LAB ROTATION 02
3rd Week Assignment

Şeyma Bayrak, Advisor: Philipp Hövel

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1 Extended FHN Model with Noise Factor

$$\varepsilon\dot{x} = (x - \frac{x^3}{3} - y) + n_x \qquad \dot{y} = x + a - \gamma y + n_y \tag{1}$$

where n_x and n_y are additional Gaussian White Noise factors. Let us say that Gaussian white noise factors n_x and n_y have two components: one is the noise distribution $f(r)$ and the other one is the strength D, such that $n_x = D_x.f(r_x)$ and $n_y = D_y.f(r_y)$. For simplicity, I would like choose both of the strengths and noise distributions equally, $n_x = n_y = n = D.f(r)$.

- Does noise factors affect the fixed point ?

  The noise factors are not included in nullclines, but only in time evoulution of $x(t)$ and $y(t)$. Therefore it has no effect on fixed point. x-nullcline : $0 = (x - \frac{x^3}{3} - y)$
  y-nullcline : $0 = x + a - \gamma y$
  Let us solve the nullclines to eliminate the fixed point on MATLAB:
  $(x - \frac{x^3}{3} - y) + n = 0 \Longrightarrow y = x - \frac{x^3}{3}$
  $x + a - \gamma y + n \Longrightarrow y = \frac{x+a}{\gamma}$
  Analytical solution for fixed point (x_f, y_f):
  $x_f = fsolve(@(x)\ \ x(\gamma - 1) - \frac{x^3}{3}\gamma - a)$ and $y_f = \frac{x_f + a}{\gamma}$

- Does noise factors affect the stability ? For the stability analysis, the egienvalues of the Jacobian matrix must be analyzed. The Jacobian Matrix of the extended FHN model is given below:

  $$\mathbf{J} = \begin{pmatrix} 1 - x_f^2 & -1 \\ \varepsilon & -\varepsilon\gamma \end{pmatrix}$$

  The Jacobian matrix does not seem different than the one eliminated for the extended FHN model without noise factors in previous assignments. Therefore, it might be reasonable to state that the noise factor does not decide the stability or instability of the system, which is decided by the eigenvalues of the Jacobian matrix.

