

# Master's Thesis Defense

## Simulation of Brain Functional and Structural Connectivity on Empirical and Randomized Complex Networks



Şeyma Bayrak

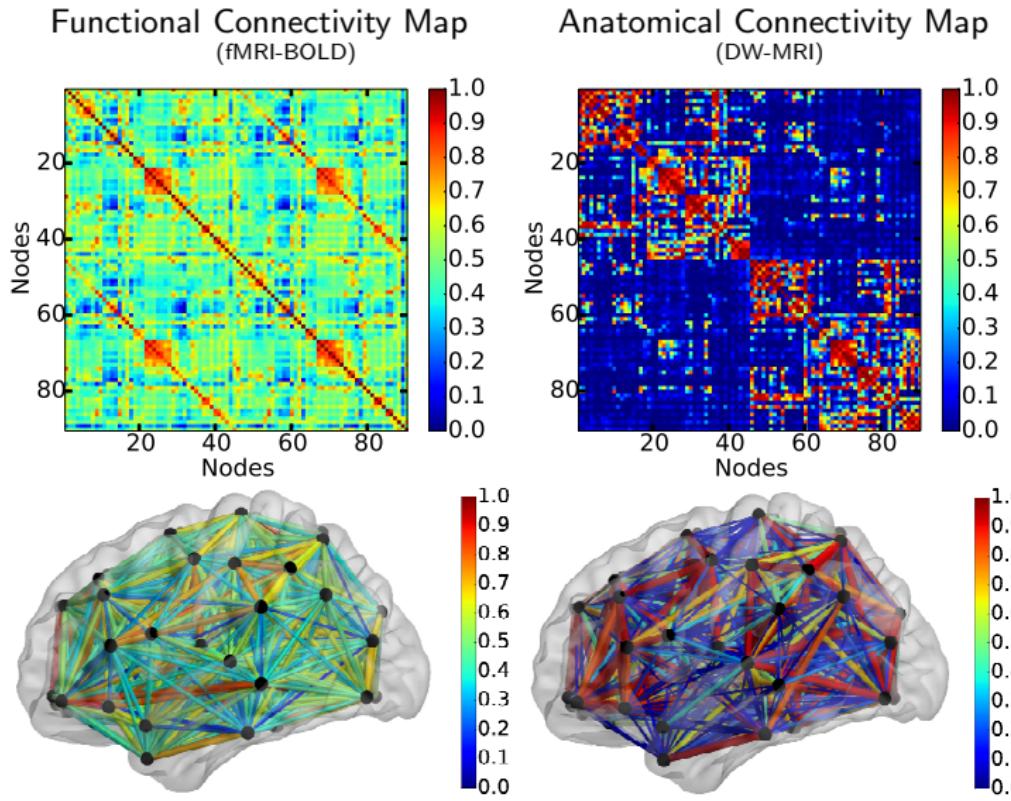
Department of Integrative Neuroscience, Otto-von-Guericke-Universität Magdeburg  
Bernstein Center for Computational Neuroscience Berlin,  
Nachwuchsgruppe, *Nonlinear Dynamics and Control in Neuroscience*

December 15, 2014

- i) How does functionally correlated behavior arise from structural connectivity?
- ii) What is characteristic for the brain network? Does it differ from random graphs ?
- resting-state
- Experimental results combined with modeling approaches
- Graph theory / network science, nonlinear dynamics, computational neuroscience

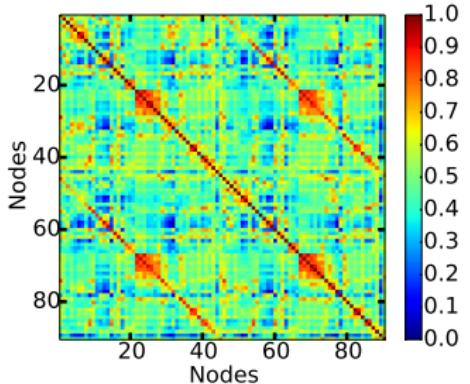
1. Empirical Data
2. The Brain Graph
3. Randomization Methods
4. Network Characterizations
5. Neuronal Activity Model  
*(FitzHugh-Nagumo Oscillators [1, 2] )*
6. BOLD Activity Model  
*(Balloon-Windkessel Hemodynamic Model [3] )*
7. Results
8. Future Directions
9. Discussion

# 1.Empirical Data

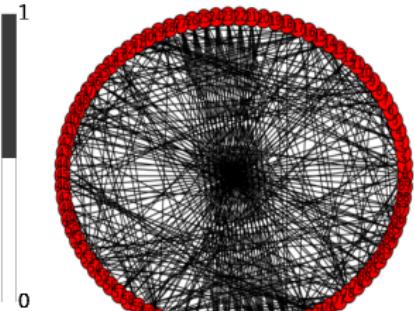
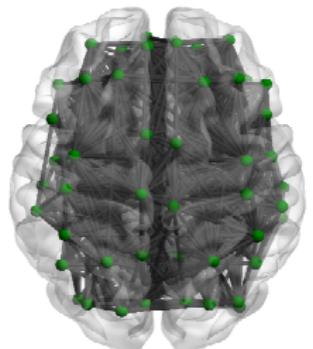
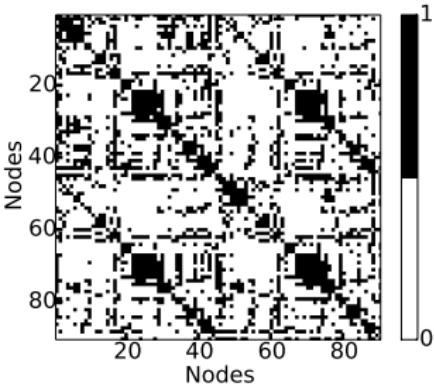


## 2.The Brain Graph

FCM, (fMRI-BOLD)



Adjacency Matrix (AM),  $r=0.55$



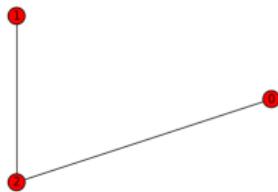
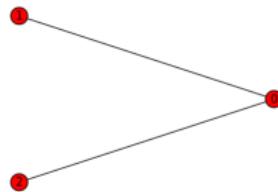
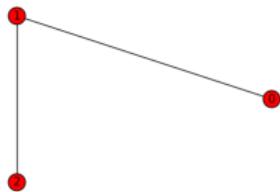
### 3.Randomization Methods

