and implement strategies to enhance model performance and accuracy.
- Identify and apply linear regression techniques to solve various real-world business problems, leveraging its predictive capabilities across different domains.

Great Learning

Happy Learning !