LAB ROTATION 02
2nd Week Assignment

Şeyma Bayrak, Advisor: Philipp Hövel

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1 Nonlinear Dynamics of Neural Networks

1.1 Simple FitzHugh-Nagumo Model

This section is a continuing part of *1st Week Assignment*, it aims to analyze the effect of parameters ε and γ in the equations (1) and (2) given below and to plot trajectories on nullcline graphs.

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y \tag{1}$$

$$\dot{y} = x + a \tag{2}$$

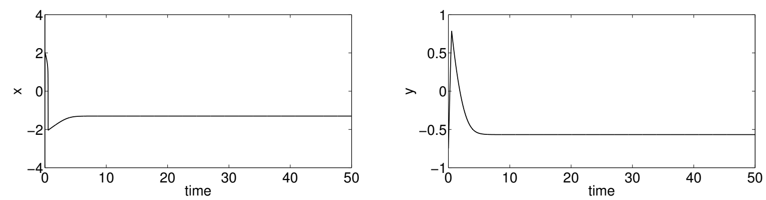

Figure 1, $a = 1.30$, $\varepsilon = 0.005$, $(x_0, y_0) = (-0.05, -0.75)$

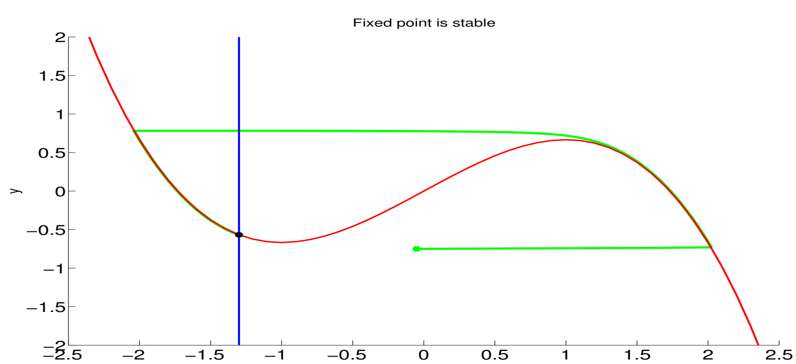

Figure 2, $a = 1.30$, $\varepsilon = 0.005$, $(x_0, y_0) = (-0.05, -0.75)$

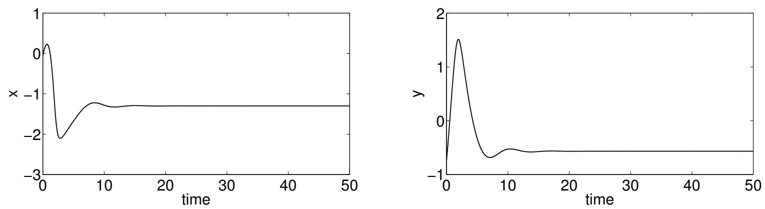

Figure 3, $a = 1.30$, $\varepsilon = 1$, $(x_0, y_0) = (-0.05, -0.75)$

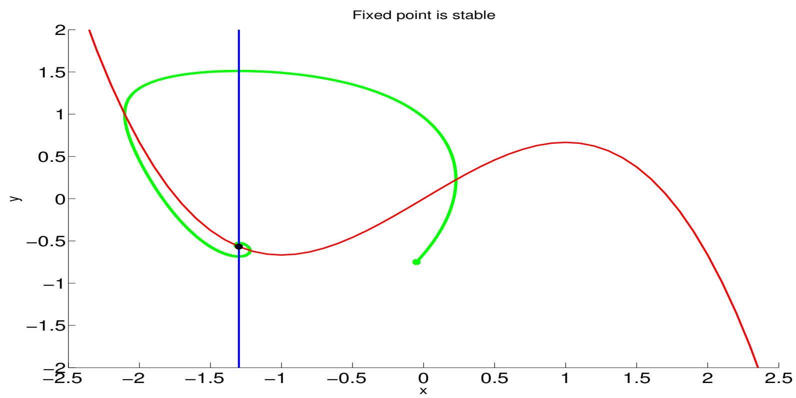

Figure 4, $a = 1.30$, $\varepsilon = 1$, $(x_0, y_0) = (-0.05, -0.75)$

- a effect: Bifurcation anaylsis done in 1st Week Assignment showed that, as long as $|a| > 1$, the system is expected to be stable. Figures 1-4 have $a = 1.30$ and they are all stable, or in other words "excitable".

- ε effect: it does not affect stability but plays a role in the time evolution and pathway of initial points x_0 and y_0 - how long it takes initial point to reach to the stable point. When ε is small, x and y reaches stability faster. (?)

