Билет 74

Интегрирование простых дробей.

1.

$$\int \frac{A}{x-a}dx = A\ln|x-a| + c$$

2.

$$\int \frac{A}{(x-a)^m},\ m>1 = \frac{-A}{(m-1)(x-a)^{m-1}} + c$$

3.

$$I = \int \frac{Mx+N}{x^2+px+q}dx,\ p^2-4q<0$$

$$x^2+px+q = \left(x^2+px+\frac{p^2}{4}\right) + \left(q-\frac{p^2}{4}\right) = \left(x+\frac{p}{2}\right)^2 + \left(q-\frac{p^2}{4}\right)$$

$$t := x+\frac{p}{2} \Rightarrow x = t-\frac{p}{2},\ dx = dt$$

$$a^2 := q-\frac{p^2}{4},\ q-\frac{p^2}{4}>0$$

$$I = \int \frac{Mt-\frac{Mp}{2}+N}{t^2+a^2}dt = \frac{M}{2}\int \frac{2t}{t^2+a^2}dt + \left(N-\frac{Mp}{2}\right)\int \frac{dt}{t^2+a^2}$$

$$\varphi := t^2+a^2 \Rightarrow d\varphi = 2tdt$$

$$K = \int \frac{2t}{t^2+a^2}dt = \int \frac{d\varphi}{\varphi} = \ln|\varphi| + c = \ln|t^2+a^2| + c = \ln|x^2+px+q| + c$$

$$J_1 = \int \frac{dt}{t^2+a^2} = \frac{1}{a}\operatorname{arctg}\left(\frac{t}{a}\right) + c = \frac{1}{a}\operatorname{arctg}\left(\frac{2x+p}{2a}\right) + c$$

$$I = \frac{M}{2}K + \left(N-\frac{Mp}{2}\right)J_1$$

$$I = \frac{M}{2}\cdot\ln|x^2+px+q| + \left(N-\frac{Mp}{2}\right)\cdot\frac{1}{a}\operatorname{arctg}\left(\frac{2x+p}{2a}\right) + c$$

4.

$$I = \int \frac{Mx+N}{(x^2+px+q)^m}dx,\ p^2-4q<0, m>1$$

$$x^2+px+q = \left(x^2+px+\frac{p^2}{4}\right)+\left(q-\frac{p^2}{4}\right) = \left(x+\frac{p}{2}\right)^2+\left(q-\frac{p^2}{4}\right)$$

$$t := x+\frac{p}{2} \Rightarrow x = t-\frac{p}{2},\ dx = dt$$

$$a^2 := q-\frac{p^2}{4},\ q-\frac{p^2}{4}>0$$

$$I = \int \frac{Mt-\frac{Mp}{2}+N}{(t^2+a^2)^m}dt = \frac{M}{2}\int \frac{2t}{(t^2+a^2)^m}dt + \left(N-\frac{Mp}{2}\right)\int \frac{dt}{(t^2+a^2)^m}$$

$$\varphi := t^2+a^2 \Rightarrow d\varphi = 2tdt$$

$$K = \int \frac{2t}{(t^2+a^2)^m}dt = \int \frac{d\varphi}{\varphi^m} = \frac{-1}{(m-1)\varphi^{m-1}} = \frac{-1}{(m-1)(x^2+px+q)^{m-1}}$$

$$J_m = \int \frac{dt}{(t^2+a^2)^m} \text{ (билет 72)}$$

$$I = \frac{M}{2}K + \left(N-\frac{Mp}{2}\right)J_m$$

$$I = \frac{M}{2}\cdot\frac{-1}{(m-1)(x^2+px+q)^{m-1}} + \left(N-\frac{Mp}{2}\right)\cdot J_m + c$$