Table : Abbreviations for the brain graph and the randomly constructed graphs

Abbreviation	Description	method
$R_{BG}$	the brain graph	NETWORKX [4]
$R_{ER}$	Erdős-Rényi, $G(N,L)$	NETWORKX [4]
$R_{DES}$	double-edge-swap	NETWORKX [4]
$R_{PDD}$	preserved-degree-distribution	BCT [5]
$R_{CM}$	configuration model	NETWORKX [4]
$R_{PR}$	partial randomization	BCT [5]

### 3. The Randomization Methods

$R_{ER}$ , Erdős-Rényi-Type Randomization [6],  $G(N, L)$   
( $P$  : a binomial distribution for number of edges per node)



$$L = \binom{N}{2} P$$

## 4. Network Characterizations

Table : Statistical measures to characterize networks

### Network Measures

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Network Density,  $\kappa$

---

Average Clustering Coefficient,  $C$

---

Transitivity,  $T$

---

Connected Components

---

Small-Worldness,  $S$

---

Global and Local Efficiency,  $E$  and  $E_{loc}$

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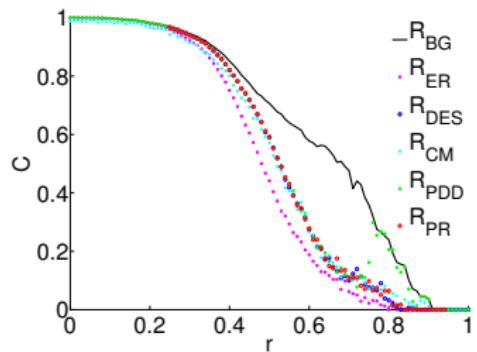
Degree Distribution,  $p(k)$

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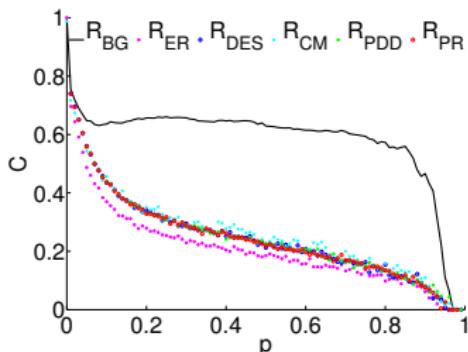
## 4. Network Characterizations

$$\text{Average Clustering Coefficient, } C = \frac{1}{n} \sum_{i \in N} C_i = \frac{1}{n} \sum_{i \in N} \frac{2t_i}{k_i(k_i-1)}$$

FCM related graphs



ACM related graphs

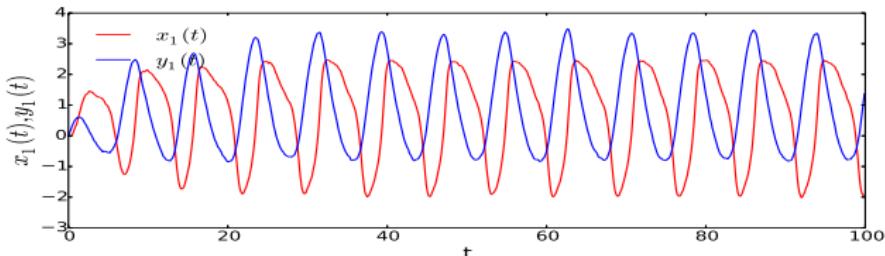
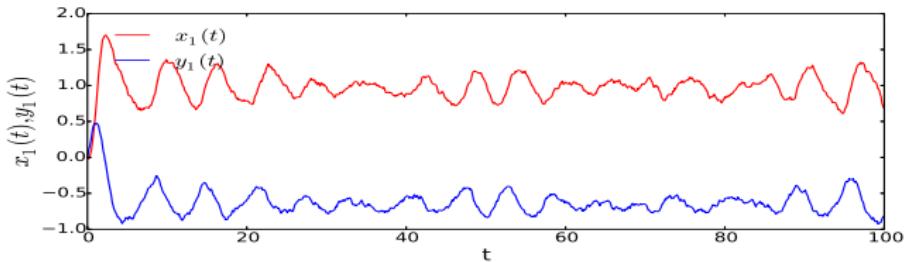


# 5. Neuronal Activity Model

FitzHugh-Nagumo Oscillations, Network Dynamics [7,8]

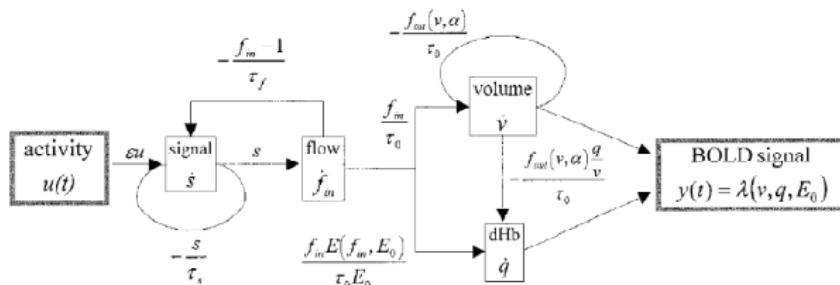
$$\dot{x}_i = \tau \left( y_i + \gamma x_i - \frac{x_i^3}{3} \right) - c \sum_{j=1}^N a_{ij} x_j(t - \Delta t_{ij}) + Dn_x \quad (1a)$$