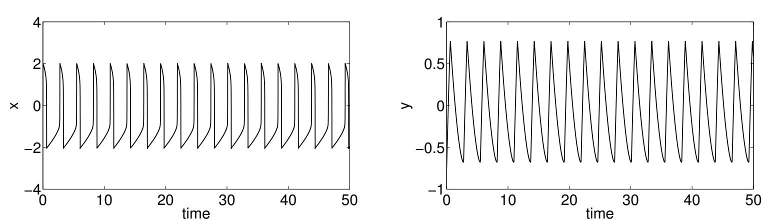

Figure 5, $a = 0.90$, $\varepsilon = 0.005$, $(x_0, y_0) = (-0.05, -0.75)$

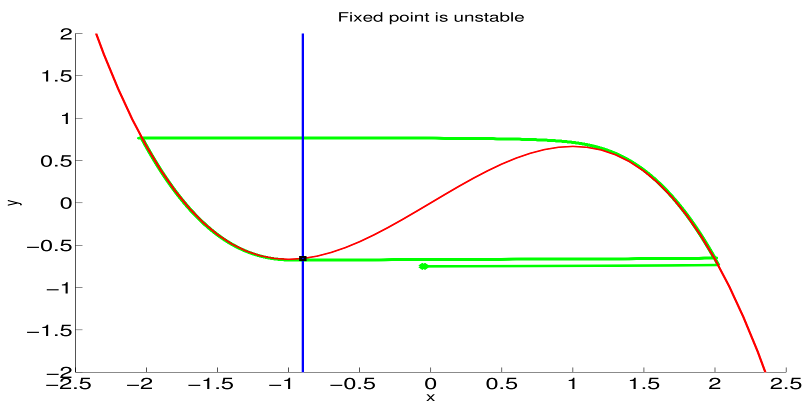

Figure 6, $a = 0.90$, $\varepsilon = 0.005$, $(x_0, y_0) = (-0.05, -0.75)$

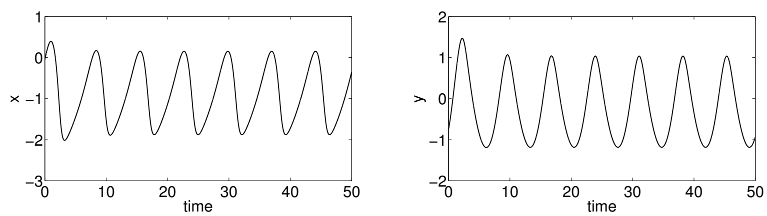

Figure 7, $a = 0.90$, $\varepsilon = 1$, $(x_0, y_0) = (-0.05, -0.75)$

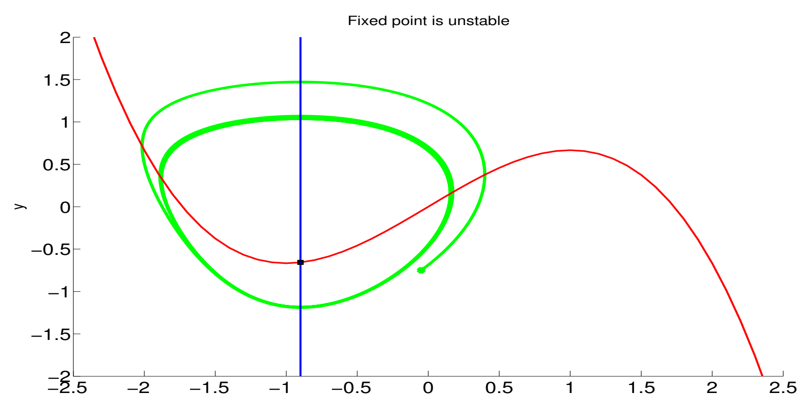

Figure 8, $a = 0.90$, $\varepsilon = 1$, $(x_0, y_0) = (-0.05, -0.75)$

- a effect: When $|a| < 1$, the system is unstable. Figures 5 and 7 have oscillations with $a = 0.90$.

- ε effect: It affects the period of oscillations, when ε is small, oscillatory behavior of x and y happens more frequently. (What about Figure 6 and 8?)

1.2 Extended FitzHugh-Nagumo Model

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y \tag{3}$$

$$\dot{y} = x + a - \gamma y \tag{4}$$

Fixed point can be now *stable, unstable, saddle.*
Saddle point means that the signs of real parts of the eigenvalues of Jacobian matrix are different. We would like to eliminate saddle points.(?)

$$\mathbf{J} = \begin{pmatrix} (1 - x_f)^2 & -1 \\ \varepsilon & -\varepsilon\gamma \end{pmatrix}$$

$$\lambda_{1,2} = \frac{trJ \pm \sqrt{tr^2J - 4detJ}}{2} = \frac{trJ \pm trJ(\sqrt{1 - \frac{4detJ}{tr^2J}})}{2}$$

In order to keep the real part of eigenvalue to be dominated by the first term of the equation above (trJ), the term in square root must be either positive and smaller than 1 or it must be negative, which contributes to λ only with a complex part. This is done by assuming $det(J) > 0$. Then the sign of the eigenvalues is controlled directly by trJ, it is either positive for λ_1 and λ_2 or negative.

$$det(J) = \varepsilon(\gamma(x_f^2 - 1) + 1) > 0 \Longrightarrow 0 < \gamma < 1$$

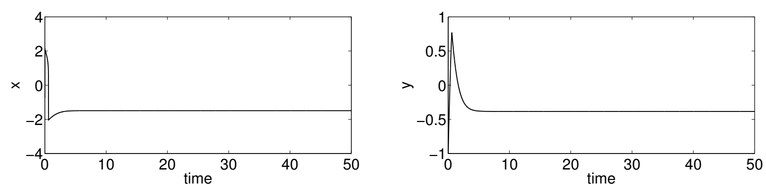

Figure 9, $a = 1.30$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

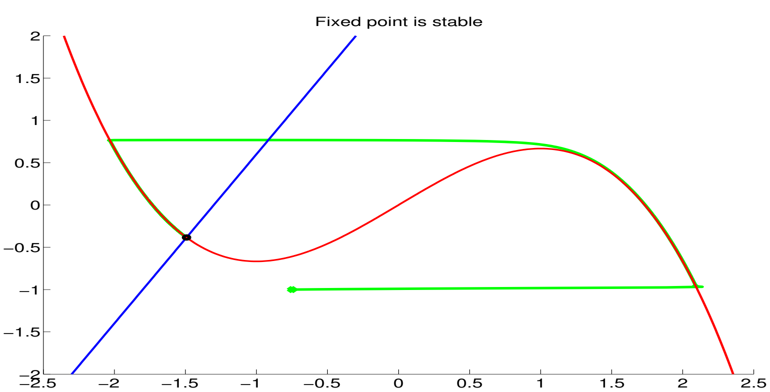

Figure 10, $a = 1.30$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

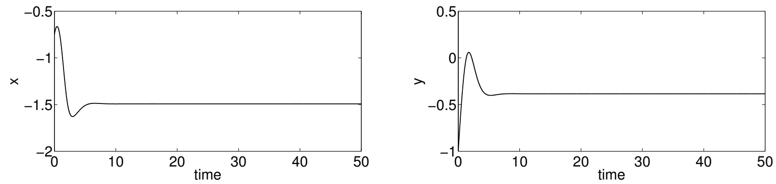

Figure 11, $a = 1.30$, $\varepsilon = 1$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

