# Computer Vision - Assignment 3

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## Exercise 1

$$P_1 = [I\ \ 0],\ \ P_2 = [A\ \ t] = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$l = [e_2]_x(Ax) \implies F = [t]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}$$

We can use the fundamental matrix to map points in image one to lines in image two. The line will be:

$$l = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} \sim \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix}$$

The points can only be projections of the same 3D point if they are on the epipolar line $l$:

$$x1 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix},\ x2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix},\ x3 = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

$$l^T x1 = 0 \implies \text{on the line}$$

$$l^T x2 = 0 \implies \text{on the line}$$

$$l^T x3 = -1 \implies \text{not on the line}$$

## Exercise 2

$$P_1 = [I\ \ 0],\ \ P_2 = [A\ \ t] = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

We have that $C_1 = (0, 0, 0, 1)$ and $C_2 = \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix}$ in homogeneous coordinates. Projecting these will give us the epipoles:

$$e_2 \sim P_2 C_1 = t = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$e_1 \sim P_1 C_2 = -A^{-1} t = -\begin{pmatrix} 1 & -1/2 & -1 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$F = [e_2]_x A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix}, \ |F| = 0$$

$$e_2^T F = \begin{pmatrix} 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$F e_1 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

## Exercise 3

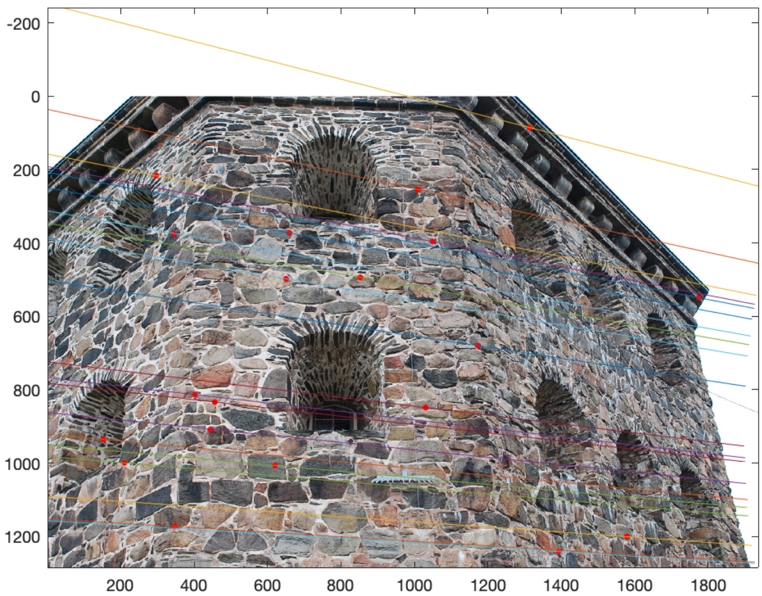
$$\bar{x}_1 \sim N_1 x_1 \text{ and } \bar{x}_2 \sim N_2 x_2$$

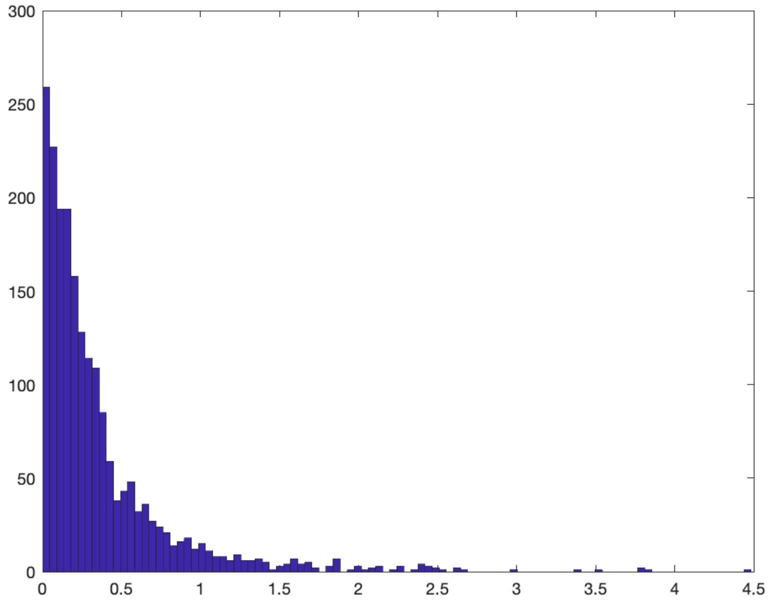
$$\bar{x}_2^T \bar{F} \bar{x}_1 = x_2^T F x_1 = 0$$