$$\dot{y}_i = -\frac{1}{\tau}(x_i - \alpha + by_i) + Dn_y, \quad (1b)$$



# 6.BOLD Activity Model

## Balloon-Windkessel Hemodynamic Model [3]



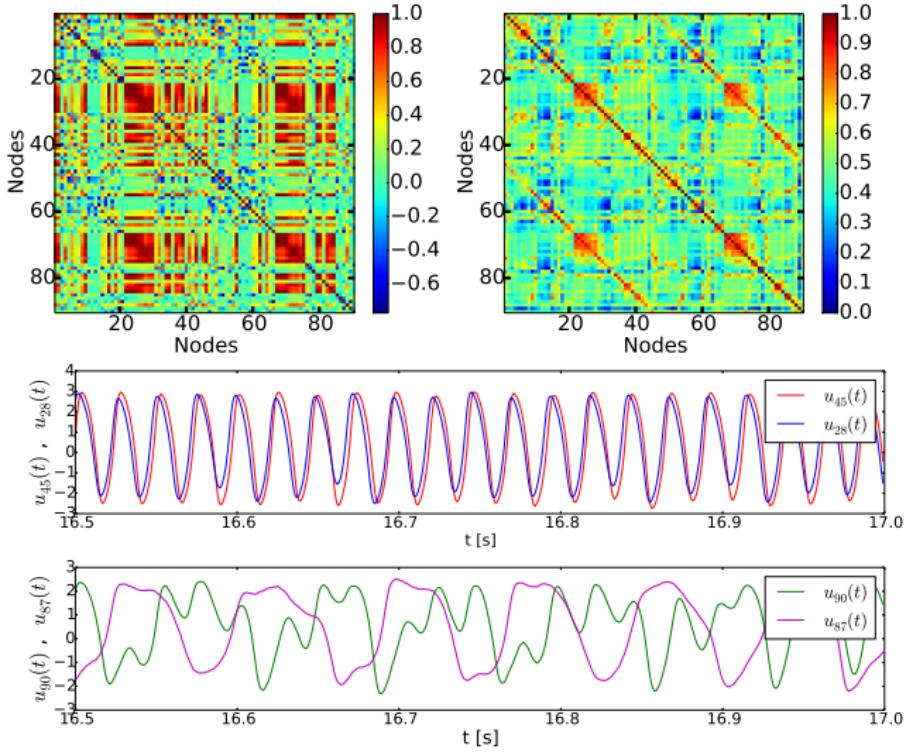
- neuronal activity  $\Rightarrow$  regional changes in surrounding
- CBV, CBF,  $O_2$ -level,  $Hb$  and  $dHB$  level in capillaries
- Friston et. al.: mediating between non-linear time-series and BOLD activity

## Parameter Analysis

- $c$ , the coupling strength (FHN network model)
- $v$ , the axonal propagation velocity,  $\Delta t_{ij} = d_{ij}/v$  (FHN network model)
- $r$  or  $p$ , threshold or probability (network topology)

# 7. Results

## Neuronal Activity Simulations of FCM Related Brain Graphs to fMRI-BOLD Data

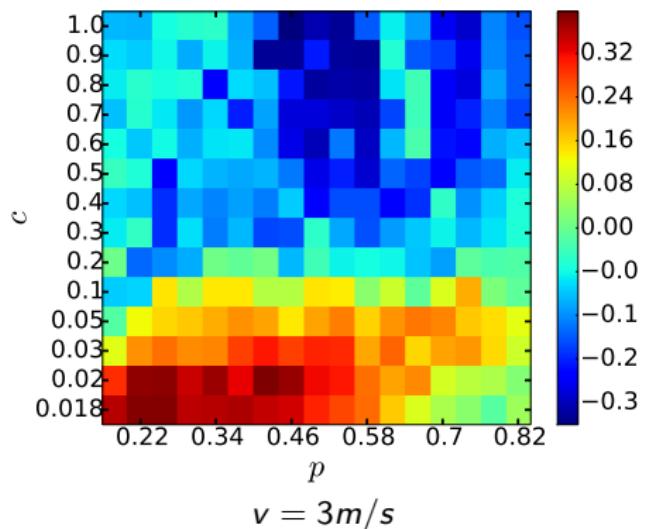


$$c = 0.2, \quad v = 7 \text{ m/s}, \quad r = 0.60, \quad \rho_{e,s} = 0.43, \quad \rho_{45,28} = 0.88, \quad \rho_{90,87} = 0.13$$



