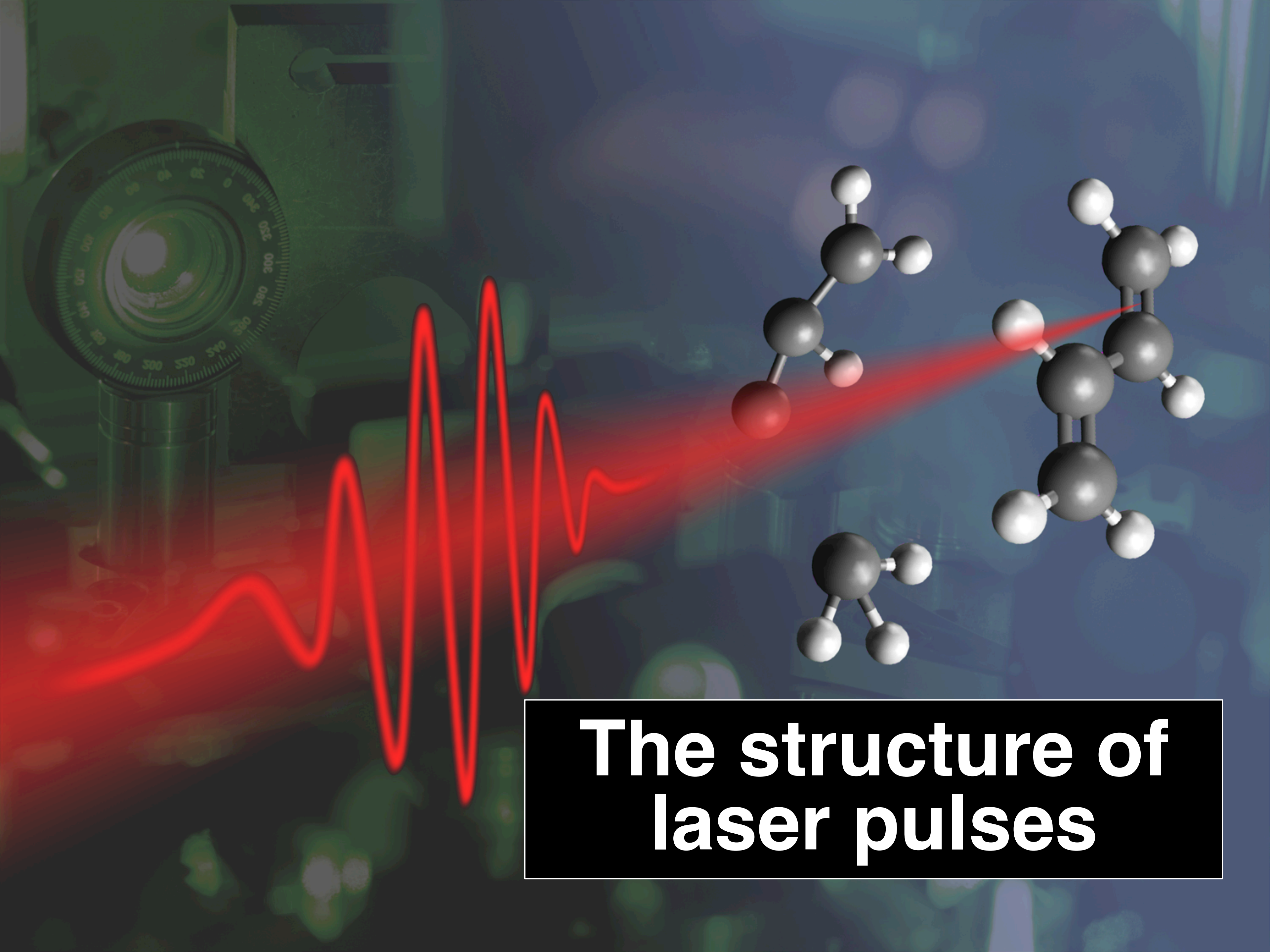

The structure of
laser pulses

The structure of laser pulses

Pulse characteristics

- Temporal and spectral representation
- Fourier transforms
- Temporal and spectral widths
- Instantaneous frequency
- Chirped pulses

Gaussian and chirped Gaussian pulses

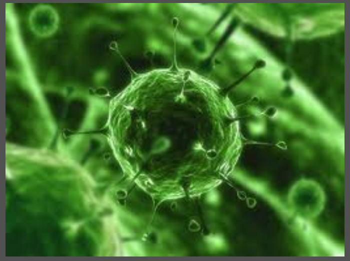
- Temporal properties
- Spatial properties
- Linear systems and filters
- The chirp filter

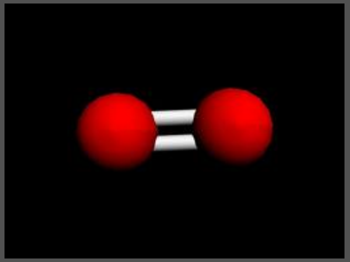
Why pulsed lasers?

Power = Energy / time

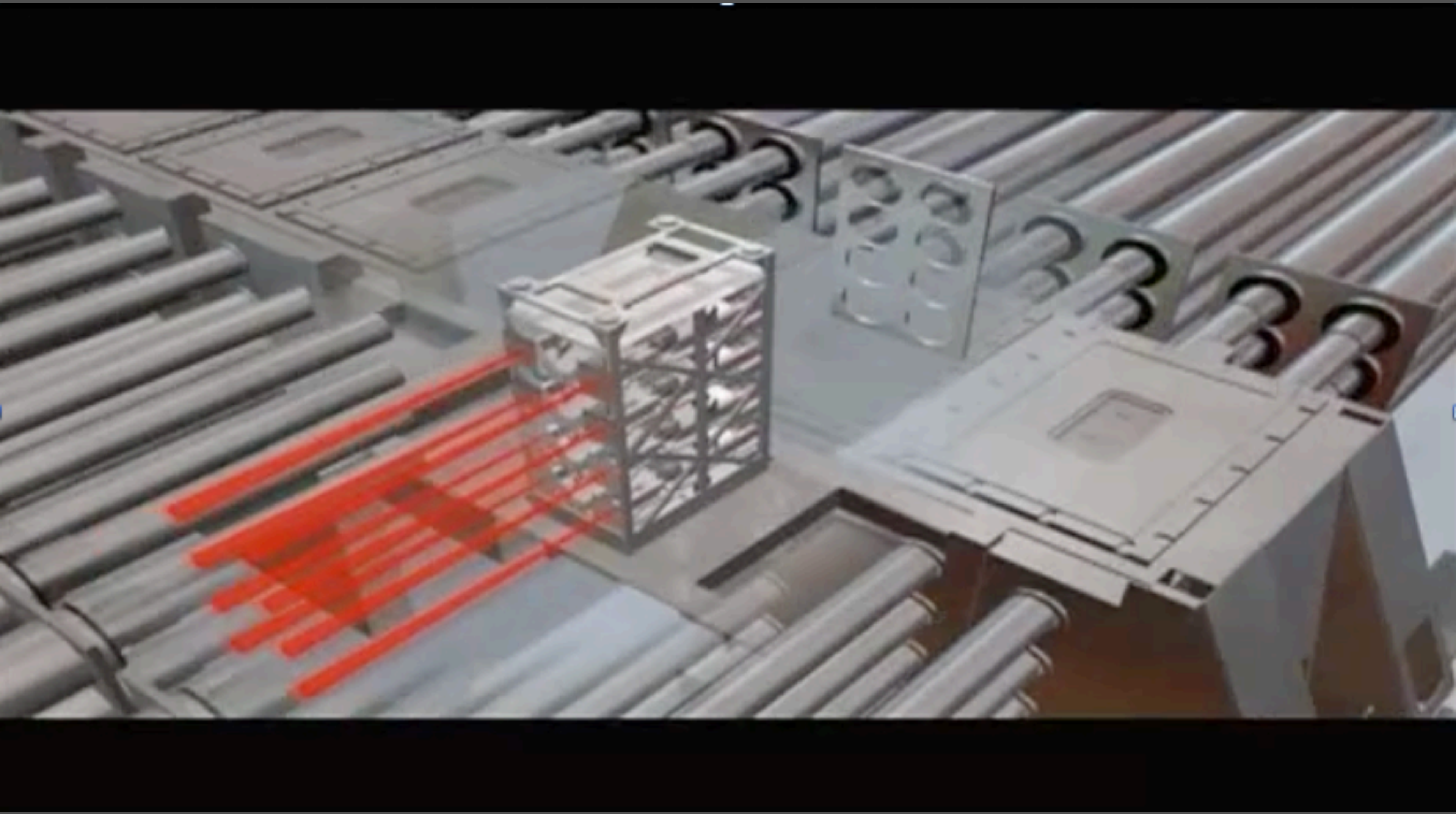
Concentrating the optical energy in a (short) pulse increases the power of the laser.

1 μs	microsecond	10^{-6} s	0.3 km
1 ns	nanosecond	10^{-9} s	0.3 m
1 ps	picosecond	10^{-12} s	0.3 mm
1 fs	femtosecond	10^{-15} s	0.3 μm
1 as	attosecond	10^{-18} s	0.3 nm

