Optimizing COVID-19 pandemic model outcomes based on government spending models

A Report Submitted in Partial Fulfillment of the Requirements for SYDE/BME 411 Optimization and Numerical Methods

Group 21

Abhishek Shah - 20658927
Avery Soule - 20652663
Chris Karanassios - 20661716
Sachin Fernando - 20661723

Faculty of Engineering
Department of Systems Design Engineering
University of Waterloo
Fall 2020

Course Instructors: Nasser Lashgarian Azad, PhD, Associate Professor
Siby Samuel, Assistant Professor

Monday, December 7th, 2020

Introduction and Problem Description

Pandemic modeling is a discipline of science that greatly benefits a nation's ability to respond to the threat of infectious disease, and with SARS- CoV-2, commonly known as Covid-19 being highly prevalent in 2020, nearly all nations have to deal with its impacts. Even within Canada, over 380 thousand cases have been recorded, and more that may have gone by undetected [1]. As such, an effective pandemic model that works to predict active cases, patient recoveries, and more, would provide great benefits to Canada in order to better work towards overcoming Covid-19. One of the possible uses of a pandemic model would be optimizing where funding is spent by the government in order to yield the best results.

The objective of this project is to create a simplified pandemic model, directly linked with certain budget allocations which can be grouped into three major cost groupings, i.e. the design variables, X, Y and Z. X includes investment in spread prevention programs such as legislature changes, contact tracing and social distancing awareness campaigns. Y primarily reflects investment in vaccine development. Lastly, Z reflects financial commitments to hospitals and other treatment/detection centers responsible for dealing with potentially infected individuals. The implementation of the pandemic model is based on a modified version of the model described in [2], which like many previous attempts is based on the Susceptible, Infected, Removed (SIR) approach. The end goal is to minimize the number of Covid-19 related deaths in Canada, given a budget of $1 billion.

The following sections present our problem's modelling steps and formulation process, assumptions and justification, our optimization technique of choice, followed by the results, the analysis of the results, and ending with conclusions drawn and future research prospects.

Modeling & Formulation

As mentioned previously, in this report a pandemic model is created in order to simulate the effects of Covid-19 in Canada. The main inspiration for the structure of the model was from [2], where the authors modified the SIR model. The SIR model is described in the book "Infectious Diseases of Humans: Dynamics and Control" by R.M. Anderson and R.M. May [3], which presents the concept that a certain population of interest can be separated into three classes, susceptible, infected, and removed. Each class is governed by differential equations, presented later in this report. The current values for the population in each class is subjected to several variables that dictate the nature of the disease being modeled. These would include mortality rate, recovery rate, and transmission rate. A block diagram showcasing these states and their corresponding variables are shown in Figure 1. A few additional states were also defined to better simulate COVID-19.

The goal for this model is to optimize budget allocations in order to minimize, which are a subset of a greater budget allocation from the government. The design vector can be written as: $\bar{x} = [X\ Y\ Z]^T$ where X is the investment in spread prevention programs (ex. legislature changes, contact tracing, social distancing), Y is the investment in vaccine development, and Z is the investment in hospital / treatment centers. Thus, the objective function is $C(X, Y, Z) = \int_0^\infty \frac{dC}{dt} dt$, and is subject to the following constraints: $X + Y + Z = B$, which represents that the sum of design variables must be less than or equal to the proposed government budget, and $X, Y, Z \geq 0$, which is the non-zero requirement.

Drawing inspiration from [2], the equations governing the model of Figure 1 are also shown in Figure 2, where N represents the entire population from all groups. The remainder of this section covers each of the model variables, their meaning, and the justification as to why certain values were used.