1.1 Gaussian White Noise Vector on MATLAB

A random vector having real numbers, probability distribution with zero mean and finite variance, elements of the vector are indipendent.

```
wgn Generate white Gaussian noise.
    Y = wgn(M,N,P) generates an M-by-N matrix of white
    Gaussian noise. P specifies the power of the output
    noise in dBW. The unit of measure for the output of
    the wgn function is Volts. For power calculations,
    it is assumed that there is a load of 1 Ohm.
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1.2 Related Figures to Observe the Noise Effect

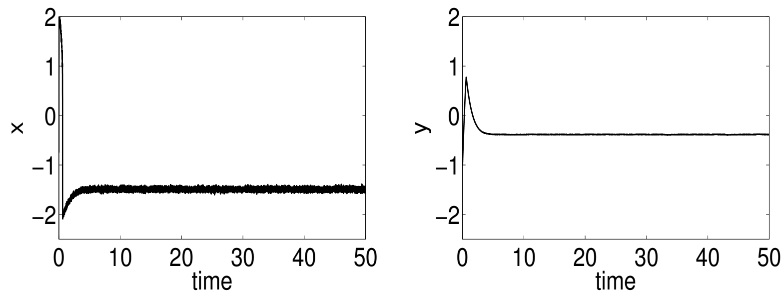

Figure 1, $D = 0.05$, $a = 1.30$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$.

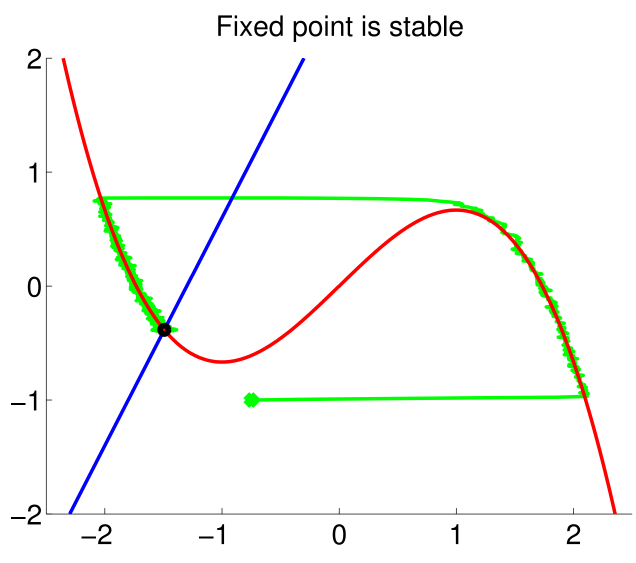

Figure 2, $D = 0.05$, $a = 1.30$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$.

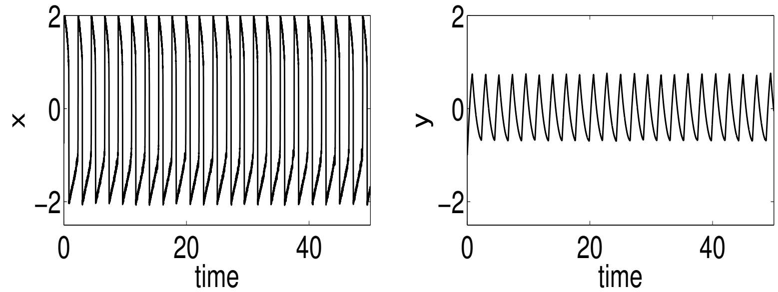

Figure 3, $D = 0.05$, $a = 0.50$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

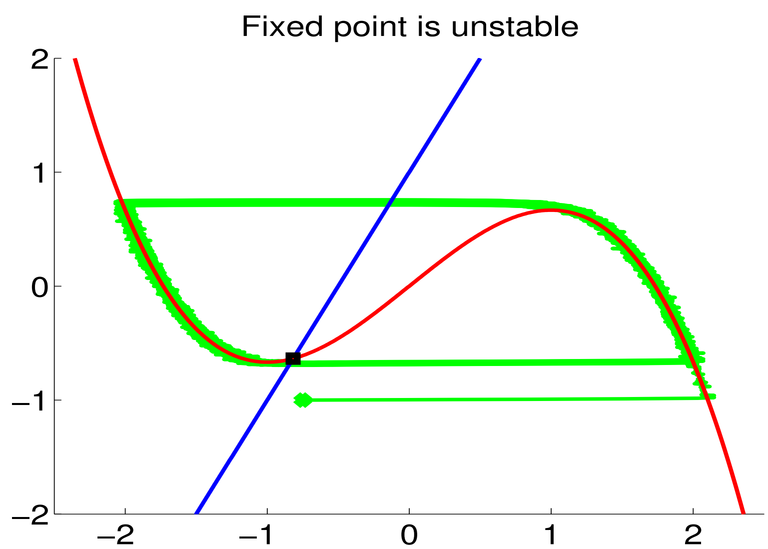

Figure 4, $D = 0.05$, $a = 0.50$, $\varepsilon = 0.005$, $\gamma = 0.5$, $(x_0, y_0) = (-0.75, -1)$

- Noise strength D brings fluctuations to the time evolution of x and y, however its effect is more on $x(t)$ compared to the $y(t)$, since as equation (1) states, $\dot{x}$ is constructed by time-constant-like value ε differently than $\dot{y}$.