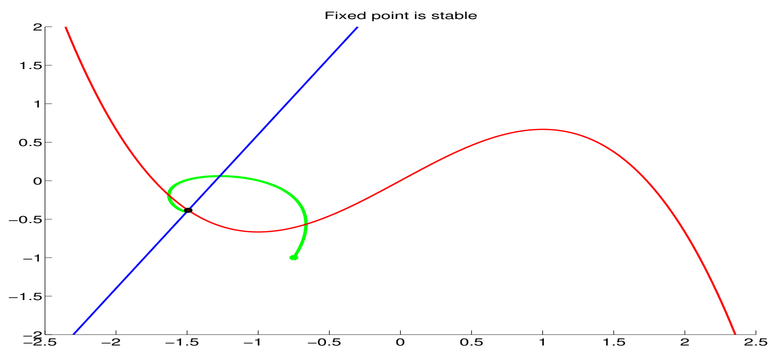

Figure 12, $a = 1.30$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

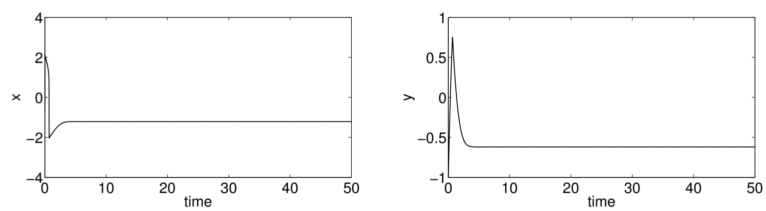

Figure 13, $a = 0.90$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

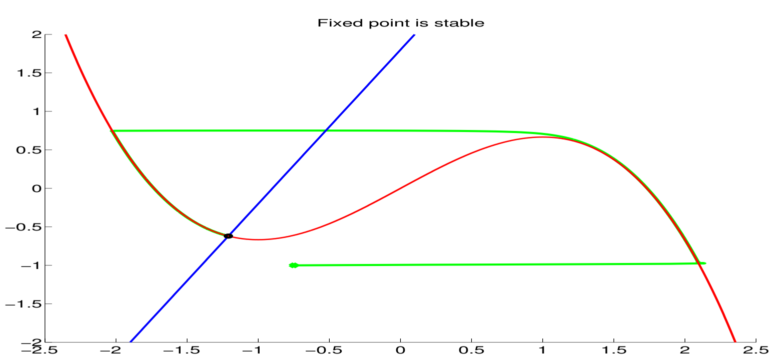

Figure 14, $a = 0.90$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

- a effect: When $|a| < 1$, the system was unstable in simple FHN model, however the stability in extended FHN model is now controlled by not only a but also γ. Figures 13 and 14 shows stability with $|a| < 1$ and $\gamma = 0.5$ in opposite to the Figures 5-8.

- ε effect: It affects again the time period of initial points to reach to the stability and pathway of trajectory. Smaller ε brings stability faster in Figure 13 compared to Figure 11.

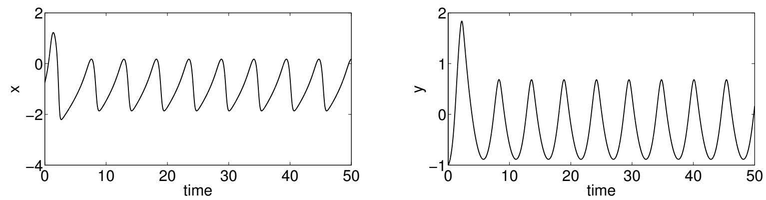

Figure 15, $a = 0.90$, $\varepsilon = 0.4$, $\gamma = 0.05$, $(x_0, y_0) = (-0.75, -1)$

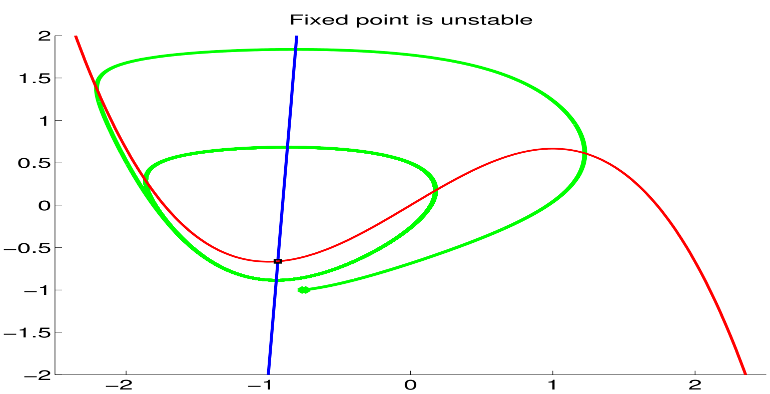

Figure 16, $a = 0.90$, $\varepsilon = 0.04$, $\gamma = 0.05$, $(x_0, y_0) = (-0.75, -1)$

- γ effect: This parameter controls the stability or instability of the system. We already assumed γ between 0 and 1 as it was discussed in bifurcation analysis of extenden FHN model. When γ is close to 0, then the system acts unstable and x-y seem to oscillate. (?) Figure 13 ($\gamma = 0.5$) turns into a series of oscillatory behavior for x and y in Figure 15 ($\gamma = 0.05$). Here I did not necessariyl keep ε at the same values, but we are already sure that ε has nothing to do with stability.

2 Correlation Matrix of Functional Connectivity

The provided data file *A.txt* is an example of fMRI signals reflecting functional connectivities of a mammalian brain at resting state. *A.txt* is a simple NxN matrix, where $N = 64$, meaning that the brain is assumed to have $N = 64$ functionally separated nodes. The connectivity matrix can be applied to a threshold value and then converted into a new matrix, which has 1 for the elements of matrix greater than the threshold and 0 for the smaller ones.