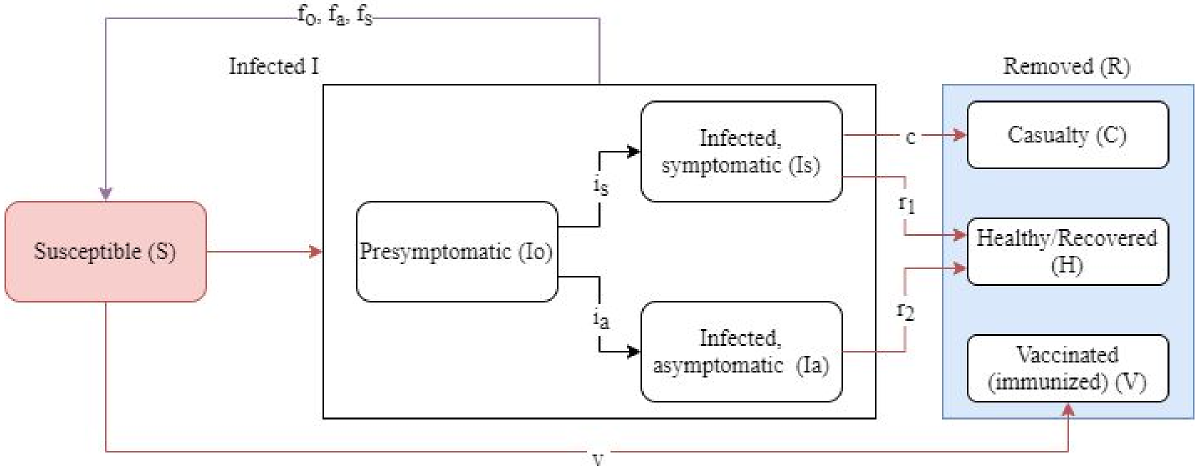


Figure 1: Covid-19 Pandemic Model

$$dI_o/dt = z \cdot S/N(I_o f_o + I_s f_s + I_a f_a) - I_o i_s - I_o i_a$$
$$dS/dt = -z \cdot S/N(I_o f_o + I_s f_s + I_a f_a) - S \cdot v$$
$$dI_s/dt = I_o i_s - I_s c - I_s r_1$$
$$dI_A/dt = I_o i_a - I_a r_2$$
$$dC/dt = I_s c$$
$$dH/dt = I_s r_1 + I_a r_2$$
$$dV/dt = S \cdot v$$

Figure 2: Model equations

In this diagram, Susceptible is the default healthy population, before the virus begins to spread. In our model, S is treated as Canada's population pre-pandemic, with a value of 38 million [4]. From this susceptible population, each person can either become infected, or they may become immune. Gaining immunity is dictated by variable v, moving individuals from the S to the V (vaccinated) state. Immunity is also assumed indirectly for individuals who recover after infection (H). Once infected, they move to the central Infected block, where they are categorised as Presymptomatic. Subsequently, they either begin to show symptoms, becoming Infected Symptomatic (Is) or they never develop symptoms, becoming Infected Asymptomatic (Ia) governed by probabilities i_s and i_a respectively. Individuals within the infected block are able to infect the susceptible population. These variable infection rates are f_o, denoting the people that have not yet moved to Is or Ia, f_a denoting the infection rate of Ia people, and f_s, denoting the infection rate of Is people. Lastly, both Is and Ia individuals can move to the Removed Block, where Ia people are assumed to not be at risk for becoming a casualty, while Is people either become a casualty, with a rate c, or become healthy, with a rate r_1. Note that 'rate', 'weight' and 'probability' are used synonymously in this report to refer to the transitionary probability of an individual from one state to another on a given day.

The collection of 'states' that the Canadian population find themselves in comprise the model as a whole, and given budget allocations that contribute to the values of each weight, can be plotted over the course of the simulation time period. This time period begins on 'day 1' where all patients but one are susceptible, and the last day is when the susceptible population reaches zero (everyone is either immunized, recovered, or a casualty). It is logical to think that there is a correlation between budget spending in each of the three areas mentioned above, and the impact of each variable. As such, justifications must be made as to how each variable is determined, and what impact the budget has.

A key consideration is the variable **z** which denotes the number of individuals interacted with per day (also referred to as an individual's nearest neighbours in this report). Based on the pre-pandemic average of individuals that people come in contact with per day, 13.4 ([5]), a relationship between the amount invested in spread prevention awareness (X) and the number of nearest neighbours can be proposed. The first relevant assumption of this variable was that less investment resulted in a linearly proportional increase in the number of people exposed (i.e. less concern for the problem means a larger social 'bubble'). Secondly, to account for the transient nature of society's response, an exponential model of social isolation was used. Combining these equations allowed for a relationship between the design variable X and the numbers of nearest susceptible neighbours for an individual. Appendix A provides information on how z was calculated.

The following section provides the definitions, derivations and relevant assumptions of the weights characterizing the dynamic behaviour of the model. The numerical relationships of the weights described in this section are numerically defined in Appendix A.

$\mathbf{f_0}$ defines the probability that an individual in the presymptomatic phase (also defined as the incubation period) infects their nearest neighbours. This can primarily be thought of as being dependent on investment in spread prevention programs (X) which have the effect of spreading awareness of individual susceptibility. Studies have shown transmission probabilities in the incubation period ranging from 0% to approximately 15% [6]. Additionally, based on federal government commitments to COVID-19 related awareness campaigns [7], an upper bound of $50 million was selected. The linear relationship between these factors is based on research indicating an approximately linear decrease in case rate following the implementation of social distancing [8]. Thus, function implementation utilised X as an input and scaled probability of infection between 0.01% (corresponding to $0 investment) and 15% (corresponding to $15 million).

$\mathbf{f_s}$ represents the probability of a symptomatic individual infecting their susceptible neighbour on a given day. This was assumed to be dependent on largely awareness campaigns in the initial stage (X), and effective treatment resources (ex. testing) to prevent further spread in the latter stages of the pandemic (Z). A study on SARS, a related coronavirus with a similar reproductive number to COVID-19, showed disease spread to 15% of close contacts [6]. As for its relation to budget, national health spending was shown to increase 23% in 2020 compared to the prior year. Provided a maximum budget of $1 billion as stated in the problem statement, the lower bound was thus set to $770 million, representing the baseline national health spending allocation. Because this weight was a product of the effect of two variables, the resulting probability was a weighted linear relationship between X and Z. Consider α as the weight of X and (1-α) as the corresponding weight of Z. In the initial phase (assumed to be three months or 180 days), awareness is especially important and thus α was made to equal 0.6. As the pandemic progresses, the majority of the population is already familiar with the necessary precautions and thus the importance of X wanes, resulting in α equalling 0.1 after the three month mark.

f_a represents the probability of an asymptomatic individual infecting their nearest susceptible neighbours. Data has shown asymptomatic individuals being approximately 42% less likely to transmit the virus than their symptomatic counterparts. Thus this weight was made to be 42% less than the value of **f_s** given the same inputs of X and Z.

i_s and **i_a** represent the probability of a presymptomatic individual becoming either symptomatic or asymptomatic respectively. International studies have found an approximate 20% of the proportion of infected individuals showing symptoms [9]. As such, the weight given to **i_s** was 0.20 while **i_a** equalled 0.80 as its complement. These weights were inherent to the nature of the disease and thus were independent of the design variables X, Y or Z.

