LAB ROTATION 02
1st Week Assignment

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1 Nonlinear Dynamics of Neural Networks

1.1 Simple FitzHugh-Nagumo Model

$$\varepsilon\dot{x} = x - \frac{x^3}{3} - y \tag{1}$$

$$\dot{y} = x + a \tag{2}$$

x: activator, reproducing behavior of voltage during the course of a spike
y: inhibitor, inhibiting production of x
a: bifurcation parameter - threshold parameter: determines whether the system is excitable ($a > 1$) or exhibits periodic firing (autonomous oscillations) ($a < 1$)

Bifurcation Analysis and Nullclines / Fixed Points:

- $\dot{x} = \dot{y} = 0, \quad y - nulcline: \quad y = x - \frac{x^3}{3}, \quad x - nullcline: \quad x = -a$
- *equilibrium (fixed) point* :$(x^*, y^*) = (-a, -a + \frac{a^3}{3})$
- *Jacobian Matrix*: the stability analysis of fix points with eigenvalues $Re(\lambda \geq 0)$
- $\lambda_{1,2} = \frac{1-a^2 \pm \sqrt{(1-a^2)^2 - 4\varepsilon}}{2\varepsilon}$, $\varepsilon = 0.005$, for $|a| > 1$ the fixed point is stable, for $|a| < 1$ unstable

Fixed point (x^*, y^*): intersection of nullclines
Stability : resting towards (x^*, y^*)
Change of stability to instability: periodic oscillations

1.2 Related Figures

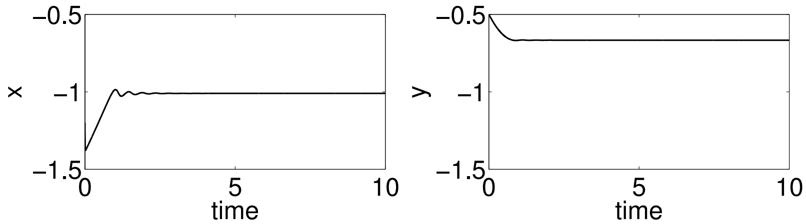

Figure 1, Simple FHN model, x and y plots as time series. $x_0 = -1.2$, $y_0 = -0.5$, $a = 1.01$ (excitable), and $\varepsilon = 0.005$

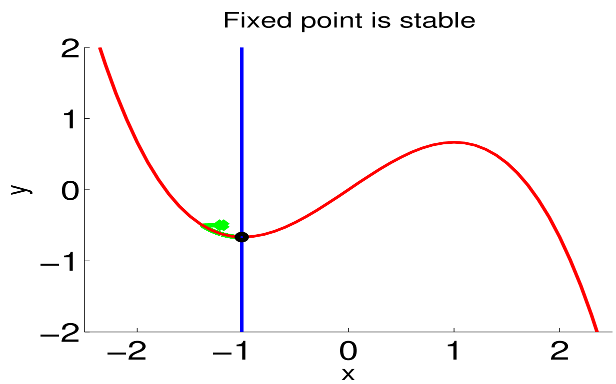

Figure 2, Simple FHN model, x versus y in state space $x_0 = -1.2$, $y_0 = -0.5$, $a = 1.01$ (excitable), and $\varepsilon = 0.005$

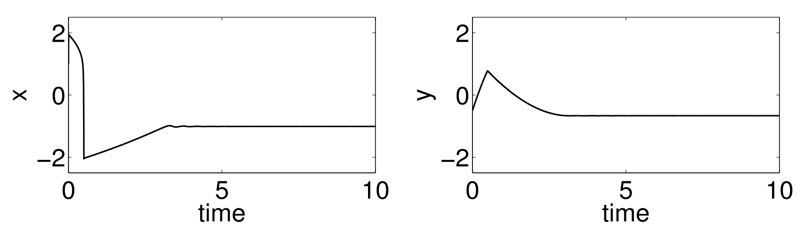

Figure 3, Simple FHN model, x and y plots as time series. $x_0 = 1$, $y_0 = -0.5$, $a = 1.01$ (excitable), and $\varepsilon = 0.005$