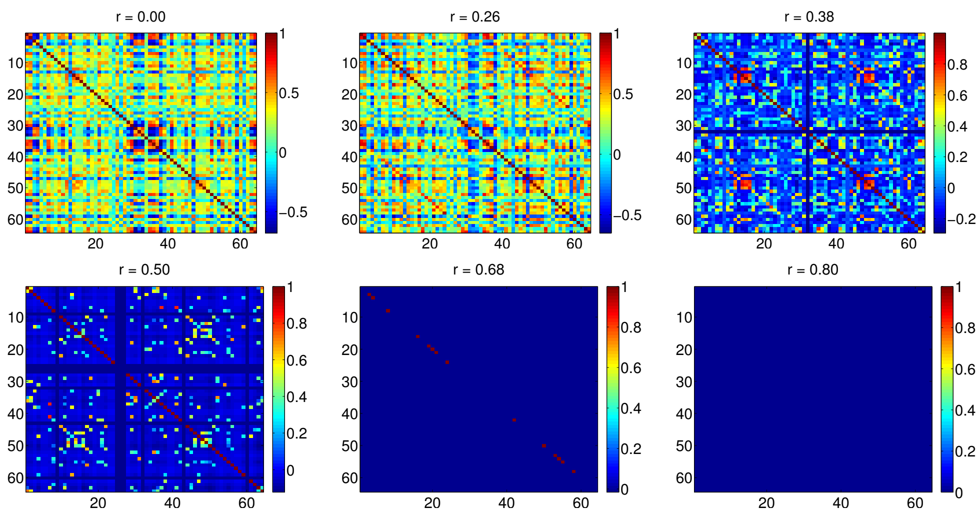

Figure 17, r given on top of subfigures stands for the threshold values applied to the functional connectivity matrix f_{ij} (here f_{ij} corresponds to *A.txt*). Both of the x and y axis have scales from 0 to 64.

3 Isolated FitzHugh-Nagumo Neural Model

$[VUK13]$ paper simulates the neural network dynamics of one node with u_i (attractor) and v_i (inhibitor) by using FHN model as in the following eqautions;

$$\dot{u}_i = g(u_i, v_i) - c \sum_{j=1}^{N} f_{ij} u_j (t - \Delta t_{ij}) + n_u \tag{5}$$

$$\dot{v}_i = h(u_i, v_i) + n_v \tag{6}$$

where c is coupling strength, f_{ij} is the connectivity matrix, Δt_{ij} is time delay due to finite signal propagation velocity between nodes, n_u is the noise factor. Δt_{ij} is calculated as $\Delta t_{ij} = \frac{d_{ij}}{\nu}$, distance matrix divided by velocity and noise factor is includes the noise strength D. Note that $i, j = 0, 1, 2, ..., N$

The functions g and v are modeled very similar to FitzHugh-Nagumo model introduced before:

$$\dot{u} = g(u, v) = \tau(v + \gamma u - \frac{u^3}{3}) \quad (7)$$

$$\dot{v} = h(u, v) = -\frac{1}{\tau}(u - \alpha + bv - I) \quad (8)$$

This section aims to observe the noise strength D on an isolated system meaning no coupling (simply by $c = 0$). Figures below indicate the attractor behavior of the first node u_1 over time with different noise strengths.

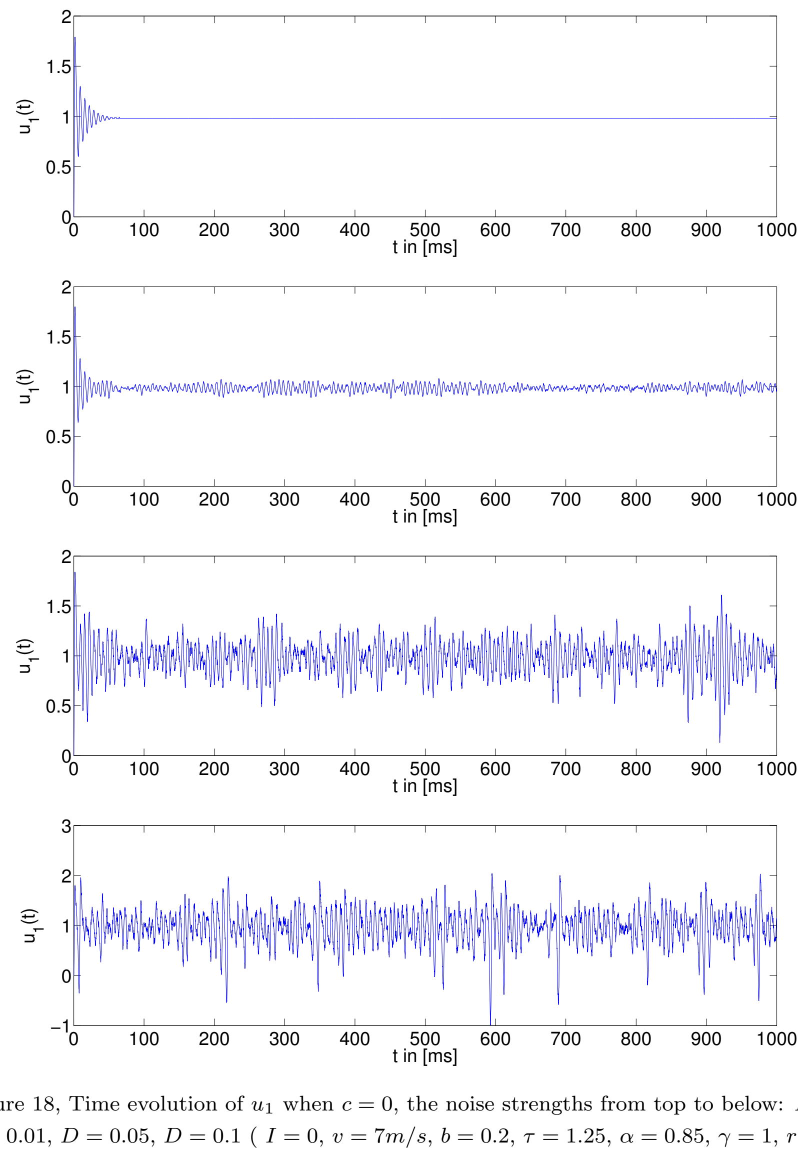

Figure 18, Time evolution of u_1 when $c = 0$, the noise strengths from top to below: $D = 0$, $D = 0.01$, $D = 0.05$, $D = 0.1$ ($I = 0$, $v = 7m/s$, $b = 0.2$, $\tau = 1.25$, $\alpha = 0.85$, $\gamma = 1$, $r = 0.5$)

