Random Testing to Manage a Safe Exit from a COVID-19 Lockdown

Shirley He

Date: 23 June 2020

1 Introduction

Random testing policies have the potential to provide 'real-time' estimates of the proportion of infected individuals within a population. In comparison to delayed indicators that rely on the number of confirmed infections of symptomatic individuals, or COVID-related deaths, we demonstrate how random testing policies produce a faster discovery of the true infection rate and allow policymakers to choose the level of intervention required to maintain a 'safe' transmission rate, while reducing the economic cost of a prolonged fully restrictive lockdown. Focusing on a scenario for Spain, we compare uniform and targeted random testing policies across subgroups by measuring the total value of statistical lives saved in US Dollars.

2 Random Testing Model

Our random testing simulation is based on the preprint paper by Markus Müeller [1], who argues that random testing policies support a safe economy reboot with a higher proportion of an infected population, compared to when the chosen testing policy targets symptomatic individuals. The model is built around a feedback loop and a simplified diagram of the it can be found in Figure 1.

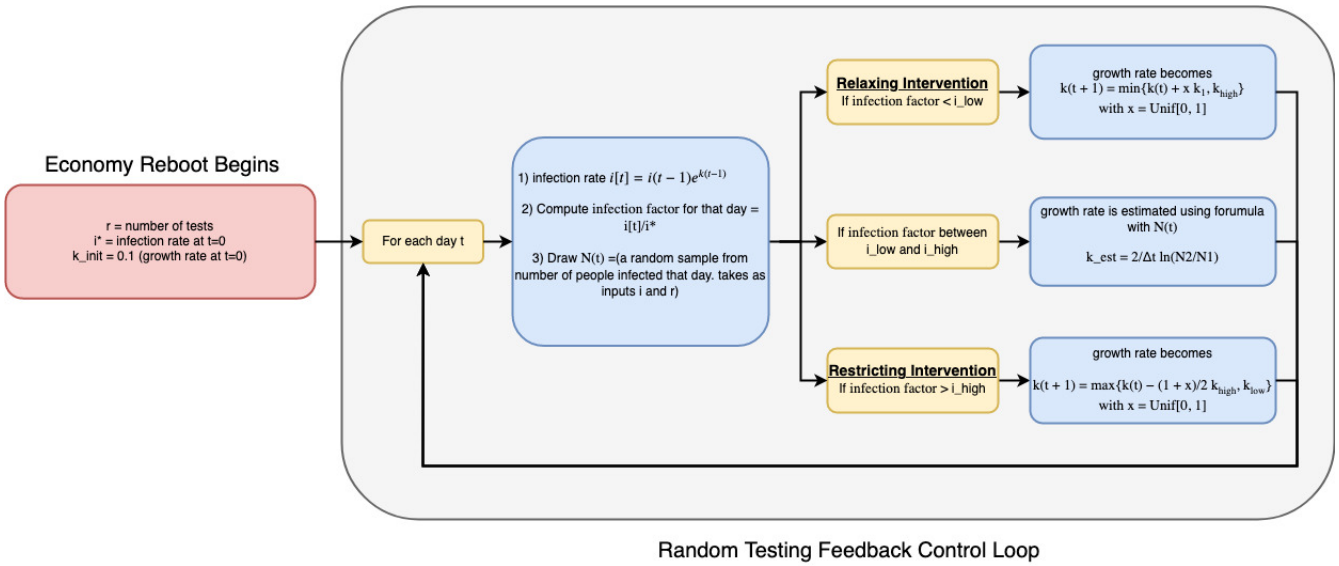

Figure 1: Feedback Control Loop

The simulation begins on the first day of an economy reboot and takes as input, the number of randomly tested people per day r, the chosen level of currently infected people in the population that is 'safe' for the economy to restart i^*, and an initial growth rate of the number of people infected as input k_{init}.