$$\implies (N_2 x_2)^T \bar{F} N_1 x_1 = x_2^T N_2^T \bar{F} N_1 x_1 \implies F = N_2^T \bar{F} N_1$$

## Computer Exercise 1

$$F = \begin{pmatrix} -3.3901e-08 & -3.7201e-06 & 0.0058 \\ 4.6674e-06 & 2.8936e-07 & -0.0267 \\ -0.0072 & 0.0263 & 1 \end{pmatrix}$$



## Exercise 4

Note: when finding the $e_2$, we compute the nullspace of $F^T$ and not $F$ as I wrote in the solution below.

$$F = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad P_1 = [I \; 0], \quad P_2 = [\,[e_2]_\times F \;\; e_2\,]$$

Projecting $e_2$ through $F$ should yield $(0, 0, 0)$ since $P_1 = [I \; 0]$. $e_2$ will thus be the nullspace of $F$:

$$(x, y, z) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \Rightarrow \begin{cases} y = 0 \\ x + z = 0 \\ x + z = 0 \end{cases} \Rightarrow \begin{cases} x = t \\ y = 0 \\ z = -t \end{cases}$$

$$x = t$$

$$e_2 = (1, 0, -1)^T, \quad [e_2]_\times = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Rightarrow P_2 = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

Now, we can project the points and verify the epipolar constraint

$$X = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}, \quad \hat{X} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} \Rightarrow X_{p1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad X_{p2} = \begin{pmatrix} 2 \\ -10 \\ 0 \end{pmatrix}$$

$$\hat{X}_{p1} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \quad \hat{X}_{p2} = \begin{pmatrix} 4 \\ -6 \\ 2 \end{pmatrix}$$

$$X_{p2}^T F X_{p1} = (2 \; -10 \; 0) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 10 - 10 = 0$$

$$\hat{X}_{p2}^T F \hat{X}_{p1} = (4 \; -6 \; 2) \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = 12 - 18 + 6 = 0$$

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We know that $Fe_1 = 0$ and $P_2 \begin{bmatrix} e_1 \\ 0 \end{bmatrix} = 0$, so we find $e_1$:

$$Fe_1 = 0 \Rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \Rightarrow \begin{cases} y + z = 0 \\ x = 0 \\ y + z = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = -t \\ z = t \end{cases} \Rightarrow e_1 \sim \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