Now, let us have a look at the attractor behavior of all the isolated nodes $u_i(t)$, $i = 1, 2, 3, ..., N$ in $t = 500ms$ with different noise strengths. (?=compare with section 1 and stability of figures below)

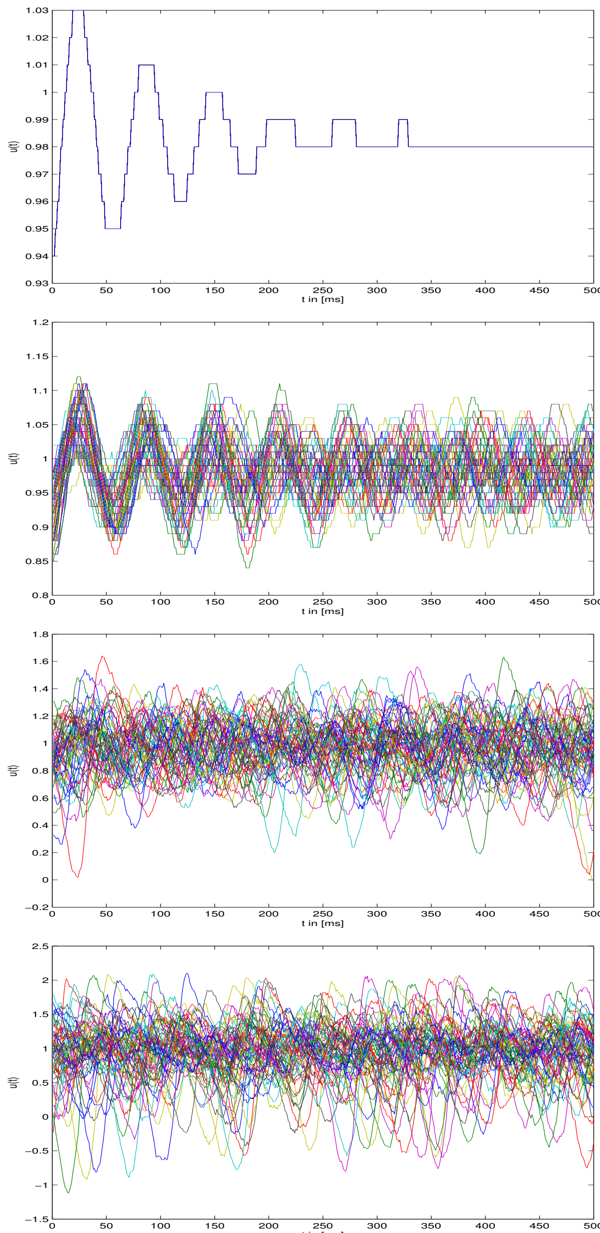

Figure 19, Time evolution of $u_i(t)$ when $c = 0$, the noise strengths from top to below: $D = 0$, $D = 0.01$, $D = 0.05$, $D = 0.1$. ($I = 0$, $v = 7m/s$, $b = 0.2$, $\tau = 1.25$, $\alpha = 0.85$, $\gamma = 1$, $r = 0.5$)

4 Distance Distribution between Cortical Regions

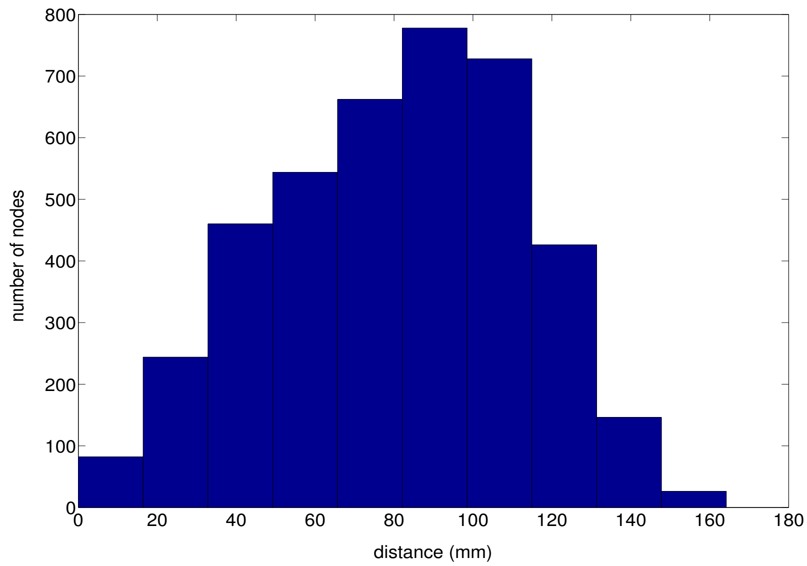

Figure 20, Distance distribution between the $N = 64$ nodes, source: d_{ij} =*FSL_ROIs_distance_matrix.dat*, the distances between nodes are Euclidean.

