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# Kinetic Theory

## TOPIC 1

### Kinetic Theory of Gases and Gas Laws

- 01** The mean free path  $l$  for a gas molecule depends upon diameter,  $d$  of the molecule as

[NEET (Oct.) 2020]

(a)  $l \propto \frac{1}{d^2}$  (b)  $l \propto d$  (c)  $l \propto d^2$  (d)  $l \propto \frac{1}{d}$

**Ans. (a)**

The mean free path  $l$  for a gas molecule is given as

$$l = \frac{1}{\sqrt{2} n \pi d^2} \Rightarrow l \propto \frac{1}{d^2}$$

where,  $d$  = diameter of molecule of gas.

- 02** An ideal gas equation can be written as  $p = \frac{\rho RT}{M_0}$

where,  $\rho$  and  $M_0$  are respectively, [NEET (Oct.) 2020]

- (a) mass density, mass of the gas  
(b) number density, molar mass  
(c) mass density, molar mass  
(d) number density, mass of the gas

**Ans. (a)**

Ideal gas equation is given as

$$p = \frac{\rho RT}{M_0} \Rightarrow p \cdot \frac{M_0}{\rho} = RT$$

$$\Rightarrow pV = RT$$

where,  $V = \frac{M_0}{\rho}$

Hence,  $\rho$  and  $M_0$  are mass density and mass of gas, respectively.

- 03** The mean free path  $\lambda$  for a gas, with molecular diameter  $d$  and number density  $n$  can be expressed as [NEET (Sep.) 2020]

(a)  $\frac{1}{\sqrt{2} n \pi d^2}$  (b)  $\frac{1}{\sqrt{2} n^2 \pi d^2}$   
(c)  $\frac{1}{\sqrt{2} n^2 \pi^2 d^2}$  (d)  $\frac{1}{\sqrt{2} n \pi d}$

**Ans. (a)**

The mean free path  $\lambda$  for a gas, with molecular diameter  $d$  and number density  $n$  is given by the relation

$$\lambda = \frac{1}{\sqrt{2} n \pi d^2}$$

Hence, correct option is (a).

- 04** A cylinder contains hydrogen gas at pressure of 249 kPa and temperature 27°C. Its density is ( $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ )

[NEET (Sep.) 2020]

(a) 0.2 kg/m<sup>3</sup> (b) 0.1 kg/m<sup>3</sup>  
(c) 0.02 kg/m<sup>3</sup> (d) 0.5 kg/m<sup>3</sup>

**Ans. (a)**

Given,

$$\text{pressure } p = 249 \text{ kPa} = 249 \times 10^3 \text{ Pa}$$

$$\text{Temperature, } T = 27^\circ\text{C}$$

$$= 273 + 27 \text{ K} = 300 \text{ K}$$

$$\text{Density, } \rho = ?$$

As, from ideal gas equation,

$$pV = nRT$$

$$\Rightarrow pV = \frac{m}{M} RT \quad \left[ \text{as } n = \frac{m}{M} \right]$$

$$\Rightarrow pVM = mRT$$

$$\Rightarrow \rho M = \frac{m}{V} RT = \rho RT \quad \left[ \text{as } \frac{m}{V} = \rho \right]$$

$$\Rightarrow \rho = \frac{pM}{RT} = \frac{249 \times 10^3 \times 2 \times 10^{-3}}{8.3 \times 300}$$

$$[\because \text{for hydrogen gas, } M = 2 \text{ g} = 2 \times 10^{-3} \text{ kg}]$$

$$\Rightarrow \rho = 0.2 \text{ kg/m}^3$$

Hence, correct option is (a).

- 05** Increase in temperature of a gas filled in a container would lead to [NEET (National) 2019]

- (a) increase in its kinetic energy  
(b) decrease in its pressure  
(c) decrease in intermolecular distance  
(d) increase in its mass

**Ans. (a)**

As the temperature of gas in the container is increased, the kinetic energy also increases. This is because the average kinetic energy of a gas is given by

$$KE = \frac{f}{2} nRT \quad \dots (i)$$

where,  $f$  = degree of freedom,

$n$  = number of moles of gas molecules,

$R$  = universal gas constant, and

$T$  = absolute temperature of the gas.

From Eq. (i),

$$KE \propto T$$

option (b) is incorrect as increase in temperature will lead to increase in pressure as  $p \propto T$ . Other options (c) and (d) are also incorrect as molecular distance increases while mass remains the same for increase in the temperature.

- 06** At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere?

(Given: mass of oxygen molecule,  $m = 2.76 \times 10^{-26}$  kg, Boltzmann's constant  $k_B = 1.38 \times 10^{-23}$  J K<sup>-1</sup>) [NEET 2018]

- (a)  $5.016 \times 10^4$  K  
(b)  $8.326 \times 10^4$  K  
(c)  $2.508 \times 10^4$  K  
(d)  $1.254 \times 10^4$  K

**Ans. (b)**

**Key Concept** The minimum velocity with which the body must be projected vertically upwards, so that it could escape from the Earth's atmosphere, is its escape velocity ( $v_e$ ).

$$\text{As, } v_e = \sqrt{2gR}$$

Substituting the value of  $g$  ( $9.8 \text{ ms}^{-2}$ ) and radius of Earth ( $R = 6.4 \times 10^6 \text{ m}$ ), we get

$$v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} \\ \approx 11.2 \text{ km s}^{-1} = 11200 \text{ m s}^{-1}$$

Let the temperature of molecule be  $T$  when it attains  $v_e$ .