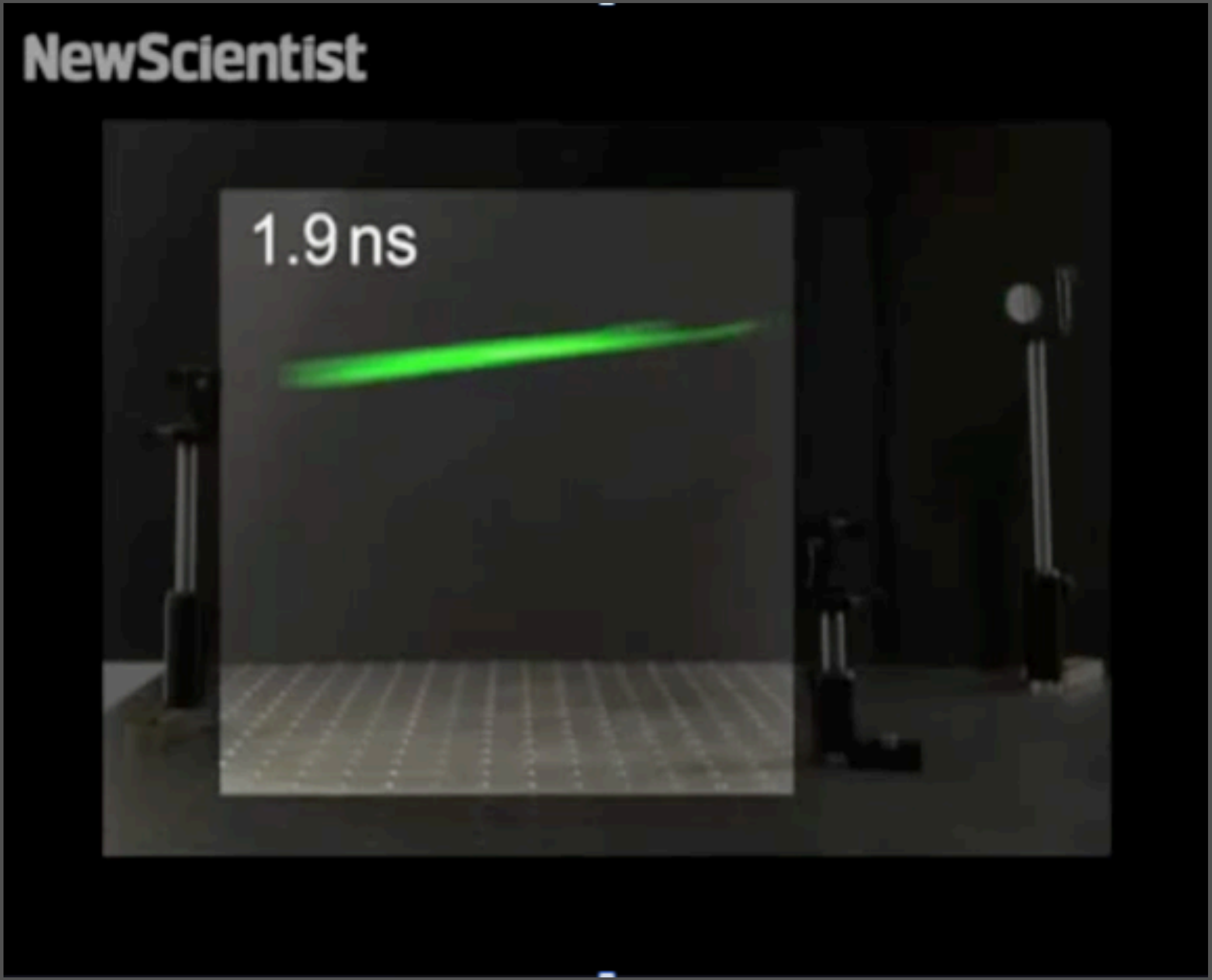




Ultrashort laser pulses are the shortest events produced by mankind

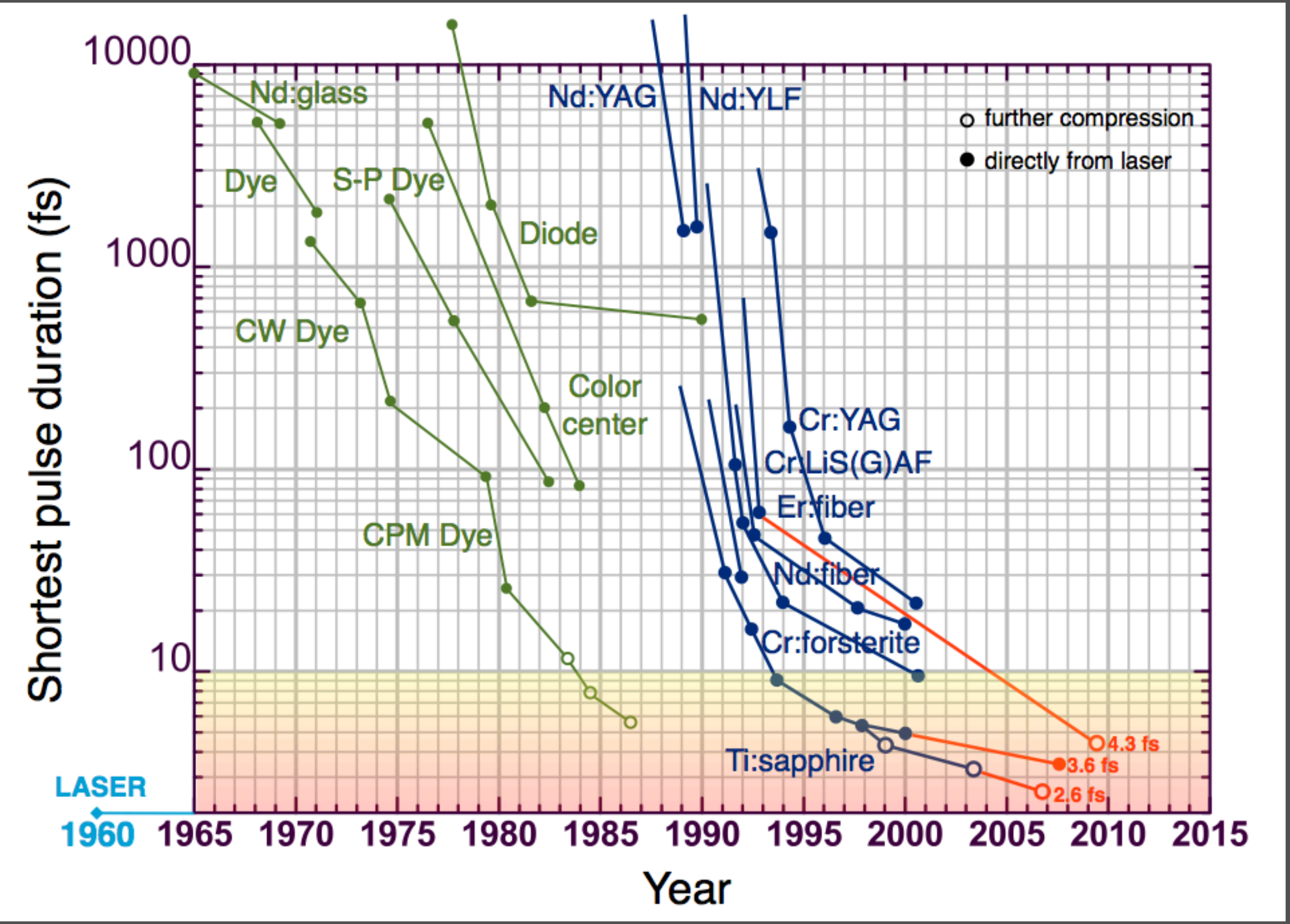


https://lasers.llnl.gov/media/video-gallery



https://youtu.be/Uq0H4-nvBB8

Progress in short-pulse generation



How to represent a basic optical pulse?

1. Choose the **central frequency ν_0**

$$\exp(i2\pi\nu_0 t) = \exp(i\omega_0 t)$$

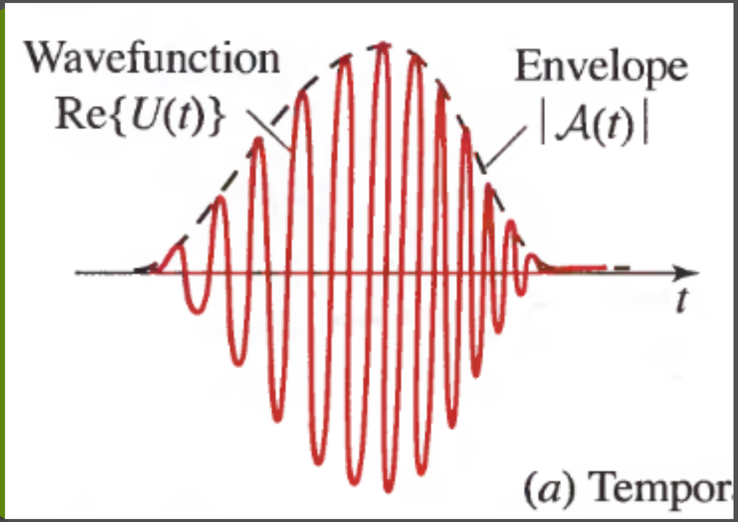
2. Define the **complex envelope *A(t)***

$$A(t) = |A(t)|\exp[i\varphi(t)]$$

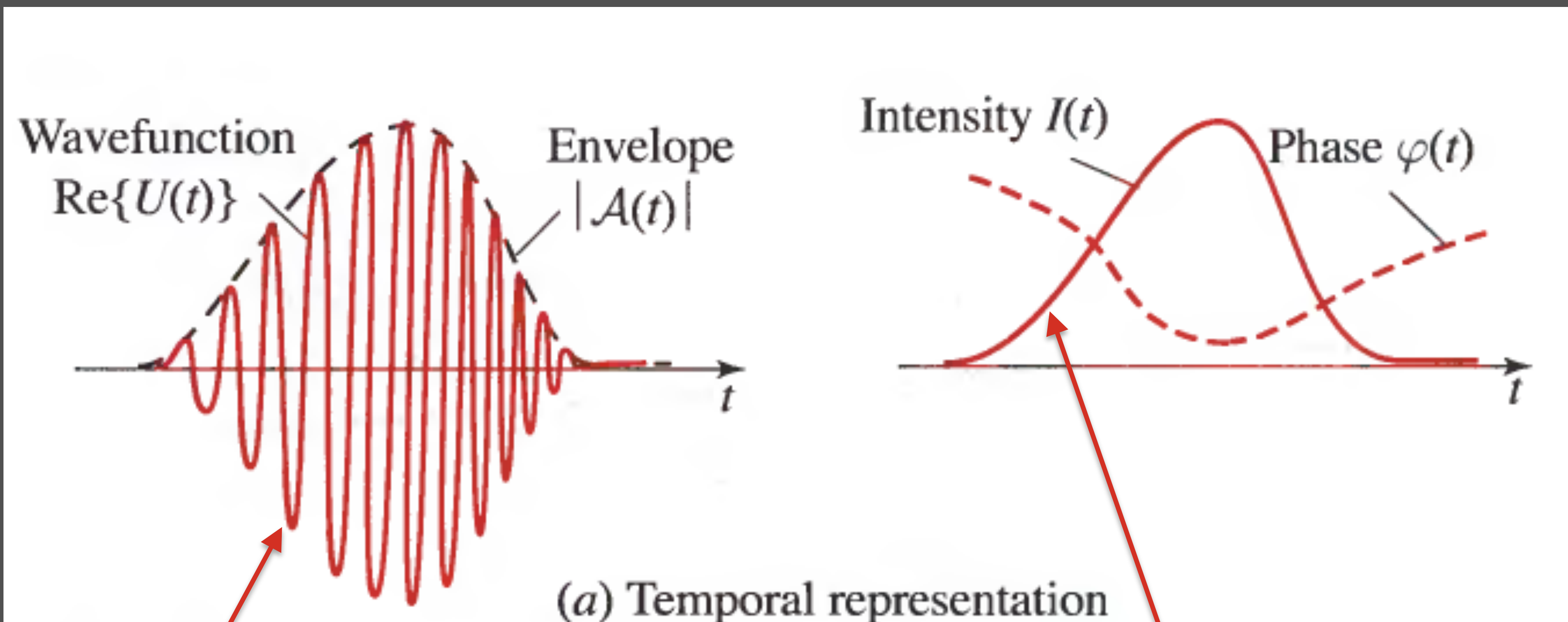
amplitude phase

3. Multiply them

$$U(t) = A(t)e^{i\omega_0 t}$$
$$= |A(t)|e^{i(\omega_0 t + \varphi(t))}$$



Temporal representation of an optical pulse



$$\mathrm{Re}\{U(t)\} = |A(t)|\cos[\omega_0 t + \varphi(t)]$$

$$I(t) = |U(t)|^2 = |A(t)|^2$$

Intensity and types of pulses

The usual definition applies: $I(t) = |U(t)|^2 = |A(t)|^2$

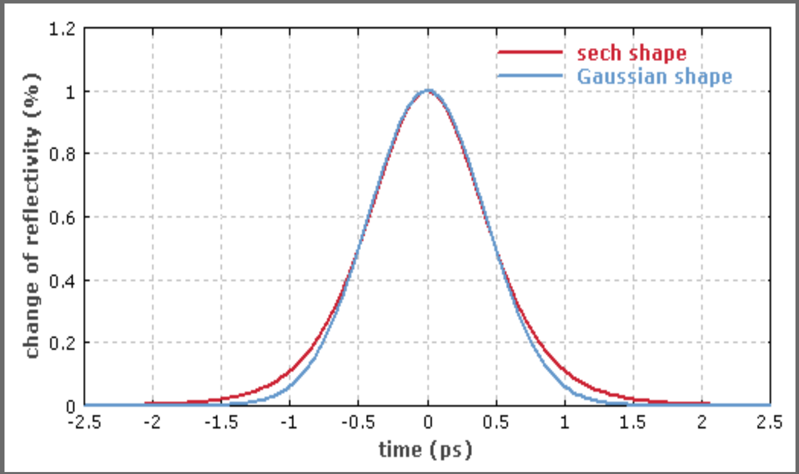
Some frequent types of laser pulses (τ is called the **width**):