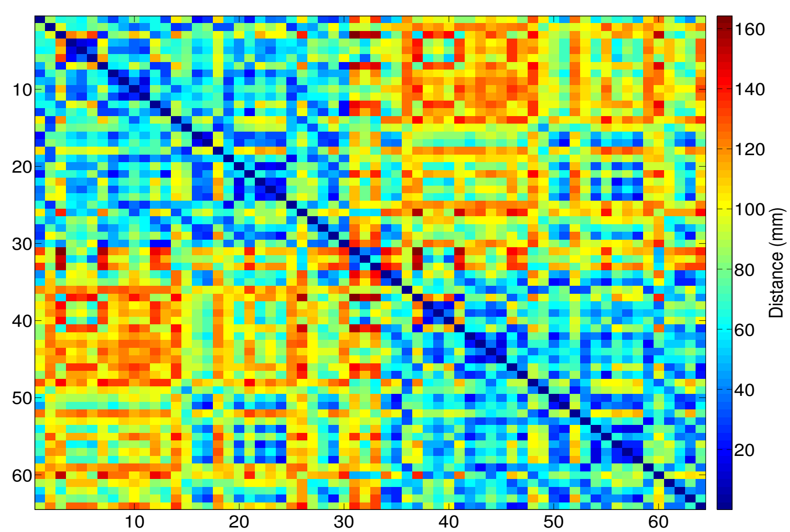

Figure 21, Euclidean distance matrix d_{ij} in color code

5 Correlation Distribution of fMRI Data

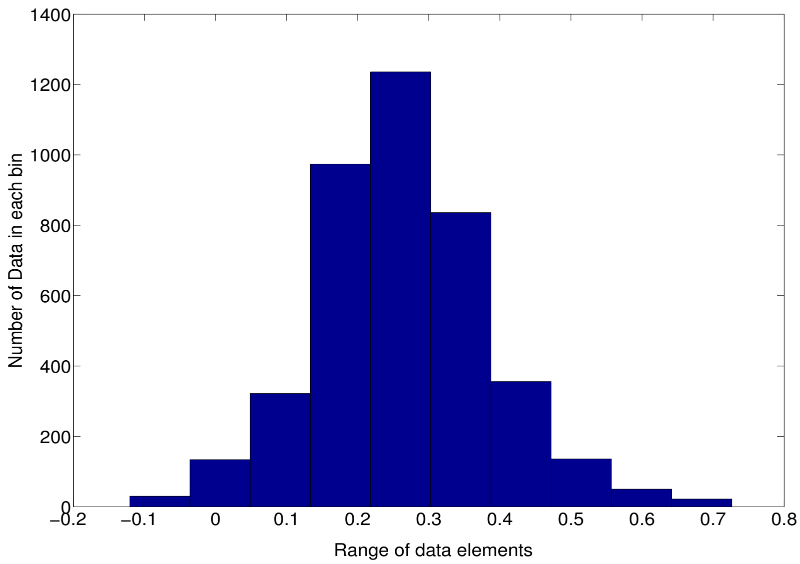

Figure 22, Data distribution in functional connectivity matrix as a result of fMRI signaling, f_{ij}=$A.txt$, where $i, j = 1, 2, ..., N = 64$

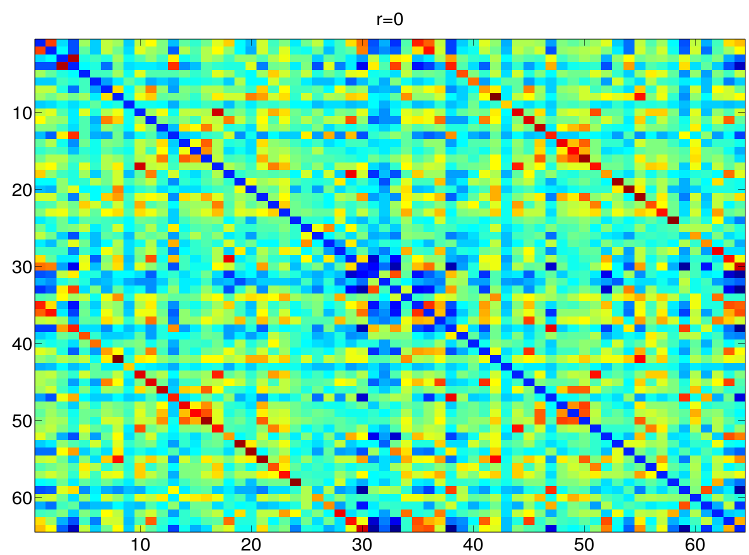

Figure 23, Functional connectivity matrix f_{ij} in color code. f_{ij}=$A.txt$, there is no threshold applied on it, $r = 0$

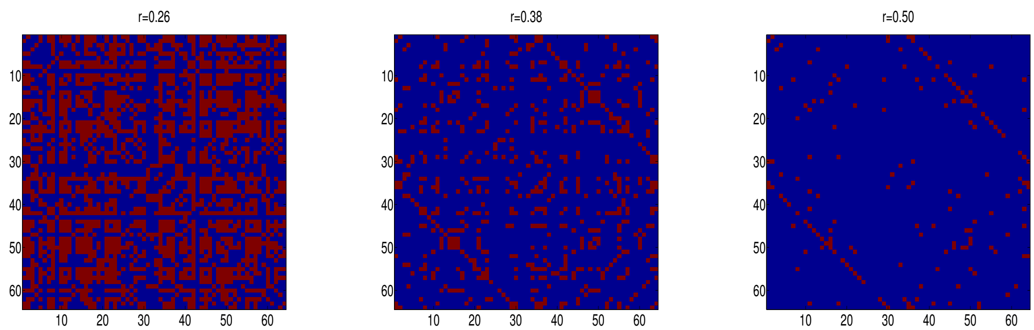

Figure 24, Functional connectivity matrices f_{ij} with applied thresholds in color code, red color for the ones, and blue for the zeros. Figure on the left: f_{ij}=*A_r.0.26.dat*, in the middle: f_{ij}=*A_r.0.38.dat*, on the right: f_{ij}=*A_r.0.50.dat*