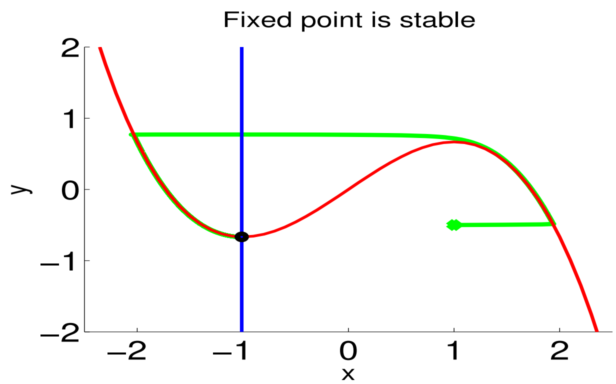

Figure 4, Simple FHN model, x versus y in state space $x_0 = 1$, $y_0 = -0.5$, $a = 1.01$ (excitable), and $\varepsilon = 0.005$

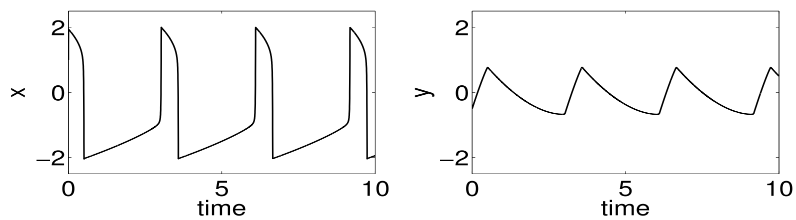

Figure 5, Simple FHN model, x and y plots as time series. $x_0 = 1$, $y_0 = -0.5$, $a = 0.97$ (oscillatory), and $\varepsilon = 0.005$

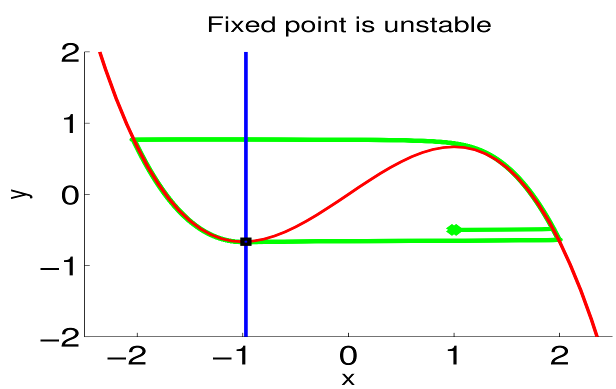

Figure 6, Simple FHN model, x versus y in state space $x_0 = 1$, $y_0 = -0.5$, $a = 0.97$, and $\varepsilon = 0.005$

1.3 Extended FitzHugh-Nagumo Model

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y \tag{3}$$

$$\dot{y} = x + a - \gamma y \quad (4)$$

Extended FHN model : now $\dot{y}$ depends on an additional linear inhibitory term γy **Bifurcation Analysis and Nullclines / Fixed Points**:

- $\dot{x} = \dot{y} = 0, \quad x - nulcline: \quad x - \frac{x^3}{3} - y = 0, \quad y - nullcline: \quad y = \frac{x}{\gamma} + \frac{a}{\gamma}$
- *analytic solution for the equilibrium (fixed) point* : $@fsolve(x(x - \frac{1}{\gamma}) - \frac{x^3}{3} - \frac{a}{\gamma}) \longrightarrow x(a,\gamma) = x_f$
- $Jacobian\ matrix\ eigenvalues: \lambda_{1,2} = \frac{(1-x_f^2-\gamma\varepsilon) \pm \sqrt{(1-x_f^2-\gamma\varepsilon)^2 - 4(\gamma\varepsilon x_f^2 + \varepsilon - \gamma\varepsilon)}}{2}$