2.1 Estimating the growth rate

To reach a decision on the change in policy strategies, data is acquired by daily testing of random sets of people for infections. We assume tests are carried out at a limited rate r. Let $i(t, \Delta t)$ be the fraction of positive infections detected among the $r\Delta t \gg 1$ tests carried out in the time interval $[t, t + \Delta t]$. By the law of large numbers, it is a Gaussian random variable with mean

$$\langle i(t, \Delta t)\rangle = \frac{\overline{U(t)}}{N}, \quad \overline{U(t)} \equiv \int_t^{t+\Delta t} \frac{\mathrm{d}t'}{\Delta t} U\left(t'\right) \tag{1}$$

and standard deviation

$$\left\langle [i(t, \Delta t)]^2 \right\rangle_{\mathrm{c}}^{1/2} = \sqrt{\frac{\langle i(t)\rangle}{r\Delta t}} = \sqrt{\frac{\overline{U(t)}}{Nr\Delta t}} \tag{2}$$

If an intervention is not triggered, the growth rate will be estimated using the current infection rate, the number of tests, and a random draw of infected people that day.

That is, the current value of $k(t)$ is estimated as $k^{\mathrm{fit}}(t)$ by fitting these test data to an exponential, where only data since the last policy change should be used. The data is acquired over a time window Δt, which is the period after a policy change at time t_ι. Then to estimate the growth rate and the infection level at time $t_{\iota+1}$ before another policy change is implemented, there are four steps.

Step 1: Measurement

We split the time window $\Delta T_\iota \equiv [t_\iota, t_\iota + \Delta t]$ of length Δt after the policy change into the time interval

$$\Delta T_{\iota,1} \equiv \left[t_\iota, t_\iota + \frac{\Delta t}{2}\right] \tag{3}$$

and the time interval

$$\Delta T_{\iota,2} \equiv \left[t_\iota + \frac{\Delta t}{2}, t_\iota + \Delta t\right] \tag{4}$$

Testing delivers the number of currently infected people

$$N_{\iota,1}(\Delta t) = r\Delta t i\left(t_\iota, \frac{\Delta t}{2}\right) \tag{5}$$

for the time inverval (4) and

$$N_{\iota,2}(\Delta t) = r\Delta t i\left(t_\iota + \frac{\Delta t}{2}, \frac{\Delta t}{2}\right) \tag{6}$$

for the time interval (5), where we recall that r denotes the number of people tested per unit time. Given those two measurements over the time window $\Delta t/2$, we obtain the estimate

$$k_\iota^{\mathrm{fit}}(\Delta t) = \frac{2}{\Delta t} \ln\left(\frac{N_{\iota,2}(\Delta t)}{N_{\iota,1}(\Delta t)}\right) \tag{7}$$

with the standard deviation

$$\delta k(\Delta t) = \frac{2}{\Delta t}\sqrt{\frac{1}{N_{\iota,1}(\Delta t)} + \frac{1}{N_{\iota,2}(\Delta t)}} \tag{8}$$

Step 2: Condition for new policy intervention

A new policy intervention is taken once the magnitude $\mid k_\iota^{\text{fit}}\left(\Delta t\right) \mid$ with $k_\iota^{\text{fit}}\left(\Delta t\right)$ given by Eq. (8) exceeds $\alpha\delta k(\Delta t)$ with $\delta k(\Delta t)$ given by Eq. (9) Here, α controls the accuracy to which the actual k has been estimated at the time of the next intervention. The condition

$$k_\iota^{\text{fit}}\left(\Delta t\right) \mid = \alpha\delta k(\Delta t) \tag{9}$$

for a new policy intervention thus becomes

$$\left|\ln\left(\frac{N_{\iota,2}(\Delta t)}{N_{\iota,1}(\Delta t)}\right)\right| = \alpha\sqrt{\frac{1}{N_{\iota,1}(\Delta t)} + \frac{1}{N_{\iota,2}(\Delta t)}} \tag{10}$$

Step 3: Comparison with modelling
We call the actual fraction of interventions as a function of time, which we assume to follow a simple exponential evolution between two successive policy interventions, i.e., the normalised solution

$$i\left(t_\iota + t'\right) = i\left(t_\iota\right)\exp\left(k_\iota t'\right) \tag{11}$$

to the growth equation on the interval $t_\iota < t' < t_{\iota+1}$.

