

ELASTICITY

If we apply any force on any body then the shape of body will change & after removing the force body again come back to its original shape. Then that property of body is called Elasticity. & the force applied on body is called deforming force.

$$\# \text{ Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A} \quad * \text{SI unit} = \text{N/m}^2 * [\text{ML}^{-1}\text{T}^{-2}]$$

$$\# \text{ Strain} = \frac{\text{change in configuration}}{\text{original configuration}} \quad (\text{length or, volume})$$

$$\# \text{ Hook's Law} \rightarrow \boxed{\text{Stress} \propto \text{Strain}}$$

$$\text{Stress} = E \times \text{Strain}$$

$E \rightarrow \text{coefficient of Elasticity.}$

It is of three types -

iii) \rightarrow young Modulus of Elasticity (y) \rightarrow

$$y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$y = \frac{F/A}{\Delta L/L}$$

$$y = \frac{FL}{A \Delta L}$$

$A \rightarrow \text{Area of cross section}$

$L \rightarrow \text{original length}$

$\Delta L \rightarrow \text{change in length.}$

|a| \rightarrow change in length of a massless wire when 'm' mass is hanged at lower end \rightarrow

$$y = \frac{FL}{A \Delta L}$$

$$\Delta L = \frac{mgL}{YA}$$

$$\Delta L = \frac{mgL}{4\pi r^2}$$

$r \rightarrow \text{radius of wire}$

|b| \rightarrow change in length due to its own weight of a uniform of mass 'm'

*
$$\Delta L = \frac{mg(1/2)}{4\pi r^2}$$

$$\Delta L = \frac{mgL}{2\pi r^2}$$

|c| \rightarrow potential energy stored in a stretched wire -

$$F = k\Delta L \text{ (like spring)}$$

so, because behave like spring of force const.

$$K = \frac{YA}{L}$$

$$U = \frac{1}{2} K \Delta L^2$$

$$\Delta L = U(\text{vol.})$$

$$U = \frac{1}{2} \left(\frac{YA}{L} \right) (\Delta L)^2$$

$$\frac{U}{V} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$\frac{U}{V} = \frac{1}{2} \frac{(\text{Stress})^2}{Y}$$

iii) \rightarrow Bulk Modulus (B) or, K (MAKELWELL) \rightarrow

$$B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{\Delta P}{\Delta V/V}$$

$$B = -\frac{(\Delta P)V}{\Delta V}$$

$$\text{Compressibility (c)} = \frac{1}{B}$$

|a| \rightarrow For same $\Delta P, V$

$$B \propto \frac{1}{\Delta V}$$

$$B_{\text{solid}} > B_{\text{Liq}} > B_{\text{gas}}$$

$$\therefore (\Delta V)_{\text{solid}} < (\Delta V)_{\text{Liq}} < (\Delta V)_{\text{gas}}$$

|b| \rightarrow There are two bulk modulus in gases \rightarrow

* Isothermal bulk modulus $= P \Rightarrow P \rightarrow \text{gas}$
 * Adiabatic $\Rightarrow \text{gas} \rightarrow \gamma = \text{constant}$

Isothermal

$$\begin{aligned} PV &= \text{const} \\ P\Delta V + V\Delta P &= 0 \\ P\Delta V &= V\Delta P \end{aligned}$$

$$-\frac{V\Delta P}{\Delta V} = P$$

$$\text{Bisothermal} = P$$

Adiabatic

$$PV^\gamma = \text{const}$$

$$\nabla P = -\frac{(\Delta P)V}{\Delta V}$$

$$\text{Badiabatic} = VP$$

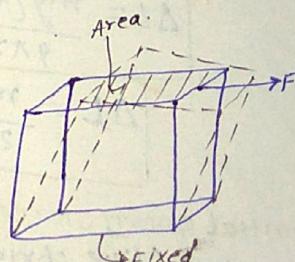
|iii| \rightarrow Rigidity Modulus / shear modulus (η)

$$\eta = \frac{\text{Shear Stress}}{\text{shear strain}}$$

$$\eta = \frac{F/A}{\theta} \quad (\theta \rightarrow \text{shear angle})$$

$$\eta = \frac{F}{A\theta}$$

NOTE \rightarrow * Twisting of solid cylinder.
 * Angle of shear always take in radian.