Gaussian $I(t) \propto \exp\left(-2t^2/\tau^2\right)$

Lorentzian $I(t) \propto \dfrac{1}{1+t^2/\tau^2}$

Hyperbolic secant $I(t) \propto \operatorname{sech}^2(t/\tau)$

$$= \left(\frac{2}{e^{t/\tau}+e^{-t/\tau}}\right)^2$$



A pulse can be described in the time or frequency domains

They are fully equivalent. To switch between one description and the other we use **Fourier transforms** (FT):

time

$$U(t) = |A(t)| e^{i(\omega_0 t + \varphi(t))}$$

amplitude phase

frequency

$$V(\omega) = \int U(t) \exp(-i\omega t) = |V(\omega)| \exp[i\psi(\omega)]$$

Fourier transform

spectral amplitude

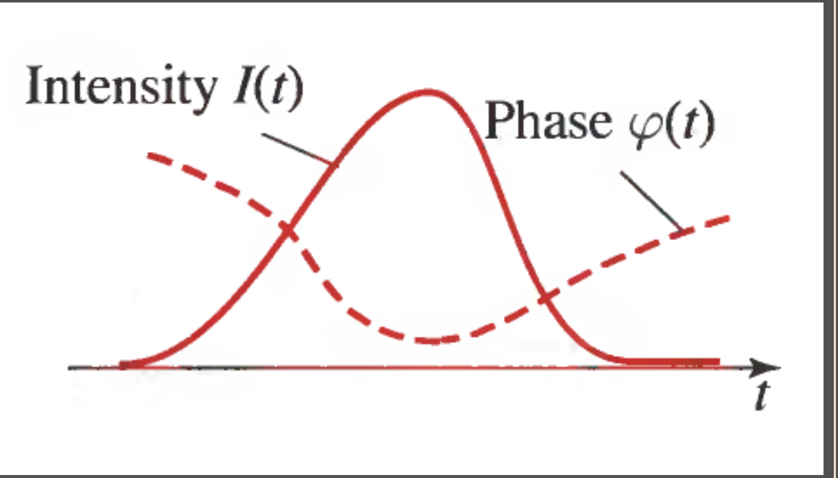
spectral phase

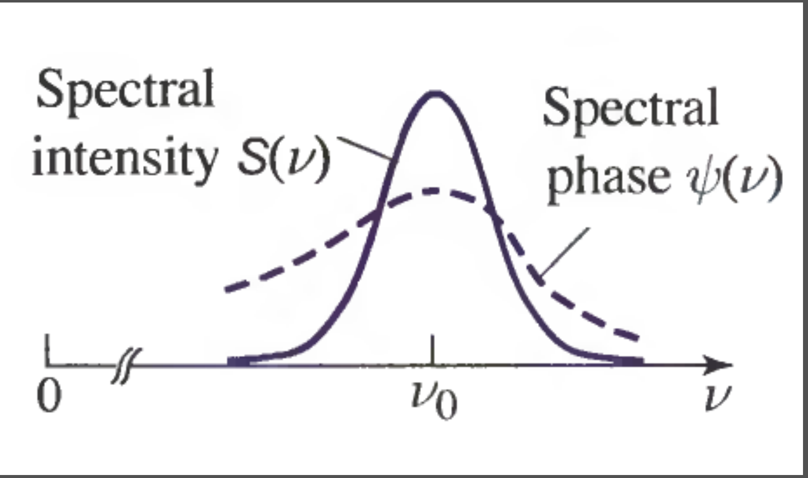
FT's are always applied to **amplitude** (or electric field), not intensity.

This concept is essential for understanding laser pulses.

Temporal and spectral descriptions

time	frequency
$U(t) = \lvert A(t) \rvert \exp[i\varphi(t)]$	$\lvert V(\omega) \rvert \exp[i\psi(\omega)]$





The spectral amplitude and intensity

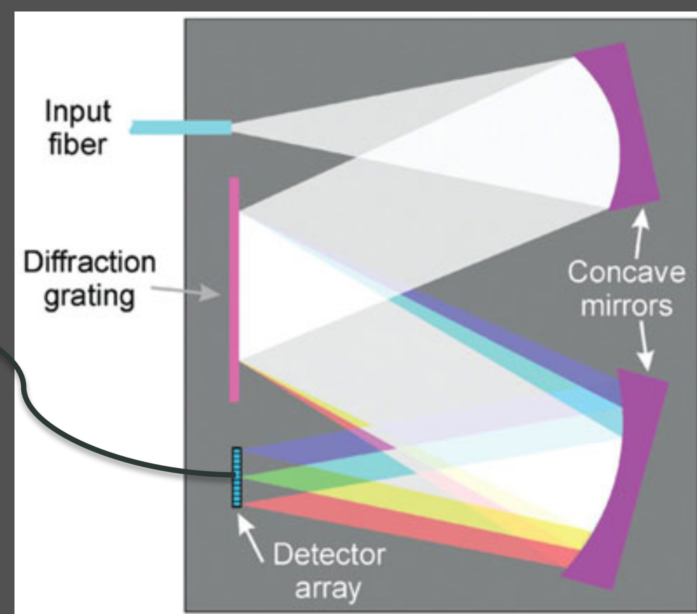
Just like for the time description we can also define a **spectral intensity**:

$$I(t) = |U(t)|^2$$

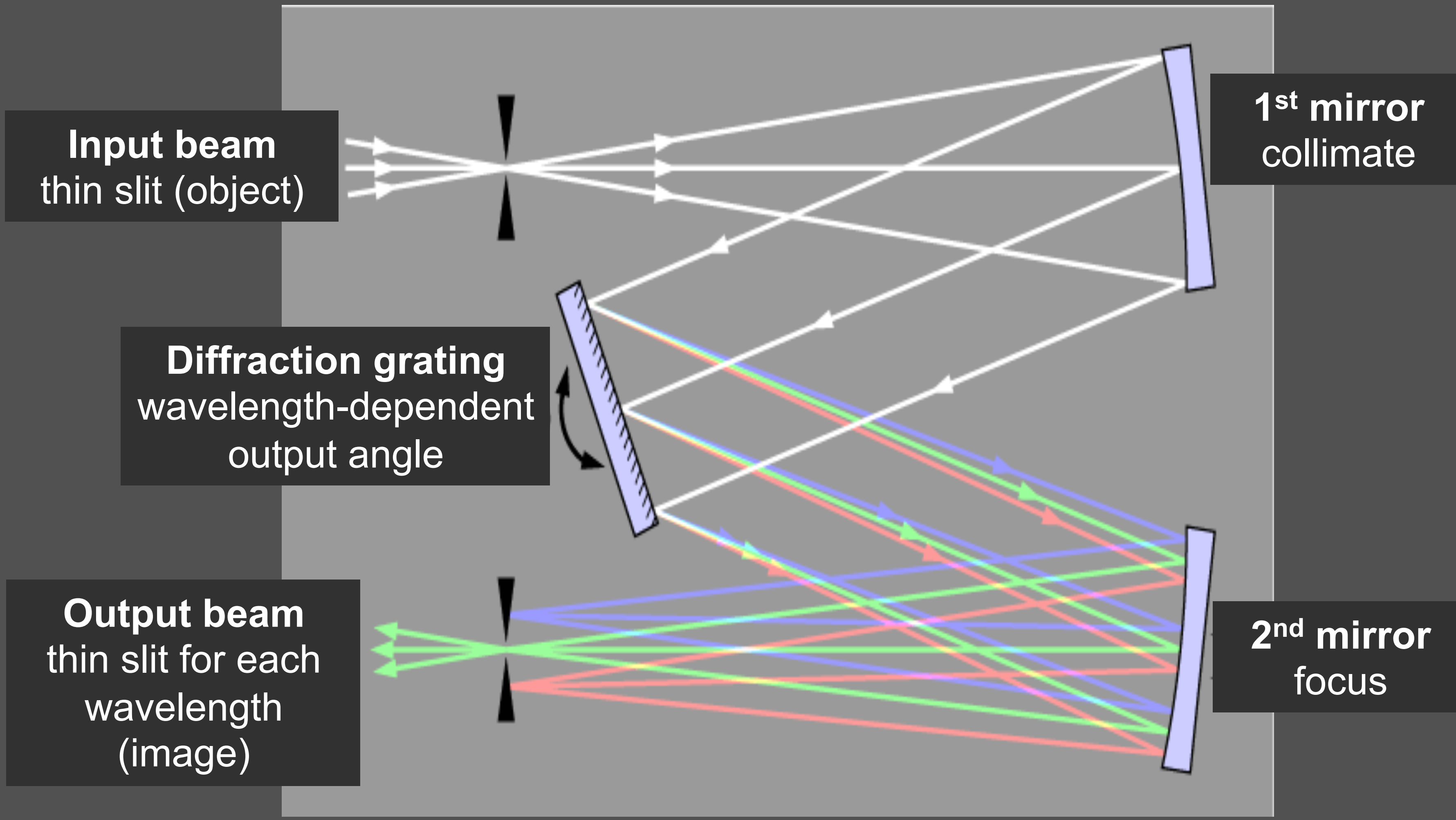
$$S(\omega) = |V(\omega)|^2$$

The spectral intensity has a concrete physical meaning: it is the value measured by a **spectrometer**

$S(\omega)$



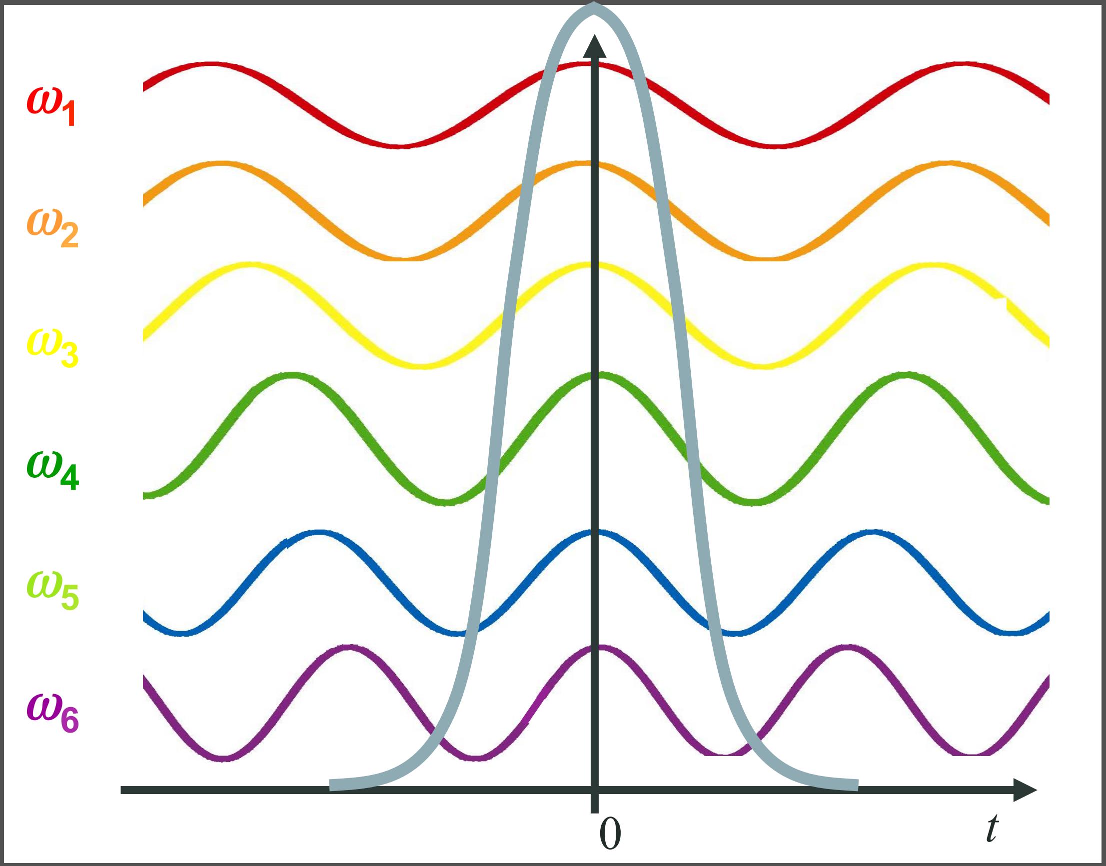
How does a spectrometer work?



What is the spectral phase?

The spectral phase is the phase of each of the frequencies that compose the function.

$$\left|V(\omega)\right| \exp\left[i\psi(\omega)\right]$$



In this example we have $\psi(\omega) = 0$.

This leads to the formation of a pulse at $t = 0$ (**constructive interference**) and zero elsewhere (**destructive interference**).