The expected number of newly detected infected people in the time interval (4) is

$$\begin{aligned}\langle N_{\iota,1}(\Delta t)\rangle &= r\int_0^{\Delta t/2} \mathrm{d}t' i\left(t_t + t'\right) \\ &= ri\left(t_t\right)\frac{e^{k_t}\Delta t/2 - 1}{k_\iota}\end{aligned} \tag{12}$$

Similarly, the predicted number of infected people in the time interval (5) is

$$\begin{aligned}\langle N_{\iota,2}(\Delta t)\rangle &= r\int_{\Delta t/2}^{\Delta t} \mathrm{d}t' i\left(t_\iota + t'\right) \\ &= ri\left(t_\iota\right)\frac{e^{k_\iota\Delta t/2}\left(e^{k_\iota}\Delta t/2 - 1\right)}{k_\iota}\end{aligned} \tag{13}$$

Step 4: Estimated time for a new policy intervention
We now approximate $N_{\iota,1}$ and $N_{\iota,2}$ by replacing them with their expectation value equations (13) and (14) respectively, and anticipating he limit $k_\iota\Delta t/2 \ll 1$. We insert

$$N_{\iota,1} \approx N_{\iota,2} \approx ri\left(t_\iota\right)\Delta t/2 \approx ri^*\Delta t/2 \tag{14}$$

into (11) and solve for Δt. The solution is the time until the next intervention

$$\Delta t_\iota \equiv t_{\iota+1} - t_\iota = \frac{(4\alpha)^{2/3}}{\left(k_L^2 ri^*\right)^{1/3}} \tag{15}$$

from which we deduce the relative increase

$$\begin{aligned}\frac{i\left(t_{t+1}\right)}{i\left(t_t\right)} &\equiv \exp\left(k_t\Delta t_c\right) \\ &= \exp\left(\mathrm{sgn}\left(k_t\right)(4\alpha)^{2/3}\left(\frac{|k_t|}{ri^*}\right)^{1/3}\right)\end{aligned} \tag{16}$$

of the fraction of currently infected people over the time window. This relative increase is close to 1 if the argument of the exponential on the right-hand side is small.

We will show below that the characteristics

$$\Delta t_1 = \frac{(4\alpha)^{2/3}}{\left(k_1^2 r i^*\right)^{1/3}} \tag{17}$$

and

$$\frac{i\left(t_2\right)}{i\left(t_1\right)} = \exp\left((4\alpha)^{2/3}\left(\frac{k_1}{r i^*}\right)^{1/3}\right) \tag{18}$$

of the first time interval $[t_1, t_2]$ set the relevant scales for the entire process. From the equations (16) and (17), we infer that the higher the testing frequency r, the smaller the typical variations in the fraction of currently infected people, and thus in the case numbers.

2.2 Interventions

Intervention policies are made based on an infection factor that is computed each day and the growth rate of infection numbers k. The infection factor is

$$i(t)/i^* \tag{19}$$

where i^* is the chosen level of currently infected people in the population that is 'safe' for the economy to restart. Müller et al recommends i^* on the first day of the economy reboot to be $i^* = i_c/4 = 0.0007$, where i_c is the limit of the health system.

A tightening restriction is imposed if the growth rate k exceeds a tolerable upper threshold of k_+ and the upper threshold of i/i^* is met. The upper threshold for i/i^* is $i_{\text{high}} = 3$. If $i/i^* > i_{\text{high}}$ an intervention is made even if k is still smaller than $\alpha\delta k$. $\alpha\delta k(t)$ is the statistical uncertainty (standard deviation) of the fitting of $k^{fit}(t)$, the test data to an exponential, to estimate the current value of $k(t)$ in Eq. (7). It will take at least 2-3 days to make a fit that is reasonably trustworthy.