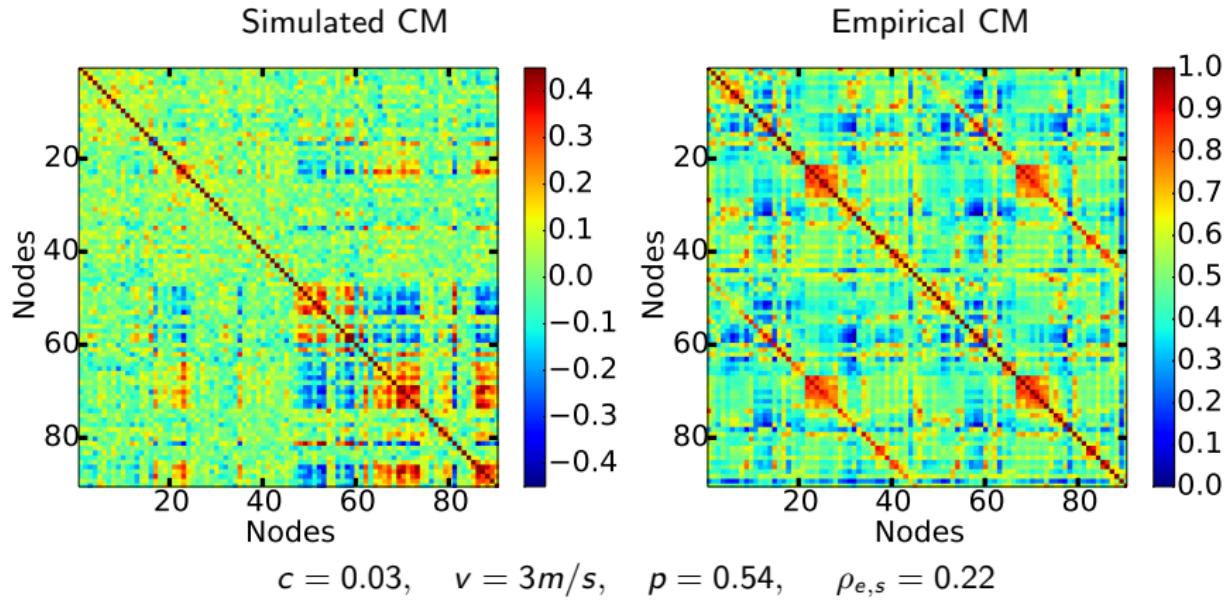
## 7. Results

### Comparing BOLD Simulations of Anatomical Brain Graphs to fMRI-BOLD Data



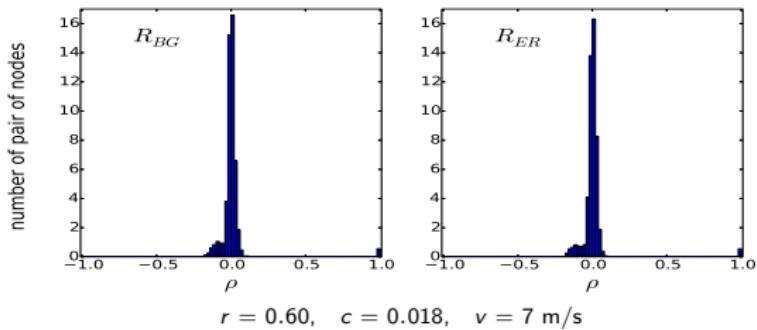
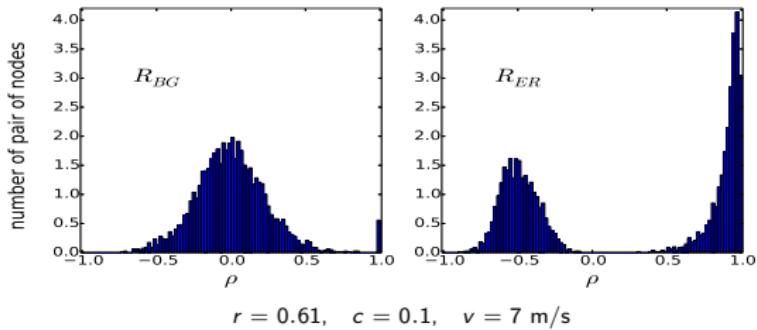
## 7. Results

### Comparing BOLD Simulations of Anatomical Brain Graphs to fMRI-BOLD Data



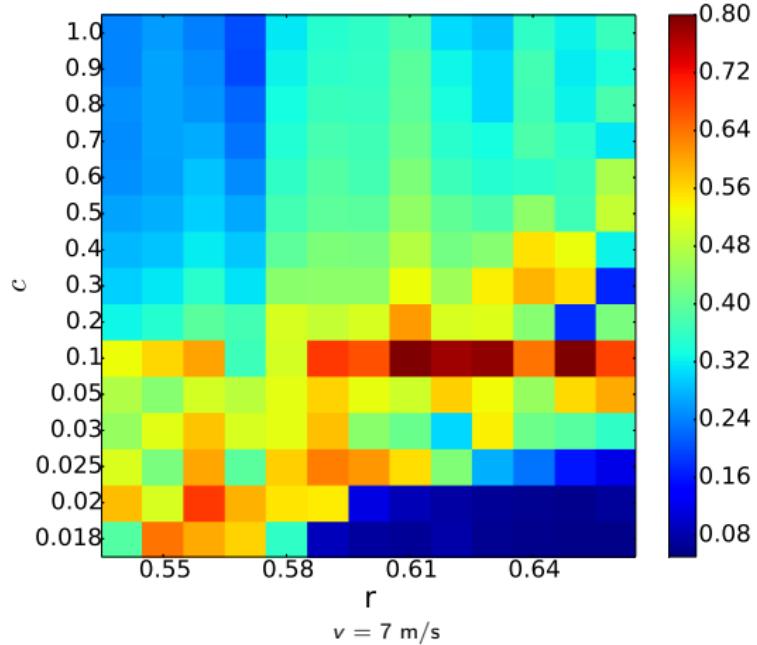
## 7. Results

### Comparison of FCM Related Brain Graph to the Random Networks



## 7. Results

Comparison of FCM Related Brain Graph to the Random Networks



$v = 7 \text{ m/s}$

## 8. Future Directions

- ... low coupling strengths ?
- ... comparison of temporal BOLD activities of  $R_{BG}$  and  $R_{ER}$  ?

# Thanks



Dr. Philipp Hövel<sup>1</sup>

Dr. Vesna Vuksanović<sup>1</sup>

Prof. Jochen Braun<sup>2</sup>

- [1] TU Berlin, Institut für Theoretische Physik, BCCN-Nachwuchsgruppe: Nonlinear Dynamics and Control in Neuroscience
- [2] OVGU Magdeburg, Institute of Biology, Faculty of Science, Cognitive Biology Group

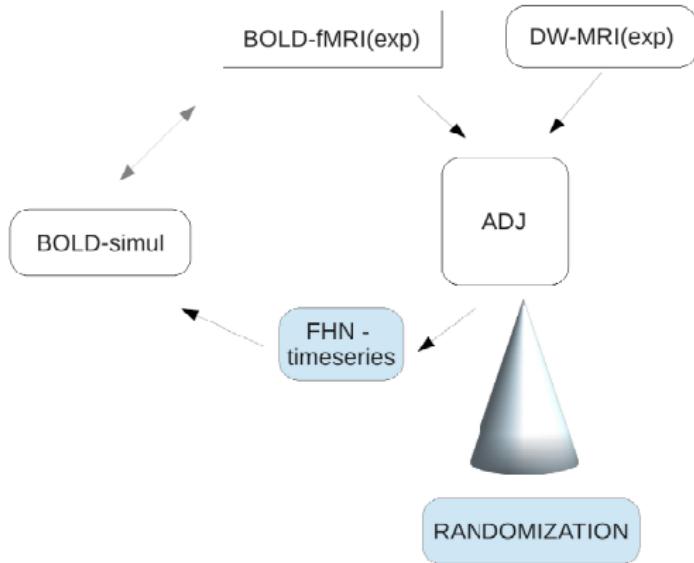
# References

- 1 Nagumo, J. and Arimoto, S. and Yoshizawa., S. 1962. An active pulse transmission line simulating nerve axon, *Proc. IRE*, 50:2061–2070
- 2 FitzHugh, R. 1961. Impulses and physiological states in theoretical models of nerve membrane, *Biophys. J.*, 1:445–466
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- 6 Erdős, P. and Rényi, A. 1959. On Random Graphs. I, *Publicationes Mathematicae*, 6:290297
- 7 Vuksanović, V. and Hövel, P. 2014. Large-scale neural network model for functional networks of the human cortex *Selforganization in Complex Systems: The Past, Present, and Future of Synergetics, Proc. of the Int. Symp.*
- 8 Ghosh, A. and Rho, Y. and McIntosh, A. R. and Kotter, R. and Jirsa, V. K. 2008. Cortical network dynamics with time delays reveals functional connectivity in the resting brain, 2:115–120

# 9.Discussion

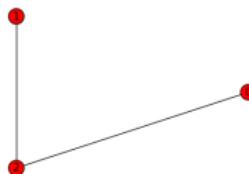
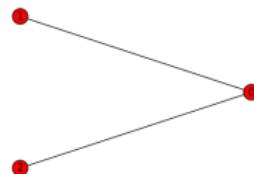
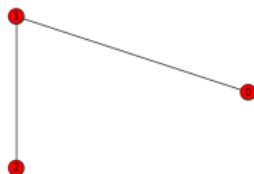
Research Proposals on the Schema... i) How does functionally correlated behavior among cortical and subcortical brain regions emerge from the structural connectivity?

ii) Does the topological properties as well as temporal dynamics of the brain graph differ from that of random graphs?

