DEPARTMENT OF MATHEMATICAL SCIENCES, FAU.

MAS 6215 Algebraic Number Theory: syllabus and outline

Fall, 2019

1. General

Course information.

• Semester: Fall 2019.

• Course: MAS-6215 Algebraic Number Theory (CRN: 15772)

 \bullet Lecture Time: 12:30 PM - 1:50 PM on Wednesday and Friday.

• Lecture Venue: SE314.

• Credit hours: 3.

• Instructor: Shi Bai, SE230, sbai@fau.edu

• Office hours: Wed, Fri 2 - 3 pm; and by appointment.

Content. Number theory is one of the oldest and most beautiful branches of mathematics. Yet it continues to be a dynamic research area and has numerous applications in the real-world (such as cryptography). Algebraic number theory is a major branch of number theory that studies algebraic structures related to algebraic integers and algebraic number fields. This course (3-credits) will cover the structure of the ring of integers and algebraic number fields; splitting of primes in extensions; action of the Galois group; bounds on the size of the ideal class group and the structure of the group of units. A tentative lecture plan includes:

- Preliminaries on commutative algebra
- Algebraic number fields and rings of integers
- Dedekind domain, ideals and factorization
- Class groups and Minkowski's bound
- Units and Dirichlet's theorem
- Cyclotomic extension and Fermat's Last Theorem
- Additional topics may include: local fields; algorithmic aspects and applications

Pre-requisites. The prerequisites are MAS 5311 and 5312. We assume general familiarity with modern algebra: the course presupposes adequate background and knowledge of rings, fields and basic Galois theory at the level of MAS 5311 and 5312. Apart from the prerequisites, the course will be self-contained; we shall develop all the machinery that we need.

Objective. Students taking this course will develop abilities to solve number-theoretic problems using algebraic methods; In particular, students will be comfortable with the algebraic structures involved such as algebraic number fields, their rings of integers, ideals, class group, unit groups; and understand their algebraic manipulations.

Textbooks. We will mainly use the following two books,

- S. Alaca and K. S. Williams, *Introductory Algebraic Number Theory*, Cambridge University Press, 2004. (Electronic version available through FAU library website).
- J.S. Milne, Algebraic Number Theory.

http://www.jmilne.org/math/CourseNotes/ant.html

We will also use Stein's book for computational aspects of the course.

• W. Stein, Algebraic Number Theory, a Computational Approach. 2012. http://wstein.org/books/ant/

Supplementary reading. For additional reading the following books may be considered,

- Albrecht Fröhlich and Martin J. Taylor, *Algebraic Number Theory*, Cambridge University Press, 1991.
- Jürgen Neukirch, Algebraic Number Theory, Grundlehren der mathematischen Wissenschaften, 1st Edition, 1999.
- Serge Lang, Algebraic Number Theory, Graduate Texts in Mathematics, 2nd Edition, 1994.
- Daniel A. Marcus, Number Fields (Universitext), Springer, 1995.
- Richard A. Mollin, *Algebraic Number Theory*, Chapman and Hall/CRC, 2nd Edition, 2011.
- Harry Pollard and Harold G. Diamond, *The Theory of Algebraic Numbers*, Dover Publications, 3rd Edition, 1998.
- Ian Stewart and David O. Tall, Algebraic Number Theory and Fermat's Last Theorem, 3rd Edition, 2002.

2. Evaluation

The grade for the course will be determined by the following scheme:

Assignments (50%), Midterm Exam (25%), Group Project (25%).

Assignments. There will be 6-7 assignments for the course. All these assignments contribute 50% to your overall score. There will be no assignments for the first week, the mid-exam week and the last week before the group project presentation. Assignments should be clearly handwritten or printed on paper or sent by email in PDF formats.

Mid Exam. There will be a midterm exam, which counts for 25% of the grade. The tentative date of the midterm exam is October. The midterm exam will cover the basic definition and algebraic structures taught during the first half semester.

Project. A group project on relevant topics will be given during the semester, which counts for 25% of the class grade. The evaluation consists of a short report; some literature review and a final project presentation.

Grading scale. At the end of the semester, the following scale for FAU grade will be used.

% 92-100 87-91 84-86 81-83 78-80 69-71 66-68 63-65 60-62 0-5975-77 72 - 74Grade Α A-B+В B-C+ \mathbf{C} C-D+D D-F

3. Course Policies

Students are expected to be familiar and comply with the standard university policies. In addition, the following policies on assignments, exams and academic integrity should conformed.

<u>Attendance</u>. Attendance is required. Students are expected to be regular in class attendance and to fully participate in the course. Grade penalties will be imposed for unexcused absences.

<u>Late assignment</u>. Students will receive 0 for any assignment that is not handed in before the due date and time, without prior approval.

<u>Collaboration policy on assignments</u>. Collaboration on the assignments is permitted. If you do collaborate, your write-ups must be done independently and you must acknowledge your collaborators in your write-up. Failure to do so constitutes plagiarism.

<u>Exam policies</u>. All exams will be taken as scheduled. Make-up examinations will not be given for missed tests, unless prior arrangements have been made under exceptional circumstances with advance notice. In the case of a valid excuse, documentation should be supplied before any remedy can be considered. The exams should be completed independently; no collaboration is allowed during the exams.

<u>Disability policy</u>. In compliance with the Americans with Disabilities Act Amendments Act (ADAAA), students who require reasonable accommodations due to a disability to properly execute coursework must register with Student Accessibility Services (SAS) and follow all SAS procedures. SAS has offices across three of FAU's campuses – Boca Raton, Davie and Jupiter – however disability services are available for students on all campuses. For more information, please visit the SAS website at www.fau.edu/sas.

Academic integrity. Students at Florida Atlantic University are expected to maintain the highest ethical standards. Academic dishonesty is considered a serious breach of these ethical standards, because it interferes with the university mission to provide a high quality education in which no student enjoys an unfair advantage over any other. Academic dishonesty is also destructive of the university community, which is grounded in a system of mutual trust and places high value on personal integrity and individual responsibility. Harsh penalties are associated with academic dishonesty. For more information, see University Regulation 4.001 at the link: http://www.fau.edu/regulations/chapter4/4.001_Code_of_Academic_Integrity.pdf

Counseling and Psychological Services (CAPS) Center. Life as a university student can be challenging physically, mentally and emotionally. Students who find stress negatively affecting their ability to achieve academic or personal goals may wish to consider utilizing FAU's Counseling and Psychological Services (CAPS) Center. CAPS provides FAU students a range of services – individual counseling, support meetings, and psychiatric services, to name a few – offered to help improve and maintain emotional well-being. For more information, go to http://www.fau,edu/counseling/.

Disclaimer. This syllabus is subject to reasonable changes at the discretion of the instructor.