$\theta \rightarrow$ angle of twist

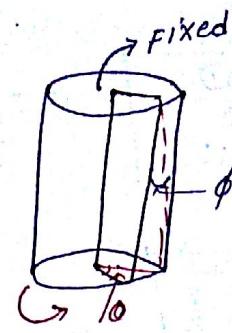
$r \rightarrow$ radius

$L \rightarrow$ Length

$\phi \rightarrow$ shear angle

$$AB \Rightarrow r\theta = L\phi$$

$$\text{Shear angle } \phi = \frac{r\theta}{L} \quad **$$

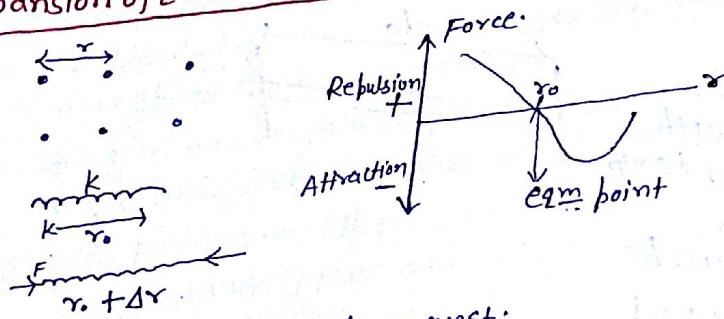


- ** # A wire has length L_1 when tension is T_1 & length is found to be L_2 when tension is T_2 . Find its natural length.

$$FL = \frac{L_1 T_2 - L_2 T_1}{T_2 - T_1}$$

- # A massless wire of length 'L' Radius 'r' is suspended vertically & pulled by force 'F' then change in length is 'L' another wire made of same material having length $2L$ & radius ' $2r$ ' & pulled by force $2F$ then change in length will be $\boxed{L' = L}$ *

Expansion of Elasticity by Interatomic force



If $K \rightarrow$ Interatomic force const.
then, $F = K\Delta r \quad \text{--- (I)}$

$$\text{strain} = \frac{\Delta r}{r_0} \quad \text{--- (II)}$$

Cross section area 'A'

$$\text{no. of atom } n = \frac{A}{r_0^2}$$

$$\begin{aligned} \text{Total Force } F_T &= nF \\ &= \frac{A}{r_0^2} K\Delta r \end{aligned}$$

$$\frac{F_T}{A} = \frac{K\Delta r}{r_0^2}$$

$$\text{stress} = \frac{K\Delta r}{r_0^2} \quad \text{--- (III)}$$

$$*\text{ stress} = Y \times \text{strain}$$

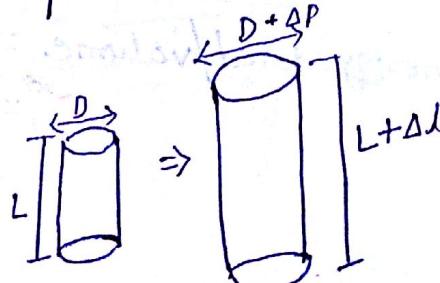
$$\frac{K\Delta r}{r_0^2} = Y \times \frac{\Delta r}{r_0}$$

$$*\boxed{K = Y r_0}$$

$r_0 \rightarrow$ Interatomic
distance.

Poisson's Ratio (σ) :-

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$



$$\epsilon = \frac{\Delta D}{D} = \frac{\Delta L}{L}$$

$$0 < \epsilon < \frac{1}{2}$$

* If volume is const on stretching

Then, $V = \text{const}$

$$\frac{\pi D^2}{4} L = \text{const.}$$

$$D^2 L = \text{const.}$$

$$\epsilon = \frac{1}{2}$$

MPPMT

young Bulk shear poisson.

