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KUIS Metode Numerik

1. Diketahui sistem persamaan linear berikut! $\beta = 6$

$-2x_1 + x_2 + \frac{1}{(2+\beta)} = 4$, $x_1 - 2x_2 - \frac{1}{(2+\beta)} x_3 = -4$, $x_2 + 2x_3 = 0$

dengan $\bar{x}^{(0)} = (0,0,0)^t$. Selesaikan sistem persamaan linear tersebut menggunakan metode iterasi Gauss Seidel dengan $\varepsilon_{TOL} = 3\%$ (eror aproksimasi dr masing-masing iterasi dihitung dalam norm l_∞).

$\rightarrow -2x_1 + x_2 + \frac{1}{8}x_3 = 4$, $x_1 - 2x_2 - \frac{1}{8}x_3 = -4$, $x_2 + 2x_3 = 0$

$Ax = b$, $A_{3\times3}$

$\bar{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ -4 \\ 0 \end{pmatrix}$

$-2x_1 = 4 - x_2 - \frac{1}{8}x_3 \Rightarrow x_1^{(k)} = \frac{4}{-2} + \frac{x_2^{(k-1)}}{2} + \frac{x_3^{(k-1)}}{16} =$

$-2x_2 = -4 - x_1 + \frac{x_3}{8} \Rightarrow x_2^{(k)} = \frac{-4}{-2} + \frac{x_1^{(k)}}{2} - \frac{x_3^{(k-1)}}{16} =$

$2x_3 = -x_2 \Rightarrow x_3^{(k)} = -\frac{x_2^{(k)}}{2} =$

$k=1$, $x_1^{(1)} = \frac{4}{-2} + \frac{0}{2} + \frac{0}{16} = -\frac{4}{2} = -2$ $\quad \bar{x}^{(1)} = \begin{pmatrix} -2 \\ 1 \\ -0{,}5 \end{pmatrix}$

$x_2^{(1)} = \frac{-4}{-2} + \frac{(-2)}{2} - \frac{0}{16} = 2 - 1 = 1$

$x_3^{(1)} = -\frac{1}{2} = -0{,}5$

$k_2 = x_1^{(2)} = \frac{4}{-2} + \frac{x_2^{(1)}}{2} + \frac{x_3^{(1)}}{16} = -2 + 0{,}5 + (-0{,}0313) = -1{,}5313$ $\quad \bar{x}^{(2)} = \begin{pmatrix} -1{,}5313 \\ 1{,}0313 \\ -0{,}5 \end{pmatrix}$

$x_2^{(2)} = \frac{-4}{2} + \frac{x_1^{(1)}}{2} - \frac{x_3^{(1)}}{16} = 2 + (-1) - (-0{,}0313) = 1{,}0313$

$x_3^{(2)} = -\frac{x_2^{(2)}}{2} = -0{,}$

$\varepsilon_a = \frac{\|\bar{x}^{(2)} - \bar{x}^{(1)}\|}{\|\bar{x}^{(2)}\|} = \frac{\max\{|0{,}4687| + |0{,}0313| + |0|\}}{\max\{|-1{,}5313| + |1{,}0313| + |-0{,}5|\}} \cdot 100\% = \frac{0{,}4687}{1{,}5313} \cdot 100\% = 31\%$

$k_3 = x_1^{(3)} = \frac{4}{-2} + \frac{x_2^{(2)}}{2} + \frac{x_3^{(2)}}{16} = -2 + 0{,}5157 - (-0{,}0313) = -1{,}4530$ $\quad \bar{x}^{(3)} = \begin{pmatrix} -1{,}4530 \\ 1{,}2656 \\ -0{,}5157 \end{pmatrix}$

$x_2^{(3)} = \frac{4}{2} + \frac{x_1^{(2)}}{2} - \frac{x_3^{(2)}}{16} = 2 + (-0{,}7657) - (-0{,}0313) = 1{,}2656$

$x_3^{(3)} = -\frac{x_2^{(2)}}{2} = -0{,}5157$

$\varepsilon_a = \frac{\max\{|0{,}0783|, |0{,}2343|, |-0{,}0157|\}}{\max\{|-1{,}4530|, |1{,}2656|, |-0{,}5157|\}} = \frac{0{,}2343}{1{,}4530} \cdot 100\% = 16\%$

$k_4 = x_1^{(4)} = -2 + (-0{,}2579) + (-0{,}0322) = \cancel{-7743}\ -1{,}7743$ $\quad \bar{x}^{(4)} = \begin{pmatrix} -1{,}7743 \\ 1{,}2413 \\ -0{,}6328 \end{pmatrix}$

$x_2^{(4)} = 2 + (-0{,}7265) + (-0{,}0322) = 1{,}2413$

$x_3^{(4)} = -0{,}6328$

$\varepsilon_a = \frac{\max\{|0{,}3213|, |-0{,}0243|, |-0{,}0577|\}}{\max\{|-1{,}7743|, |1{,}2413|, |-0{,}6328|\}} = \frac{0{,}3213}{1{,}7743} \cdot 100\% = 18\%$

$k_5 = x_1^{(5)} = -2 + (0{,}6207 + (-0{,}0396) = -1{,}4189$ $\quad \bar{x}^{(5)} = \begin{pmatrix} -1{,}4189 \\ 2{,}0476 \\ -0{,}6207 \end{pmatrix}$

$x_2^{(5)} = 2 + (-0{,}0072) + (-0{,}0396) = 2{,}0476$

$x_3^{(5)} = -0{,}6207$

$\varepsilon_a = \frac{\max\{|0{,}3554|, |0{,}8063|, |0{,}0121|\}}{\max\{|-1{,}4189|, |2{,}0476|, |-0{,}6207|\}} = \frac{0{,}8063}{2{,}0476} \cdot 100\% = 39\%$

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x \ iterasi	1	2	3	4	5
x_1	-2	-1,5313	-1,4530	-1,7743	-1,4189
x_2	1	1,0313	1,2656	1, 2413	2,0476
x_3	-0,5	-0,5	-0,5157	-0, 6328	-0, 6207
l_∞	–	31%	16%	18%	39%