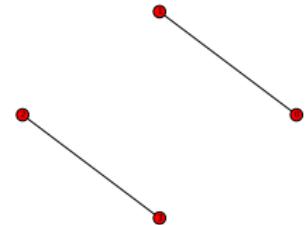
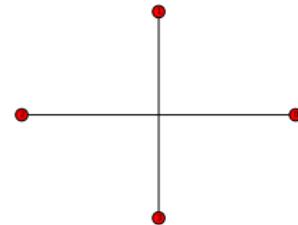
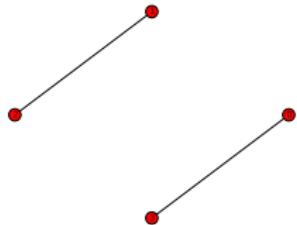


## Randomization Methods...

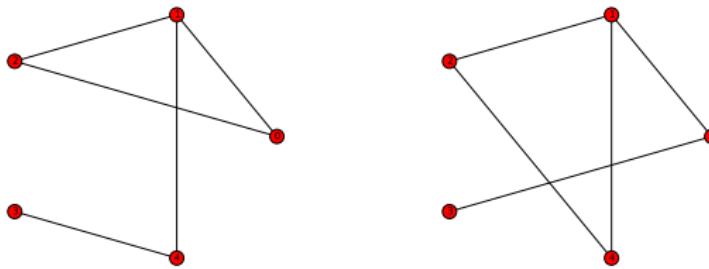
Erdős-Rényi-Type Randomization,  $G(N, L)$



Double-Edge-Swap Type Randomization,  $k_i$



## Randomization Methods... Preserved-Degree-Distribution Type Randomization,

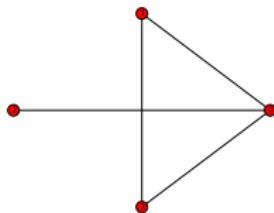
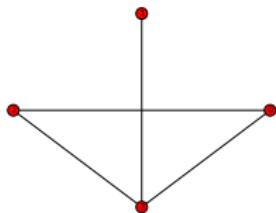


$p(k)$

$$p(k=1) = \frac{1}{5}, p(k=2) = \frac{3}{5}, \text{ and } p(k=3) = \frac{1}{5}$$

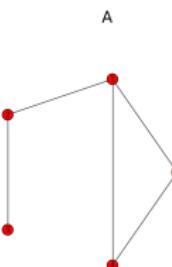
## Randomization Methods...

### Configuration Model Randomization



(left):  $k_0 = 2, k_1 = 1, k_2 = 2, k_3 = 3$  (right):  $k_0 = 3, k_1 = 2, k_2 = 1, k_3 = 2$ . The degree sequence in non-increasing order in both graphs:  $\{3, 2, 2, 1\}$

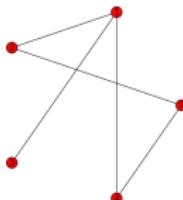
### Partial Randomization



B



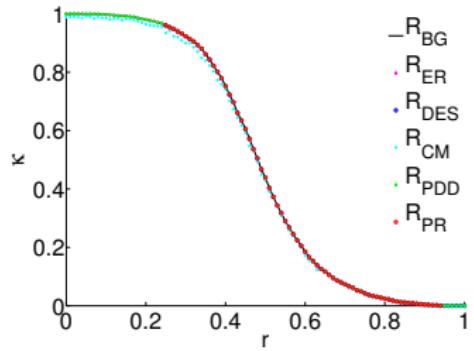
A random



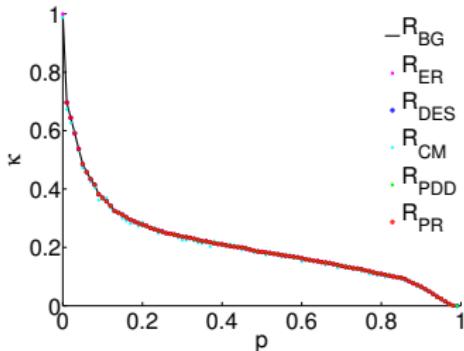
## Network Characterizations...

$$\text{Network Density, } \kappa = \frac{2L}{N(N-1)}$$

FCM related graphs



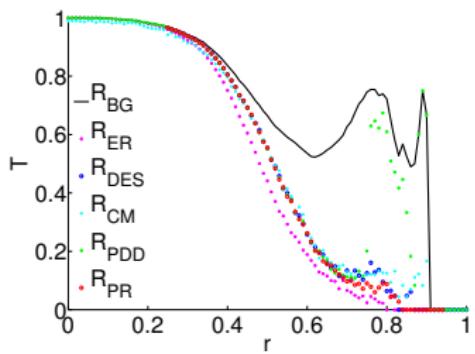
ACM related graphs



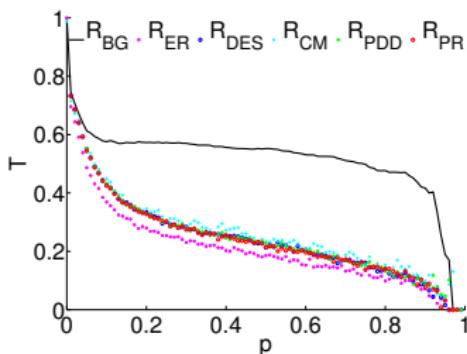
## Network Characterizations...

$$\text{Transitivity, } T = \frac{2 \sum_{i \in N} t_i}{\sum_{i \in N} k_i(k_i - 1)}$$