Relation b/w γ , B , η , ϵ

$$i \rightarrow \gamma = 3B(1 - 2\epsilon)$$

$$ii \rightarrow \gamma = 2n(1 + \epsilon)$$

$$iii \rightarrow \frac{g}{V} = \frac{1}{B} + \frac{3}{\eta}$$

BHU

$$\epsilon = -1 < \epsilon < 0.5 \text{ (Theoretical limit)}$$

$$\epsilon = 0.2 \text{ to } 0.4 \text{ (Experimental limit)}$$

Thermal stress \rightarrow

Let, a rod of length l_0 is clamped at its end with rigid support & then temp. is increased by $\Delta\theta$

\therefore Its length will be

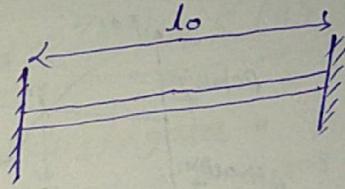
$$l = l_0 (1 + \alpha \Delta \theta)$$

$$l - l_0 = l_0 \alpha \Delta \theta$$

$$\Delta l = l_0 \alpha \Delta \theta$$

$$\frac{\Delta l}{l_0} = \alpha \Delta \theta$$

$$\text{Thermal stress} = \gamma \alpha \Delta \theta$$



Let $A \rightarrow$ area of cross section

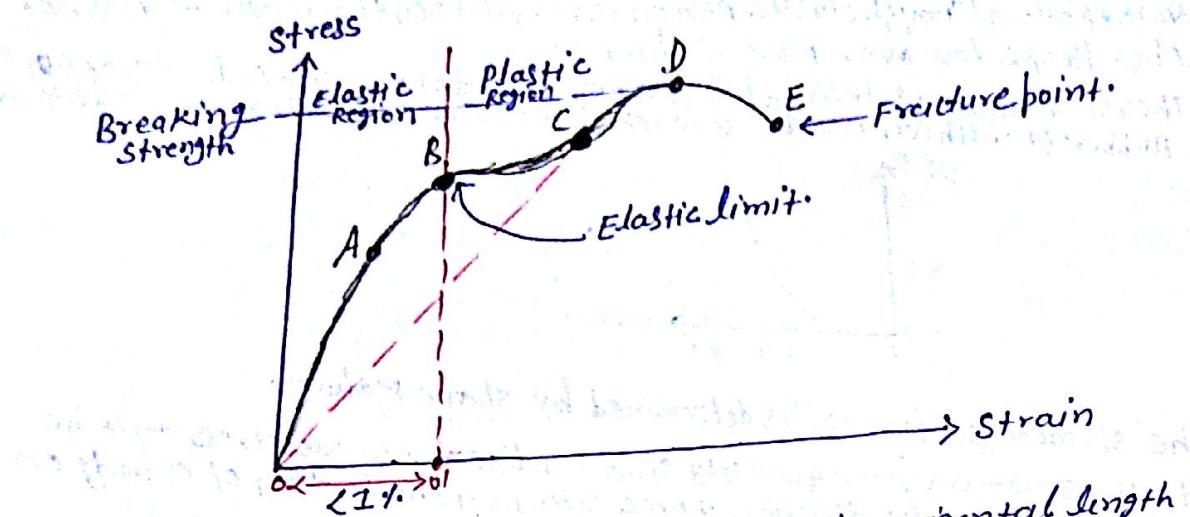
then, thermal stress $= F/A = \gamma \alpha \Delta \theta$

$$F = \gamma A \alpha \Delta \theta$$

Energy stored/volume $= \frac{1}{2} \text{ Stress} \times \text{Strain}$

$$= \frac{1}{2} \gamma (\alpha \Delta \theta)^2$$

Stress - stress curve →



* OA → Follows Hooke's Law & wire return in original length when weight/Force is removed.

* AB → Doesn't follow Hooke's Law.

* BC → When weight removed, some permanent strain remain.

* CD → Little extra stress cause large strain.

* D → Max stress without breaking.

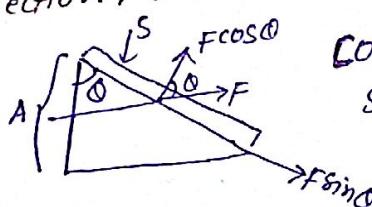
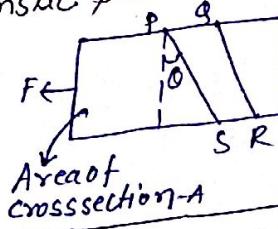
** If metal has very small plastic region there called brittle material.