- The fluctuations of $x(t)$ and $y(t)$ in figures 1 and 3 can be additionally followed as their state space paths (green) in figure 2 and 4.

- The parameter a plays role to determine the stability of the system, but one should not forget that a is not the only parameter determining stability, but also γ. The γ effect will be introduced in the followig figures below.

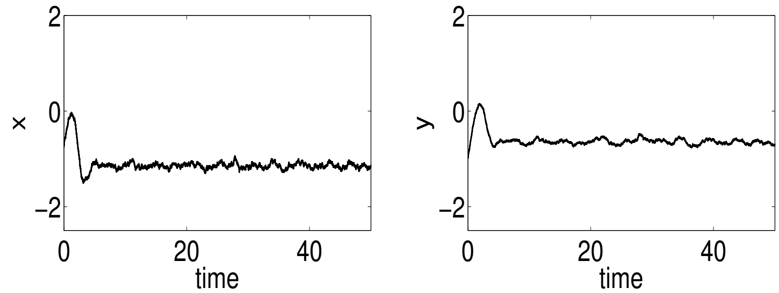

Figure 5, $D = 1$, $a = 0.50$, $\varepsilon = 0.5$, $\gamma = 1$, $(x_0, y_0) = (-0.75, -1)$

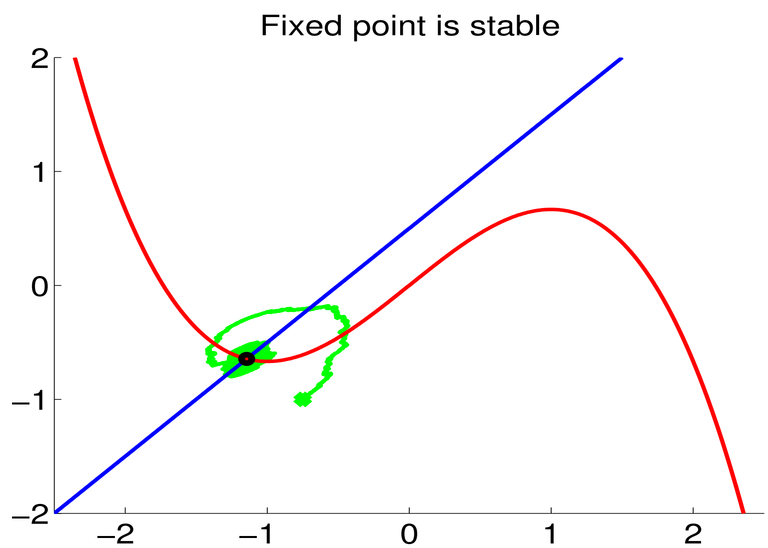

Figure 6, $D = 1$, $a = 0.50$, $\varepsilon = 0.5$, $\gamma = 1$, $(x_0, y_0) = (-0.75, -1)$

- Even though the parameter a is kept as small as in figure 4, the increased γ parameter converts the unstable system into a stable form.
- The ε can be thought as the width of the time window that we are interested in. In order to visualize system more understandable, I chose a bigger ε in figures 5 and 6, it has no effect on stability analysis as proven in previous assignments.
- A bigger noise strength makes the fluctuation amplitudes bigger, but it has no effect on stability. Noise effect