FCM related graphs

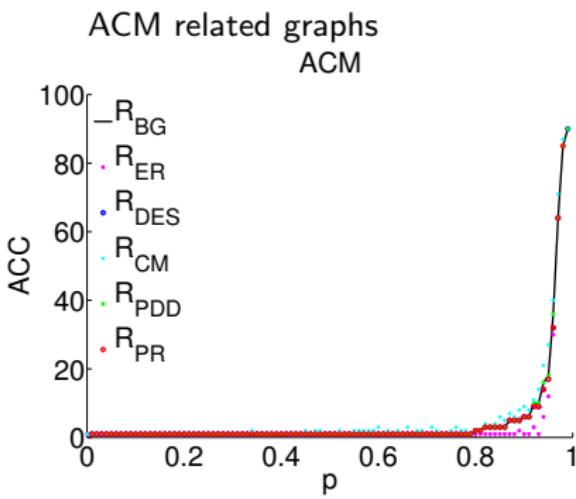
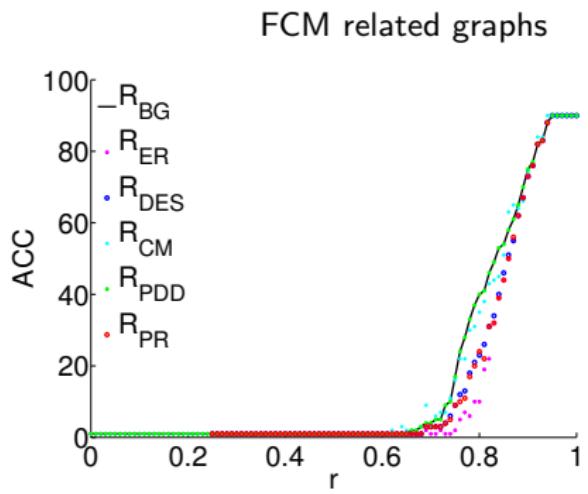


ACM related graphs



## Network Characterizations...

### Average number of connected components

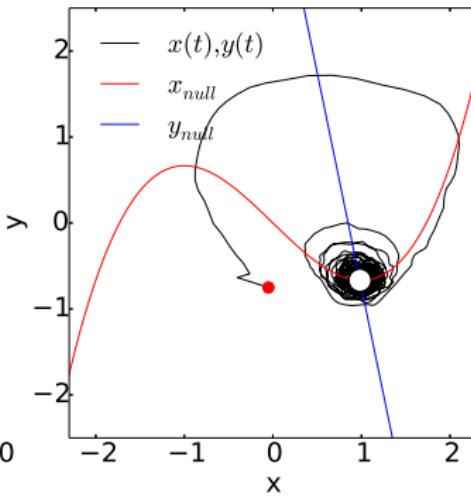
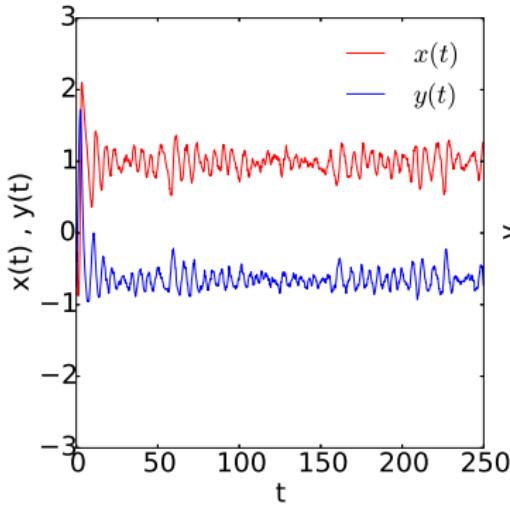


# Appendix

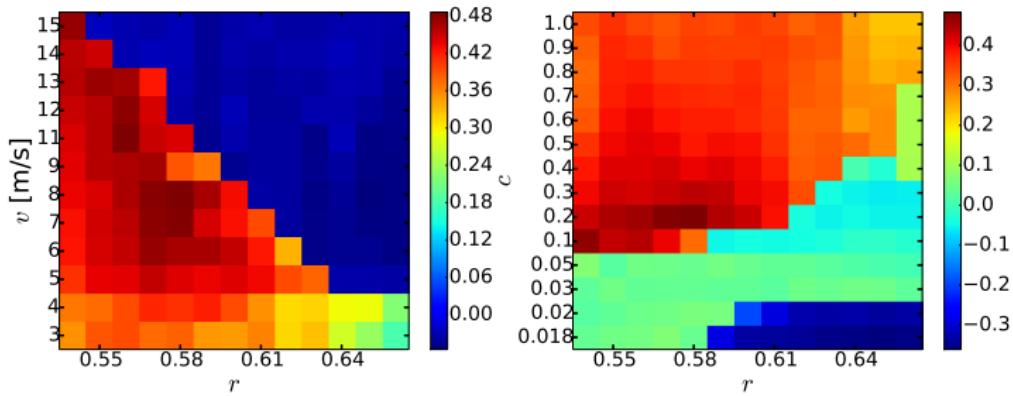
FitzHugh-Nagumo Oscillations, Local Dynamics with Gaussian White Noise Source  
(noise induced oscillations, system in excitable regime)