** If metal has large plastic region called ductile material.

→ Elastic Fatigue → When weight on wire is applied & removed continuously then after sometime it losses its elastic property called Elastic Fatigue.

→ Elastic After Effect → Time taken by material to regain its original shape when deforming force is removed, there are some material like quartz, phosphor bronze regain its original shape immediately after deforming force is removed. i.e. these material has no elastic effect.

The force 'F' is applied on the face of rectangular block as shown in fig. define the tensile & shear stress at section PQRS.



$$\cos \theta = A/S$$

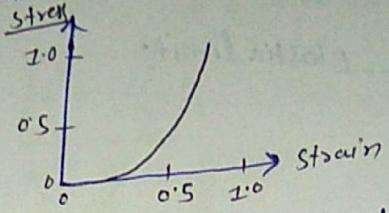
$$S = A/\cos \theta$$

* Tensile stress = $\frac{F \cos \theta}{A/\cos \theta} = \frac{F \cos^2 \theta}{A}$

* Shear stress = $\frac{F \sin \theta}{A/\cos \theta} = \frac{F \sin^2 \theta}{2A}$

* Rubber can be pulled to several times to its original length & still returns to original shape.

- * Stress & strain curve for elastic tissue of Aorta, present in Heart.
Note that, although elastic region is very large, the material does not obey Hooke's Law over most of the region.
- * There is no well defined plastic region substance like tissue of Aorta, Rubber etc. which can be stretched to cause large strains called Elastomer.



* The stretching of a coil is determined by shear Modulus.

- * Stress is not a vector quantity since, unlike force, the stress can't be assigned a specific direction. Force acting on the portion of a body on a specific side of a section has definite direction.

'SURFACE TENSION'

- It is the property of surface of liquid by which liquid tries to minimise its surface area.
- If surface tension (σ , T) is force acting per unit length on a line assumed on the surface of liquid on any one side of the line.



$$T \text{ or } \sigma = \frac{F}{L}$$

⇒ Act br to line assumed.

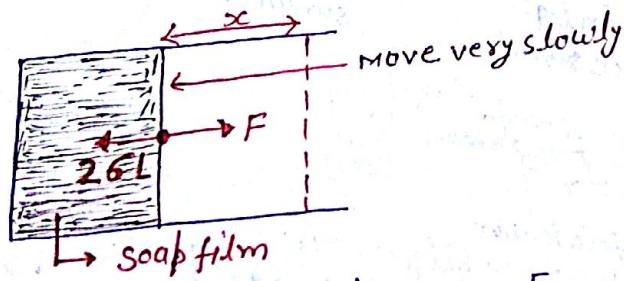
* unit → N/m

$$\text{Dimension} = \frac{\text{Imp}}{L} = [M\bar{T}^2]$$

- ** It is the property of surface of liquid & does not depend on length of line used.
- * Surface tension ↓ with rise in temp. & becomes zero at a critical temp. where interface b/w liquid & vapour disappear.
- ** It depends on impurities & size. When impurities contaminate on the surface.
- ** Generally, surface tension ↑ with highly soluble impurities like (NaCl in water) & ↓ with sparingly soluble impurities.

Work Done by surface Tension →

When surface area change



$$F = 2\sigma L$$

$$W_{\text{net}} = 0$$

$$W_F + W_\sigma = 0$$

$$W_F = -W_\sigma$$

$$W_\sigma = -Fx$$

$$= -2\sigma L x$$

$$= -\sigma L (2x) = -\sigma x 2(Lx)$$

$$\text{Work done by surface tension} = \boxed{-\sigma(AS)} \\ (\text{change in area})$$

∴ Work done on the surface is $W = \sigma x (AS)$

& this work done is the energy of surface.

All energy associated with the surface due to surface tension is also called 'surface energy'. (Bcoz of this reason)

Liquid drops are spherical.)

Force Required to Raise a Masses.