The weight **v** represents the probability that a susceptible individual is vaccinated on a given day. A simplifying assumption in the model was that vaccination ensures full immunity for the lifetime of that individual. Key considerations for this factor was the delay necessary before a vaccine is available and the subsequent rollout once it is ready for distribution. The distribution model is based on the standard three for developed countries: stage 1 covers 3% of the population (health/social care workers), stage 2 reaches 20% of the population (over 65s and high risk) and stage 3 reaches the remaining priority groups [10]. Based on a one year distribution plan, an exponential function was calculated to fit these points. In terms of budget, Canada has spent approximately \$200 million on vaccine development [11]. Given the current timeline, this upper bound in spending was associated with a one year vaccine development timeline with less spending assumed to cause a linearly proportional delay in the timeline. This cost-time function was inserted into the previously mentioned probability-time function to fully define this weight.

c represents the probability of an infected symptomatic individual becoming a casualty, with minimal hospital treatment. This assumption is in place, since as hospital funding increases, the quality of care provided increases, and subsequently, the chances of being a casualty decreases. The base value of c represents the case where hospital funding for Covid-19 is a hypothetical \$0. A paper by Baud et al discusses adjusted mortality rates for Covid-19 during the first few months of 2020, where mortality was up to 20% in Wuhan, China [12.]. If the budget for hospitals were to be increased to its maximum, the value of c would decrease, however it is logically impossible to find an explicit correlation between mortality and funding, as there are many factors that affect an individual's chances of survival. There are however studies showing the improvements over time to the survival rate of individuals in critical care. A study conducted on three hospitals in New York shows how adjusted mortality rate, from 25% in March (similar to our defined c value), decreased down to around 7% by August [13]. It was deemed appropriate to set this value of 7% as the lowest mortality rate for the average individual, assuming maximum hospital budget was spent. From these two points, 20% given no investment, and 7% given maximum investment, it was assumed that there is a linear relationship where budget is concerned. There is insufficient data to allow us to propose a nonlinear relationship.

r_1 is the recovery chance of symptomatic individuals, and it is logical to assume that if one does not succumb to Covid-19, they will recover. As a result, r_1 was defined as 1- c.

r_2 is the recovery chance for asymptomatic individuals, and we are assuming that asymptomatic individuals cannot become casualties, as they are exhibiting no symptoms that may lead to death. However, since they can still infect others, their recovery chance is set to be delayed by 14 days - where these days refer to the approximate end range of when one would exhibit symptoms [14].

Given a fixed budget of \$1 billion CDN, the constraints are as follows on the left hand side of Figure 3. Additionally, using baseline (non-pandemic) national spending as the lower bounds and conservative estimates based on current/projected data as the upper bounds, the design variables X, Y, and Z are also limited as seen below on the right hand side:

$$X + Y + Z = B$$
where B = \$1000 million,
and X, Y, Z ≥ 0

$$0 \leq X \leq \$50 \text{ million}$$
$$0 \leq Y \leq \$200 \text{ million}$$
$$\$770 \text{ million} \leq Z \leq \$1000 \text{ million}$$

Fig. 3: Constraints (left) and bounds (right) governing design variables

Choice of Applied Optimization Technique

The implementation of the optimization problem was done using Python's open-source SciPy library used for scientific and technical computing. One of the key modules provided by SciPy is the optimization toolbox. Since the focus of the optimization problem relies on minimizing the total amount of casualties within the pandemic model, the objective function is dependent on three independent variables X, Y, Z that must equal budget B (\$1 billion CDN) and stay within the specified bounds. The optimization problem is multivariate and contains both an equality constraint and bound constraints.

While there are multiple optimization techniques that are provided by the SciPy optimization toolbox, the chosen technique was Sequential Least Squares Quadratic Programming (SLSQP). This technique in particular was chosen as it deals with minimization of nonlinear multivariate functions that have equality and inequality constraints, along with bound constraints [15]. SLSQP is a Sequential Quadratic Programming (SQP) optimization algorithm [16] as the optimization problem is treated as a sequence of constrained least-squares problems, but this least-squares problem is equivalent to Quadratic Programming [17].

The application of SQP is performed on nonlinear optimization problems (NLP) that have the form shown in Figure 4a, where f(x) is the multivariate objective function for which a local minimum must be found and $c_i(x)$ describe the equality and inequality constraints [18]. For certain variables in the function, there are specified bounds that it is further constrained to.