$$\dot{x} = \tau \left( y + \gamma x - \frac{x^3}{3} \right) + Dn_x \quad (2a)$$

$$\dot{y} = -\frac{1}{\tau}(x - \alpha + by) + Dn_y, \quad (2b)$$

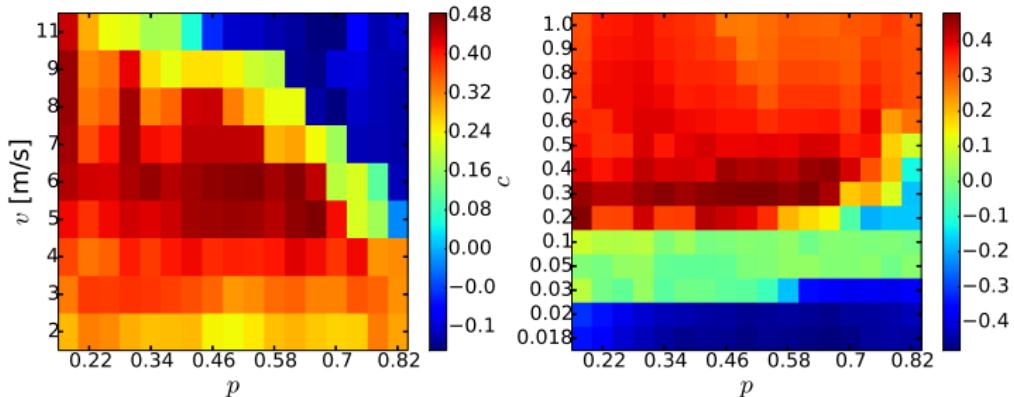


## Results... Neuronal Activity Simulations of FCM Related Brain Graphs to fMRI-BOLD Data



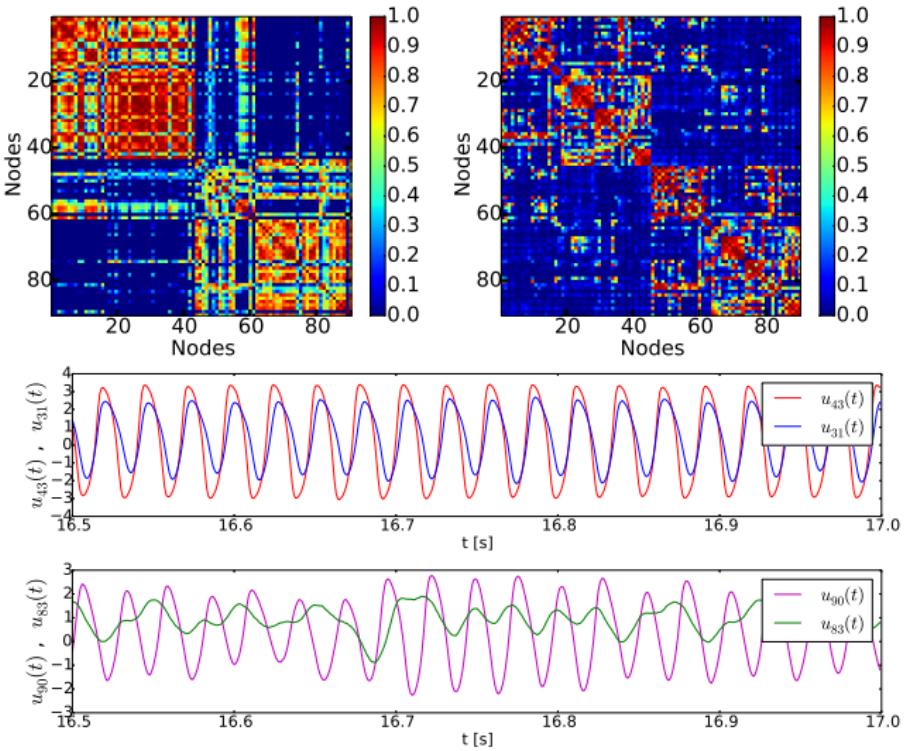
## Results...

### Neuronal Activity Simulations of ACM Related Brain Graphs to DW-MRI Data



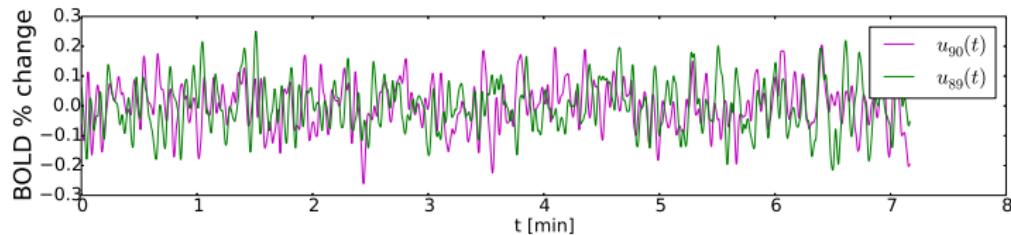
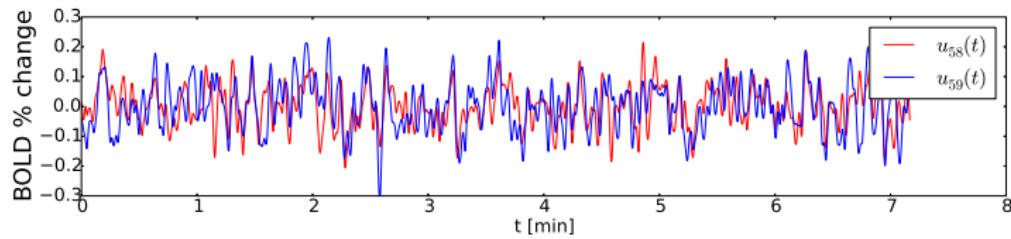
## Results...

### Neuronal Activity Simulations of ACM Related Brain Graphs to DW-MRI Data



## Results...

Comparing BOLD Simulations of Anatomical Brain Graphs to fMRI-BOLD Data



$$\rho_{58,59} = 0.48$$

$$\rho_{90,89} = 0.11$$

# Appendix

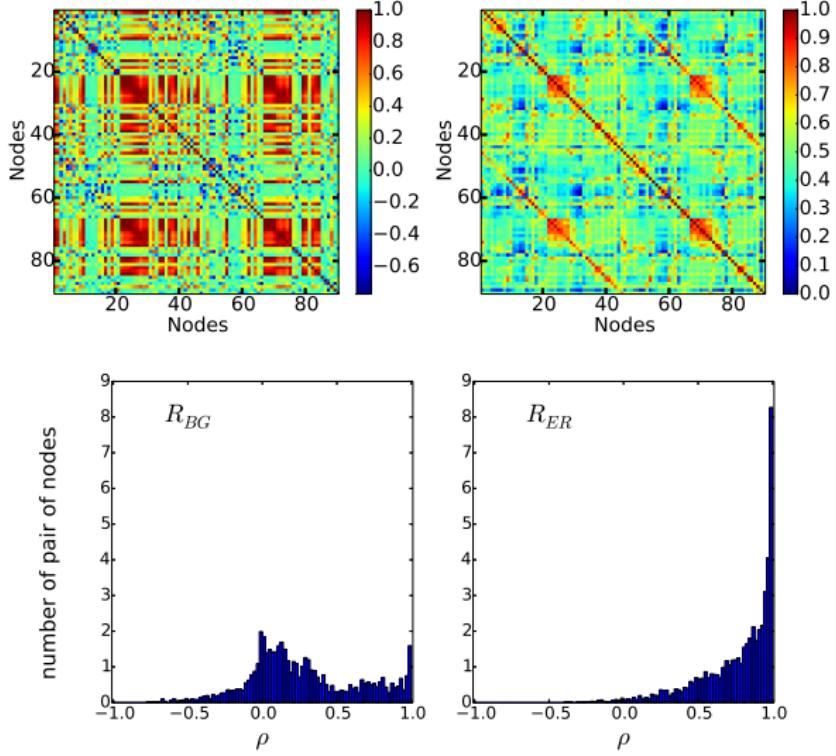
## Statistical Method...