Fourier transforms

The principle is that you can express any function *A(t)* as a superposition of **harmonic functions** of different amplitude and phase:

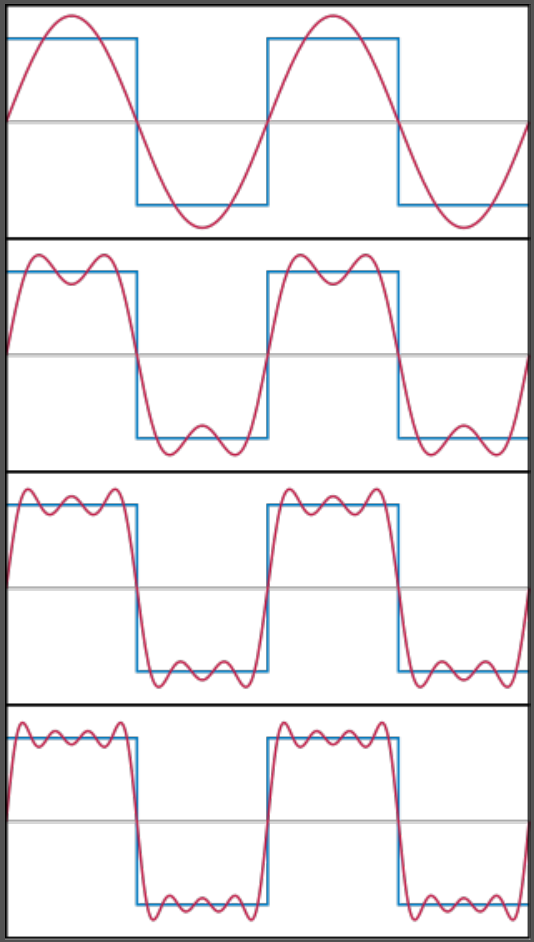
$$f(t) = \int \boxed{\tilde{f}(\omega)} \boxed{\exp(i\omega t)}$$

Coefficient **Harmonic function**

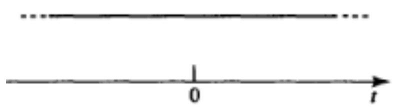
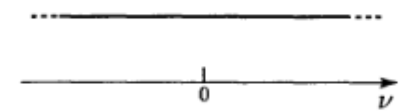
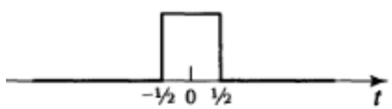
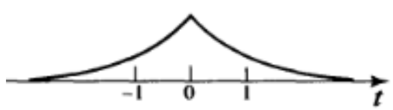
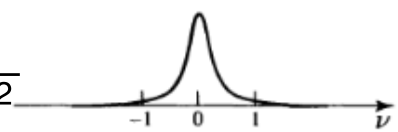
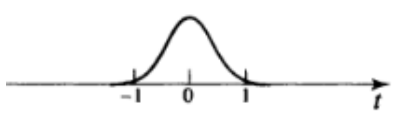
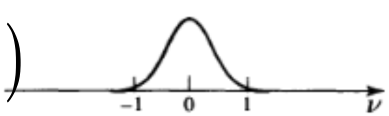
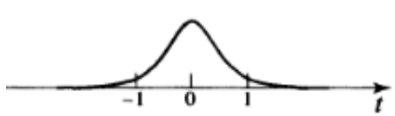
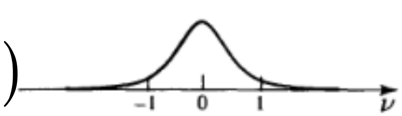
The (complex) coefficients can be determined by calculating the integral

$$\tilde{f}(\omega) = \int f(t)\exp(-i\omega t)$$

Inverse Fourier transform



Examples of important FT's

	$f(t)$		$\tilde{f}(\nu)$	
uniform		1	$\delta(\nu)$	
impulse		$\delta(t)$	1	
rectangular		$\mathrm{rect}(t)$	$\mathrm{sinc}(\nu)$	
exponential		$\exp(-\lvert t\rvert)$	$\dfrac{2}{1+(2\pi\nu)^2}$	
Gaussian		$\exp(-\pi t^2)$	$\exp(-\pi\nu^2)$	
hyperbolic secant		$\mathrm{sech}(\pi t)$	$\mathrm{sech}(\pi\nu)$	

Interesting properties of FT's

- Linearity
- Scaling
- Time translation τ
- Frequency transl. ω_0
- Parseval's theorem

$$\mathcal{F}(f_1 + f_2) = \mathcal{F}(f_1) + \mathcal{F}(f_2)$$

$$\mathcal{F}[f(t)] = \tilde{f}(\omega):$$

$$\mathcal{F}[f(t/\tau)] = \tilde{f}(\tau\omega)/\tau$$

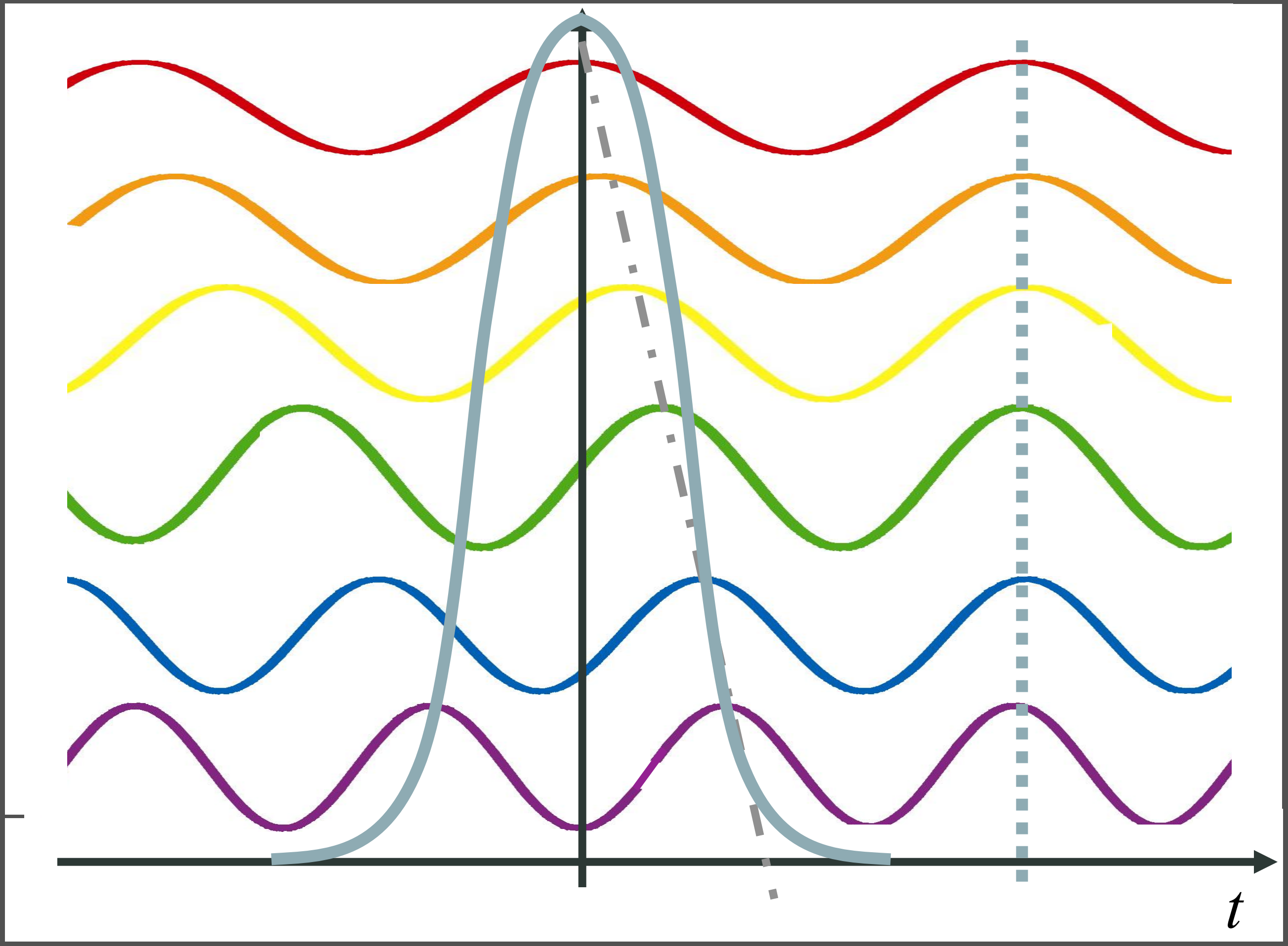
$$\mathcal{F}[f(t-\tau)] = \tilde{f}(\omega)e^{-i\omega\tau}$$