According to the question,

$$v_{\text{rms}} = v_e$$

where,  $v_{\text{rms}}$  is the rms speed of the oxygen molecule.

$$\Rightarrow \sqrt{\frac{3k_B T}{m_{O_2}}} = 11.2 \times 10^3$$

$$\text{or } T = \frac{(11.2 \times 10^3)^2 (m_{O_2})}{(3k_B)}$$

Substituting the given values, i.e.,

$$R_B = 1.38 \times 10^{-23} \text{ J K}^{-1} \text{ and}$$

$$m_{O_2} = m = 2.76 \times 10^{-26} \text{ kg}$$

We get,

$$T = \frac{(11.2 \times 10^3)^2 (2.76 \times 10^{-26})}{(3 \times 1.38 \times 10^{-23})} \\ = 8.3626 \times 10^4 \text{ K}$$

- 07** The molecules of a given mass of a gas have r.m.s. velocity of  $200 \text{ ms}^{-1}$  at  $27^\circ\text{C}$  and  $1.0 \times 10^5 \text{ Nm}^{-2}$  pressure. When the temperature and pressure of the gas are respectively,  $127^\circ\text{C}$  and  $0.05 \times 10^5 \text{ Nm}^{-2}$ , the rms velocity of its molecules in  $\text{ms}^{-1}$  is [NEET 2016]

- (a)  $\frac{400}{\sqrt{3}}$  (b)  $\frac{100\sqrt{2}}{3}$  (c)  $\frac{100}{3}$  (d)  $100\sqrt{2}$

**Ans. (a)**

It is given that

$$v_{\text{rms}} = 200 \text{ ms}^{-1}, T_1 = 300 \text{ K}, P_1 = 10^5 \text{ N/m}^2$$

$$T_2 = 400 \text{ K}, P_2 = 0.05 \times 10^5 \text{ N/m}^2$$

As, rms velocity of gas molecules,

$$\therefore v_{\text{rms}} \propto \sqrt{T} \quad \left( \because v_{\text{rms}} = \sqrt{\frac{3RT}{m}} \right)$$

For two different cases

$$\Rightarrow \frac{(v_{\text{rms}})_1}{(v_{\text{rms}})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\Rightarrow \frac{200}{(v_{\text{rms}})_2} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow (v_{\text{rms}})_2 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} \text{ ms}^{-1}$$

- 08** A given sample of an ideal gas occupies a volume  $V$  at a pressure  $p$  and absolute temperature  $T$ . The mass of each molecule of the gas is  $m$ . Which of the following gives the density of the gas? [NEET 2016]

- (a)  $p/(kT)$  (b)  $pm/(kT)$   
(c)  $p/(kTV)$  (d)  $mkT$

**Ans.(b)**

As we know that

$$\text{Pressure, } p = \frac{1}{3} \cdot \frac{nm}{V} v_{\text{rms}}^2$$

$\therefore nm =$  mass of the gas,  $V =$  volume of the gas

$\therefore \frac{mn}{V} =$  density of the gas. Thus,

$$p = \frac{1}{2} \rho v_{\text{rms}}^2 = \frac{1}{3} \rho \frac{3RT}{M_0} = \frac{\rho RT}{M_0} \\ \left( \because v_{\text{rms}} = \sqrt{\frac{3RT}{M_0}} \right)$$

$$\rho = \frac{pM_0}{RT} = \frac{pmN_A}{kN_A T}$$

$$[\because R = N_A k \text{ and } M_0 = mN_A]$$

$$\rho = \frac{pm}{kT}$$

- 09** 4.0 g of a gas occupies 22.4 L at NTP. The specific heat capacity of the gas at constant volume is  $5.0 \text{ J K}^{-1} \text{ mol}^{-1}$ . If the speed of sound in this gas at NTP is  $952 \text{ ms}^{-1}$ , then the heat capacity at constant pressure is (Take gas constant  $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ ) [CBSE AIPMT 2015]

- (a)  $8.0 \text{ J K}^{-1} \text{ mol}^{-1}$  (b)  $7.5 \text{ J K}^{-1} \text{ mol}^{-1}$   
(c)  $7.0 \text{ J K}^{-1} \text{ mol}^{-1}$  (d)  $8.5 \text{ J K}^{-1} \text{ mol}^{-1}$

**Ans. (a)**

Given,  $M = 4 \text{ gm}$ ,  $V = 22.4 \text{ L}$ ,

$$C_V = 5 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$v_{\text{sound}} = 952 \text{ m/s}, C_p = ?$$

$$\text{As, velocity of sound, } v_{\text{sound}} = \sqrt{\frac{\gamma pV}{M}}$$

$$\Rightarrow \gamma = \frac{M}{\rho V} v_{\text{sound}}^2 = \frac{C_p}{C_V}$$

So, heat capacity at constant pressure,

$$C_p = C_V \left[ \frac{M}{\rho V} \right] v_{\text{sound}}^2 \\ = 5 \left[ \frac{4 \times 10^{-3}}{10^{-5} \times 22.4 \times 10^{-3}} \right] (952)^2 \\ = \frac{20}{22.4} \times (952)^2 \times 10^{-5} \\ = 809.200 \times 10^{-5} \\ = 8.09 \text{ J/mol K}$$