Bifurcation point depends now not only a but also γ.
Fixed point can be now *stable, unstable, saddle.*
Let us make determinant of Jacobian matrix positive to eliminate saddle points (this part will be explained in detail in next assignment):
$\mathbf{det}(J) = \varepsilon(\gamma(x_f^2 - 1) + 1) > 0$, then we should choose $0 < \gamma < 1$

1.4 Related Figures

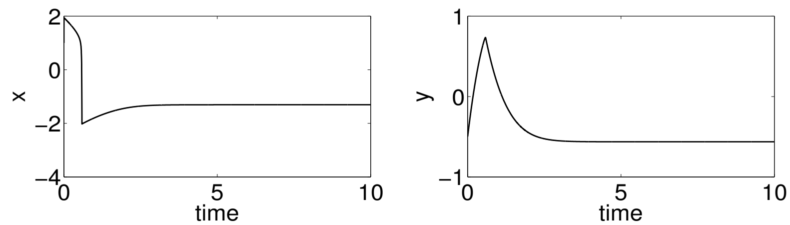

Figure 7, Extended FHN model, x and y plots as time series. $x_0 = 1$, $y_0 = -0.5$, $\gamma = 0.9$, $a = 0.80$ (excitable), and $\varepsilon = 0.005$

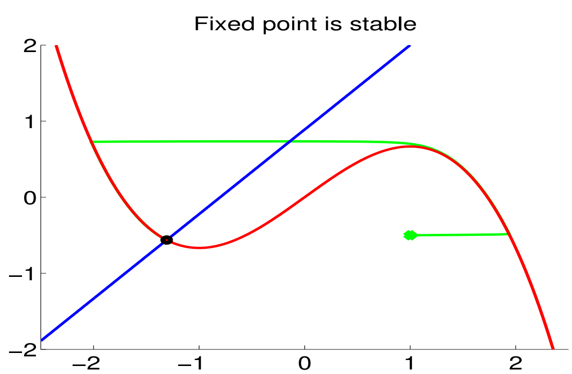

Figure 8, Extended FHN model, x and y in state space. $x_0 = 1$, $y_0 = -0.5$, $\gamma = 0.9$, $a = 0.80$ (excitable), and $\varepsilon = 0.005$

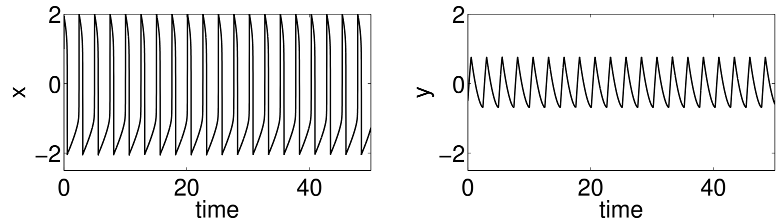

Figure 9, Extended FHN model, x and y plots as time series. $x_0 = 1$, $y_0 = -0.5$, $\gamma = 0.1$, $a = 0.80$, and $\varepsilon = 0.005$

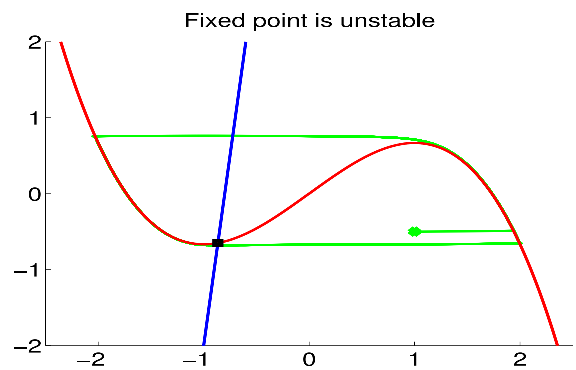

Figure 10, Extended FHN model, x and y in state space. $x_0 = 1$, $y_0 = -0.5$, $\gamma = 0.1$, $a = 0.80$, and $\varepsilon = 0.005$