$$\mathcal{F}[f(t)e^{i\omega_0 t}] = \tilde{f}(\omega - \omega_0)$$

$$\int \left|f(t)\right|^2 dt = \int \left|\tilde{f}(\omega)\right|^2 d\omega$$

Example: time translation

$$\mathcal{F}[f(t-\tau)] = \tilde{f}(\omega)e^{-i\omega\tau}$$



$\psi(\omega_1) = 0$

$\psi(\omega_2) = 0.2\,\pi$

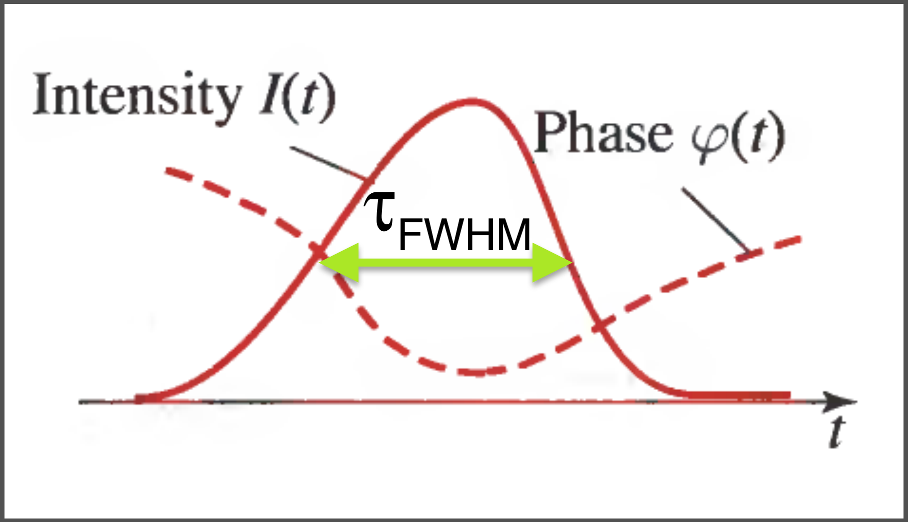
$\psi(\omega_3) = 0.4\,\pi$

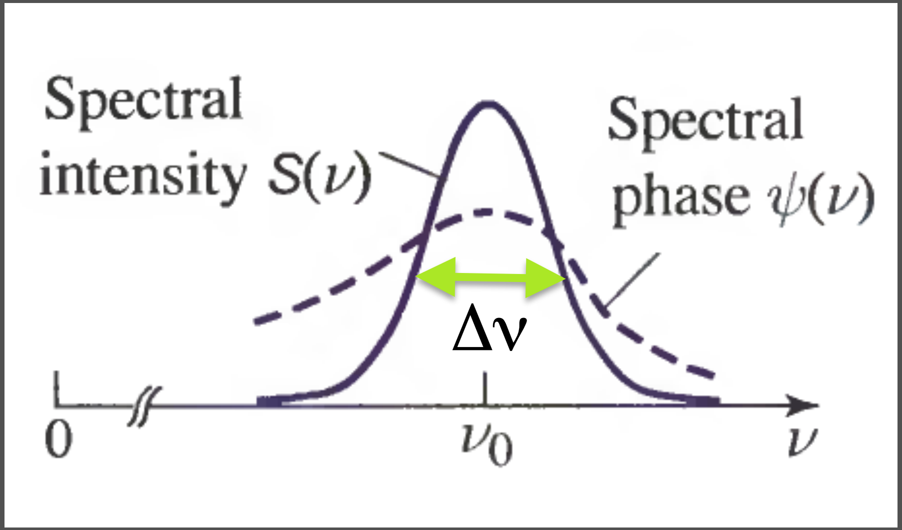
$\psi(\omega_4) = 0.6\,\pi$

$\psi(\omega_5) = 0.8\,\pi$

$\psi(\omega_6) = \pi$

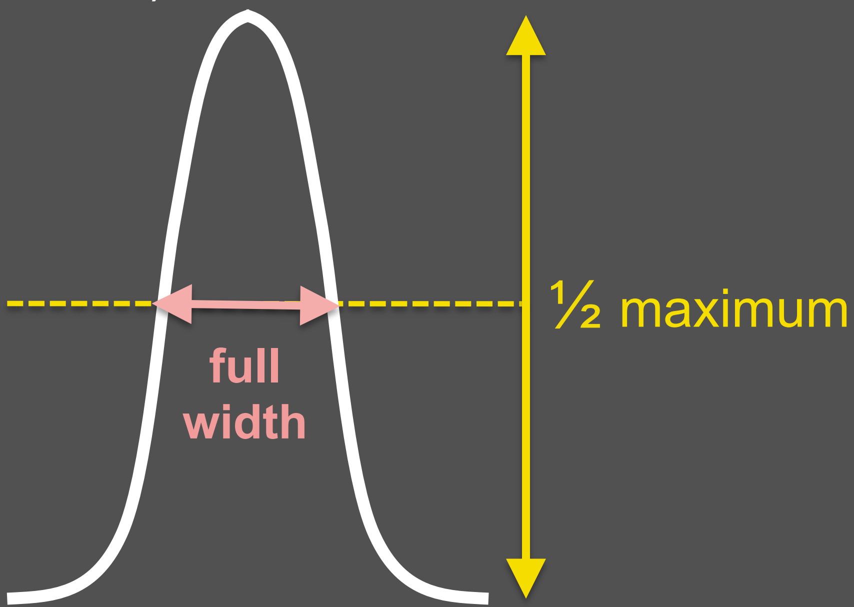
Temporal and spectral widths





= widths of the temporal and spectral intensities.

A commonly used measure is the **Full Width at Half Maximum** (FWHM):



Relation between temporal and spectral widths

Because of the Fourier transform relationship the following relationship holds ($\Delta\omega = 2\pi\Delta\nu$):

$$\boxed{\tau_{FWHM} \times \Delta\nu \geq\sim 1}$$

- The exact value depends of the pulse shape (e.g. 0.44 for a Gaussian temporal shape – *Homework!*)
- This is similar to the result for a mode-locked laser
- It is also similar to a Heisenberg uncertainty principle

This is a fundamental result – especially for very short pulses

Exercise: spectral width in frequency and in wavelength

Normally it is more practical to express a spectral width in terms of a wavelength range (e.g. nm) than in terms of a frequency range (e.g. THz).

a) Show that for $\Delta\nu \ll \nu$ the following relationship between $\Delta\lambda$ and $\Delta\nu$ holds:

$$\boxed{\Delta\lambda \approx \frac{\lambda_0^2}{c}\Delta\nu}$$

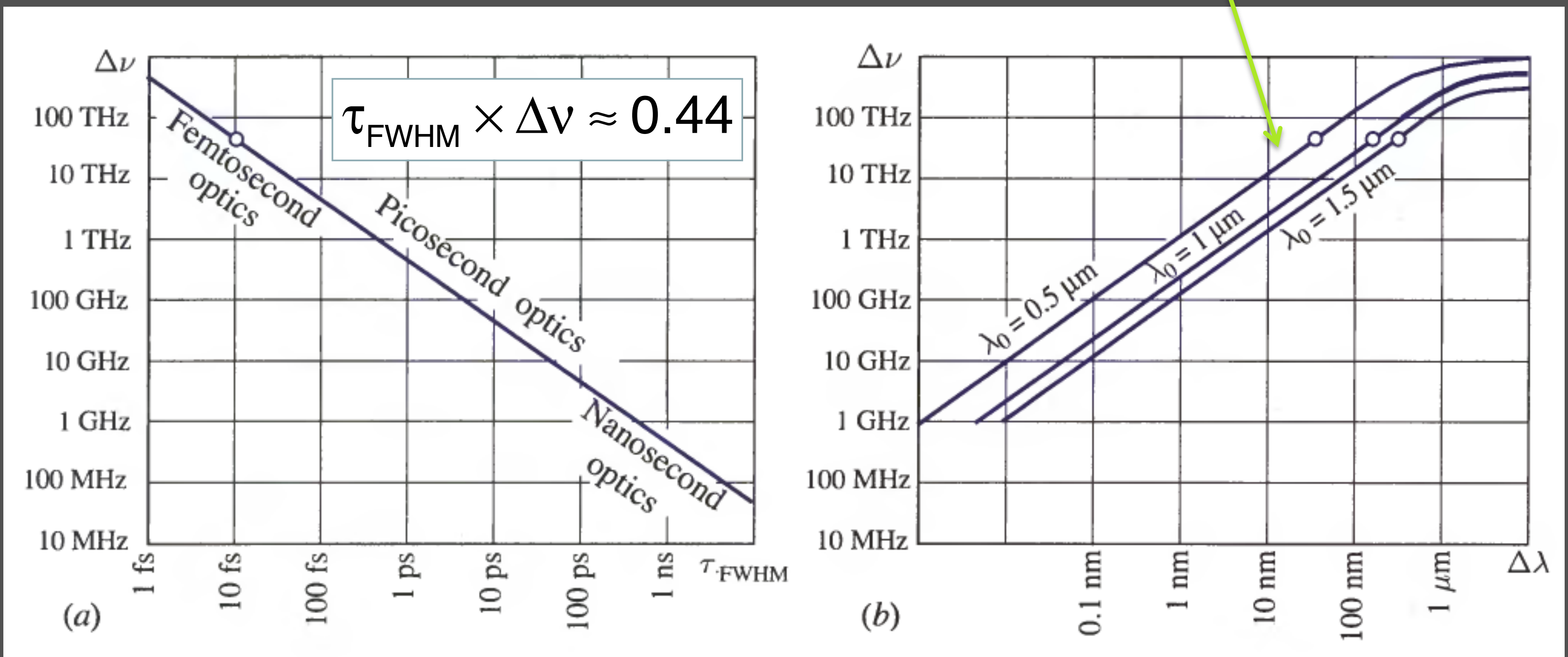
b) For a spectral width $\Delta\nu$ = 10 THz, calculate $\Delta\lambda$ at 0.55 μm and 1.1 μm.
(Answer: 10 nm; 40 nm)

Relation between widths for a Gaussian pulse

In terms of the wavelength width:

$$\Delta\lambda \approx \frac{\lambda_0^2}{c}\Delta\nu$$

depends on the wavelength!



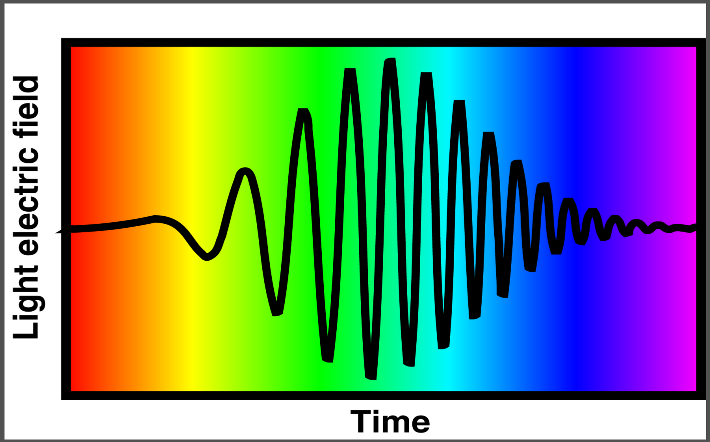
Instantaneous frequency – what is the “instant color” of a pulse?

$$U(t) = |A(t)| e^{i(\omega_0 t + \varphi(t))}$$

The **instantaneous frequency** is defined as the **time derivative** of the **temporal phase**:

$$\omega_i = \frac{d}{dt}\left[\omega_0 t + \varphi(t)\right] = \omega_0 + \frac{d\varphi}{dt}$$

$$\nu_i = \nu_0 + \frac{1}{2\pi}\frac{d\varphi}{dt}$$



A particular case of interest is the **linearly varying phase**:

$$\varphi(t) = \alpha t$$

$$\Rightarrow \omega_i = \omega_0 + \alpha$$

This simply corresponds to a **fixed frequency shift**.

Chirped pulses

Another very important case is when **the instantaneous frequency varies with time.** Such pulses are called **chirped**.

$$\varphi''(t) = \frac{d^2\varphi}{dt^2} \begin{cases} > 0 & \text{up} - \text{chirped} \\ < 0 & \text{down} - \text{chirped} \end{cases}$$

The simplest case is a **linearly varying instantaneous frequency**. This corresponds to a **quadratic temporal phase**:

$$\varphi(t) = \omega_0 t + \boxed{at^2 / \tau^2}$$

$$\omega_i = \varphi' = \omega_0 + (2a / \tau^2)t$$

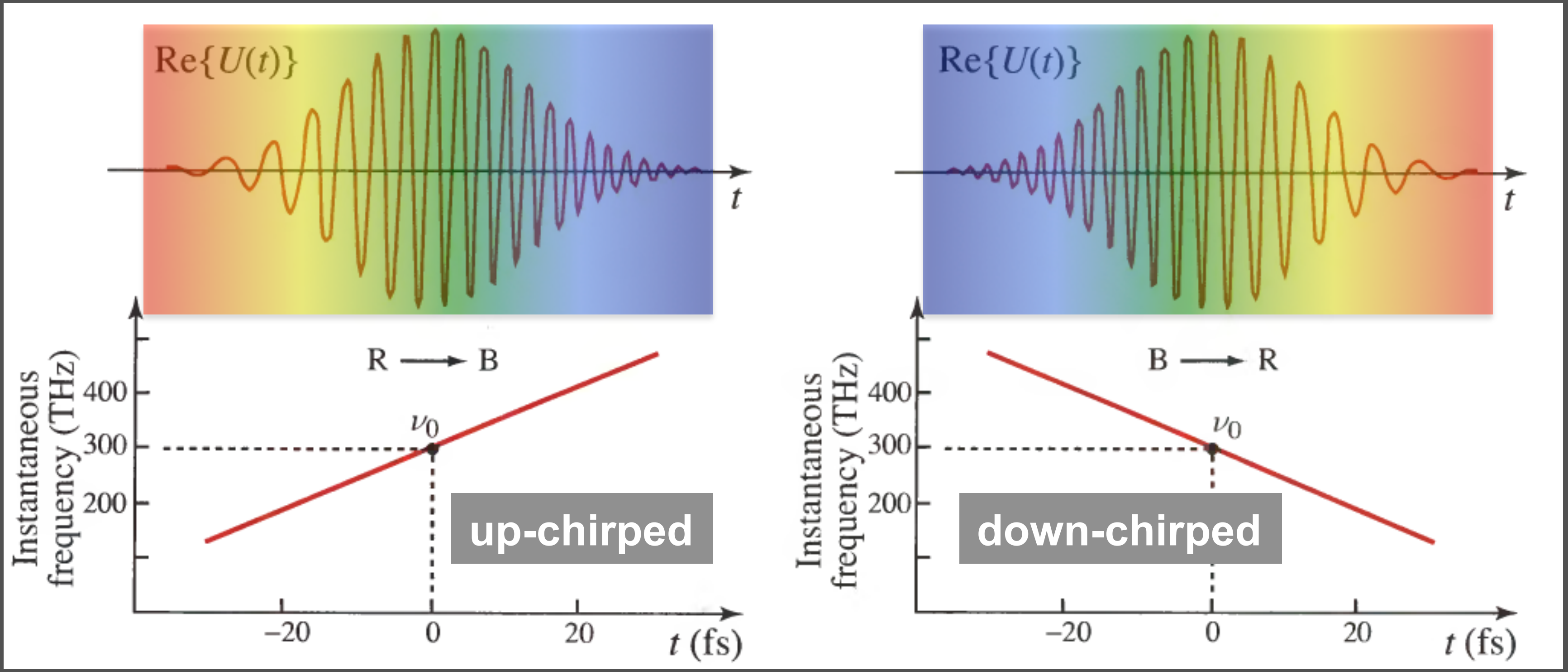
$$\frac{d^2\phi}{dt^2} \equiv \varphi'' = 2a / \tau^2$$

chirp parameter

$$a = \tfrac{1}{2}\varphi''\tau^2$$

Pulses with linear chirp

τ = 20 fs, ν_0 = 300 THz



Linearly chirped pulses are the basis for the technique of **chirped pulse amplification (CPA)** which is used in modern high power lasers.