- 10** Two vessels separately contain two ideal gases A and B at the same temperature, the pressure of A being twice that of B. Under such conditions, the density of A is found to be 1.5 times the density of B. The ratio of molecular weight of A and B is [CBSE AIPMT 2015]

- (a)  $\frac{2}{3}$  (b)  $\frac{3}{4}$   
(c) 2 (d)  $\frac{1}{2}$

**Ans. (b)**

Vessel-I

Ideal Gas A  
 $T$

$$p_A = 1.5 p_B$$

$$p_A = 2p_B$$

Vessel-II

Ideal Gas B  
 $T$

$$p_B$$

$$p_B$$

According to ideal gas equation, we have

$$\text{Pressure, } p = \frac{\rho RT}{M}, \text{ where } M \text{ is molecular}$$

weight of ideal gas.

$$\text{Such that, } \frac{p}{\rho} = \frac{RT}{M}$$

$$\Rightarrow M = \frac{\rho RT}{p}$$

where,  $R$  and  $T$  are constants.

$$\text{So, } M \propto \frac{\rho}{p}$$

$$\Rightarrow \frac{M_A}{M_B} = \frac{\rho_A}{\rho_B} \times \frac{p_B}{p_A}$$

$$= 1.5 \times \frac{1}{2} = 0.75 = \frac{3}{4}$$

- 11** The mean free path of molecules of a gas, (radius  $r$ ) is inversely proportional to [CBSE AIPMT 2014]

(a)  $r^3$  (b)  $r^2$  (c)  $r$  (d)  $\sqrt{r}$

**Ans. (d)**

Mean free path ( $l$ ) is given by

$$l = \frac{1}{\sqrt{2}n\pi d^2} \Rightarrow l \propto \frac{1}{d^2}$$

So,  $l \propto \frac{1}{r^2}$   
 $\left[ \begin{array}{l} r = \text{radius of the} \\ \text{molecules of gas} \end{array} \right]$

- 12** The molar specific heats of an ideal gas at constant pressure and volume are denoted by  $C_p$  and  $C_v$

respectively. If  $\gamma = \frac{C_p}{C_v}$  and  $R$  is the

universal gas constant, then  $C_v$  is equal to [NEET 2013]

(a)  $\frac{1+\gamma}{1-\gamma}R$  (b)  $\frac{R}{(\gamma-1)}$  (c)  $\frac{(\gamma-1)}{R}$  (d)  $\gamma R$

**Ans. (b)**

As we know that

$$C_p - C_v = R$$

$$C_p = R + C_v$$

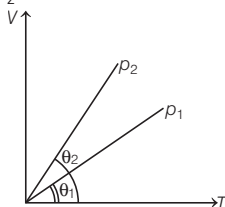
and  $\frac{C_p}{C_v} = \gamma$  (given)

$$\text{So, } \frac{R + C_v}{C_v} = \gamma \Rightarrow \gamma C_v = R + C_v$$

$$\Rightarrow \gamma C_v - C_v = R$$

$$\Rightarrow C_v = \frac{R}{\gamma - 1}$$

- 13** In the given ( $V$ - $T$ ) diagram, what is the relation between pressures  $p_1$  and  $p_2$ ? [NEET 2013]



- (a)  $p_2 = p_1$   
 (b)  $p_2 > p_1$   
 (c)  $p_2 < p_1$   
 (d) Cannot be predicted

**Ans. (c)**

According to question,

Slope of the graph  $\propto \frac{1}{\text{Pressure } p}$

So,  $p_2 < p_1$

- 14** During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its

temperature. The ratio of  $\frac{C_p}{C_v}$  for

the gas is

(a)  $\frac{4}{3}$  (b) 2 (c)  $\frac{5}{3}$  (d)  $\frac{3}{2}$

[NEET 2013]

**Ans. (d)**

According to question,

$$p \propto T^3 \quad \dots(i)$$

$$\left( \begin{array}{l} p = \text{pressure} \\ T = \text{temperature} \end{array} \right)$$

and we know that

$$pV = nRT \text{ and } pV \propto T \quad \dots(ii)$$

So, putting Eq. (ii) in (i),

$$p \propto (pV)^3$$

$$\Rightarrow p^2 V^3 = \text{constant}$$

$$\Rightarrow p V^{3/2} = \text{constant} \quad \dots(iii)$$

$\Rightarrow$  Comparing Eq. (iii) with

$$pV^\gamma = \text{constant.}$$

We have  $\gamma = 3/2$ .

- 15** At  $10^\circ\text{C}$  the value of the density of a fixed mass of an ideal gas divided by its pressure is  $x$ . At  $110^\circ\text{C}$  this ratio is [CBSE AIPMT 2008]

(a)  $x$  (b)  $\frac{383}{283}x$  (c)  $\frac{10}{110}x$  (d)  $\frac{283}{383}x$

**Ans. (d)**

**Concept** Use ideal gas equation to find the ratio between density of a fixed mass of an ideal gas and its pressure.

Ideal gas equation

$$pV = nRT$$

$$\frac{pV}{m} = \frac{1}{M} RT \quad \left( \because n = \frac{m}{M} \right)$$

$$\frac{p}{\rho} = \frac{RT}{M} \quad \left( \because \frac{V}{m} = \frac{1}{\rho} \right)$$

$$\therefore \frac{p}{\rho} \propto \frac{1}{T}$$

Molecular mass  $M$  and universal gas constant  $R$  remains same for a gas.