* |a| → square plate of side 'a' from liquid surface.

$$F = T(4L) = 4Ta$$

* |b| → square frame of wire side 'a'.

$$\begin{aligned} F &= \text{Force on one surface} \times 4 \\ &= (T \times 2a) \times 4 \\ &= 8Ta \end{aligned}$$

* |c| → Disc of Radius 'R'

$$F = Tl = T(2\pi r)$$

* |d| → A Ring of inner & outer radii R_1, R_2 ($R_1 < R_2$)

$$\begin{aligned} F &= T \cdot 2\pi R_1 + T \cdot 2\pi R_2 \\ &= 2\pi T (R_1 + R_2) \end{aligned}$$

Free surface →

Liquid drop

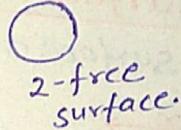


Bubble



Bubble in liq.
(one free surface)

Bubble in air.



* Work done to make a liquid drop.

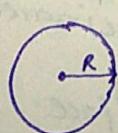
$$W = 6 \times 4\pi r^2$$

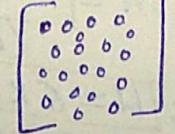
↑ area.
surface tension

* Work done to blow a bubble in air

$$W = 6 \times (4\pi r^2) \times 2 = 12 \times 4\pi r^2$$

* Work done to split a liquid drop.



\Rightarrow  (n identical drops.
(each of radius r'))

$$\rho \times \frac{4}{3}\pi R^3 = \rho \times \left(\frac{4}{3}\pi r^3\right)^n \Rightarrow R = n^{1/3}r$$

$$W = (n^{1/3} - 1) 6 \times 4\pi r^2$$

If this process is considered adiabatic then temp. of system will fall so,

$$\Delta Q = \frac{3T}{SSG} \left(\frac{1}{r} - \frac{1}{R} \right)$$

When,
AMU
2016

Bigger drop \rightarrow smaller drops
surface ↑
Surface energy ↑
temp. of system ↑

In this case,

$$\frac{\text{Initial surface energy}}{\text{Final surface energy}} = \frac{1}{N^{2/3}}$$

* Work done to ↑ radius of a liq. drop from R_1 to R_2 .

$$W = 4\pi T (R_2^2 - R_1^2)$$

* Work done to ↑ radius of a soap bubble from R_1 to R_2 .

$$W = 8\pi T (R_2^2 - R_1^2)$$

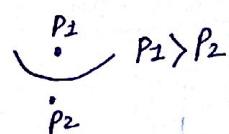
* Two identical liq. drop of radius r_1 & r_2 combine to form a single spherical drop. In Isothermal condition then ratio of new drop.

$$r = \sqrt{r_1^2 + r_2^2}$$

Excess Pressure \rightarrow The extra pressure inside a drop or bubble.



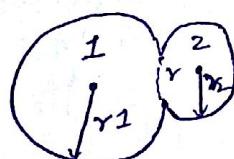
$$P = P_{in} - P_{out} = \begin{cases} \frac{2\sigma}{r} & \text{For one free surface.} \\ \frac{4\sigma}{r} & \text{For two free surfaces.} \end{cases}$$



$$P_1 > P_2$$

$$P_1 - P_2 = \frac{2\sigma}{r}$$

* Two soap/air bubble when made in contact to form double bubble.



$$r = \frac{r_1 r_2}{r_1 - r_2}$$

AIMS
** When stopper/knob is opened how size of (1) & (2) will change.

$$\text{as, } r_1 < r_2$$

$$P_1 > P_2$$

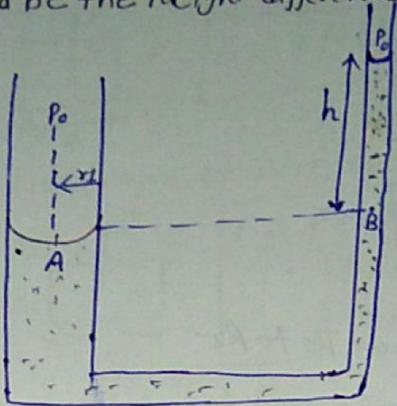
$$\Delta P \propto \frac{1}{r}$$

$$P_1 = P_0 + \frac{4\sigma}{r_1}$$

$$P_2 = P_0 + \frac{4\sigma}{r_2}$$

Hence, When stopper is opened smaller bubble reduces its size while bigger bubble expand its size.

** There is U-tube having their arms of radius $r_1 \neq r_2$ ($r_1 > r_2$) then what would be the height difference of Hg . In both arm angle of contact θ .