SQP itself is an iterative algorithm as it models the NLP for a given iterate x^i, where i is the iteration version, by a quadratic programming subproblem. The iterations first start with a given set of parameters x^0 and then solving the quadratic programming subproblem to obtain a solution [19]. This quadratic programming subproblem replaces the objective function with the general quadratic approximation shown in Figure 4b. In order to find a convergence point, the Lagrange multiplier estimates a solution for (x^i and λ^i) that satisfies the second order sufficiency conditions [20]. This newly constructed solution is then used to construct a new iteration x^{i+1}. This is done in such a way that the sequence x^i converges to a local minimum x* as $i \rightarrow \infty$ [18]. The advantages of using the SQP is that the initial point x^0 does not need to satisfy all the constraints of the NLP [21] and that the size of the problem (i.e. the number of constraints) do not need to be moderately large [19].

a)
$$\begin{array}{llll} \text{minimize} & f(x) & & \\ \text{subject to} & c_i(x) & = & 0 \quad \forall i \in \mathcal{E} \\ & c_i(x) & \leq & 0 \quad \forall i \in \mathcal{I} \end{array}$$

b)
$$q_k(d) = \nabla f(x_k)^T d + \frac{1}{2} d^T \nabla^2_{xx} \mathcal{L}(x_k, \lambda_k) d$$

Figure 4. a) SQP problem optimization form b) Quadratic Approximation [20]

Results

Using the SLSQP method in the, “minimize” SciPy optimizer function produced optimal investment values of $50 million in spread prevention programs (X = 50, max value), $0 investment in vaccine development (Y = 0, min value), and $950 million in financial investment targeted towards preparing hospitals and other medical centers to care for infected individuals (Z = 950, out of 1000). Using an initial condition of 1 infected with no symptoms developed yet (state I_0) and 38 million susceptible individuals (state S, the rough population of Canada) [4], the optimal solution produced 1,541,690 casualties. This assumed global minimum was found after trialing a wide range of valid and non-valid starting points based on the one billion dollar budget constraint to actively search for local versus global minimums. In all trials, the optimization process converged on one of two solutions. Solutions were most often reached after two gradient evaluations, between 10 to 20 function evaluations, and just over one minute of computation time using a 2.8GHz Intel Core i7 processor. The second produced solution or a local minimum in our feasible region (X = 0, Y = 0, Z = 1000) resulted in 2,150,734 casualties. Also worth noting, initial conditions were quite sensitive to a change in the X variable starting condition indicating the assumed global minimum has a steep descent with respect to spread prevention programs near X = 50.

As seen in Figure 5, one clear pandemic “wave” is observed to take place with approximately ⅔ of the population becoming sick in the first year. After this initial wave, the remaining susceptible population maintains a constant value until vaccines are introduced after three years. Based on the previously explained weight functions, the plot in Figure 6a displays how each probability changes over time for the optimal solution found. Most notably, the vaccination probability (v) can be seen to exponentially increase at the three year mark, a reasoned time delay when no investment in vaccine companies is made. Casualty rate (c) quickly drops until settling on a steady state non-zero value referencing the initial difficulties with identifying and dealing with a new virus while the recovery rate of symptomatic individuals (r_1) reflects the opposite increase in rate. The increase in f_a and f_s are due to the revised weightings of X and Z that occur at the three month (180 day) mark. After this point investment in treatment (Z) is assumed to be more important than investment in spread prevention (X) due to a large portion of the population already being aware of the problem and the associated prevention strategies.