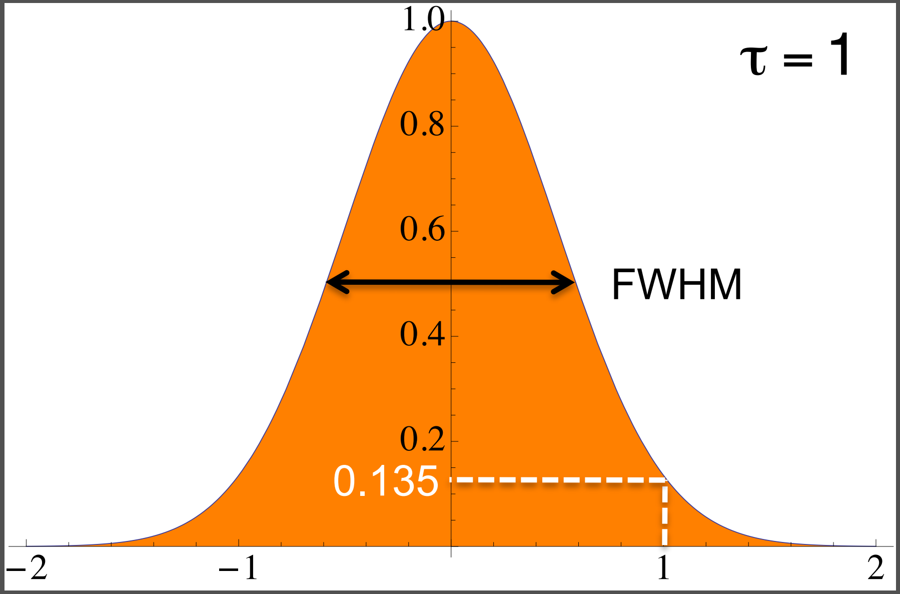
Gaussian pulses

Describing pulses using the Gaussian function is very practical, even if in reality a pulse may not be *exactly* Gaussian.

$$A(t) = A_0 \exp(-t^2/\tau^2)$$

$$I(t) = I_0 \exp(-2t^2/\tau^2)$$

Amplitude and intensity of a **transform-limited Gaussian pulse** (n.b. the phase is constant)



$t = \tau$:

$$I(t) = I_0 \exp(-2)$$

$$\approx 0.135\, I_0$$

Exercise – transform-limited Gaussian pulse properties

Demonstrate the following relations for a **transform limited Gaussian pulse**

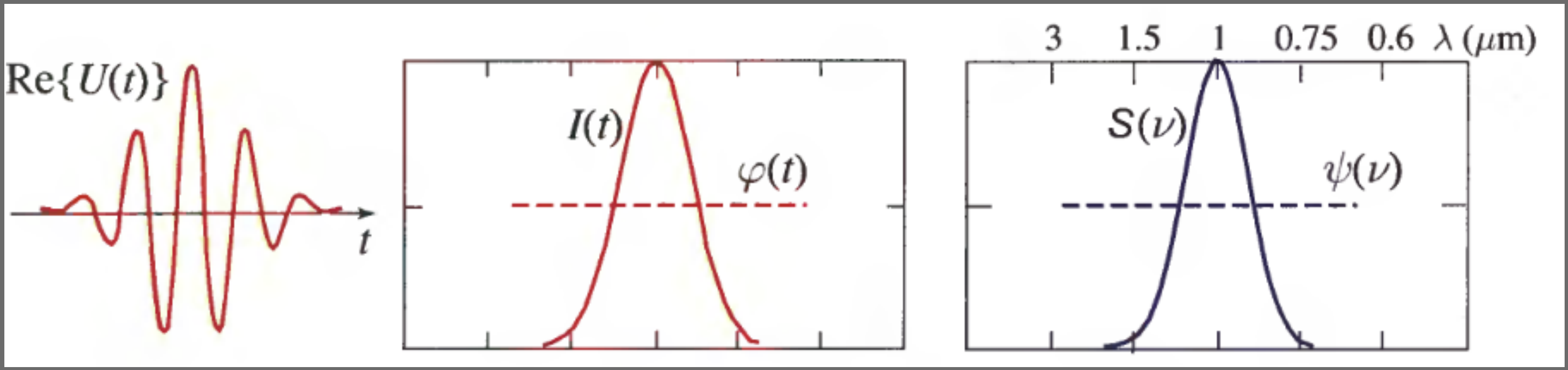
$$A(t) = A_0 \exp(-t^2/\tau^2)$$

$$I(t) = I_0 \exp(-2t^2/\tau^2)$$

$$\tau_{FWHM} = \sqrt{2\ln 2}\,\tau \approx 1.18\tau$$

$$S(\omega) \propto \exp\left[-\frac{\tau^2}{2}(\omega-\omega_0)^2\right]$$

$$\Delta\nu = 0.375/\tau = 0.44/\tau_{FWHM}$$



Why "transform-limited"?

A **transform-limited Gaussian pulse** obeys the relation

$$\tau_{\mathrm{FWHM}} \times \Delta\nu = \frac{2\ln 2}{\pi} \approx 0.44$$

This is the **minimum duration × bandwidth** product allowed by the Fourier transform. The transform-limited Gaussian pulse is the "ideal" pulse. We can also say that it is "*Fourier-transform limited*".

For a **chirped Gaussian pulse** we **always** have

$$\tau_{\mathrm{FWHM}} \times \Delta\nu > \frac{2\ln 2}{\pi}$$

Chirped Gaussian pulses

More generally, a Gaussian pulse may have the form:

$$A(t) = A_0 \exp(-\alpha t^2)$$
$$\alpha = (1 - ia)/\tau^2$$

This is called a **chirped Gaussian pulse**

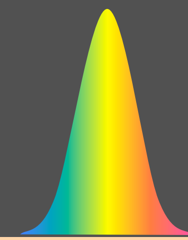
$$A(t) = A_0 \exp\left(-t^2/\tau^2\right)\exp\left(iat^2/\tau^2\right)$$

Intensity $I(t) = |A(t)|^2 = A_0 \exp(-2t^2/\tau^2)$

Phase $\varphi(t) = at^2/\tau^2$

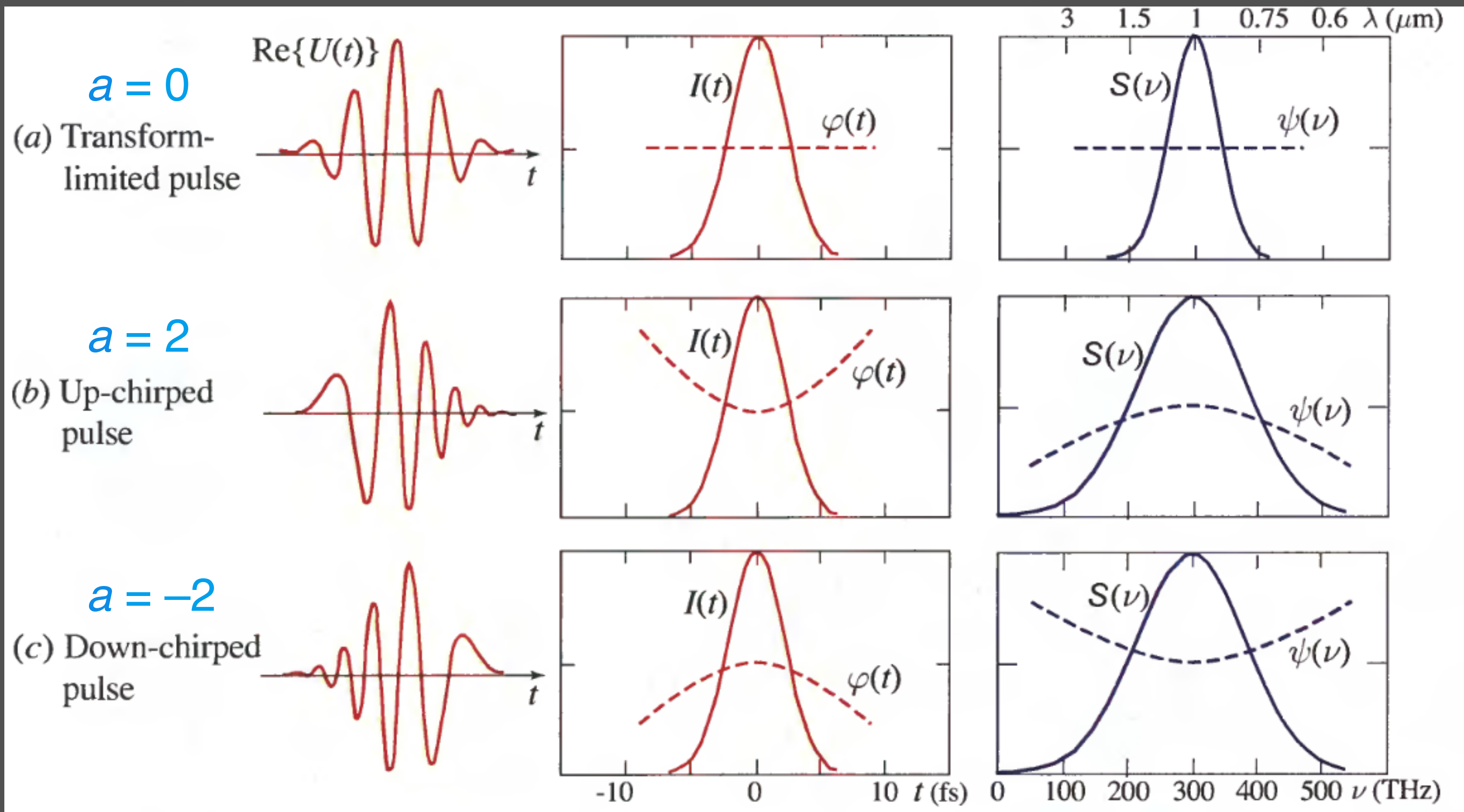
Instant. frequency $\omega_i(t) = \varphi'(t) = 2at/\tau^2$

Transform-limited and chirped Gaussian pulses: properties

	transform limited  $A(t) = A_0 \exp(-t^2/\tau^2)$	chirped  $A(t) = A_0 \exp\left(-(1-ia)t^2/\tau^2\right)$
$I(t)$	$I_0 \exp\left(-2t^2/\tau^2\right)$	$I_0 \exp\left(-2t^2/\tau^2\right)$
τ_{FWHM}	$\sqrt{2\ln 2}\,\tau \approx 1.18\tau$	$\sqrt{2\ln 2}\,\tau \approx 1.18\tau$
$S(\omega)$	$\propto \exp\left[-\frac{\tau^2}{2}(\omega-\omega_0)^2\right]$	$\exp\left[-\frac{\tau^2(\omega-\omega_0)^2}{2\boxed{\left(1+a^2\right)}}\right]$
$\Delta\nu$	$0.44/\tau_{\text{FWHM}}$	$\frac{0.44}{\tau_{\text{FWHM}}}\boxed{\sqrt{1+a^2}}$
ω_i	ω_0	$\omega_0 + \boxed{\left(2a/\tau^2\right)}t$