2 Our Model...

$$\dot{x_i} = g(x_i, y_i) - c\sum_{j=1}^{N} f_{ij}x_j(t - \Delta t_{ij}) + n_x \qquad \dot{y_i} = h(x_i, y_i) + n_y \tag{2}$$

where c is the coupling strength, n_x and n_y are noise factors as introduced in section 1. To visualize the attractor and inhobitor behaviors easier, I chose $c = 0$ for the further figures. Functions g and h are shown below.

$$g(x, y) = \dot{x} = \tau\left(y + \gamma x - \frac{x^3}{3}\right) \qquad h(x, y) = \dot{y} = -\frac{1}{\tau}(x - \alpha + by) \tag{3}$$

y-nullcline: $y = \frac{x^3}{3} - \gamma x$
x-nullcline: $y = \frac{\alpha - x}{b}$
Fixed point analytical solutions:
$x_f = fsolve((x)@ \quad (\frac{x^3}{3} + x(\frac{1}{b} - \gamma) - \frac{\gamma}{b}))$ and $y_f = \frac{\alpha - x_f}{b}$
Jacobian Matrix:

$$\mathbf{J} = \begin{pmatrix} (\gamma - x_f^2).\tau & \tau \\ -\frac{1}{\tau} & -\frac{b}{\tau} \end{pmatrix}$$

The eigenvalues of the Jacobian Matrix:

$$\lambda_{1,2} = \frac{-\left(\frac{b}{\tau} + (\gamma - x_f^2).\tau\right) \pm \sqrt{\left(-\frac{b}{\tau} - (\gamma - x_f^2).\tau\right)^2 - 4.(1 - b(x_f^2 - \gamma))}}{2} \tag{4}$$

Let us first plot the time evolution of x and y as stated in equation (4) and also the nullclines as trajectory, and then see the differences between our model and original extended FHN model.