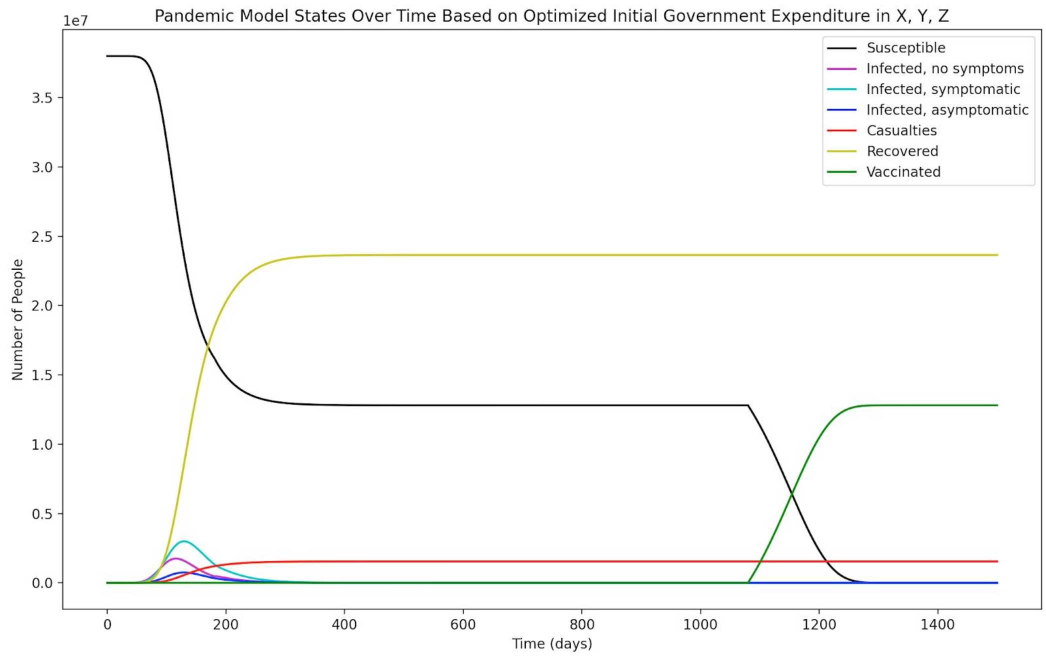
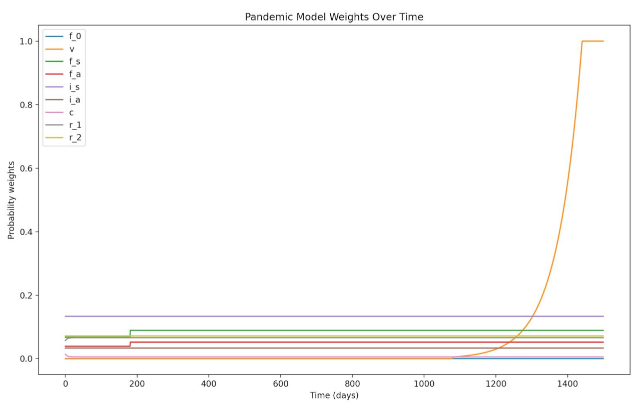
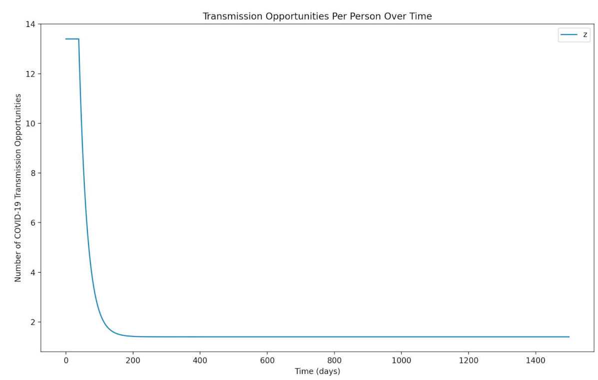


Figure 5: Number of individuals in each pandemic model state plotted over time based on the optimized spending values of X equal to the maximum $50 million, Y at $0, and Z at $950 million.



a)



b)

Figure 6: a) Pandemic model weights displayed over time for the optimal spending values of X equal to the maximum $50 million, Y at $0, and Z at $950 million. b) Number of daily transmission opportunities each person in the population experiences on average throughout the pandemic.

The final parameter is the average number of daily transmisible opportunities each person experiences (z) used to represent the social distancing abilities of the population. As seen in Figure 6b, the z factor remains at the reasoned 13.4 value until a threshold of sick individuals is surpassed at which point the population responds by decreasing on an exponential curve until settling on its lower bound of 1.4. It is assumed the strict social distancing practices are held in place until the distribution of vaccines at the three year mark. The above mentioned behaviour and related assumptions will be further analyzed and limitations will be discussed in the following section.

Analysis

Model Accuracy for Covid-19 Application

Given the previously presented results, it is clear with the current assumed investment-parameter relationships and parameter estimations that this model is not suitable to predict or optimize investment values for Canada in its current form. Notably, ⅔ of the population was modeled to become infected after the first year with a peak of nearly 3 million infected symptomatic individuals. Reflecting on Canada's current Covid-19 data shows one initial distinct peak capping out at 35000 active cases and roughly 150 thousand total cases, followed now by a second much larger wave which at the time of submission of this report has led to a total of 400,000 cases [22]. When contrasting with the presented model, it is clear parameter values, the investment to parameter relationships, and additional assumptions made, led to the significant error seen in the results. While significant changes to this model would be required to make it a functional prediction tool for COVID-19, the following section discusses useful observations that can still be made from the optimization results.

Learnings from the model

Three key learnings about infectious viruses such as the corona virus can be made from looking at the results of the described model. Firstly, we notice in both minimums, vaccine contributions are minimized to zero. Taking a closer look at the model, it can be seen that vaccine production, regardless of how much investment was made, would not take effect until long after the initial pandemic wave was over. Since all the model's casualties were in this first wave, the vaccine investment had no effect on the outcome. In order to accurately model vaccines' contribution to saving lives, the model must be able to support the ongoing multiwave pandemic situation that is caused by governments stiffening and relaxing restrictions. When analyzing the parameter implementation in the current model, we can see the z parameter implementation is a large limitation for this model's accuracy on this front. Despite this inaccuracy, the presented results still make it clear that in the case of highly contagious viruses, vaccine investments are mainly targeted for long term prevention on the susceptible population but will not typically be helpful for first waves.

Secondly, the investment in medical resources and prevention/awareness programs drastically reduced the number of deaths by spreading out the infection curve as an effective short term measure. Solutions not near the global minimum all were characterised by comparatively very steep infection curves resulting in a significantly larger percentage of the population becoming infected in a comparatively shorter period of time, and as a consequence more casualties. This points to a general takeaway that flattening the curve leads to more effective control of the virus and a comparatively smaller wave can be expected.