So, for two different situations i.e. at two different temperatures and densities

$$\therefore \frac{p_1/\rho_1}{p_2/\rho_2} = \frac{T_2}{T_1} \Rightarrow \frac{x}{(p_2/\rho_2)} = \frac{383\text{K}}{283\text{K}}$$

$$\Rightarrow \frac{p_2}{\rho_2} = \frac{283}{383}x$$

- 16** The molar specific heat at constant pressure of an ideal gas is  $(7/2)R$ .

The ratio of specific heat at constant pressure to that at constant volume is

[CBSE AIPMT 2006]

(a)  $7/5$  (b)  $8/7$  (c)  $5/7$  (d)  $9/7$

**Ans. (a)**

We have given molar specific heat at instant pressure

$$C_p = \frac{7}{2}R$$

Mayer's relation can be written as :

Molar specific heat at constant pressure  
 - Molar specific heat at constant volume  
 = Gas constant,

$$\text{i.e. } C_p - C_v = R \Rightarrow C_v = C_p - R$$

$$= \frac{7}{2}R - R = \frac{5}{2}R \left[ \because C_p = \frac{7}{2}R \right]$$

Hence, required ratio is

$$\gamma = \frac{C_p}{C_v} = \frac{\left(\frac{7}{2}\right)R}{\left(\frac{5}{2}\right)R} = \frac{7}{5}$$

- 17** The equation of state for 5 g of oxygen at a pressure  $p$  and temperature  $T$ , when occupying a volume  $V$ , will be [CBSE AIPMT 2004]

$$(a) pV = \left(\frac{5}{32}\right)RT \quad (b) pV = 5RT$$

$$(c) pV = \left(\frac{5}{2}\right)RT \quad (d) pV = \left(\frac{5}{16}\right)RT$$

**Ans. (a)**

Number of moles,

$$n = \frac{m}{\text{molecular weight}} = \frac{5}{32}$$

As, from ideal gas equation

$$pV = nRT \Rightarrow pV = \frac{5}{32}RT$$

- 18** An ideal gas at  $27^\circ\text{C}$  is compressed adiabatically to  $\frac{8}{27}$  of its original

volume. The rise in temperature is

$$\left( \gamma = \frac{5}{3} \right)$$

[CBSE AIPMT 1999]

(a)  $475^\circ\text{C}$  (b)  $402^\circ\text{C}$   
 (c)  $275^\circ\text{C}$  (d)  $375^\circ\text{C}$

**Ans. (d)**

In an adiabatic process

$p$  = pressure

$V$  = volume

$$\gamma = \text{atomicity of gas}$$

$$pV^\gamma = \text{constant} \quad \dots(i)$$

Now from ideal gas equation,

$$pV = RT \quad (\text{for one mole})$$

$$\text{or } p = \frac{RT}{V} \quad \dots(ii)$$

$$(R = \text{gas constant})$$

From Eqs. (i) and (ii), we have

$$\left(\frac{RT}{V}\right)V^\gamma = \text{constant}$$

$$TV^{\gamma-1} = \text{constant} \quad \dots(iii)$$

So for two different cases of temperature and volume

$$\text{So, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\text{or } \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \dots(iv)$$

$$\text{Given, } T_1 = 27^\circ \text{C}$$

$$= 27 + 273 = 300 \text{ K}$$

$$\text{Given, } \frac{V_2}{V_1} = \frac{8}{27}, \gamma = \frac{5}{3}$$

Substituting in Eq. (i), we get

$$\frac{T_2}{300} = \left(\frac{27}{8}\right)^{5/3-1}$$

$$\text{or } \frac{T_2}{300} = \left[\left(\frac{3}{2}\right)^3\right]^{2/3}$$

$$\text{or } \frac{T_2}{300} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\therefore T_2 = \frac{9}{4} \times 300 = 675 \text{ K} = 402^\circ \text{C}$$

Thus, rise in temperature

$$= T_2 - T_1 = 402 - 27 = 375^\circ \text{C}$$

- 19** A diatomic gas initially at  $18^\circ \text{C}$  is compressed adiabatically to one-eighth of its original volume. The temperature after compression will be

[CBSE AIPMT 1996]

- (a)  $18^\circ \text{C}$  (b)  $668.4 \text{ K}$   
(c)  $395.4^\circ \text{C}$  (d)  $144^\circ \text{C}$

**Ans. (b)**

According to adiabatic process the relation between temperature and volume is given by

$$TV^{\gamma-1} = \text{constant}$$

So, for two different cases

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad \dots(i)$$

Given, initial temperature

$$T_1 = 18^\circ \text{C} = 291 \text{ K}$$

Let initial volume  $V_1 = V$

$$\text{and final volume } V_2 = \left(\frac{1}{8}\right)V$$

Putting these values in Eq. (i)

$$T_2 = 291 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 291 \left(\frac{V}{\left(\frac{1}{8}\right)V}\right)^{7/5-1}$$

$$(\gamma = \frac{7}{5} \text{ for diatomic gas})$$

$$= 291 \times 2.297 = 668.4 \text{ K}$$

- 20** The pressure of a gas is raised from  $27^\circ \text{C}$  to  $927^\circ \text{C}$ . The root mean square speed [CBSE AIPMT 1994]