2 Execution of Python and MATLAB Scripts

- Create a matrix with threshold:

```
python        threshold_matrix.py  <arg1>  <arg2>
python        threshold_matrix.py   A.txt    0.5
```

  that means the data matrix *A.txt* (or n_{ij}) is converted with threshold value 0.5 into another matrix (new matrix - f_{ij}). The elements of new matrix are $[0, 1]$, the values below the given threshold are 0, otherwise 1. The new matrix is saved as *A_r0.5.dat*

- Simulation of neural activity: time evolution of activator and inhibitor

```
python fhn_time_delays.py <arg1> <arg2> <arg3> <arg4>
```

  $< arg1 >$: f_{ij}, functional connectivity matrix (thresholded form)
  $< arg2 >$: d_{ij}, matrix; Euclidean distances between nodes in brain
  $< arg3 >$: c, coupling constant
  $< arg4 >$: D, noise strength

- $[VUK13]$ - some theoretical approach to the command-line above:

$$\dot{u}_i = g(u_i, v_i) - c\sum_{j=1}^{N} f_{ij} u_j(t - \Delta t_{ij}) + n_u \tag{5}$$

$$\dot{v}_i = h(u_i, v_i) + n_v \tag{6}$$

  where c is coupling strength, f_{ij} is the connectivity matrix ($i, j = 1, 2, ..., N$, here $N = 64$), Δt_{ij} is time delay due to finite signal propagation velocity between nodes, n_u is the noise factor. Δt_{ij} is calculated as $\Delta t_{ij} = \frac{d_{ij}}{\nu}$, distance matrix divided by velocity and noise factor is includes the noise strength D.

  The functions g and v are modeleds very similar to FitzHugh-Nagumo model introduced before:

$$\dot{u} = g(u, v) = \tau\left(v + \gamma u - \frac{u^3}{3}\right) \tag{7}$$

$$\dot{v} = h(u, v) = -\frac{1}{\tau}\left(u - \alpha + bv - I\right) \tag{8}$$

- The outcome of the $fhn\_time\_delays.py$:

  where $I = 0$ in further calculations.

$$simfile = \begin{bmatrix} 0 & u_{11} & v_{11} & u_{21} & v_{21} & . & . & . & u_{N1} & v_{N1} \\ dt & u_{12} & v_{12} & u_{22} & v_{22} & . & . & . & u_{N2} & v_{N2} \\ 2dt & u_{13} & v_{13} & u_{23} & v_{23} & . & . & . & u_{N2} & v_{N3} \\ 3dt & . & . & . & . & . & . & . & u_{N3} & v_{N3} \\ 4dt & . & . & . & . & . & . & . & u_{N4} & v_{N4} \\ . & . & . & . & . & . & . & . & . & . \\ t_{max} & . & . & . & . & . & . & . & u_{NN} & v_{NN} \end{bmatrix}$$

- Observe attractors's activity as time series

```
MATLAB >>    calcBOLD.m >>    calcBOLD('simfile')
```

The program *calcBOLD.m* firstly eliminates all the u_i time series from the input i.e. $A\_r0.50.dat$ and plots the total time versus all the u_i series for each node, $i = 1, 2, ..., 64$.