Restrictions are released if k is below a lower threshold of k_- and if infection numbers are below critical. The lower threshold for i/i^* is $i_{\text{low}} = 0.2$. If $i/i^* < i_{\text{low}}$ a relaxing intervention is made, irrespective of the estimate of k. The optimal target of the growth rate is $k = 0$, corresponding to a marginally stable state, where infections neither grow nor decrease exponentially with time.

3 Value of Statistical Life (VSL)

To accompany the random testing model, we apply a recommended value of statistical life (VSL) measured in US dollars. This allows the model to evaluate the optimal number of tests for each subgroup in monetary terms. The cost function brings together the infection level from the random testing model, COVID fatality rates, and the VSL, to minimize the total VSL lost due to COVID-related fatalities.

The age-specific mortality rates associated with COVID-19 used in this cost function are from the Spanish Ministry of Health, *Red Nacional de Vigilancia Epidemiológica* [2].

We also compare the results from two VSL values to account for the sensitivity of the optimization results to the specific VSL chosen. Firstly we apply VSL values that are stratified by age group from Greenstone's evaluation of the economic benefits of social distancing during COVID-19 [3]. These VSL values are derived from projected age-specific reductions in death and age-varying estimates of the United States Government's value of a statistical life. Secondly, we apply the OECD's recommended VSL values for policy making, which do not vary across age groups [4].

4 Approach

4.1 Uniform Testing Simulations

To identify the limits of this model and to choose an optimal number of tests relative to the population of Spain, I run the simulation for 30 days and compare the effects of different number

of tests, r. I take averages over 100 runs and indicate where a relaxing or restricting intervention occurs in Figures 2 and 3. Using the infection level output from the model, I perform t-tests on different levels of r to determine when a higher r no longer results in a statistical improvement for reducing the infection level. Based on the 95% confidence level, the optimal number of tests for Spain's population is 20,000 or 40,000. The number of 20,000 and 40,000 daily tests over a period of 30 days can be interpreted as a recommendation for Spain to test 1.3% and 2.6% of their population respectively in this time frame.