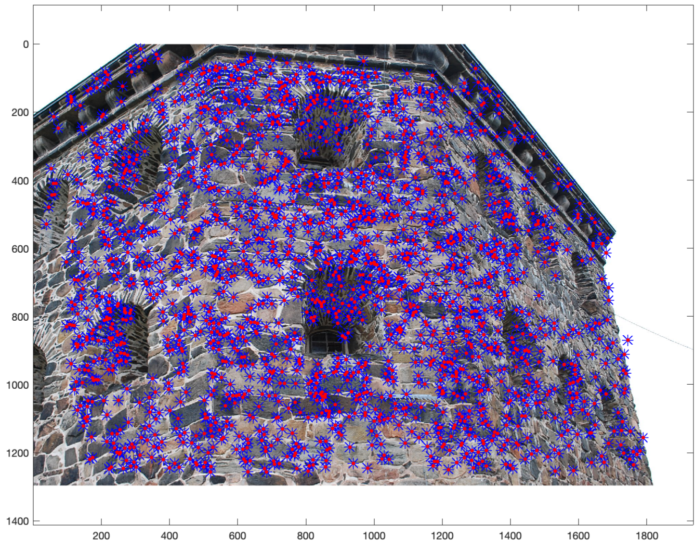
$$z = t$$

and thus that gives $e_2 = (0, -1, 1, 0)$

## Computer Exercise 2

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

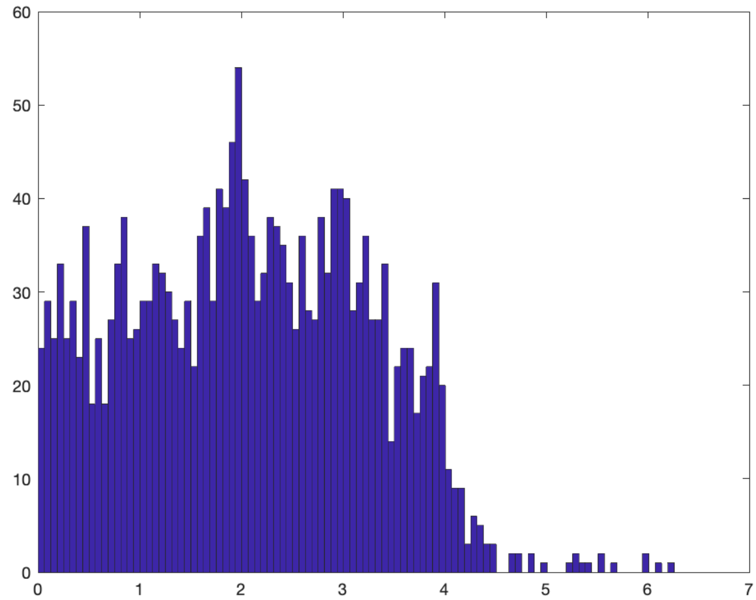
$$P_2 = \begin{pmatrix} -0.0016 & 0.0057 & 0.2163 & 0.9763 \\ 0.0070 & -0.0257 & -0.9763 & 0.2163 \\ 4.5642e-06 & 1.0872e-06 & -0.0273 & 0.0001 \end{pmatrix}$$

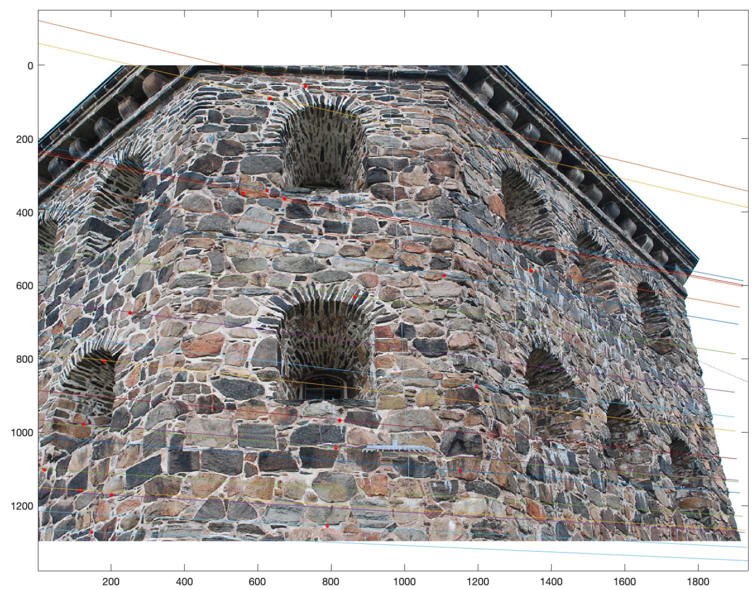



Blue points are original, red are projected back from triangulated 3D points

## Computer Exercise 3

$$E = \begin{pmatrix} -8.8885 & -1.0058e+03 & 3.7708e+02 \\ 1.2525e+03 & 78.3677 & -2.4482e+03 \\ -4.7279e+02 & 2.5502e+03 & 1 \end{pmatrix}$$