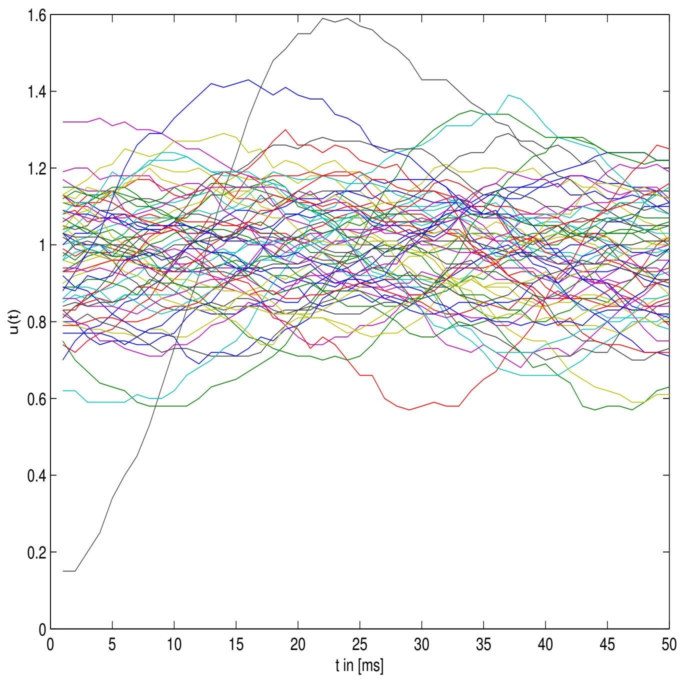

Figure 11, Extended FHN model, x and y in state space. Threshold applied to n_{ij} matrix is $r = 0.5$, (the resulting f_{ij} matrix is i.e. $A\_r0.50.dat$) coupling constant $c = 0$, noise strength $D = 0.05$ and velocity of signal propagation $v = 7m/s$. The modelled (with FHN model) neural activity $u(t)$ for each node (N=64 node in total)on the y axes.

- Simulated Bold activity with Baloon-Windkessel model

  The resulting time series of the modelled neural activity $u(t)$ is used to infer the BOLD signal versus time observed in fMRI data via Baloon-Windkessel model. The purpose of the project is to be able observe how well the simulated BOLD signal correlates with the emprical fMRI signal. Here is an example of how the simulated BOLD signal might look like.

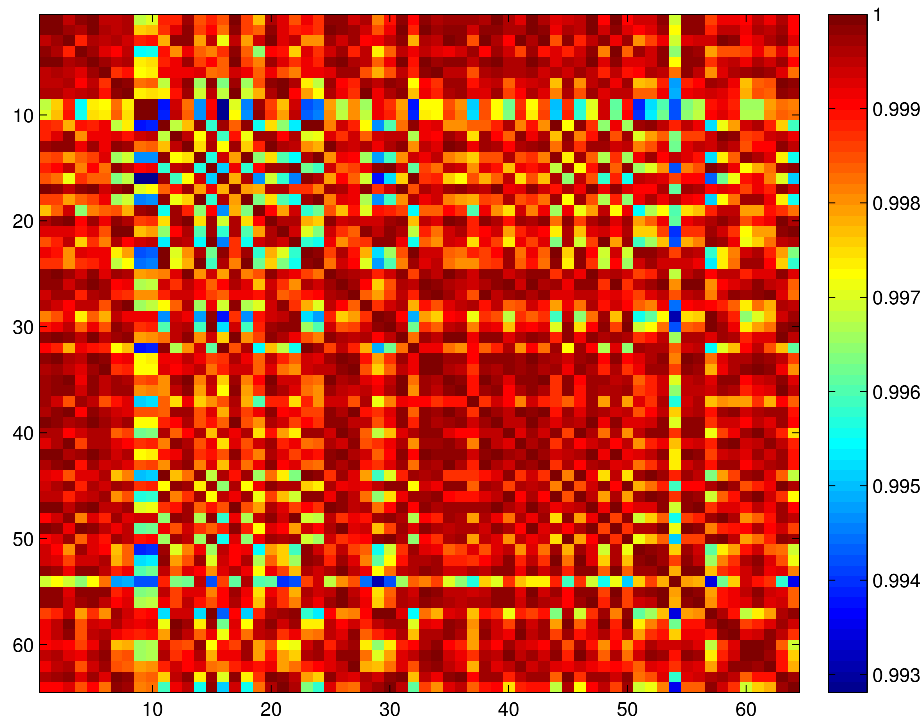

Figure 12, correlation matrix of simulated BOLD, coupling strength $c = 0$ and noise strength $D = 0.01$, $v = 7m/s$.