Finally, if the assumption that the number of daily transmission opportunities and government awareness programs are strongly correlated is indeed true, the final key point that investing in awareness and tracking is a very high value investment, can be made. This is reflected by the steep decline to the global maximum when changing X values.

Optimization discussion

Modality was verified by running the model function systematically over various initial conditions. The three key setpoints defined for testing purposes were values close to the lower, midpoint and upper bound of each design variable respective to their limitations. The remaining two variables were either kept at the midpoint or at the extreme of their bounds. Additionally, these conditions were tested under varied boundary conditions. Doing so allowed us to be aware of convergent solutions outside the limitations set forth in our problem definition. For example when the lower limits of the design variables were set to 0, the solution converged to X, Y, Z = [0, 0, 1000]. This provided evidence of the existence of multiple minimum convergence points and thus multimodality. Figure 8a and 8b points to the existence of said local minimums occurring for different combinations of X, Y and Z.

On a similar note to modality, it can be concluded that the feasible region is non-convex given the presence of two separate minimums. Given these separate minimums it is not possible for all points on the linear interpolation between the two to be overestimates of the real value of the function. In fact, all points between the two will underestimate the function value given no minimums exist between them and thus by definition the feasible region we are dealing with is non-convex. This can be more mathematically reinforced by looking at the two minimum points, X = 50, Y = 0, Z = 950 resulting in 1,541,690 casualties, X = 0, Y = 0, Z = 1000 resulting in 2,150,734 casualties, and contrasting them with the results of running a linearly interpolated line between these points As shown in Figure 8c, all the points that lie between the two mins have higher casualties establishing the non convex nature of this problem.

As SQP can handle nonlinear constraints, this property allows it to apply its optimization algorithm without requiring a feasible initial point that satisfies all constraints [21]. This is evident by using our initial point as x^0 = [50, 50, 950]. This initial point does not comply with the equality constraint previously mentioned, however SQP minimizes the objective function to x* = [50, 0, 950] which satisfies all of the constraints listed in Figure 3.When using initial points that greatly exceed the bounds, the algorithm minimizes to one of the two miniums previously mentioned. This implies that the optimization method is globally convergent [21], as there is more than one local solution.

Limitations

One of the major challenges in modeling the COVID-19 pandemic is accessing the large quantity of recent and changing information that is required to estimate parameter values and determine what is significant versus insignificant to the models performance. This reports presented model was constrained by hardware resources for the models level of complexity in addition to time for synthesizing the relevant resources to come up with accurate parameter values. In addition, the ties to government spending were more abstract and conceptual than what would be desirable for a predictive model. A more thorough investigation into the categorization of spending and its effects on pandemics is highly recommended as a future research focus.

As an example, ‘z’, the transmission opportunities per day individuals faced was thought to be related to spending in X (awareness/prevention programs) in addition to the current state of the pandemic. In the presented model, ‘I_s’ was used to indicate the public's expected willingness to reduce the number of interactions they had per day in addition to the amount of resources the government put forwards to make the population aware. While these may be substantiated observations and studies of this COVID-19 pandemic, the specific relationship is much more difficult to quantify.

As a final limitation, simplifications were also made to computationally lighten the model. One example of this was assuming recovered individuals would not be able to become infected again. This of course assumes during the models execution that individuals develop immunity after recovering, and no new strains of the virus invalidating this immunity is introduced.