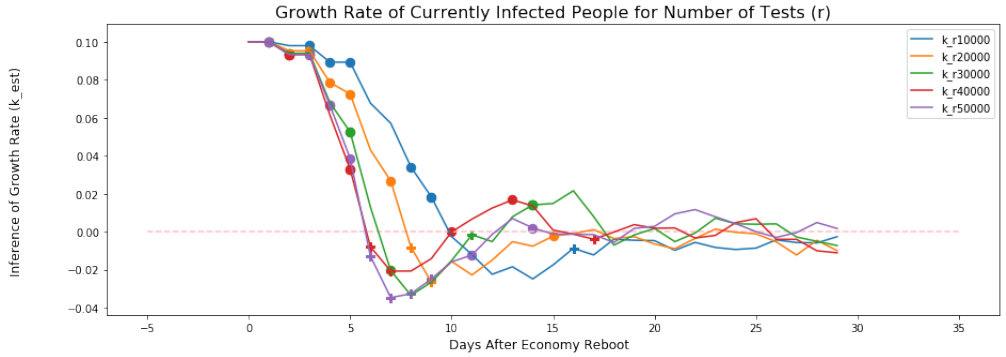

Figure 2: Comparing the Effect of the Random Testing Numbers on the Growth Rate

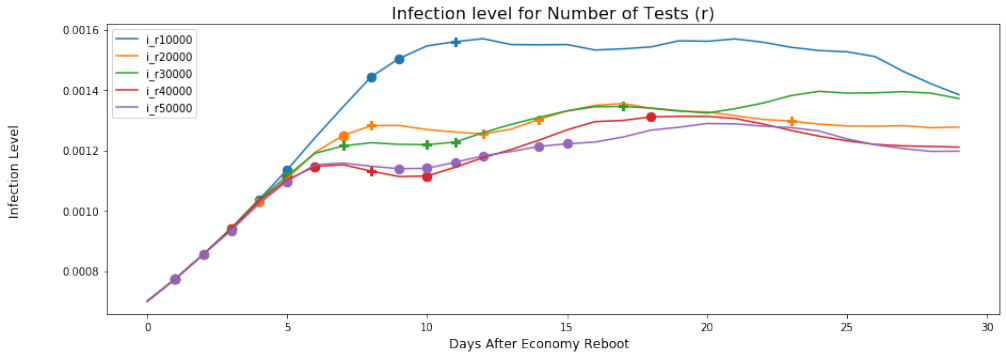

Figure 3: Comparing the Effect of the Random Testing Numbers on the Infection Level

4.2 Subgroup Testing Simulations

In addition to comparing the total number of daily tests administered across the population, I evaluate a targeted policy for the population stratified by age groups.

The motivation for assessing the effect of a heterogeneous random testing policy is based on evidence that the elderly population have a significantly higher risk of requiring intensive care than other groups [5], while allowing working-age individuals to return to work could disproportionately reduce cost to the economy [6]. Based on this knowledge, we choose to stratify Spain's population in the model by age groups.

The chosen age-subpopulations are:

- Children: 0-18 years
- Adults: 19-65 years
- Elderly: 65+ years

For each of the three subgroups, two extra parameters are included in the model: the proportion of the total tests and a caution parameter. The caution parameter represents the strength of interventions applied to each subgroup while the proportion of total tests distributed to each subgroup affects on speed discovery for the true infection rate in the model.

The full range of the additional parameters that were tested were:

$$subpopulation = [\prime child\prime, \prime adult\prime, \prime elderly\prime]$$

$$testproportion = [0.1, 0.2, ..., 0.9, 1]$$

$$caution = [0.6, 0.8, 1.0, 1.2, 1.4]$$

The choice of optimal parameters were determined by a grid search optimization method to minimize the total VSL lost due to COVID fatalities. The cost function used the daily infection level from the random testing model and applied the COVID-19 fatality rates by age in Spain published by the Spanish Ministry of Health [2]. These fatality rates broken down by subgroup can be found in Table 1.

Subgroup	Case Fatality Rate
Child	0
Adult	0.03
Elderly	0.22

Table 1: COVID-19 Case Fatality Rates for Spain by Age Group

Using the daily infection level and case fatality rates, we calculated the daily number of COVID-related deaths for a chosen number of tests in our model. Then we applied a monetary value to each death using two recommendations for VSL, to estimate the total value of statistical lives lost in millions of USD.

To test the sensitivity of our results to the choice of VSL, we compare the difference in optimized subgroup parameters and total VSL lost for two different values for VSL. Firstly, we follow Greenstone's method for estimating the projected mortality benefits of social distancing and utilise the valuations in the United States per subgroup. These values can be found in Figure 4, column (4b).

	Population	Direct Deaths				Overflow Deaths				All		
		No Policy	Mitigation Distancing	Difference		No Policy	Mitigation Distancing	Difference				
Age group	(1) US pop in millions	(2a) Pct of pop	(2b) Pct of pop	(2c) Pct of pop	(2d) Death count	(3a) Pct of pop	(3b) Pct of pop	(3c) Pct of pop	(3d) Death count	(4a) Death count	(4b) VSL in million USD	(4c) Benefits in trillion USD
0-9	39.8	0.001	0.001	0.001	265	0.001	0.000	0.000	177	442	14.7	0.01
10-19	41.4	0.004	0.002	0.002	827	0.002	0.001	0.001	554	1,381	15.3	0.02
20-29	45.0	0.020	0.010	0.010	4,487	0.009	0.003	0.005	2,405	6,892	16.1	0.11
30-39	42.7	0.052	0.026	0.027	11,364	0.023	0.009	0.014	6,091	17,455	15.8	0.28
40-49	40.2	0.098	0.048	0.050	20,032	0.045	0.017	0.028	11,048	31,080	13.8	0.43
50-59	42.9	0.391	0.192	0.200	85,635	0.179	0.069	0.111	47,598	133,234	10.3	1.38
60-69	36.4	1.435	0.704	0.732	266,364	0.656	0.250	0.405	147,585	413,949	6.7	2.76
70-79	21.3	3.327	1.631	1.696	362,001	1.514	0.578	0.936	199,692	561,694	3.7	2.06
80+	12.4	6.067	2.974	3.093	382,484	2.791	1.066	1.725	213,339	595,824	1.5	0.89
US Total					**1,133,460**				**628,491**	**1,761,951**		**7.94**