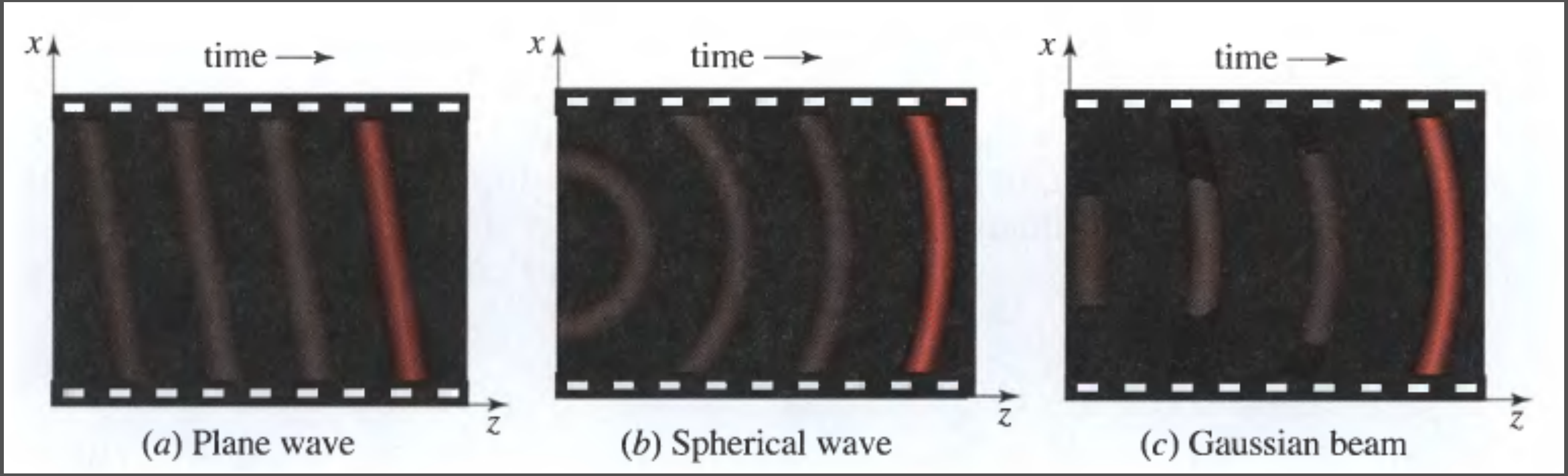
Transform-limited and chirped Gaussian pulses: properties

Example for a $\lambda_0 = 1\ \mu m$, $\tau_{FWHM} = 5$ fs pulse. Chirped pulse with $a = \pm 2$.



Spatial properties of laser pulses

In free space, or in a linear, homogeneous and non-dispersive medium, an optical pulse obeys the wave equation. Using the same considerations as we did for wave optics the solutions are of the type:

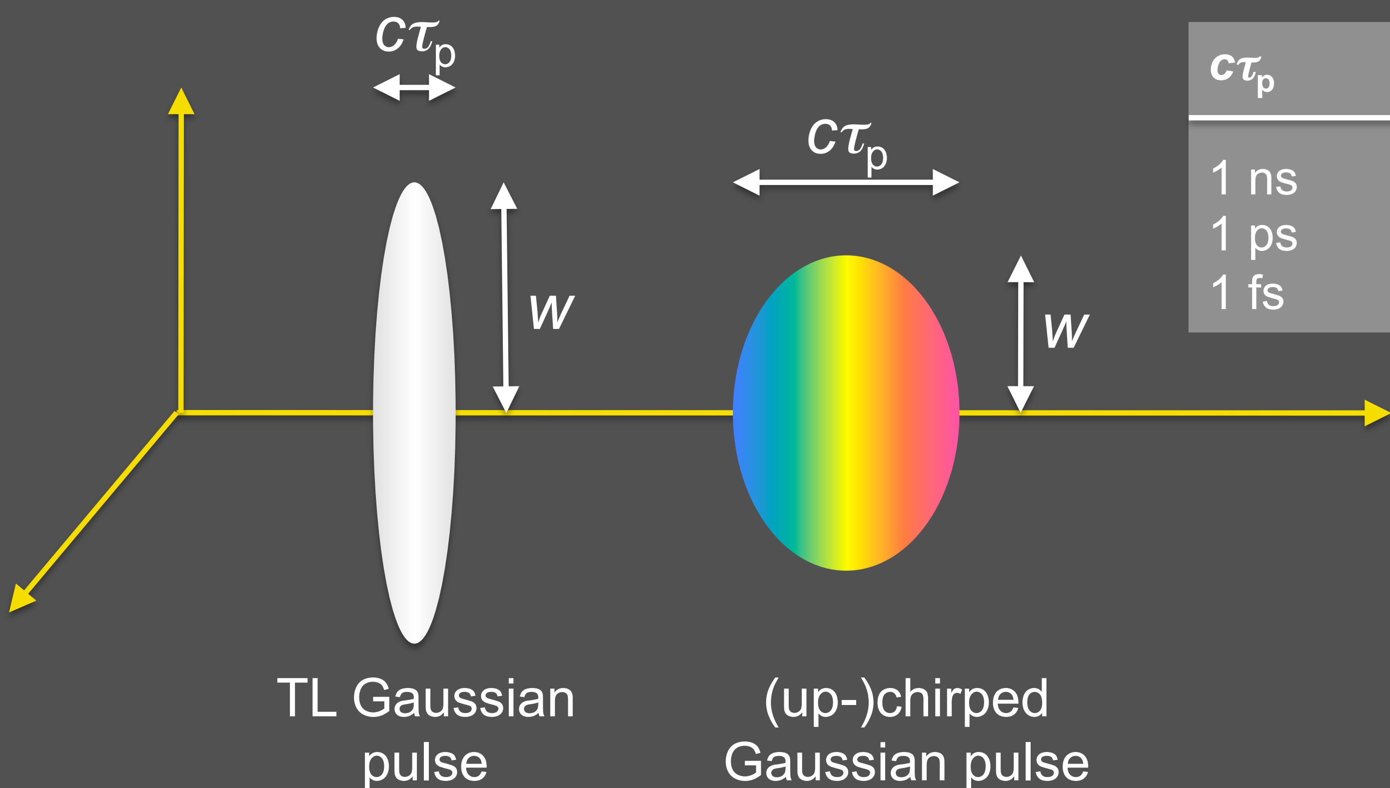


$$U(r,t) = A(t - z/c)\, e^{i\omega_0 (t - z/c)}$$
$$I(t) = \left|A^2(t)\right|$$

$$A(\rho, z, t) = g(t - z/c)\, \frac{iz_0}{z + iz_0} \exp\left(-i \frac{\pi}{\lambda_0} \frac{\rho^2}{z + iz_0}\right)$$

What does a Gaussian pulse look like?

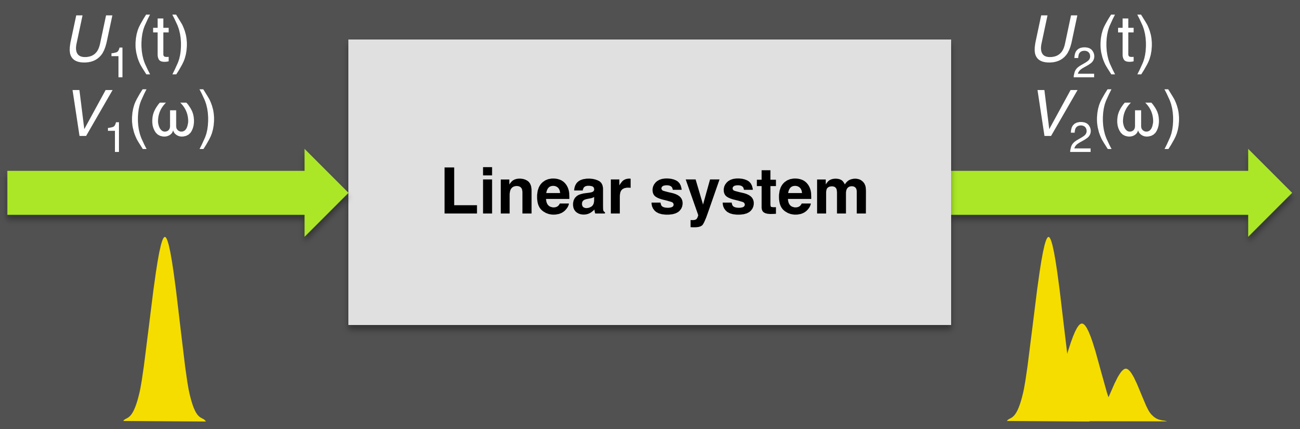
This is a pulse that is **Gaussian both in space and in time**.



$c\tau_p$	L
1 ns	0.3 m
1 ps	0.3 mm
1 fs	0.3 μm

Linear systems*

The transmission of an optical pulse through an arbitrary linear optical system can be described in terms of the **theory of linear systems**.
(*see *Fund. Photonics* Appendix B)



$$\boxed{V_2(\omega) = H(\omega)V_1(\omega)}$$

Transfer function of the system

Linear filtering of an optical pulse

For an optical pulse centered around ω_0 we only need $H(\omega)$ at frequencies inside the spectral width $\Delta\omega$.

If $\Delta\omega \ll \omega_0$ it is more practical to work with the complex envelope *A(t)* instead of the wavefunction *U(t)*.

$$U(t) = A(t)e^{i\omega_0 t} \rightarrow V(\omega) = \tilde{A}(\omega - \omega_0) \equiv \tilde{A}(\delta\omega)$$

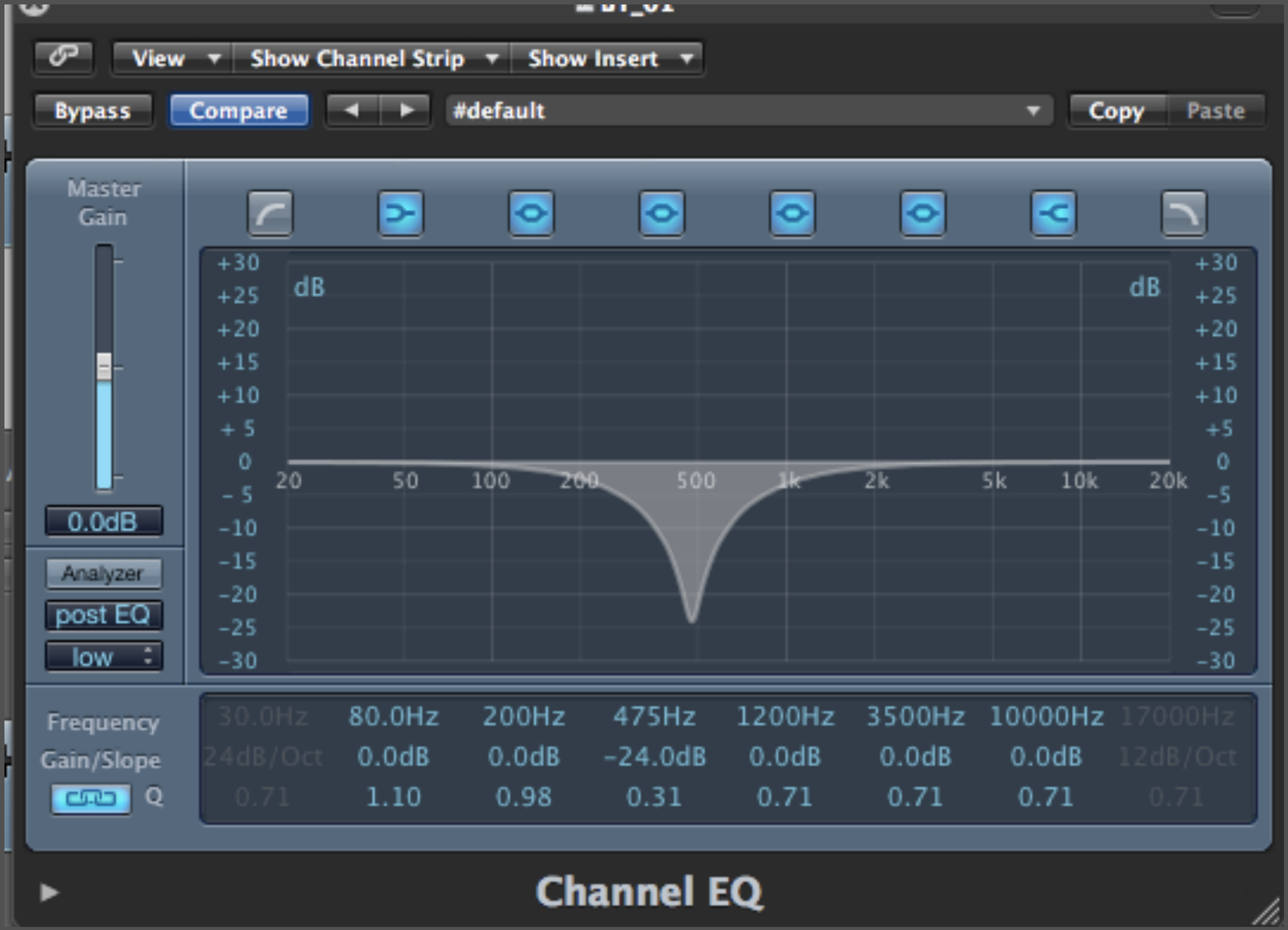
$$\tilde{A}_2(\delta\omega) = \underbrace{\tilde{H}_e(\delta\omega)}_{\text{envelope transfer function}}\tilde{A}_1(\delta\omega)$$