(a) is  $\sqrt{\left(\frac{927}{27}\right)}$  times the earlier value

(b) remains the same

(c) gets halved

(d) gets doubled

**Ans. (d)**

RMS speed is defined as the square root of the mean of the squares of the random velocities of the individual molecules of a gas. From Maxwellian distribution law, RMS speed is given by

$$c_{\text{rms}} = \sqrt{\left(\frac{3kT}{m}\right)}$$

$$\Rightarrow c_{\text{rms}} \propto \sqrt{T}$$

For two different cases i.e. at two different temperatures

$$\therefore \frac{(c_{\text{rms}})_1}{(c_{\text{rms}})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\text{Here, } T_1 = 27^\circ \text{C} = 300 \text{ K}$$

$$T_2 = 927^\circ \text{C} = 1200 \text{ K}$$

$$\therefore \frac{(c_{\text{rms}})_1}{(c_{\text{rms}})_2} = \sqrt{\frac{300}{1200}} = \frac{1}{2}$$

$$\Rightarrow (c_{\text{rms}})_2 = 2(c_{\text{rms}})_1$$

Hence, root mean square speed will be doubled.

- 21** Relation between pressure ( $p$ ) and energy ( $E$ ) of a gas is

[CBSE AIPMT 1991]

- (a)  $p = \frac{2}{3}E$  (b)  $p = \frac{1}{3}E$   
(c)  $p = \frac{3}{2}E$  (d)  $p = 3E$

**Ans. (a)**

Pressure exerted by gas molecules is

$$p = \frac{1}{3} \rho \bar{v}^2 \quad \dots(ii)$$

where,  $\rho$  = density of gas

$\bar{v}$  = average velocity of gas molecules

$$\text{or } p = \frac{2}{3} n \cdot \frac{1}{2} m \bar{v}^2 \quad (\because p = mn)$$

Now,  $\frac{1}{2} m \bar{v}^2$  = average kinetic energy of a gas molecule ( $\overline{\text{KE}}$ )

$$\text{Therefore, } p = \frac{2}{3} n \overline{\text{KE}}$$

If  $N$  is total number of gas molecules in volume  $V$ , then

No of gas molecules per unit volume

$$n = \frac{N}{V}$$

$$\therefore p = \frac{2}{3} \cdot \frac{N}{V} \left(\frac{1}{2} m \bar{v}^2\right)$$

$$\text{or } pV = \frac{2}{3} N (\overline{\text{KE}}) \quad \left[\text{KE} = \frac{1}{2} m \bar{v}^2\right]$$

Also, from Eq. (i),

$$p = \frac{2}{3} \cdot \frac{1}{2} \rho \bar{v}^2$$

Now,  $\frac{1}{2} \rho \bar{v}^2$  = average kinetic energy of

the gas per unit volume.

$$\text{Therefore, } p = \frac{2}{3} E$$

- 22** Three containers of the same volume contain three different gases. The masses of the molecules are  $m_1, m_2$  and  $m_3$  and the number of molecules in their respective containers are  $N_1, N_2$  and  $N_3$ . The gas pressure in the containers are  $p_1, p_2$  and  $p_3$  respectively. All the gases are now mixed and put in one of these containers. The pressure  $p$  of the mixture will be

[CBSE AIPMT 1991]

$$(a) p < (p_1 + p_2 + p_3)$$

$$(b) p = \frac{p_1 + p_2 + p_3}{3}$$

$$(c) p = p_1 + p_2 + p_3$$

$$(d) p > (p_1 + p_2 + p_3)$$

**Ans. (c)**

According to Dalton's law of partial pressure, the total pressure exerted by a mixture of gases, which do not interact with each other, is equal to sum of the partial pressures which each would exert, if alone occupied the same volume at the given temperature. When gases are put in one container, then pressure  $p$  of the mixture will be

$$p = p_1 + p_2 + p_3$$

- 23** One mole of an ideal gas requires 207 J heat to rise the temperature by 10 K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10 K, the heat required is (Given the gas constant  $R = 8.3 \text{ J/mol-K}$ )

[CBSE AIPMT 1990]

- (a) 198.7 J (b) 29 J  
(c) 215.3 J (d) 124 J

**Ans. (d)**

Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree.

$$\text{As } (dQ)_p = \mu C_p dT \quad \dots(i)$$

(At constant pressure)

$$\text{and } (dQ)_v = \mu C_v dT \quad \dots(ii)$$

(At constant volume)

$$\text{Given, } (dQ)_p = 207 \text{ J}$$

$$R = 8.3 \text{ J/mol-K}$$

$$dT = 10 \text{ K}$$

Putting value in Eq. (i)

$$207 = 1 \times C_p \times 10$$

$$\therefore C_p = 20.7 \text{ J/kg}$$

$$\text{As } C_p - C_v = R = 8.3$$

$$C_v = 20.7 - 8.3 = 12.4 \text{ J}$$

$$\therefore (dQ)_v = 1 \times 12.4 \times 10 = 124 \text{ J}$$

- 24** For a certain gas the ratio of specific heats is given to be  $\gamma = 1.5$ , for this gas [CBSE AIPMT 1990]