2.1 Related Figures

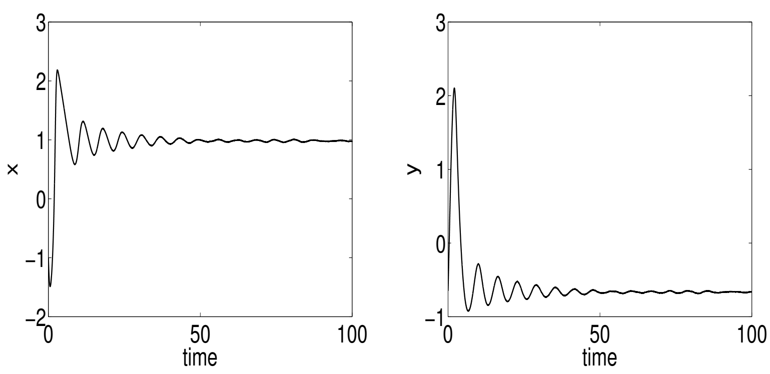

Figure 7, Left: time evolution of x and y separately, right: state space of x-y with trajectory. Parameters according to the equation (3): $\alpha = 0.85$, $\tau = 1.25$, $\gamma = 1.0$, $b = 0.2$, $x_0 = -1$, $y_0 = -0.65$, $D = 0.05$. The slightly fluctuations are due to the noise factor.

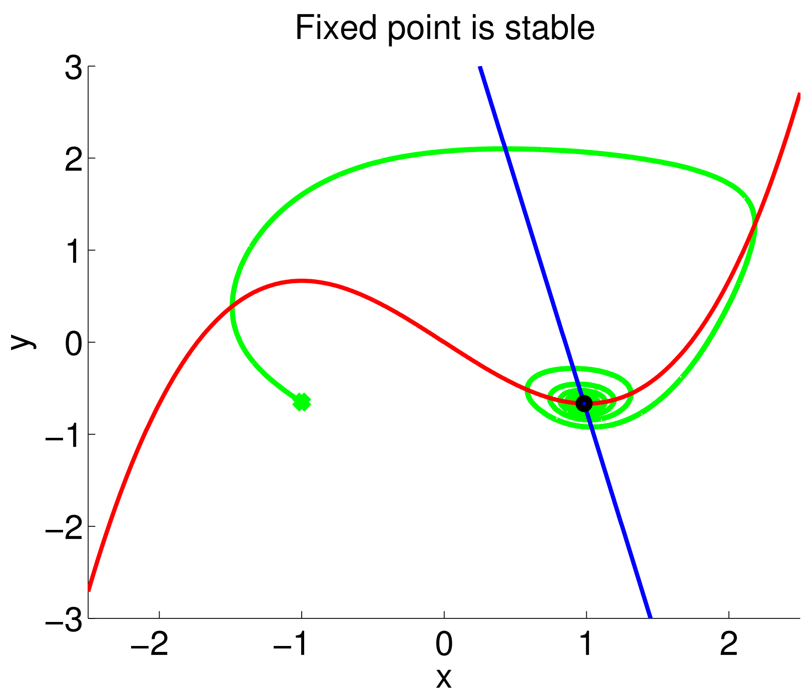

Figure 8, $\alpha = 0.85$, $\tau = 1.25$, $\gamma = 1.0$, $b = 0.2$, $x_0 = -1$, $y_0 = -0.65$, $D = 0.05$

- How to integrate the parameters in order to get a similar graph as in original FHN model ? Obviously, the incline of blue line, which is the parameter b actually, should be reversed. Additionally, the red curve, the nullcline $y = -\frac{x^3}{3} + \gamma x$ should be mirrored on y axis:

  $b \longrightarrow -b$
  $y\_nullcline \longrightarrow -y\_nullcline$
  The new Jacobian Matrix :

$$\mathbf{J} = \begin{pmatrix} (\gamma - x_f^2).\tau & -\tau \\ -\frac{1}{\tau} & -\frac{b}{\tau} \end{pmatrix}$$

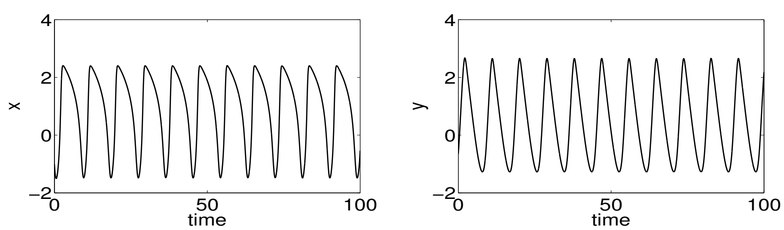

Figure 9, Parameters:$\alpha = 0.85$, $\tau = 1.25$, $\gamma = 1.0$, $b = -0.2$, $x_0 = -1$, $y_0 = -0.65$, $D = 0.05$

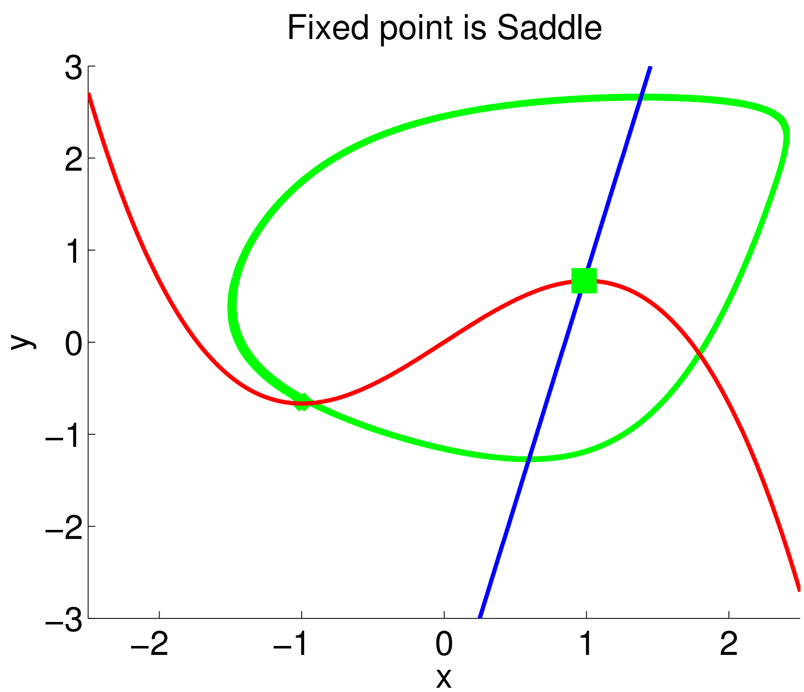

