$c = v_i \alpha_i$

$$\alpha_i = \frac{\exp(k_i^T q)}{\sum_j \exp(k_j^T q)}$$

(a)

when $k_i^T q$ gets particularly big.

eg. $k_i = q = e_i^{\times\phi}$ then $\exp(\cdot) = 1$ others

$\exp(c)$ is large $k_i^T q \gg k_j^T q \quad j \neq i$

$\{k_1 \cdots k_n\}$ $\quad \forall i \neq j \quad k_i^T k_j = 0$

$\forall i \quad \|k_i\| = 1$

(b)

$q = N(k_a + k_b) \quad N \to +\infty$

$$\alpha_i \approx \begin{cases} \frac{1}{2} & i = a, b \\ 0 & \text{others} \end{cases}$$

$$q = (\mu_a + \mu_b)\frac{1}{4\sqrt{\alpha}}$$ (c) i

$$k_i^T q = \varepsilon_i^T q \;\; \mu_i^T (\mu_a + \mu_b)\frac{1}{4\sqrt{\alpha}}$$

$$+ \varepsilon_i^T q$$

$$\|\varepsilon_i^T q\| \leq \|q\|_2^2 \|\varepsilon_i\|_2^2 \leq \frac{d}{4\sqrt{\alpha}}\sqrt{\alpha} = d^4\sqrt{\alpha} \to 0$$

$$k_i^T q \to \mu_0^T (\mu_a + \mu_b)\frac{1}{4\sqrt{\alpha}}$$

(c) ii

$$k_i^T q = \mu_i^T (\mu_a + \mu_b)\frac{1}{4\sqrt{\alpha}}$$

$$+ \varepsilon_i^T q$$

$$\mathrm{var}(\varepsilon_i^T q) = -\left(E(\varepsilon_i)^T q\right)^2 + E\left(q^T \varepsilon \varepsilon_i^T q\right)$$

$$= q^T E(\varepsilon \varepsilon_i^T) q = q^T \Sigma_i q$$

$$= \frac{1}{\sqrt{2}} (\mu_a^T + \mu_b^T) \left(\alpha I + \frac{1}{2} \mu_a \mu_a^T \right) (\mu_a + \mu_b)$$

$$= \sqrt{2}\, \mu_a^T \mu_a + \frac{1}{2\sqrt{2}} (\mu_a^T \mu_a)^2$$

$$\mathrm{Var}(\varepsilon_i^T q) \to \frac{1}{2\sqrt{2}} \delta_{ia}$$

$$\mathrm{Var}(k_i^T q) = \frac{1}{2\sqrt{2}} \delta_{ia}$$

$$E(k_i^T q) = \mu_i^T (\mu_a + \mu_b) \frac{1}{4\sqrt{2}}$$

$$E(\alpha_i) = \begin{cases} \frac{1}{2} & i = a, b \\ 0 & \text{others} \end{cases} \quad \alpha \to 0$$

α_a could swing between 0 and 1

suddenly $c = V_a$, suddenly $c = V_b$

$q_1 \quad q_2$

$$q_1 = \mu_a \frac{1}{\sqrt[4]{\alpha}} \quad q_2 = \mu_b \frac{1}{\sqrt[4]{\alpha}}$$

$$k_i^T q_1 = \frac{1}{\sqrt[4]{\alpha}} \delta_{ia} + \varepsilon_i^T q_1$$

$$k_i^T q_2 = \frac{1}{\sqrt[4]{\alpha}} \delta_{ib} + \varepsilon_i^T q_b \to \frac{1}{\sqrt[4]{\alpha}} \delta_{ib}$$

$$\mathrm{var}(\varepsilon_i^T q_1) = \frac{1}{\sqrt{\alpha}} \mu_a^T \Sigma_a \mu_a$$

$$= \sqrt{\alpha} + \frac{1}{2\sqrt{\alpha}} \to +\infty$$

C always has $\frac{1}{2} v_b$, $\frac{1}{2} v_a$ might stay or swings to average of all vs

$$x_1 = u_d + u_b, \quad x_2 = u_a, \quad x_3 = u_c + u_b$$

$$K = \frac{1}{\beta}\left(u_c u_a^T + u_a u_c^T + u_b u_b^T\right)$$

$$k_1 = u_b, \quad k_2 = u_c, \quad k_3 = u_a$$

$$Q = \frac{1}{\beta}\left(u_b u_a^T + u_a u_b^T + u_c u_c^T\right)$$

$$q_1 = u_a, \quad q_2 = u_b, \quad q_3 = u_c$$

$$c_1 = v_3, \quad c_2 = v_1, \quad c_3 = v_2$$

$$V = \frac{1}{\beta}\left(u_b u_b^T - u_c u_c^T\right)$$

$$v_1 = u_b, \quad v_2 = 0, \quad v_3 = u_b - u_c$$