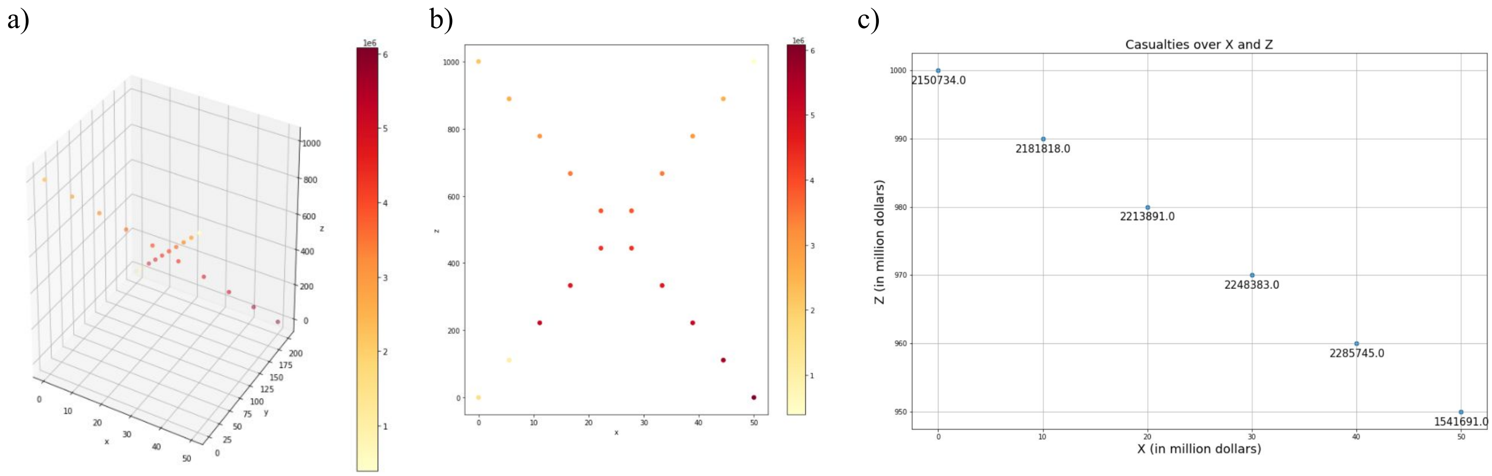


Figure 8: a & b) Modality Plots c) Linear Interpolation between the two minimums

Conclusion

The goal of this report was to create a Covid-19 model to understand what the impact of different budget allocation strategies was, and to attempt to optimize this spending such that the number of casualties is minimized. As it was determined by our analysis, the two points that minimize casualties were [0, 0, 1000] and [50, 0, 950] with the latter being the feasible region global minimum for social distancing awareness, vaccine development, and hospital and treatment centre funds respectively. This shows that, for our current model, it is not preferable to invest in vaccine development, as it takes too long to introduce, and the number of susceptible individuals remaining is relatively low for the vaccine to impact the spread of COVID-19.

While the results are indicative of what would be best for this model, more work is needed to create a realistic representation of Canada's current situation, in order to be able to better utilize any of these findings. There are several aspects that can be expanded upon for future versions of this project.

Suggestions for Future

There are three main areas where this project could be expanded upon in future research. First, in-depth research can be conducted in order to obtain more justifiable weights for the model parameters, which would provide an even more realistic pandemic model. In this project, a lot of assumptions had to be made, and replacing assumptions on the basis of more concrete data could result in a better budget optimization. Additionally, there's the possibility of building upon an existing Covid-19 model that is being currently used/developed in research. The second area that could be improved upon is the means of optimization, specifically since in this project, the built in optimization capabilities of Python had to be used. It would be possible to design a more intricate approach to optimizing budget allocation that is tailored to suit the model structure. The third area would be to function less predictively, and instead first collect data regarding Canada's funding responses, as well as gather statistics concerning the relationship between budgeting and casualty rates, and work backward with this information to attempt to optimize Canada's spending retrospectively. These are all areas of improvement that can be considered if a similar project were to be undertaken but at a larger scale.

References

[1] "Coronavirus disease 2019 (COVID-19): Epidemiology update", *Government of Canada*, 2020. [Online]. Available: https://health-infobase.canada.ca/covid-19/epidemiological-summary-covid-19-cases.html. [Accessed: 03- Dec- 2020]

[2] A. Kleczkowski, K. Oleś, E. Gudowska-Nowak, and C. A. Gilligan, “Searching for the most cost-effective strategy for controlling epidemics spreading on regular and small-world networks,” *Journal of the Royal Society, Interface*, 07-Jan-2012. [Online]. Available: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3223629/. [Accessed: 03-Dec-2020].

[3] R. Anderson and R. May, *Infectious diseases of humans*. Oxford: Oxford Univ. Press, 2010.

[4] "Population estimates, quarterly", *Statistics Canada*, 2020. [Online]. Available: https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1710000901. [Accessed: 04- Dec- 2020]

[5] J. Mossong, N. Hens and M. Jit, "Social Contacts and Mixing Patterns Relevant to the Spread of Infectious Diseases", *PLoS Medicine*, vol. 5, no. 3, p. e74, 2008.

[6] M. Slifka and L. Gao, "Is presymptomatic spread a major contributor to COVID-19 transmission?", *Nature Medicine*, vol. 26, no. 10, pp. 1531-1533, 2020.

[7] L. Berthiaume, "Feds plan $30M ad buy to help media deal with COVID-19 fallout", *Coronavirus*, 2020. [Online]. Available: https://www.ctvnews.ca/health/coronavirus/feds-plan-30m-ad-buy-to-help-media-deal-with-covid-19-fallout-1.4868393. [Accessed: 03- Dec- 2020]

[8] Y. Alimohamadi, K. Holakouie-Naieni, M. Sepandi and M. Taghdir, "Effect of Social Distancing on COVID-19 Incidence and Mortality in Iran Since February 20 to May 13, 2020: An Interrupted Time Series Analysis", *Risk Management and Healthcare Policy*, vol. 13, pp. 1695-1700, 2020.

[9] C. Heneghan, J. Brassey and T. Jefferson, "COVID-19: What proportion are asymptomatic?", The Centre for Evidence-Based Medicine, 2020. [Online]. Available: https://www.cebm.net/covid-19/covid-19-what-proportion-are-asymptomatic/. [Accessed: 04- Dec- 2020]

[10] D. Bailey, "Coronavirus: How soon can we expect a working vaccine?", *BBC News*, 2020. [Online]. Available: https://www.bbc.com/news/health-54027269. [Accessed: 04- Dec- 2020]

[11] M. Abedi, "Canada to spend $192M on developing COVID-19 vaccine", *Global News*, 2020. [Online]. Available: https://globalnews.ca/news/6717883/coronavirus-canada-vaccine-spending/. [Accessed: 04- Dec- 2020]

[12] D. Baud, X. Qi, K. Nielsen-Saines, D. Musso, L. Pomar and G. Favre, "Real estimates of mortality following COVID-19 infection", *The Lancet Infectious Diseases*, vol. 20, no. 7, p. 773, 2020.

