

Freeman's bar potential (with pattern speed W):

$$\Phi = \frac{1}{2}(A^2x^2 + B^2y^2)$$

Define:

$$V^2 = \sqrt{A^2 + B^2 + W^2 + 2AB}\sqrt{A^2 + B^2 + W^2 - 2AB}$$

$$P = (A^2 + B^2)/2; \quad Q = (A^2 - B^2)/2$$

$$R = (V^2 + W^2)/2; \quad S = (V^2 - W^2)/2$$

The orbital equation:

$$x(t) = \frac{a(-W(-P-R) - W(-Q+R))e^{-t\sqrt{-P-R}}}{A^2(-Q+R)} + \frac{b(-W(-P-R) - W(-Q+R))e^{t\sqrt{-P-R}}}{A^2(-Q+R)} + \frac{c(-W(-P+S) - W(-Q-S))e^{-t\sqrt{-P+S}}}{A^2(-Q-S)} + \frac{d(-W(-P+S) - W(-Q-S))e^{t\sqrt{-P+S}}}{A^2(-Q-S)}$$

$$y(t) = \frac{a(-W^2\sqrt{-P-R} + \sqrt{-P-R}(-Q+R))e^{-t\sqrt{-P-R}}}{B^2(-Q+R)} + \frac{b(W^2\sqrt{-P-R} - \sqrt{-P-R}(-Q+R))e^{t\sqrt{-P-R}}}{B^2(-Q+R)} + \frac{c(-W^2\sqrt{-P+S} + \sqrt{-P+S}(-Q-S))e^{-t\sqrt{-P+S}}}{B^2(-Q-S)} + \frac{d(W^2\sqrt{-P+S} - \sqrt{-P+S}(-Q-S))e^{t\sqrt{-P+S}}}{B^2(-Q-S)}$$

$$V_x(t) = -\frac{Wa\sqrt{-P-R}e^{-t\sqrt{-P-R}}}{-Q+R} + \frac{Wb\sqrt{-P-R}e^{t\sqrt{-P-R}}}{-Q+R} - \frac{Wc\sqrt{-P+S}e^{-t\sqrt{-P+S}}}{-Q-S} + \frac{Wd\sqrt{-P+S}e^{t\sqrt{-P+S}}}{-Q-S}$$

$$V_y(t) = ae^{-t\sqrt{-P-R}} + be^{t\sqrt{-P-R}} + ce^{-t\sqrt{-P+S}} + de^{t\sqrt{-P+S}}$$

Where constants a, b, c, d depends on the initial condition:

$$a = \frac{-\sqrt{2}A^2B^2V^2y_0 - \sqrt{2}A^2V^2Wu_0 + 2A^2V^2Wx_0\sqrt{-2P-2R} - A^2V^2v_0\sqrt{-2P-2R} + \sqrt{2}B^4V^2y_0 + \sqrt{2}B^2V^4y_0 + \sqrt{2}B^2V^2W^2y_0 - \sqrt{2}B^2V^2Wu_0 + B^2V^2v_0\sqrt{-2P-2R} - \sqrt{2}V^4Wu_0 + V^4v_0\sqrt{-2P-2R} - \sqrt{2}V^2W^3u_0 + V^2W^2v_0\sqrt{-2P-2R}}{4V^4\sqrt{-2P-2R}}$$

$$b = \frac{\sqrt{2}A^2B^2V^2y_0 + \sqrt{2}A^2V^2Wu_0 + 2A^2V^2Wx_0\sqrt{-2P-2R} - A^2V^2v_0\sqrt{-2P-2R} - \sqrt{2}B^4V^2y_0 - \sqrt{2}B^2V^4y_0 - \sqrt{2}B^2V^2W^2y_0 + \sqrt{2}B^2V^2Wu_0 + B^2V^2v_0\sqrt{-2P-2R} + \sqrt{2}V^4Wu_0 + V^4v_0\sqrt{-2P-2R} + \sqrt{2}V^2W^3u_0 + V^2W^2v_0\sqrt{-2P-2R}}{4V^4\sqrt{-2P-2R}}$$

$$c = \frac{\sqrt{2}A^2B^2V^2y_0 + \sqrt{2}A^2V^2Wu_0 - 2A^2V^2Wx_0\sqrt{-2P+2S} + A^2V^2v_0\sqrt{-2P+2S} - \sqrt{2}B^4V^2y_0 + \sqrt{2}B^2V^4y_0 - \sqrt{2}B^2V^2W^2y_0 + \sqrt{2}B^2V^2Wu_0 - B^2V^2v_0\sqrt{-2P+2S} - \sqrt{2}V^4Wu_0 + V^4v_0\sqrt{-2P+2S} + \sqrt{2}V^2W^3u_0 - V^2W^2v_0\sqrt{-2P+2S}}{4V^4\sqrt{-2P+2S}}$$

$$d = \frac{-\sqrt{2}A^2B^2V^2y_0 - \sqrt{2}A^2V^2Wu_0 - 2A^2V^2Wx_0\sqrt{-2P+2S} + A^2V^2v_0\sqrt{-2P+2S} + \sqrt{2}B^4V^2y_0 - \sqrt{2}B^2V^4y_0 + \sqrt{2}B^2V^2W^2y_0 - \sqrt{2}B^2V^2Wu_0 - B^2V^2v_0\sqrt{-2P+2S} + \sqrt{2}V^4Wu_0 + V^4v_0\sqrt{-2P+2S} - \sqrt{2}V^2W^3u_0 - V^2W^2v_0\sqrt{-2P+2S}}{4V^4\sqrt{-2P+2S}}$$

The mean value and variance of angular momentum:

$$L = -\frac{2W\left(B^2ab\left(Q-R\right)\left(Q+S\right)^2+B^2cd\left(Q-R\right)^2\left(Q+S\right)+ab\left(P+R\right)\left(Q-S\right)\left(Q+S\right)^2+cd\left(P-S\right)\left(Q-R\right)^2\left(Q+R\right)\right)}{B^2\left(Q-R\right)^2\left(Q+S\right)^2}$$

$$\text{Var}(L) = \frac{2W^2\left(4B^4abcd\left(Q-R\right)^3\left(Q+S\right)^3+B^4ab\left(Q-R\right)^2\left(Q+S\right)^4\left(ab+2cd\right)+B^4cd\left(Q-R\right)^4\left(Q+S\right)^2\left(2ab+cd\right)-2B^2c^2d^2\left(P-S\right)\left(Q-R\right)^4\left(Q+R\right)\left(Q+S\right)+a^2b^2\left(P+R\right)^2\left(Q-S\right)^2\left(Q+S\right)^4+2abcd\left(P+R\right)\left(P-S\right)\left(Q-R\right)^2\left(Q+S\right)^2\left(\left(Q+R\right)^2+2\left(Q+R\right)\left(Q-S\right)+\left(Q-S\right)^2\right)+c^2d^2\left(P-S\right)^2\left(Q-R\right)^4\left(Q+R\right)^2\right)}{B^4\left(Q-R\right)^4\left(Q+S\right)^4}$$