6 Visualization of f_{ij} in 2D Anatomical Space with different rs

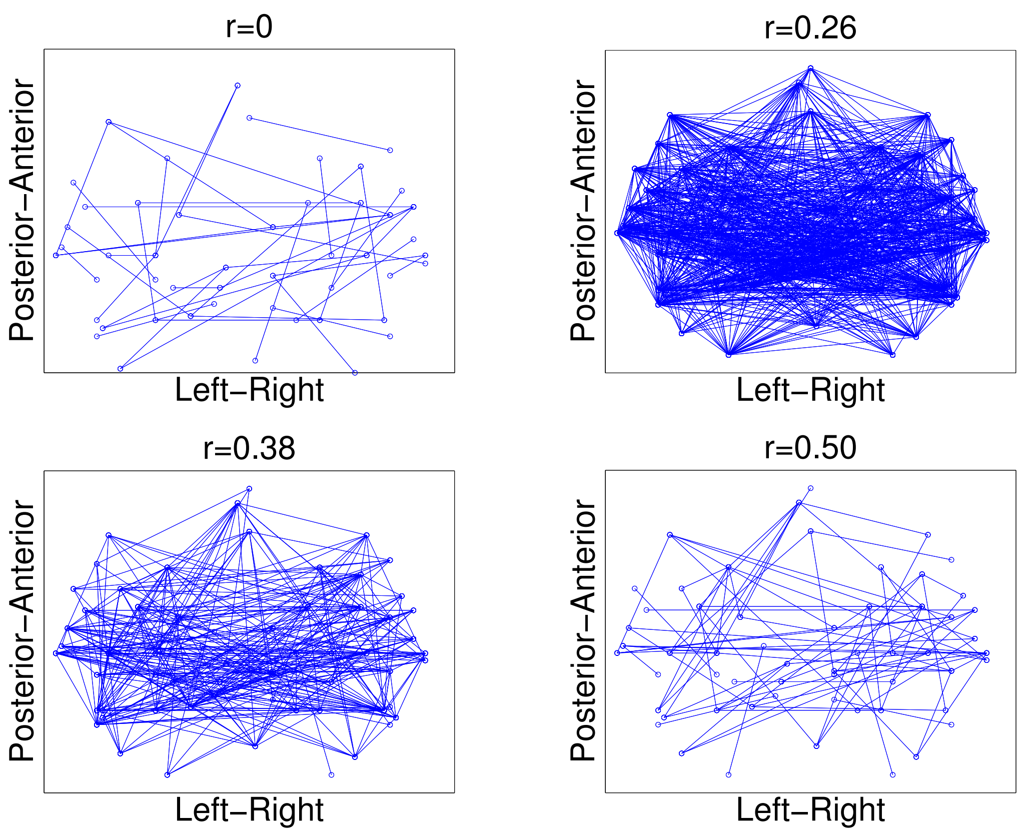

Figure 25, Visualization of threshold matrices in anatomical space by locating each region according to its x and y coordinates and drawing a link between significantly connected regions. (?= why r=0 is similar to r=0.5)

7 Visualization of f_{ij} in 3D Anatomical Space with different rs

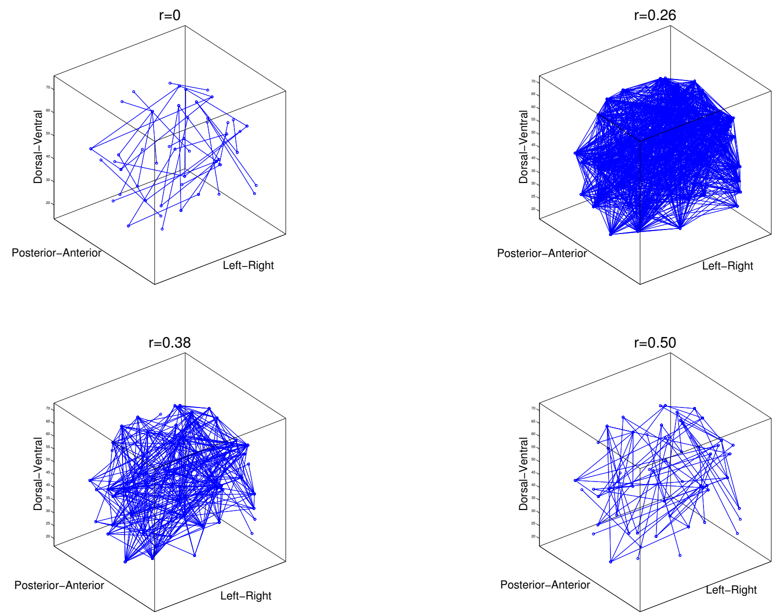

Figure 26, Visualization of threshold matrices in anatomical space by locating each region according to its x, y and z coordinates and drawing a link between significantly connected regions. (?= why r=0 is similar to r=0.5)

This part is not yet completed.