[13] L. Horwitz, S. Jones, R. Cerfolio, F. Francois, J. Greco, B. Rudy and C. Petrilli, "Trends in COVID-19 Risk-Adjusted Mortality Rates", *Cdn.mdedge.com*, 2020. [Online]. Available: https://cdn.mdedge.com/files/s3fs-public/issues/articles/horwitz11661023e_5.pdf. [Accessed: 04- Dec- 2020]

[14] H. Publishing, "If you've been exposed to the coronavirus", *Harvard Health Publishing*, 2020. [Online]. Available: https://www.health.harvard.edu/diseases-and-conditions/if-youve-been-exposed-to-the-coronavirus. [Accessed: 05- Dec- 2020]

[15] "Optimization (scipy.optimize) — SciPy v1.5.4 Reference Guide", *Docs.scipy.org*, 2020. [Online]. Available: https://docs.scipy.org/doc/scipy/reference/tutorial/optimize.html#sequential-least-squares-programming-slsqp-algorithm-method-slsqp. [Accessed: 03- Dec- 2020]

[16] "SLSQP", *Degenerate Conic*, 2016. [Online]. Available: http://degenerateconic.com/slsqp/. [Accessed: 03- Dec- 2020]

[17] NLopt algorithms - NLopt Documentation", *Nlopt.readthedocs.io*. [Online]. Available: https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#slsqp. [Accessed: 03- Dec- 2020]

[18] "Chapter 4 Sequential Quadratic Programming", *Math.uh.edu*. [Online]. Available: https://www.math.uh.edu/~rohop/fall_06/Chapter4.pdf. [Accessed: 04- Dec- 2020]

[19] D. Kraft, *A software Package for Sequential Quadratic Programming*. 1988.

[20] "Sequential Quadratic Programming", *neos Guide*. [Online]. Available: https://neos-guide.org/content/sequential-quadratic-programming. [Accessed: 06- Dec- 2020]

[21] P. Boggs and J. Tolle, *Sequential Quadratic Programming*. Chapel Hill, 1996.

[22] P. Canada, "Coronavirus disease (COVID-19): Outbreak update", *Government of Canada*, 2020. [Online]. Available: https://www.canada.ca/en/public-health/services/diseases/2019-novel-coronavirus-infection.html?&utm_campaign=gc-hc-sc-coronavirus2021-ao-2021-0005-9834796012&utm_medium=search&utm_source=google_grant-ads-107802327544&utm_content=text-en-434601690164&utm_term=%2Bcovid. [Accessed: 05- Dec- 2020].

Appendix A - Weight Equations (including z)

Weight	Equation	Variable definition
f_o	$f_0 = \frac{p_2 - p_1}{x_2 - x_1} \cdot (X - x_1) + p_1$	Probability of Presymptomatic (I_0) infection of Susceptible (S) individuals given probability bounds (p_1, p_2), cost bounds (x_1, x_2) and input cost (X).
f_a	$f_a = (1 - 0.42) \cdot f_s(t, X, Z)$	Probability of Asymptomatic (I_a) infection of Susceptible (S) individuals as 42% less likely than Symptomatic (I_s) infection given the same inputs (X, Z).
f_s	$f_{sx} = \frac{p_2 - p_1}{x_2 - x_1} \cdot (X - x_1) + p_1$ $f_{sz} = \frac{p_2 - p_1}{z_2 - z_1} \cdot (Z - z_1) + p_1$ $f_s = w_x \cdot f_{sx} + (1 - w_x) \cdot f_{sz}$	Probability of Symptomatic (I_s) infection of Susceptible (S) individuals given probability bounds (p_i) cost bounds (x_j, z_k) and input costs (X, Z).
i_s	$i_s = \frac{0.8}{6}$	Probability of Presymptomatic (I_0) transition to Symptomatic (I_s) given average 6 day incubation period.
i_a	$i_a = \frac{0.2}{6}$	Probability of Presymptomatic (I_0) transition to Asymptomatic (I_a) given average 6 day incubation period.
c	$c = 0.2 + \frac{Z \cdot 0.08}{1000} \cdot (e^{-\frac{t}{5}} - 1)$	Probability of casualty (C) from Symptomatic (I_s) state given input cost (Z).
r_1	$C(Z) = Z \cdot \frac{0.13}{1000}$ $c_{r1} = 0.2 + C \cdot e^{\frac{-t}{5}} - C$ $r_1 = \frac{1 - c_{r1}}{14}$	Probability of recovery to full health (H) from Symptomatic (I_s) state given input cost (Z) given 14 day recovery period.
r_2	$r_2 = \frac{1}{14}$	Probability of recovery to full health (H) from Asymptomatic (I_a) state given input cost (Z) given 14 day recovery period.

z	$g(X) = \frac{X}{50} \cdot (12)$ $z = 13.4 + g \cdot x^{\frac{-t}{25} + t_a} - g$	Average interactions per day that each individual in the population has that could result in the transmission of COVID-19 if one individual was contagious