Figure 4: Source: Does Social Distancing Matter? Working Paper [3])

Secondly, we apply the VSL recommended by the U.S. Environmental Protection Agency who have suggested guidelines for preparing economic analyses [7]. In comparison to the first valuation,

this recommeded value does not vary between age groups. The recommend VSL for one fatality is 10million USD.

5 Results

The optimal number of tests for Spain when the random testing is uniform, are 20,000 or 40,000. The p-values for the t-tests are $pvalue = 0.02$ and $pvalue = 0.0008$ respectively. A decision to distribute 20,000 daily tests over 30 days is equivalent to testing 1.3% of Spain's population, and 40,000 daily tests is equivalent 2.6% of Spain's population over the same time period.

The lowest total VSL lost for 20,0000 tests split across heterogeneous age subgroups using the same VSL as Greenstone is 42157.1 (Millions, USD). The corresponding optimal parameters are:

- Child: 'r': 20000, 'caution': 0.8
- Adult: 'r': 0, 'caution': 1.4
- Elderly: 'r': 0, 'caution': 1.4

We compare these to the lowest total VSL lost for 20,0000 tests split across heterogeneous age subgroups using VSL that does not vary across subgroups is 81156.3 (Millions, USD). The corresponding optimal parameters are:

- Child: 'r': 20000, 'caution': 1.2
- Adult: 'r': 0, 'caution': 1.4
- Elderly: 'r': 0, 'caution': 1.4

The results of both VSL valuations produce similar recommendations for the distribution of tests between subgroups- that is, all tests should be administered to children, and the caution parameter should be lower for children, and higher for adults and the elderly.

The large difference in the total monetary value of lives lost due to COVID fatalities in each case demonstrates how the monetary valuation of a fatality can be highly subjective, and cannot alone realistically represent the economic value of a life lost. In spite of this, the VSL is useful to allow for a relative comparison between the different combinations of parameters.

The daily infection levels of the model corresponding to these optimal parameters can be found in Figure 5.

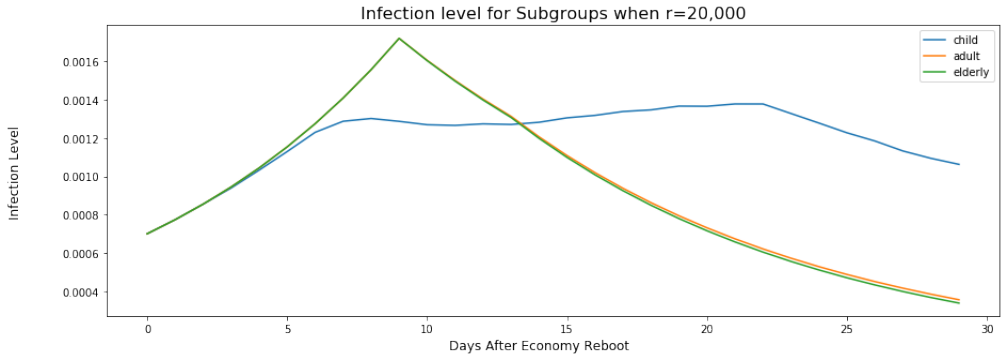

Figure 5: The Subgroup Infection Level for optimized parameters using Greenstone VSL and 20,000 tests

6 Discussion and Further Work

6.1 Exploration of Child Subgroup Testing Results

The optimization framework in this study produces a result that heavily prioritizes tests to the child subgroup. One explanation for this result is related to the high asymptomatic level of children compared to individuals from other age subgroups, because they are less likely to develop life-threatening symptoms. Based on the current information, we know that children do not contract the virus at the same rate as adults, however research has yet to show whether young children transmit the new coronavirus at a similar rate as adults.

