

Tugas Metode Numerik

1). Diketahui $f(x) = \sin(0.5\sqrt{x})$. Tentukan turunan pertama dari f di $x=1$ dan $h=0.2$. Gunakan metode:

a) selisih maju

$$f'(x) = \frac{f(x+h) - f(x)}{h} = \frac{f(1.2) - f(1)}{0.2} = \frac{0.4332 - 0.4794}{0.2} = \frac{-0.0462}{0.2} = -0.231 \rightarrow f'(x) = -0.231$$

$$f(1) = \sin(0.5\sqrt{1}) = \sin(0.5) = 0.4794$$

$$f(1.2) = \sin(0.5\sqrt{1.2}) = \sin(0.5 \times 1.095) = \sin(0.5475) = 0.5198 = 0.4332$$

b) selisih mundur

$$f'(x) = \frac{f(x) - f(x-h)}{h} = \frac{f(1) - f(0.8)}{0.2} = \frac{0.4794 - 0.5404}{0.2} = \frac{-0.061}{0.2} = -0.305 \rightarrow f'(1) = -0.305$$

$$f(1) = \sin(0.5\sqrt{1}) = \sin(0.5) = 0.4794$$

$$f(0.8) = \sin(0.5\sqrt{0.8}) = \sin(0.5 \times 0.8944) = \sin(0.4472) = 0.4323 = 0.5404$$

c) selisih pusat

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} = \frac{f(1.2) - f(0.8)}{2h}$$

$$f(1.2) = \sin(0.5\sqrt{1.2}) = \sin(0.5 \times 1.095) = \sin(0.5475) = 0.5205 = 0.4338$$

$$f(0.8) = \sin(0.5\sqrt{0.8}) = \sin(0.5 \times 0.8944) = \sin(0.4472) = 0.4323 = 0.5404$$

$$f'(1) = \frac{f(1.2) - f(0.8)}{2h} = \frac{0.4338 - 0.5404}{2 \times 0.2} = \frac{-0.1066}{0.4} = -0.2665$$

2). Diketahui $f(x) = x^2 \cos x$. Tentukan turunan kedua dari f di $x=0.4$ dan $h=0.1$. Gunakan metode:

a) selisih maju

$$f''(x) = \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2} = \frac{f(0.4+0.2) - 2f(0.4+0.1) + f(0.4)}{0.1^2} = \frac{f(0.6) - 2f(0.5) + f(0.4)}{0.01}$$

$$f(0.4) = (0.4)^2 \cos(0.4) = 0.16 \cos(0.4) = 0.16 \times 0.9211 = 0.1474$$

$$f(0.5) = (0.5)^2 \cos(0.5) = 0.25 \cos(0.5) = 0.5 \times 0.8776 = 0.2194$$

$$f(0.6) = (0.6)^2 \cos(0.6) = 0.36 \cos(0.6) = 0.36 \times 0.8253 = 0.2971$$

$$f''(0.4) = \frac{0.2971 - 2 \times 0.2194 + 0.1474}{0.1^2} = \frac{0.2971 - 0.4388 + 0.1474}{0.01} = \frac{0.0057}{0.01} = 0.57$$

b). selisih mundur

$$f''(x) = \frac{f(x) - 2f(x-h) + f(x-2h)}{h^2} = \frac{f(0,4) - 2f(0,3) + f(0,2)}{0,1^2}$$

$$f(0,3) = (0,3)^2 \cos(0,3) = 0,09 \times \cos(0,3) = 0,09 \times 0,9553 = 0,0860$$

$$f(0,2) = (0,2)^2 \cos(0,2) = 0,04 \cos(0,2) = 0,04 \times 0,9801 = 0,0392$$

$$f''(0,4) = \frac{0,1474 - 2 \times 0,0860 + 0,0392}{0,1^2} = \frac{0,1474 - 0,172 + 0,0392}{0,01} = \frac{0,0146}{0,01} = 1,46$$

c). selisih pusat

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = \frac{f(0,5) - 2f(0,4) + f(0,3)}{0,1^2}$$

$$f''(0,4) = \frac{0,2194 - 2(0,1474) + 0,0860}{0,01} = \frac{0,2194 - 0,2948 + 0,0860}{0,01} = \frac{0,0106}{0,01} = 1,06$$

3). Diketahui data Jarak tempuh suatu kendaraan sebagai berikut : Gunakan rumus numerik utk

t(s)	0	25	50	75	100	125
y(km)	0	32	58	78	92	100

rumus kecatan

$$v(t) = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}$$

$$a(t) = \frac{v_{i+1} - v_i}{t_{i+1} - t_i}$$

mengestimasi kecepatan dan percepatan kendaraan tsb setiap waktu

• Kecepatan

$$\text{- Selisih maju } (t=0) : v(0) = \frac{y(25) - y(0)}{t(25) - t(0)} = \frac{32 - 0}{25 - 0} = \frac{32}{25} = 1,28 \text{ km/s}$$

$$\text{- Selisih pusat } (t=25, 50, 75, 100) : v(25) = \frac{y(50) - y(0)}{50 - 25} = \frac{58 - 32}{50 - 25} = \frac{26}{25} = 1,04 \text{ km/s}$$

$$v(50) = \frac{78 - 58}{75 - 50} = \frac{20}{25} = 0,8 \text{ km/s}$$

$$v(75) = \frac{92 - 78}{100 - 75} = \frac{14}{25} = 0,56 \text{ km/s}$$

$$v(100) = \frac{100 - 92}{125 - 100} = \frac{8}{25} = 0,32 \text{ km/s}$$

• Percepatan

$$a(0) = \frac{1,04 - 1,28}{25 - 0} = \frac{-0,24}{25} = -0,0096 \text{ km/s}^2 \quad a(75) = \frac{0,32 - 0,56}{100 - 75} = \frac{-0,24}{25} = -0,0096 \text{ km/s}^2$$

$$a(25) = \frac{0,8 - 1,04}{50 - 25} = \frac{-0,24}{25} = -0,0096 \text{ km/s}^2 \quad a(100) = \frac{0 - 0,32}{125 - 100} = \frac{-0,32}{25} = -0,0128 \text{ km/s}^2$$

$$a(50) = \frac{0,56 - 0,8}{75 - 50} = \frac{-0,24}{25} = -0,0096 \text{ km/s}^2$$