## Exercise 6

$E = U \operatorname{diag}([1, 1, 0]) V^T$ where

$$U = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$UV^T = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix} \Rightarrow \det(UV^T) = 1/\sqrt{2} \cdot 1/\sqrt{2} + -1/\sqrt{2} \cdot -1/\sqrt{2} = 1$$

$$E = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ Verify the epipolar constraint:

$$x_2^T E x_1 = (1\ 1\ 1) \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (1\ 1\ 1) \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$= -1/\sqrt{2} + 1/\sqrt{2} + 0 = 0 \Rightarrow$$ correspondance plausible

$$UWV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}$$

$$UW^TV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{cases} x = 0 \\ y = 0 \\ z = 1 \\ w = s \end{cases}$$

Proof that $X$ must be $= \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix}$

$$P_{21} = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}, \quad P_{22} = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

$$P_{23} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}, \quad P_{24} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

$$P_{21} X(s) = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ s \end{pmatrix} \Rightarrow s = -1/\sqrt{2}, \quad \text{behind camera}$$

$$P_{22} X(s) = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ -s \end{pmatrix} \Rightarrow s = 1/\sqrt{2}, \quad \text{in front of camera}$$

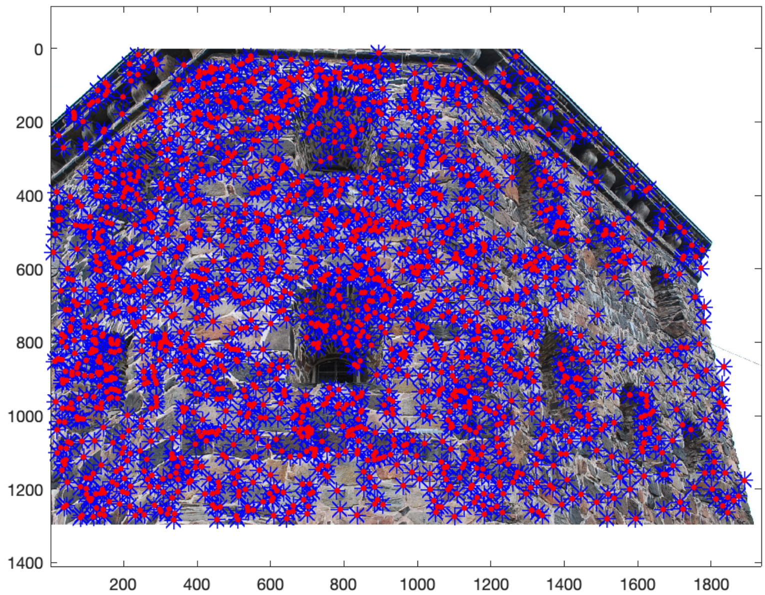
$$P_{23} X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ s \end{pmatrix} \Rightarrow s = 1/\sqrt{2}, \quad \text{in front of camera}$$

$$P_{24} X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -s \end{pmatrix} \Rightarrow s = -1/\sqrt{2} \quad \text{behind camera}$$

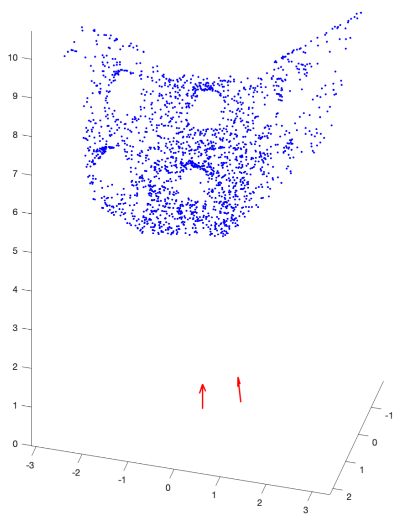
UPDATE:

The correct camera pair is $P_1 = [I\,0]$ and $P_{23}$ because both $s$ (and thus third coordinate of $x_1$) and the third coordinate of $x_2$ is positive.

## Computer Exercise 4




Blue points are original, red are the best triangulation solution projected back on image. Seems to line up nicely



The final 3D plot of the triangulated points and cameras