Exercises

1. Find the least common multiples of:

   (1) $\mathrm{lcm}(14, 15)$.
   (2) $\mathrm{lcm}(240, 610)$.
   (3) $\mathrm{lcm}(n, n+1)$.
   (4) $\mathrm{lcm}(2n-1, 2n+1)$.
   (5) $\mathrm{lcm}(2^5 \cdot 3 \cdot 5^6 \cdot 7^2 \cdot 11,\ 2^3 \cdot 5^8 \cdot 7^2 \cdot 13)$.

   (Hints: use the F.T.A and/or the relation between gcd and lcm).

2. Show that if a and b are nonzero positive integers then the $\gcd(a, b) \mid \mathrm{lcm}(a, b)$. But $(\gcd(a, b))^2 \nmid \mathrm{lcm}(a, b)$ unless a, b are coprime.

3. When are the least common multiple and the greatest common divisor equal to each other?

4. (Same question of Week 3) Find **all** integer solutions to the following equation if the solution exists or show the the solution does not exist.

   (1) $2x + 3y = 4$
   (2) $15x + 51y = 41$
   (3) $121x - 88y = 572$

   The following are constrained linear Diophantine equations.

5. How many ways are there to make \$2.00 dollars from only nickels and quarters? (Hints: converts this to a linear Diophantine equation. By its nature, solutions will be non-negative).

6. A grocer orders apples and bananas at a total cost of \$8.4. If the apples cost 25 cents each and the bananas 5 cents each, how many of each type of fruit did he order.

7. Find how many integer solutions there are to the following equation $3a + 5b = 7$ subject to $a, b \in [-10, 10]$.