λ_{ref} Language Definition

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1 Overview

2 Syntax

The syntax of λ_{ref} is given below.

$$
\begin{array}{rcl}
v & ::= & () \\
 & | & n \in \mathbb{N} \\
 & | & b \in \mathbb{B} \\
 & | & x \\
 & | & \lambda x.e
\end{array}
\qquad
\begin{array}{rcl}
e & ::= & v \\
 & | & e_1\ e_2 \\
 & | & \textbf{let}\ x = e_1\ \textbf{in}\ e_2 \\
 & | & \textbf{if}\ e_1\ \textbf{then}\ e_2\ \textbf{else}\ e_3 \\
 & | & \mathit{ref}\ e \\
 & | & !e \\
 & | & e_1 := e_2
\end{array}
$$

3 Small Step Semantics

3.1 Values

- (**unit**)

$$\frac{}{() \ \texttt{val}}$$

- (**integer**)

$$\frac{}{n \ \texttt{val}}$$

- (**boolean**)

$$\frac{}{b \ \texttt{val}}$$

- (**lambda abstraction**)

$$\frac{}{\lambda x.e \ \texttt{val}}$$

3.2 Function Application

- (**ap-l**)

$$\frac{(e_1, \sigma) \longmapsto (e_1', \sigma')}{(e_1\ e_2, \sigma) \longmapsto (e_1'\ e_2, \sigma')}$$

- (**ap-r**)

$$\frac{v\ \texttt{val} \qquad (e_2, \sigma) \longmapsto (e_2', \sigma')}{(v\ e_2, \sigma) \longmapsto (v\ e_2', \sigma')}$$

- (**ap**)

$$\frac{v\ \texttt{val}}{((\lambda x.e)\ v, \sigma) \longmapsto (e[v/x], \sigma)}$$

3.3 Let

- (**let-1**)

$$\frac{e_1 \longmapsto e_1'}{\textbf{let}\ x = e_1\ \textbf{in}\ e_2 \longmapsto \textbf{let}\ x = e_1'\ \textbf{in}\ e_2}$$

- (**let**)

$$\frac{v\ \texttt{val}}{\textbf{let}\ x = v\ \textbf{in}\ e \longmapsto e[v/x]}$$

4 If

- (**if**)

$$\frac{(e_1, \sigma) \longmapsto (e_1', \sigma')}{(\textbf{if}\ e_1\ \textbf{then}\ e_2\ \textbf{else}\ e_3, \sigma) \longmapsto (\textbf{if}\ e_1'\ \textbf{then}\ e_2\ \textbf{else}\ e_3, \sigma')}$$

- (**if-true**)

$$\frac{}{(\textbf{if true then}\ e_2\ \textbf{else}\ e_3, \sigma) \longmapsto (e_2, \sigma)}$$

- (**if-false**)

$$\frac{}{(\textbf{if false then}\ e_2\ \textbf{else}\ e_3, \sigma) \longmapsto (e_3, \sigma)}$$

4.1 References

- (**ref-e**)

$$\frac{(e, \sigma) \longmapsto (e', \sigma')}{(\mathit{ref}\ e, \sigma) \longmapsto (\mathit{ref}\ e', \sigma')}$$

- (**ref**)

$$\frac{v\ \texttt{val} \qquad l \notin \mathrm{dom}(\sigma)}{(\mathit{ref}\ v, \sigma) \longmapsto (l, \sigma[l \mapsto v])}$$

- (**deref-e**)

$$\frac{(e, \sigma) \longmapsto (e', \sigma')}{(!e, \sigma) \longmapsto (!e', \sigma')}$$

- (**deref**)

$$\frac{l\ \texttt{val} \qquad l \in \mathrm{dom}(\sigma)}{(!l, \sigma) \longmapsto (\sigma(l), \sigma)}$$

- (**set-l**)

$$\frac{(e_1, \sigma) \longmapsto (e_1', \sigma')}{(e_1 := e_2, \sigma) \longmapsto (e_1' := e_2, \sigma')}$$

- (**set-r**)

$$\frac{l\ \texttt{val} \qquad (e_2, \sigma) \longmapsto (e_2', \sigma')}{(l := e_2, \sigma) \longmapsto (l := e_2', \sigma')}$$

- (**set**)

$$\frac{l \in \mathrm{dom}(\sigma) \qquad v\ \texttt{val}}{(l := v, \sigma) \longmapsto (\texttt{()}, \sigma[l \mapsto v]}$$