8 Simulated Bold Signals - The new u_i time series with Baloon-Windkessel Model

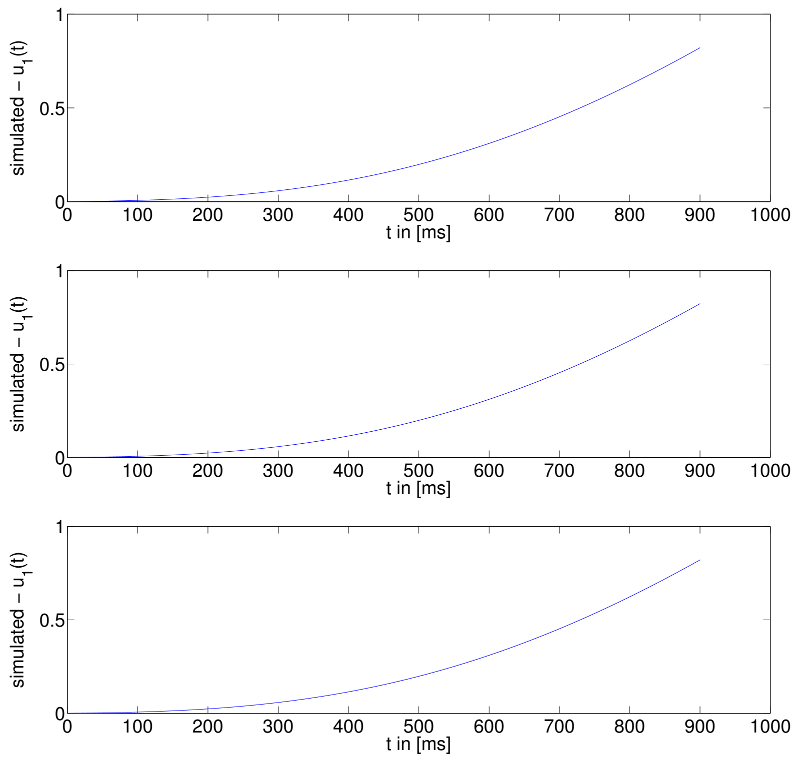

Figure 27, Time evolution of u_1 when $c = 0$, the noise strengths from top to below: $D = 0$, $D = 0.01$, $D = 0.05$, $D = 0.1$ ($I = 0$, $v = 7m/s$, $b = 0.2$, $\tau = 1.25$, $\alpha = 0.85$, $\gamma = 1$, $r = 0.5$)

- Discuss difference between Figure 25 and Figure 18 and shorter time now(?)
- When all the u_i series plotted over time, it does not seem clearly different than the $u_1(t)$ evolution. (?)

When "Butterworth lowpass filter of order 5" is applied to the simulated BOLD signalling, then the u_i time series look like as in the following figures.

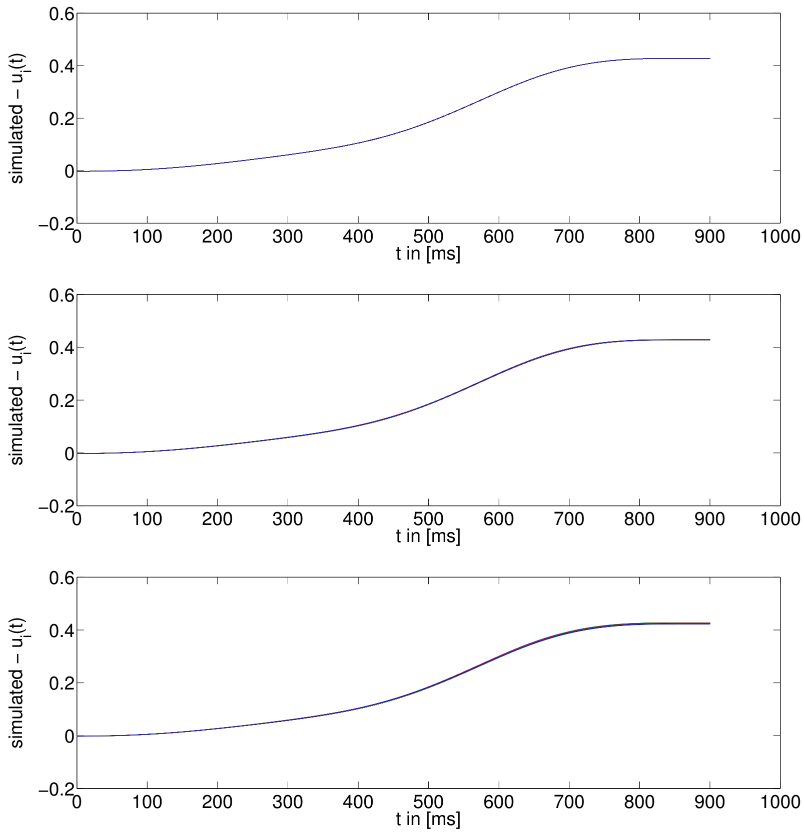

Figure 28, Time evolution of u_i when $c = 0$, the noise strengths from top to below: $D = 0$, $D = 0.01$, $D = 0.05$, $D = 0.1$ ($I = 0$, $v = 7m/s$, $b = 0.2$, $\tau = 1.25$, $\alpha = 0.85$, $\gamma = 1$, $r = 0.5$)

- Discuss filtering with Vesna (?)

9 To be continued... FHN Model

How to change parameters in [VUK] paper in order to get similar state space graph as in the extenden FHN model? Answer:

```
gamma=0.9;                    %gamma close to 1
b=-0.2;                       %change + incline to - incline
x_limit=2.5;
y_limit=2;
xE=(-x_limit:0.01:x_limit);
yE1=-xE.^3/3 + gamma*xE;  % make yE totally minus
yE2=(alpha-xE)/b;
```

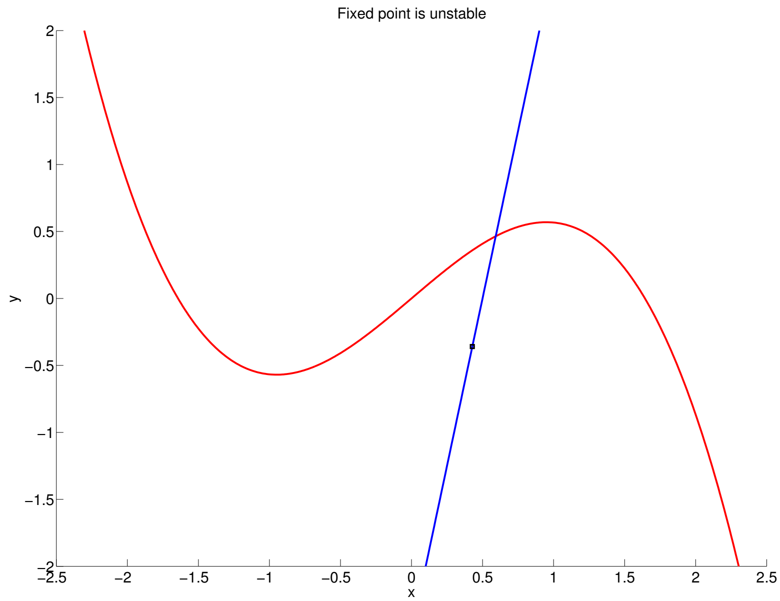

Figure 29

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