According to CDC [8], children under the age of 18 account for under 2% of reported cases of COVID-19 in China, Italy, and the United States, however the study by Dong et al [9] who report on over 2000 children with suspected or confirmed COVID-19, find that 13% of virologically-confirmed child cases had asymptomatic infection. They also believe that this rate almost certainly understates the true rate of asymptomatic infection, since many asymptomatic children are unlikely to be tested.

The caution surrounding children's ability to spread the disease to other subgroups with a higher risk of health complications raises questions about whether policy-makers should explore stronger responses directed towards the child subgroup, in order to control the spread of the virus. Supporting this claim is a study released by Christian Drosten, Germany's chief virologist [10], who "found no significant difference" in the viral loads of the new coronavirus "between any pair of age categories, including children" Researchers concluded that "children may be as infectious as adults" based on these findings and recommended that countries practice "caution against an unlimited re-opening of schools and kindergartens in the present situation".

On the other hand, a pre-published study from researchers at the University of Queensland observed a collection of families with COVID-19 across the world and found that children were the initial source of infection among the families in only 8% of households [11].

Unfortunately, the incomplete information available about the virus means that the role that children play in transmitting it remains unclear. Children are also under-represented in current studies aiming to analyse transmission of the virus, so the policy-decision to prioritize all tests towards children cannot be committed to fully. For now, the case fatality rate for the children is confirmed to be unusually low, and if they are able to transmit the virus in the same way as adults and elderly, it would be advisable to direct a large number of tests towards this subgroup.

6.2 Exploration of Adult and Elderly Subgroup Testing Results

It should be noted that results for the optimized number of tests per subgroup are associated with the lowest VSL. Comparing a range of the minimum total VSL parameter combinations recommends the majority of test to children, but also a small number of tests to the adult subgroup. This represents a societal trade off between an small increase in the total VSL lost, but a choice to provide each subgroup with some tests.

6.3 Case Fatality Rates in the Cost Function

To improve the accuracy of the cost function, the choice of case fatality rates and VSL could be modified. In these experiments, the chosen case fatality rates are provided by the Spanish Ministry of Health. It can be useful to compare the results to the case fatality rates from the Imperial College London study of Ferguson et al [12] in Table 2. Compared to the Spanish fatality rates, the fatality rate for the 'child' subgroup between 0-18 years would be above zero. This may lead to changes in the recommended distribution of tests between subgroups, with a proportion of tests being prioritized to the adult or elderly group.

Age Group (Years)	Infection Fatality Ratio
0 to 9	0.002%
10 to 19	0.006%
20 to 29	0.03%
30 to 39	0.08%
40 to 49	0.15%
50 to 59	0.6%
60 to 69	2.2%
70 to 79	5.1%
80+	9.3%

Table 2: COVID-19 Case Fatality Rates by Age Group. Source: Ferguson et al [12]

6.4 VSL in the Cost Function

We are aware that a limitation of the VSL subgroup valuations is that they are based on VSL in the United States, and may not be unrepresentative of the VSL across age groups in Spain. However, this study focuses on the relative difference between total VSL lost rather than the total number of VSL for policy recommendations, and it is reasonable to assume that the VSL valuations between children, adults, and the elderly subgroups are comparatively similar in the US and Spain.

6.5 Limits of the Model

We recognise that there are limitations to the model as a result of simplifying assumptions of the real-world and incomplete information on the new virus. Firstly, a parameter to control the time lag for the delivery of test results, and a requirement for stratified periodic sampling in the model could better represent reality. Müller's model sets the current time lag from as 0 days, however we expect this to be between 3-5 days, or up to a week. Likewise, it is known that more than one antibody test is required to be taken, two weeks apart, to provide an accurate result by reducing the number of false positives.