Figure 10, Parameters:$\alpha = 0.85$, $\tau = 1.25$, $\gamma = 1.0$, $b = -0.2$, $x_0 = -1$, $y_0 = -0.65$, $D = 0.05$

3 Simulated Functional Connectivity Matrices

This section aims to see the effect of the coupling strength, threshold value, and velocities on simulated BOLD activity. Since the program execution takes a long time for the original emprical functional connectivity matrix n_{ij} (A.txt), I preferred to generate my own smaller matrix by subtracting a 16x16 matrix from the original n_{ij} matrix, and then extract the simulated BOLD activity by using the new smaller matrix. (I also extracted 16x16 distance matrix)

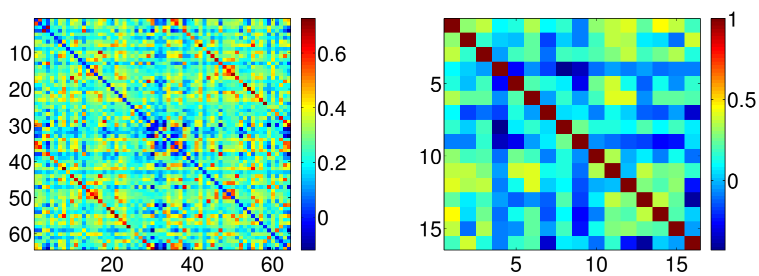

Figure 11, Left: the functional connectivity matrix for the emprical data having $N = 64$ nodes, right: that of $N = 16$ nodes

3.1 The Effects of Coupling Strength and Threshold Values on Correlation Matrices of the BOLD Activity

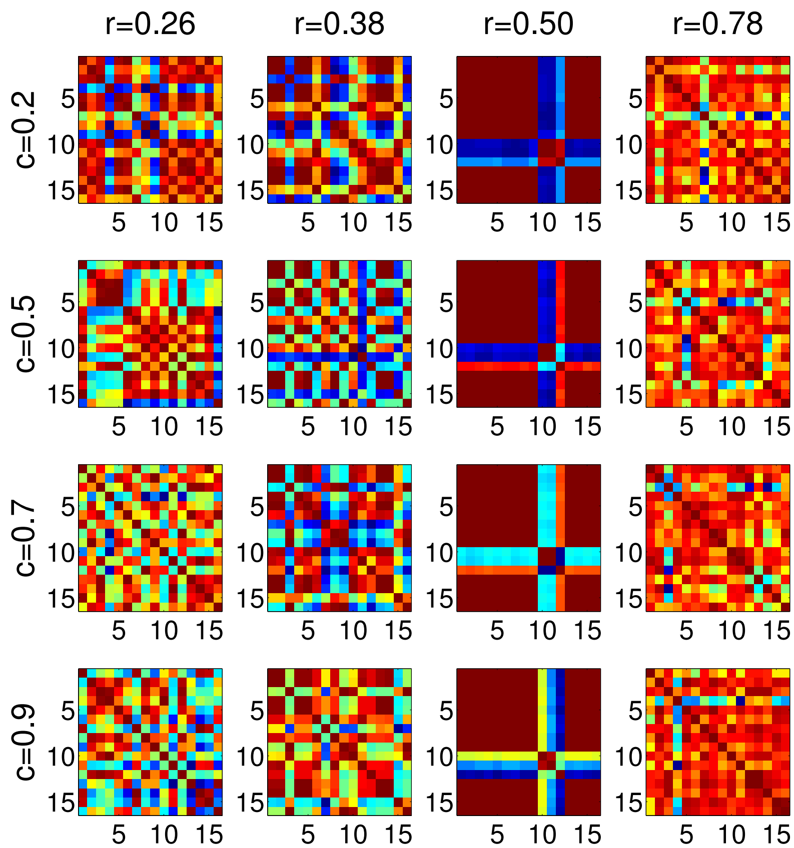

Figure 12, $t = 6000ms$, $D = 0.05$, $\gamma = 1.0$, $\alpha = 0.85$, $b = 0.2$, $\tau = 1.25$, $v = 7m/s$

3.2 Different Time Delays and Different Velocities on BOLD Activity

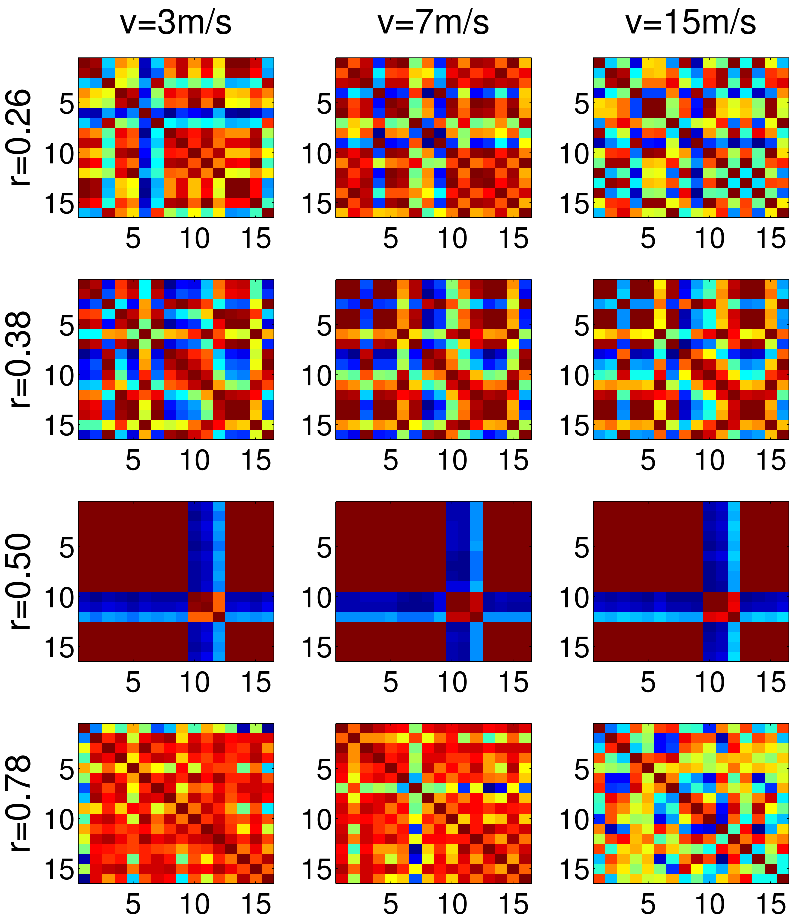

Figure 13, $t = 6000ms$, $D = 0.05$, $\gamma = 1.0$, $\alpha = 0.85$, $b = 0.2$, $\tau = 1.25$, $c = 0.2$