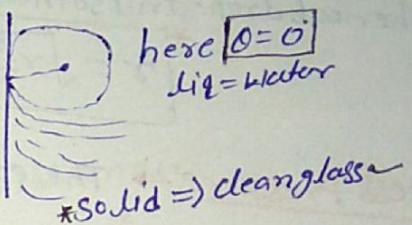
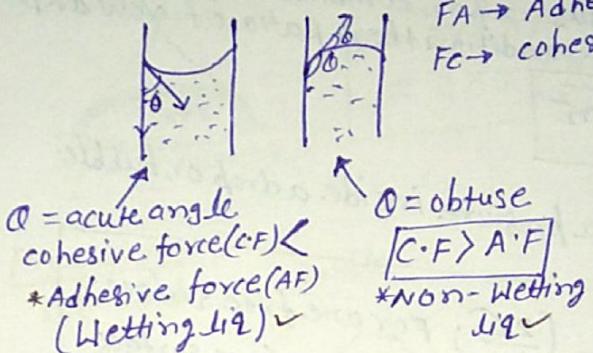


$$h = \frac{2T}{\rho g} \left(\frac{r_1 - r_2}{r_1 r_2} \right)$$

Angle of contact (θ) →

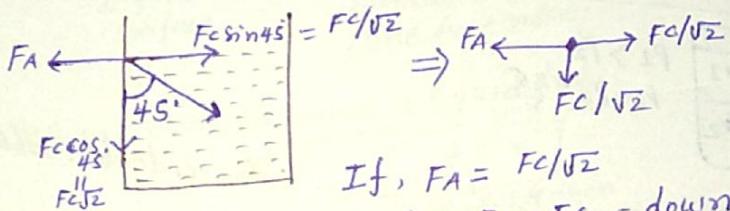
θ = At meniscus, angle b/w tangent of liq. surface in contact & tangent on solid surface inside liquid.

$F_A \rightarrow$ Adhesive force b/w solid & liquid molecule.
 $F_C \rightarrow$ cohesive force b/w liq. molecule.

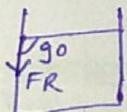


*Solid \Rightarrow clean glass

|ii| \rightarrow If $F_A = F_C/\sqrt{2}$

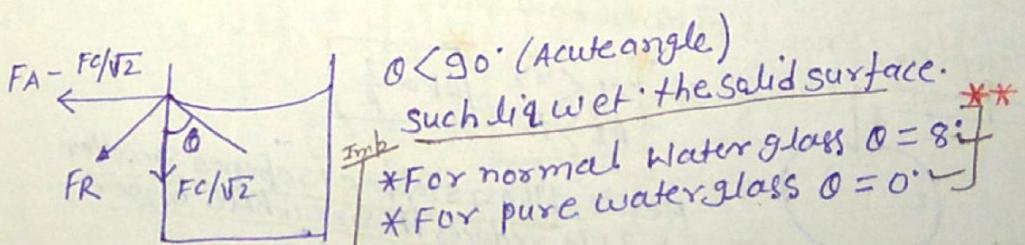


If, $F_A = F_C/\sqrt{2}$
then, $F_R = \frac{F_C}{\sqrt{2}}$ = downward



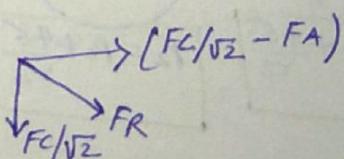
Imb \rightarrow [Water in Silver glass.]

|iii| \rightarrow If $F_A > F_C/\sqrt{2}$



|iv| \rightarrow If $F_A < F_C/\sqrt{2}$

then $\theta > 90^\circ$ (Obtuse angle.)
From such liq. surface is
not wet. Eg \rightarrow (In glass Hg)
 $\theta = 135^\circ$



