Билет 65

Дифференцирование по параметру.

Теорема

Если $y = f(x)$ задана параметрически: $y = \psi(t)$, $x = \varphi(t)$, то

$$y'_x = \frac{\psi'_t(t)}{\varphi'_t(t)}$$

Доказательство

$$\psi(t) = y = f(x) = f(\varphi(t)) \Rightarrow \psi(t) = f(\varphi(t)) \Rightarrow \psi'_t(t) = f'_x(x)\varphi'_t(t) \Rightarrow f'_x(x) = \frac{\psi'_t(t)}{\varphi'_t(t)} \Rightarrow y'_x = \frac{\psi'_t(t)}{\varphi'_t(t)} \,\square.$$

Теорема

Если $y = f(x)$ задана параметрически: $y = \psi(t)$, $x = \varphi(t)$, то

$$y''_x = \frac{\psi''_{t^2}(t)\varphi'_t(t) - \psi'_t(t)\varphi''_{t^2}(t)}{\varphi'_t(t)^3}$$

Доказательство

$$y''_x(x) = \frac{d}{dx}\frac{dy}{dx} = \frac{1}{dx}dy'_x = \frac{1}{\varphi'_t(t)dt}\left(dt\frac{dy'_x}{dt}\right) = \frac{1}{\varphi'_t(t)}\left[\frac{\psi''_{t^2}(t)\varphi'_t(t) - \psi'_t(t)\varphi''_{t^2}(t)}{\varphi'_t(t)^2}\right] = \frac{\psi''_{t^2}(t)\varphi'_t(t) - \psi'_t(t)\varphi''_{t^2}(t)}{\varphi'_t(t)^3} \,\square.$$