- (a)  $C_v = \frac{3R}{J}$  (b)  $C_p = \frac{3R}{J}$   
(c)  $C_p = \frac{5R}{J}$  (d)  $C_p = \frac{5R}{J}$

**Ans. (b)**

$$\text{Given, } \gamma = \frac{C_p}{C_v} = 1.5 = \frac{3}{2}$$

$$\therefore C_v = \frac{2}{3} C_p$$

Again from Mayer's formula

$$C_p - C_v = \frac{R}{J}$$

$$\therefore C_p - \frac{2}{3} C_p = \frac{R}{J}$$

$$\Rightarrow \frac{C_p}{3} = \frac{R}{J}$$

$$\Rightarrow C_p = \frac{3R}{J}$$

- 25** For hydrogen gas  $C_p - C_v = a$  and for oxygen gas  $C_p - C_v = b$ , so the relation between  $a$  and  $b$  is given by [CBSE AIPMT 1991]

- (a)  $a = 16b$   
(b)  $16b = a$   
(c)  $a = 4b$   
(d)  $a = b$

**Ans. (d)**

Both hydrogen and oxygen are diatomic gases and  $C_p - C_v = R$  is same for all gases, hence  $a = b$ , provided  $C_p$  and  $C_v$  are gram molar specific heats.

If it was the case of specific heat of 1 g  
 $C_p - C_v = r = \frac{R}{m}$

$$\therefore C_p - C_v = \frac{R}{2} \quad (\text{for H, } \frac{R}{2} = a)$$

$$C_p - C_v = \frac{R}{32} \quad (\text{for O}_2, \frac{R}{32} = b)$$

$$R = 2a = 32b$$

$$a = 16b$$

- 26** According to kinetic theory of gases, at absolute zero temperature [CBSE AIPMT 1990]

- (a) water freezes  
(b) liquid helium freezes  
(c) molecular motion stops  
(d) liquid hydrogen freezes

**Ans. (c)**

According to kinetic theory of gases, the pressure  $p$  exerted by one mole of an ideal gas is given by

$$p = \frac{1}{3} \frac{M}{V} c^2 \quad \text{or} \quad pV = \frac{1}{3} Mc^2$$

$$\text{or} \quad \frac{1}{3} Mc^2 = RT \quad \dots(i)$$

where  $c$  is root mean square velocity of gas.

From Eq. (i), when  $T = 0$ ,  $c = 0$

Hence, absolute zero of temperature may be defined as that temperature at which root mean square velocity of the gas molecules reduces to zero. It means molecular motion ceases at absolute zero.

- 27** At  $27^\circ\text{C}$  a gas is compressed suddenly such that its pressure becomes  $\left(\frac{1}{8}\right)$  of original pressure.

Final temperature will be  $\left(\gamma = \frac{5}{3}\right)$

[CBSE AIPMT 1989]

- (a)  $420 \text{ K}$  (b)  $300 \text{ K}$   
(c)  $-142^\circ\text{C}$  (d)  $327 \text{ K}$

**Ans. (c)**

The adiabatic relation between  $p$  and  $V$  for a perfect gas is

$$pV^\gamma = k (\text{a constant}) \quad \dots(i)$$

Again from standard gas equation

$$pV = nRT \Rightarrow V = \frac{RT}{p}$$

Putting in Eq. (i), we get

$$p \frac{R^\gamma T^\gamma}{p^\gamma} = k$$

$$\text{or } p^{1-\gamma} T^\gamma = \frac{k}{R^\gamma} = \text{another constant}$$

$$\text{i.e. } p^{1-\gamma} T^\gamma = \text{constant}$$

Comparing two different situations,

$$p_1^{1-\gamma} T_1^\gamma = p_2^{1-\gamma} T_2^\gamma$$

$$\text{Here, } p_2 = \left(\frac{1}{8}\right) p_1$$

$$T_1 = 27^\circ\text{C} = 273 + 27 = 300 \text{ K}$$

$$T_2 = ?, \gamma = \frac{5}{3}$$

$$\therefore \left(\frac{T_2}{T_1}\right)^\gamma = \left(\frac{p_1}{p_2}\right)^{1-\gamma}$$

$$\text{or } \left(\frac{T_2}{300}\right)^{5/3} = (8)^{1-5/3} = (8)^{-2/3}$$

$$\Rightarrow T_2 = 130.6 \text{ K}$$

$$\therefore T_2 = -142^\circ\text{C}$$

- 28** At constant volume temperature is increased, then [CBSE AIPMT 1989]

- (a) collision on walls will be less  
(b) number of collisions per unit time will increase  
(c) collisions will be in straight lines  
(d) collisions will not change

**Ans. (b)**

On raising the temperature, the average velocity of the gas molecules increases. As a result of which more molecules collide with the walls or number of collisions per unit time will increase.