Rise of liquid level in a capillary tube →

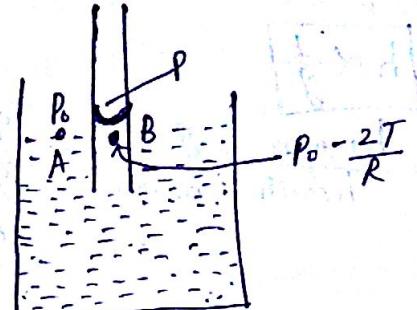
$$P_A = P_B$$

$$P_0 = P_0 - \frac{2T}{R} + \gamma g$$

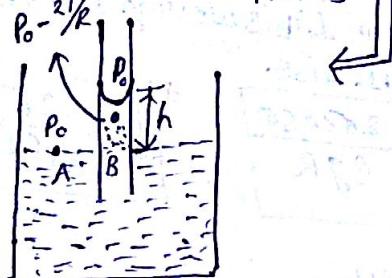
$$\frac{2T}{R} = \gamma g$$

*
$$h = \frac{2T}{R\gamma g}$$

$R \rightarrow$ Radius of curvature of meniscus ↑



$$PA \neq PB$$



Let, $r \rightarrow$ radius of curvature

$\theta \rightarrow$ angle of contact.

$$\cos \theta = r/R$$

$$R = \frac{r}{\cos \theta}$$

$$\therefore h = \frac{2T}{\frac{r}{\cos \theta} \gamma g}$$

*
$$h = \frac{2T \cos \theta}{r \gamma g}$$



i) → For liquid-gas →

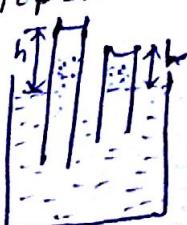
$\theta, T, S \rightarrow \text{const.}$

$$\therefore h \propto \frac{1}{r}$$

*
$$h_1 r_1 = h_2 r_2$$

BHU

ii) → If capillary has insufficient length then water can't come out from top but radius of meniscus at top does not change.



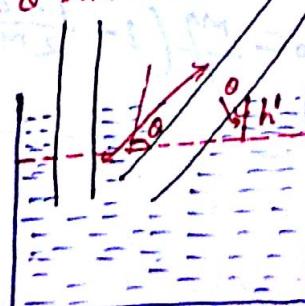
*
$$L_y = L r^{-1}$$

iii) → If capillary is inclined at angle 'θ' with vertical.

Let, $L =$ length of liq. column in inclined tube
then, $h' = h$

*
$$L \cos \theta = h$$

$$L = \frac{h}{\cos \theta}$$



$$\cos \theta = \frac{h'}{L}$$

$$L = \frac{h'}{\cos \theta}$$

\rightarrow Height raised in capillary tube is found to be 'h' on earth surface. If all exp. is taken at moon then height rise become.

$$g_{\text{moon}} = \frac{g_{\text{earth}}}{6}$$

$$h \propto \frac{1}{g}$$

$$\frac{h_{\text{moon}}}{h_{\text{earth}}} = \frac{g_e}{h_{\text{moon}}} = \frac{g_e}{\frac{g_e}{6}} \therefore h_m = 6h_e$$

NOTE → * If cohesive force (C.F) are less than adhesive force (A.F) i.e. $\theta \Rightarrow$ acute or, liq. is wetting liq. then the level of liq. in capillary tube will rise.

$$h = \frac{2\sigma \cos \theta}{\rho g R}$$

$$h = \frac{2\sigma}{\rho g r} = \frac{2\sigma}{\rho g r}$$

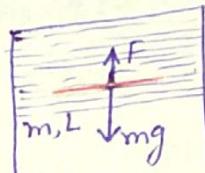
* In any capillary

$$hr = \text{const.}$$

i.e. If capillary tube is of insufficient length of liq. will not overflow but Radius of meniscus \uparrow se.

$$hr = h'r' \Rightarrow r' = \frac{hr}{h'}$$

\checkmark If needle is in equilibrium find surface tension of soap film.



$$F = mg$$

$$F \times 2L = mg \Rightarrow \sigma = \frac{mg}{2L}$$

\checkmark If capillary & beaker system is taken at the artificial satellite or, freely falling lift then height rise in capillary is equal to its full length.

A uniform cylinder of length 'L' & Mass 'M' having cross-section area Area 'A' suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liq. of density 'g' at egm . position. The extension x_0 of the spring when it is in egm . →

$$x_0 = \frac{Mg}{K} \left(1 - \frac{LA}{2M} \right)$$