How to Compare the Modeled Temporal Dynamics of Brain Graphs to that of the Random Networks ?

- construct brain graphs and their random networks at defined  $r$ - and  $p$ -ranges for FCM and ACM based empirical data
- extract FHN network modeled time-series in each graph
- find Pearson correlation coefficients  $\rho$  between pairwise combination of  $N = 90$  nodes in each graph
- distribute  $\rho$  values in histograms for each graph
- compare histograms via Bhattacharya coefficients :

$$d(H_b, H_r) = \sqrt{1 - \frac{1}{\sqrt{H_r H_b N^2}} \sum_i \sqrt{H_r(i) H_b(i)}} \quad (3)$$

# Appendix



$$\rho_{e,s} = 0.43$$

FitzHugh-Nagumo  
network model,  
simulation parameters :

$$c = 0.2$$

$$v = 7 \text{ m/s}$$

Brain Graph Topology :

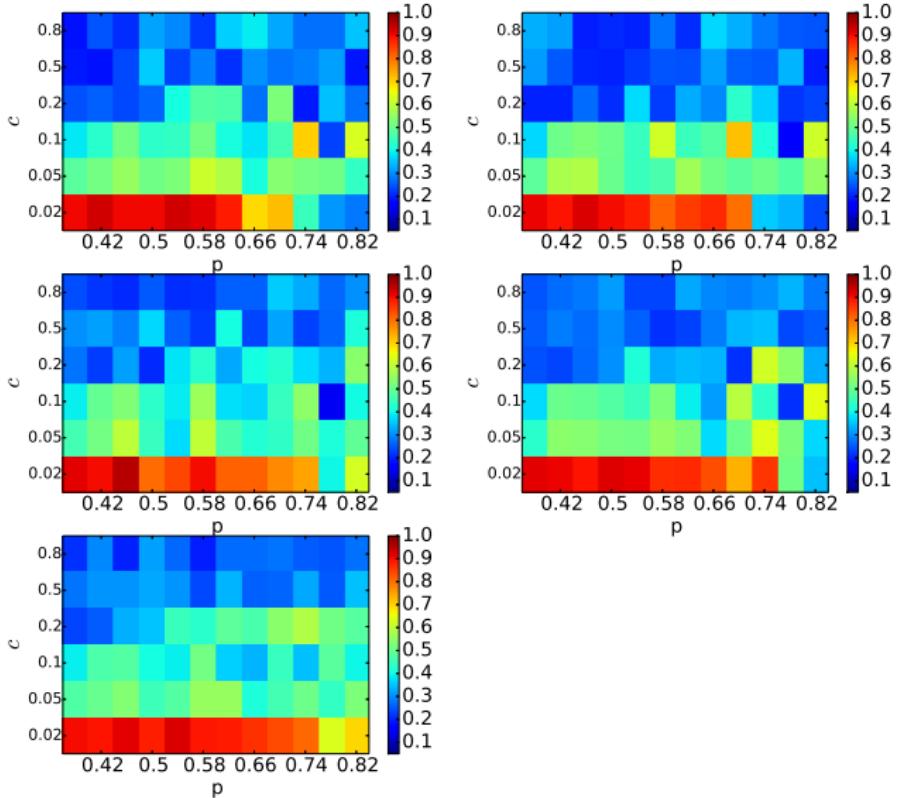
$$r = 0.60$$

Bhattacharya Coeff.:

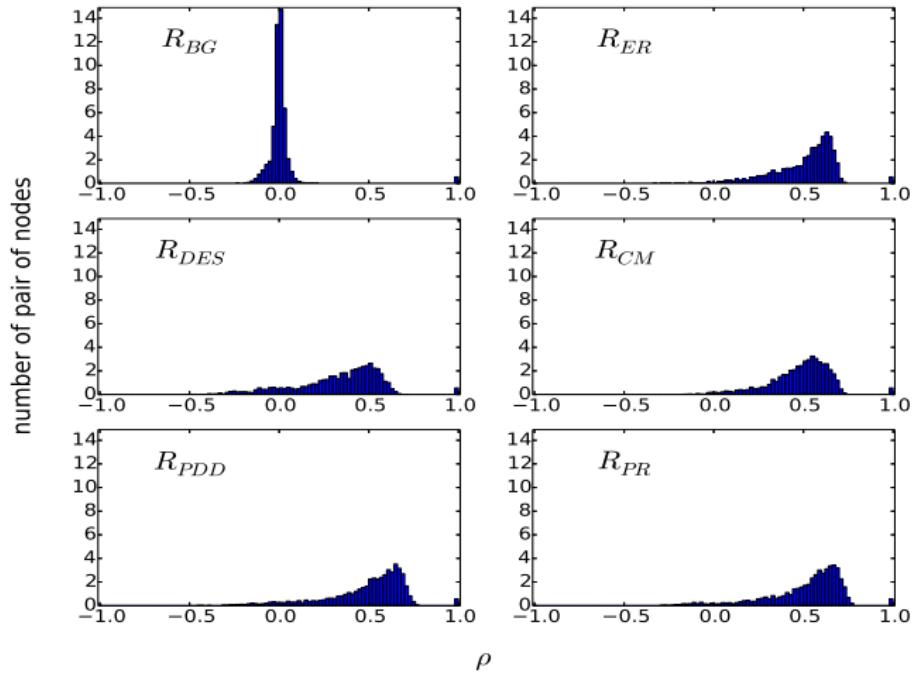
$$d(H_b, H_r) \approx 0.50$$

# Appendix

## Results... Comparison of ACM Related Brain Graph to the Random Networks



## Results... Comparison of ACM Related Brain Graph to the Random Networks



Distance Matrices...  $\Delta t_{ij} = d_{ij}/v$