## TOPIC 2

### Degree of Freedom and Law of Equipartition of Energy

- 29** Match Column I with Column II and choose the correct match from the given choices. [NEET 2021]

Column I	Column II
A. Root mean square of gas molecules	1. $\frac{1}{3}nm\bar{v}^2$
B. Pressure exerted by ideal gas	2. $\sqrt{\frac{3RT}{M}}$
C. Average kinetic energy of a molecule	3. $\frac{5}{2}RT$
D. Total internal energy of 1 mole of a diatomic gas	4. $\frac{3}{2}k_B T$

#### Codes

	A	B	C	D
(a)	3	1	4	2
(b)	2	3	4	1
(c)	2	1	4	3
(d)	3	2	1	4

#### Ans. (c)

We know that, the rms speed of the gas molecules,

$$v_{rms} = \sqrt{3RT/M}$$

Here,  $T$  is the temperature of the gas,  
 $R$  is the universal gas constant,  
 $M$  is the molar mass of the gas.

Pressure exerted by an ideal gas,

$$p = \frac{1}{3}nm\bar{v}^2$$

Here,  $n$  is the number of moles,  
 $m$  is the mass of the gas,  
 $\bar{v}$  is the average speed of the gas molecules.

The average kinetic energy of a molecule,

$$KE_{av} = \frac{3}{2}k_B T$$

Here,  $k_B$  is the Boltzmann constant,  
 $T$  is the temperature of the gas.

For diatomic gas, degree of freedom,  $f = 5$

As, total internal energy of 1 mole of diatomic gas,

$$\Delta U = \frac{nfRT}{2}$$

$$\Rightarrow \Delta U = \frac{1(5)RT}{2} \Rightarrow \Delta U = \frac{5RT}{2}$$

The correct match is A  $\rightarrow$  2, B  $\rightarrow$  1, C  $\rightarrow$  4 and D  $\rightarrow$  3.

- 30** The average thermal energy for a monoatomic gas is (where,  $k_B$  is Boltzmann constant and  $T$  is absolute temperature.)

[NEET (Sep.) 2020]

$$(a) \frac{3}{2}k_B T \quad (b) \frac{5}{2}k_B T \quad (c) \frac{7}{2}k_B T \quad (d) \frac{1}{2}k_B T$$

#### Ans. (a)

The average thermal energy of a system with degree of freedom  $f$  is equals to its average energy, which is given as

$$= \frac{f}{2}k_B \cdot T$$

For monoatomic gas,  $f = 3$

$$\therefore \text{Average thermal energy} = \frac{3}{2}k_B \cdot T$$

Hence, correct option is (a).

- 31** The value of  $\gamma \left( = \frac{C_p}{C_v} \right)$ , for hydrogen,

helium and another ideal diatomic gas X (whose molecules are not rigid but have an additional vibrational mode), are respectively equal to [NEET (Odisha) 2019]

$$(a) \frac{7}{5}, \frac{5}{3}, \frac{9}{7} \quad (b) \frac{5}{3}, \frac{7}{5}, \frac{9}{7} \\ (c) \frac{5}{3}, \frac{7}{5}, \frac{7}{5} \quad (d) \frac{7}{5}, \frac{5}{3}, \frac{7}{5}$$

#### Ans. (a)

The poisson's ratio,

$$\gamma = \frac{C_p}{C_v} \quad \dots(i)$$

where,  $C_p$  = molar heat capacity constant pressure

and  $C_v$  = molar heat capacity at constant volume

Also,  $C_p = C_v + R$  (from Mayer's relation)

$$C_v = \frac{f}{2}R \quad (\text{where, } f = \text{degree of freedom})$$

$$\Rightarrow C_p = \left( \frac{f}{2} + 1 \right) R$$

So, Eq. (i) becomes,

$$\Rightarrow \gamma = 1 + \frac{2}{f}$$

For hydrogen gas, which is diatomic, the degree of freedom is 5 (3 translational, 2 rotational).

$$\therefore \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

For helium gas, which is monoatomic, the degree of freedom is 3 (3 translational only).

$$\therefore \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

The diatomic gas X also have vibrational motion, so degree of freedom is 7 (3 translational, 2 rotational and 2 vibrational).

$$\therefore \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

- 32** A gas mixture consists of 2 moles of  $O_2$  and 4 moles of Ar at temperature  $T$ . Neglecting all vibrational modes, the total internal energy of the system is [NEET 2017]
- (a)  $4RT$  (b)  $15RT$   
 (c)  $9RT$  (d)  $11RT$

#### Ans. (d)

Total internal energy of system = Internal energy of oxygen molecules + Internal energy of argon molecules

$$= \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT = \frac{5}{2} \times 2RT + \frac{3}{2} \times 4RT \\ = 11RT$$

- 33** One mole of an ideal monatomic gas undergoes a process described by the equation  $pV^3 = \text{constant}$ . The heat capacity of the gas during this process is [NEET 2016]

$$(a) \frac{3}{2}R \quad (b) \frac{5}{2}R \quad (c) 2R \quad (d) R$$

#### Ans. (d)

As we know that for polytropic process of index  $\alpha$  specific heat capacity

$$= C_v + \frac{R}{1-\alpha}$$

$\therefore$  Process,  $pV^3 = \text{constant} \Rightarrow \alpha = 3$

$$\therefore C = C_v + \frac{R}{1-\alpha} = \frac{fR}{2} + \frac{R}{1-3}$$

$$\text{where, } C_v = \frac{fR}{2} = \frac{3R}{2}$$

$$\text{For monatomic gas, } f = 3 = \frac{3R}{2}$$

$$\Rightarrow C = \frac{3R}{2} - \frac{R}{2} = R$$

- 34** The amount of heat energy required to raise the temperature of 1 g of helium at NTP, from  $T_1$  K to  $T_2$  K is [NEET 2013]
- (a)  $\frac{3}{8}N_A K_B (T_2 - T_1)$  (b)  $\frac{3}{2}N_A K_B (T_2 - T_1)$   
 (c)  $\frac{3}{4}N_A K_B (T_2 - T_1)$  (d)  $\frac{3}{4}N_A K_B \left( \frac{T_2}{T_1} \right)$