Another area for further work could be to increase the minimum number of days to wait before a relaxing or restricting policy is implemented since the last one, to greater than 3 days.

7 Conclusion

In this study, we demonstrate how random testing policies can be used to provide 'real-time' estimates of the proportion of infected individuals within a population. Using the current available data on Spain's COVID fatality rates and recommendations for VSL, we estimate the daily number of tests required in Spain, and provide a recommendation for the distribution of tests between the child, adult, and elderly subpopulations, in order to control the spread of COVID-19 virus after a lockdown and prevent a second-wave of infections.

We hope that researchers find this tool useful for exploring random testing as a policy for tackling a virus similar to the coronavirus, and are inspired to build on the model for further use.

References

[1] Markus Mueller et al. "Using random testing in a feedback-control loop to manage a safe exit from the COVID-19 lockdown". In: (Apr. 2020). DOI: 10.1101/2020.04.09.20059360. URL: https://doi.org/10.1101/2020.04.09.20059360.

[2] Red Nacional de Vigilancia Epidemiologica. *Análisis de los casos de COVID-19 notificados a la RENAVE hasta el 10 de mayo en España*. Tech. rep. May 2020.

[3] Michael Greenstone and Vishan Nigam. "Does Social Distancing Matter?" In: *SSRN Electronic Journal* (2020). DOI: 10.2139/ssrn.3561244. URL: https://doi.org/10.2139/ssrn.3561244.

[4] *Recommended Value of a Statistical Life numbers for policy analysis*. Jan. 2012. DOI: 10.1787/9789264130807-9-en. URL: https://doi.org/10.1787/9789264130807-9-en.

[5] World Health Organisation. *Statement - Older people are at highest risk from COVID-19, but all must act to prevent community spread*. 2020. URL: https://www.euro.who.int/en/about-us/regional-director/statements/statement-older-people-are-at-highest-risk-from-covid-19,-but-all-must-act-to-prevent-community-spread (visited on 06/03/2020).

[6] Daron Acemoglu et al. *Optimal Targeted Lockdowns in a Multi-Group SIR Model*. Tech. rep. May 2020. DOI: 10.3386/w27102. URL: https://doi.org/10.3386/w27102.

[7] National Center for Environmental Economics, Office of Policy, US Environmental Protection Agency. *Guidelines for Preparing Economic Analyses*. Tech. rep. May 2014.

[8] and Stephanie Bialek et al. "Coronavirus Disease 2019 in Children — United States, February 12–April 2, 2020". In: *MMWR. Morbidity and Mortality Weekly Report* 69.14 (Apr. 2020), pp. 422–426. DOI: 10.15585/mmwr.mm6914e4. URL: https://doi.org/10.15585/mmwr.mm6914e4.

[9] Yuanyuan Dong et al. "Epidemiology of COVID-19 Among Children in China". In: *Pediatrics* 145.6 (Mar. 2020), e20200702. DOI: 10.1542/peds.2020-0702. URL: https://doi.org/10.1542/peds.2020-0702.

[10] Terry C Jones et al. "An analysis of SARS-CoV-2 viral load by patient age". In: (June 2020). DOI: 10.1101/2020.06.08.20125484. URL: https://doi.org/10.1101/2020.06.08.20125484.

[11] Smriti Mallapaty. "How do children spread the coronavirus? The science still isn't clear". In: *Nature* 581.7807 (May 2020), pp. 127–128. DOI: 10.1038/d41586-020-01354-0. URL: https://doi.org/10.1038/d41586-020-01354-0.

[12] N Ferguson et al. "Report 9: Impact of non-pharmaceutical interventions (NPIs) to reduce COVID19 mortality and healthcare demand". In: (2020). DOI: 10.25561/77482. URL: http://spiral.imperial.ac.uk/handle/10044/1/77482.