DEPARTMENT OF MATHEMATICAL SCIENCES, FAU.

MAS 6215 Algebraic Number Theory: syllabus and outline

Fall, 2017

Note: document modified in Oct.

1. General

Course information.

• Semester: Aug 21, 2017 - Dec 1, 2017

• Lecture Time: 12:00PM – 12:50PM on Monday, Wednesday and Friday.

• Lecture Venue: BU 112 (College of Business Bldg, Boca Campus).

• Instructor: Shi Bai, sbai@fau.edu

Content. Number theory is one of the oldest and most beautiful branches of mathematics. Yet it continues to be a dynamic research area and has numerous applications in the real-world (such as cryptography). Algebraic number theory is a major branch of number theory that studies algebraic structures related to algebraic integers and algebraic number fields. This course (3-credits) will cover the structure of the ring of integers and algebraic number fields; splitting of primes in extensions; action of the Galois group; bounds on the size of the ideal class group and the structure of the group of units. A tentative lecture plan includes:

- Preliminaries on commutative algebra
- Algebraic number fields and rings of integers
- Dedekind domain, ideals and factorization
- Class groups and Minkowski's bound
- Units and Dirichlet's theorem
- Cyclotomic extension and Fermat's Last Theorem
- Additional topics may include: local fields; algorithmic aspects and applications

Pre-requisites. The prerequisites are MAS 5311 and 5312. We assume general familiarity with modern algebra: the course presupposes adequate background and knowledge of rings, fields and basic Galois theory at the level of MAS 5311 and 5312. Apart from the prerequisites, the course will be self-contained; we shall develop all the machinery that we need.

Objective. Students taking this course will develop abilities to solve number-theoretic problems using algebraic methods; In particular, students will be comfortable with the algebraic structures involved such as algebraic number fields, their rings of integers, ideals, class group, unit groups; and understand their algebraic manipulations.

Textbooks. We will mainly use the following two books,

- S. Alaca and K. S. Williams, *Introductory Algebraic Number Theory*, Cambridge University Press, 2004. (Electronic version available through FAU library website).
- J.S. Milne, Algebraic Number Theory.

http://www.jmilne.org/math/CourseNotes/ant.html

We will also use Stein's book for computational aspects of the course.

• W. Stein, Algebraic Number Theory, a Computational Approach. 2012. http://wstein.org/books/ant/ Supplementary reading. For additional reading the following books may be considered,

- Albrecht Fröhlich and Martin J. Taylor, Algebraic Number Theory, Cambridge University Press, 1991.
- Jürgen Neukirch, Algebraic Number Theory, Grundlehren der mathematischen Wissenschaften, 1st Edition, 1999.
- Serge Lang, Algebraic Number Theory, Graduate Texts in Mathematics, 2nd Edition, 1994.
- Daniel A. Marcus, Number Fields (Universitext), Springer, 1995.
- Richard A. Mollin, *Algebraic Number Theory*, Chapman and Hall/CRC, 2nd Edition, 2011
- Harry Pollard and Harold G. Diamond, *The Theory of Algebraic Numbers*, Dover Publications, 3rd Edition, 1998.
- Ian Stewart and David O. Tall, Algebraic Number Theory and Fermat's Last Theorem, 3rd Edition, 2002.

2. Evaluation

The grade for the course will be determined by the following scheme:

Assignments (30%), Midterm Exam (30%), Group Project (40%).

Assignments. There will be several assignments for the course. All these assignments contribute 30% to your overall score. There will be no assignments for the first week, the mid-exam week and the last week before the group project presentation. Assignments should be clearly handwritten or printed on paper or sent by email in PDF formats.

Mid Exam. There will be a midterm exam, which counts for 30% of the grade. The tentative date of the midterm exam is late October. The midterm exam will cover the basic topics and algebraic structures taught during the first half semester.

Project. A group project on relevant topics will be given during the semester, which counts for 40% of the class grade. The evaluation consists of a short report; several computational tasks and a final project presentation.

Grading scale. At the end of the semester, the following scale for FAU grade will be used.

% 84-86 81-83 78-80 75-77 72 - 7469-71 66-68 63-65 60-62 87-91 C+Grade Α В B- \mathbf{C} D-F Α-B+C-D+D

3. Course Policies

Students are expected to be familiar and comply with the standard university policies. In addition, the following policies on assignments, exams and academic integrity should conformed.

<u>Attendance</u>. Attendance is required. Students are expected to be regular in class attendance and to fully participate in the course. Grade penalties will be imposed for unexcused absences.

<u>Late assignment</u>. Students will receive 0 for any assignment that is not handed in before the due date and time, without prior approval.

<u>Collaboration policy on assignments</u>. Collaboration on the assignments is permitted. If you do collaborate, your write-ups must be done independently and you must acknowledge your collaborators in your write-up. Failure to do so constitutes plagiarism.

Exam policies. All exams will be taken as scheduled. Make-up examinations will not be given for missed tests, unless prior arrangements have been made under exceptional circumstances with advance notice. In the case of a valid excuse, documentation should be supplied before any remedy can be considered. The exams should be completed independently; no collaboration is allowed during the exams.

Disclaimer. This syllabus is subject to reasonable changes at the discretion of the instructor.