**Ans. (a)**

We know that,  $Q = \frac{F}{2} nR \Delta T$

$\therefore$  Amount of heat required,

$$Q = \frac{3}{2} \times \frac{1}{4} \times K_B N_o \Delta T = \frac{3}{8} N_o K_B (T_2 - T_1)$$

**35** The ratio of the specific heats

$\frac{C_p}{C_v} = \gamma$  in terms of degrees of

freedom ( $n$ ) is given by

[CBSE AIPMT 2015]

- (a)  $\left(1 + \frac{1}{n}\right)$  (b)  $\left(1 + \frac{n}{3}\right)$   
(c)  $\left(1 + \frac{2}{n}\right)$  (d)  $\left(1 + \frac{n}{2}\right)$

**Ans. (c)**

The specific heat of gas at constant volume in terms of degree of freedom  $n$  is

$$C_v = \frac{n}{2} R$$

Also  $C_p - C_v = R$

$$\text{So } C_p = \frac{n}{2} R + R = R \left(1 + \frac{n}{2}\right)$$

$$\text{Now } \gamma = \frac{C_p}{C_v} = \frac{R \left(1 + \frac{n}{2}\right)}{\frac{n}{2} R} = \frac{2}{n} + 1$$

**36** The gases carbon-monoxide (CO) and nitrogen at the same temperature have kinetic energies  $E_1$  and  $E_2$  respectively. Then

[CBSE AIPMT 2000]

- (a)  $E_1 = E_2$   
(b)  $E_1 > E_2$   
(c)  $E_1 < E_2$   
(d)  $E_1$  and  $E_2$  cannot be compared

**Ans. (a)**

The gases carbon-monoxide (CO) and nitrogen ( $N_2$ ) are diatomic, so both have equal kinetic energy  $\frac{5}{2} kT$ , i.e.  $E_1 = E_2$ .

**37** The degrees of freedom of a molecule of a triatomic gas are

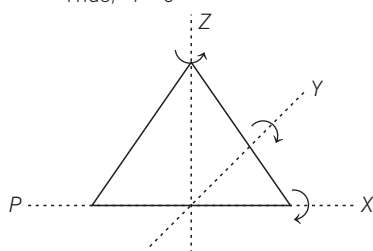
[CBSE AIPMT 1999]

- (a) 2 (b) 4  
(c) 6 (d) 8

**Ans. (c)**

The molecule of a triatomic gas has a tendency of rotating about any of three coordinate axes. So, it has 6 degrees of freedom, 3 translational and 3 rotational. At high enough temperature a triatomic molecule has 2 vibrational degree of freedom. But as temperature requirement is not given, so we answer simply by assuming triatomic gas molecule at room temperature.

Thus,  $f = 6$



(3 translational + 3 rotational) at room temperature.

**38** The number of translational degree of freedom for a diatomic gas is

[CBSE AIPMT 1993]

- (a) 2 (b) 3  
(c) 5 (d) 6

**Ans. (b)**

Number of degree of freedom of a dynamical system is obtained by subtracting the number of independent relations from the total number of coordinates required to specify the positions of constituent particles of the system.

If  $A$  = number of particles in the system,  
 $R$  = number of independent relations among the particles,

$N$  = number of degree of freedom of the system, then

$$N = 3A - R$$

Each monoatomic, diatomic and triatomic gas has three translatory degree of freedom.

**39** If for a gas,  $\frac{R}{C_v} = 0.67$ , this gas is

made up of molecules which are

[CBSE AIPMT 1992]

- (a) diatomic  
(b) mixture of diatomic and polyatomic molecules  
(c) monoatomic  
(d) polyatomic

**Ans. (c)**

$$C_v = \frac{R}{0.67} \approx 1.5 R = \frac{3}{2} R$$

This is the case of monoatomic gases.

when  $C_v = \frac{3}{2} R$

**40** A polyatomic gas with  $n$  degrees of freedom has a mean energy per molecule given by

[CBSE AIPMT 1989]

- (a)  $\frac{nkT}{N}$  (b)  $\frac{nkT}{2N}$   
(c)  $\frac{nkT}{2}$  (d)  $\frac{3kT}{2}$

**Ans. (c)**

**Concept** If there is sudden compression without exchange of heat the process will be adiabatic.

According to law of equipartition of energy for any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom and the energy associated with each molecule per degree of freedom is  $\frac{1}{2} kT$ . For a

polyatomic gas with  $n$  degrees of freedom the mean energy per molecule  $= \frac{1}{2} nkT$ .

$K$  = Boltzmann constant  
 $n$  = degree of freedom  
 $T$  = Temperature