(Note that we can also write in the time domain: the FT of a product is a convolution, and $H_e(t)$ is the *impulse response function* = the inverse FT of the envelope transfer function.)

$$A_2(t) = H_e(t) * A_1(t) = \int H_e(t - t')A_1(t')dt'$$

$$H_e(t) = FT^{-1}\left\{\tilde{H}_e(\omega)\right\}$$

Example: using an equalizer to attenuate sound frequencies



The ideal filter

An **ideal filter**

- preserves the shape of the input pulse
- multiplies by a constant
  ($G > 1$: amplifier; $G < 1$: attenuator)
- may delay the pulse by a fixed time τ_d
  (remember the *time-delay* property of FT's)

$$\tilde{H}_e(\delta\omega) = H_0 \exp\left(-i\,\delta\omega\tau_d\right)$$

$$G = \left|H_0\right|^2$$

Example: a slab of ideal material with thickness d, ref. index n and distributed attenuation coefficient α (m^{-1}):

$$\tilde{H}_e(\omega) = \underbrace{\exp\left(-\alpha d/2\right)}_{H_0}\underbrace{\exp\left(-i\,\omega n d/c_0\right)}_{\exp\left(-i\,\delta\omega\tau_d\right)}$$

A neutral density filter

The chirp filter

It's the most important type of filter in optics. Its phase is a **quadratic function of frequency**:

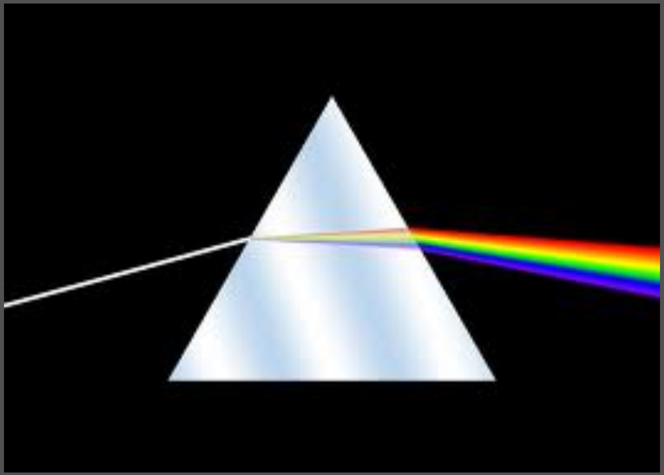
$$\tilde{H}_e(\delta\omega) = \exp\left(-i\,b\,\frac{\delta\omega^2}{4}\right)$$

$$H_e(t) = \frac{1}{\sqrt{ib/2}}\exp\left(i\,\frac{t^2}{b}\right)$$

b = **chirp coefficient** (s^2)
$b > 0$: up-chirping
$b < 0$: down-chirping

Chirp filters can be cascaded: the sum of two chirp filters with coeff. b_1 and b_2 is equal to a single filter $b = b_1 + b_2$.

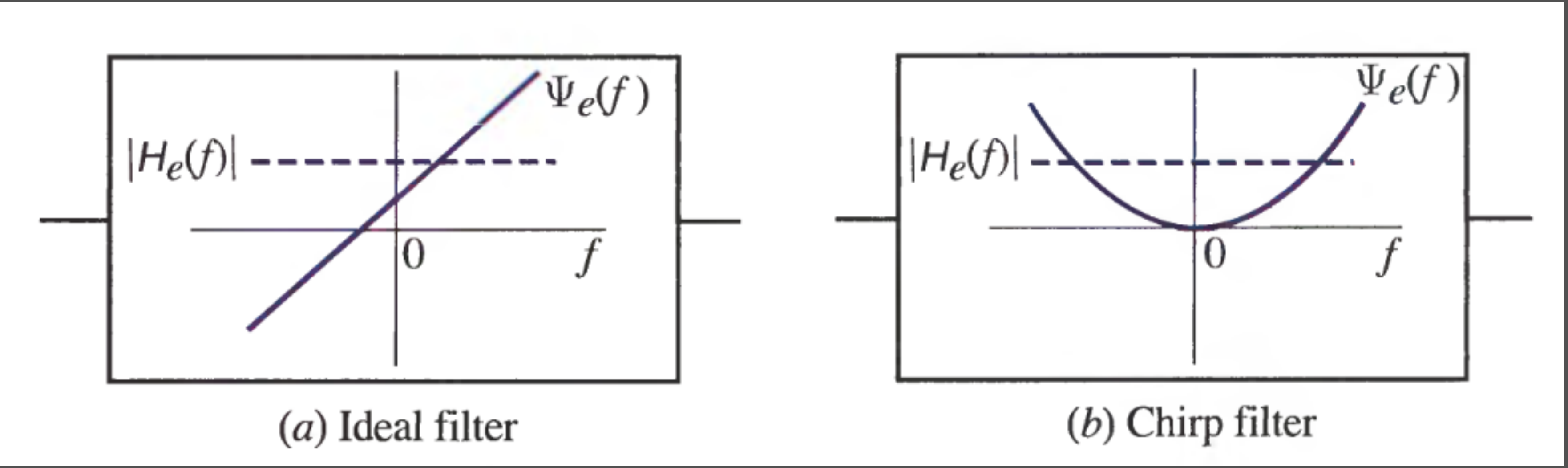
Pulse dispersion by a prism may be described in terms of chirped filters.



Ideal vs. chirp filters

$$H_0 \exp\left(-i\,\delta\omega\tau_d\right)$$

$$\exp\left(-i\,b\frac{\delta\omega^2}{4}\right)$$



Arbitrary filter ≈ chirped filter + ideal filter

If the magnitude and phase of an arbitrary filter $H_e(\omega)$ vary slowly within the spectral width of the pulse, we may assume the following:

$$\left|\tilde{H}(\omega_0 + \delta\omega)\right| \approx \left|\tilde{H}(\omega_0)\right| \equiv \left|H_0\right|$$

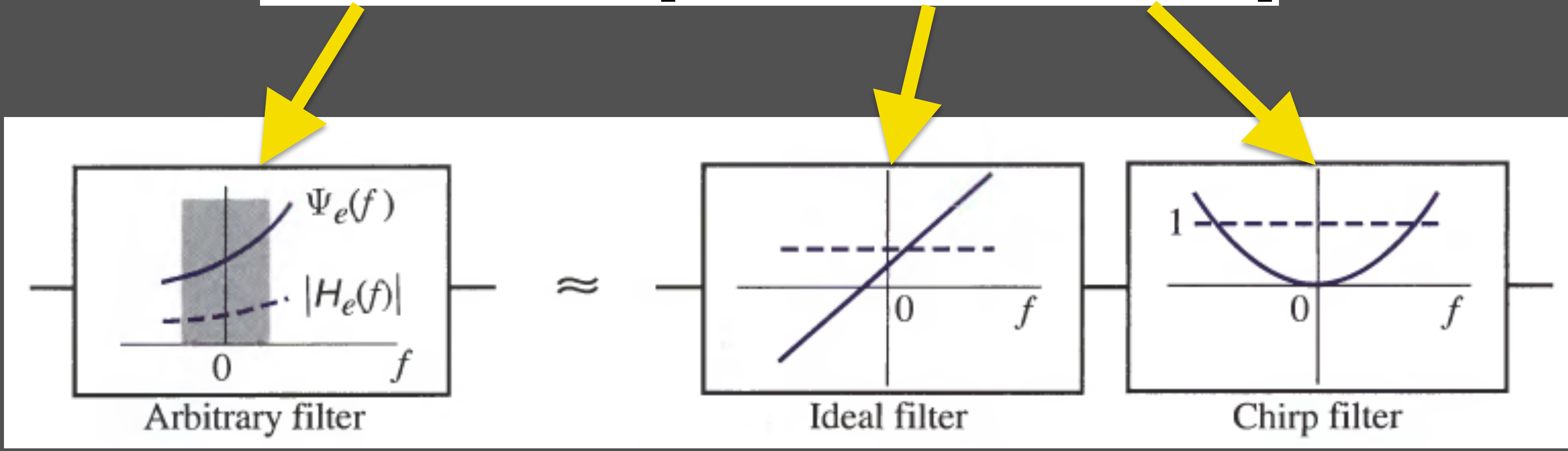
$$\psi(\omega_0 + \delta\omega) \approx \psi_0 + \psi'\delta\omega + \tfrac{1}{2}\psi''\delta\omega^2$$

(Taylor series around ω_0)

$$\tilde{H}_e(\delta\omega) \approx \left|H_0\right| \exp\left[-i\left(\underbrace{\psi_0}_{\text{constant phase}} + \underbrace{\psi'\delta\omega}_{\text{linear filter: } \tau_d = \psi'} + \underbrace{\tfrac{1}{2}\psi''\delta\omega^2}_{\text{chirped filter: } b = 2\psi''}\right)\right]$$

Arbitrary filter ≈ chirped filter + ideal filter

$$\tilde{H}_e(\delta\omega) \approx |H_0| \exp\left[-i\left(\psi_0 + \psi'\delta\omega + \tfrac{1}{2}\psi''\delta\omega^2\right)\right]$$



A treatment based on ideal + chirped filters is enough to understand several important phenomena related to the propagation and modification of short laser pulses.
(more complex systems may require higher order terms: $\psi^{(3)}$, $\psi^{(4)}$, etc)

Exercise: chirp filtering of a transform-limited Gaussian pulse

Calculate and interpret the effect of a chirp filter with chirp parameter b on a transform limited Gaussian pulse of amplitude A_{10} and width τ_1. Determine the new duration, chirp parameter and amplitude.

$$\tilde{H}_e(\delta\omega) = \exp\left(-i\, b\delta\omega^2/4\right)$$

$$A_1(t) = A_{10}\exp\left(-t^2/\tau_1^2\right)$$

Steps:

1) Calculate the Fourier-transform $\tilde{A}_1(\delta\omega)$
2) Multiply by the chirped filter to obtain $\tilde{A}_2(\delta\omega)$
3) Compare $\tilde{A}_2(\delta\omega)$ with a general chirped Gaussian pulse

Chirp filtering of a transform-limited Gaussian pulse: results

Width $$\tau_2 = \tau_1 \sqrt{1 + b^2/\tau_1^4}$$

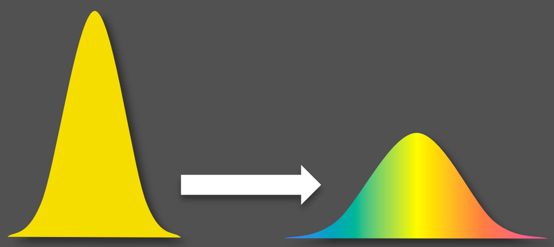
Chirp parameter $$a_2 = b/\tau_1^2$$

Amplitude $$A_{20} = \frac{A_{10}}{\sqrt{1 + ib/\tau_1^2}}$$

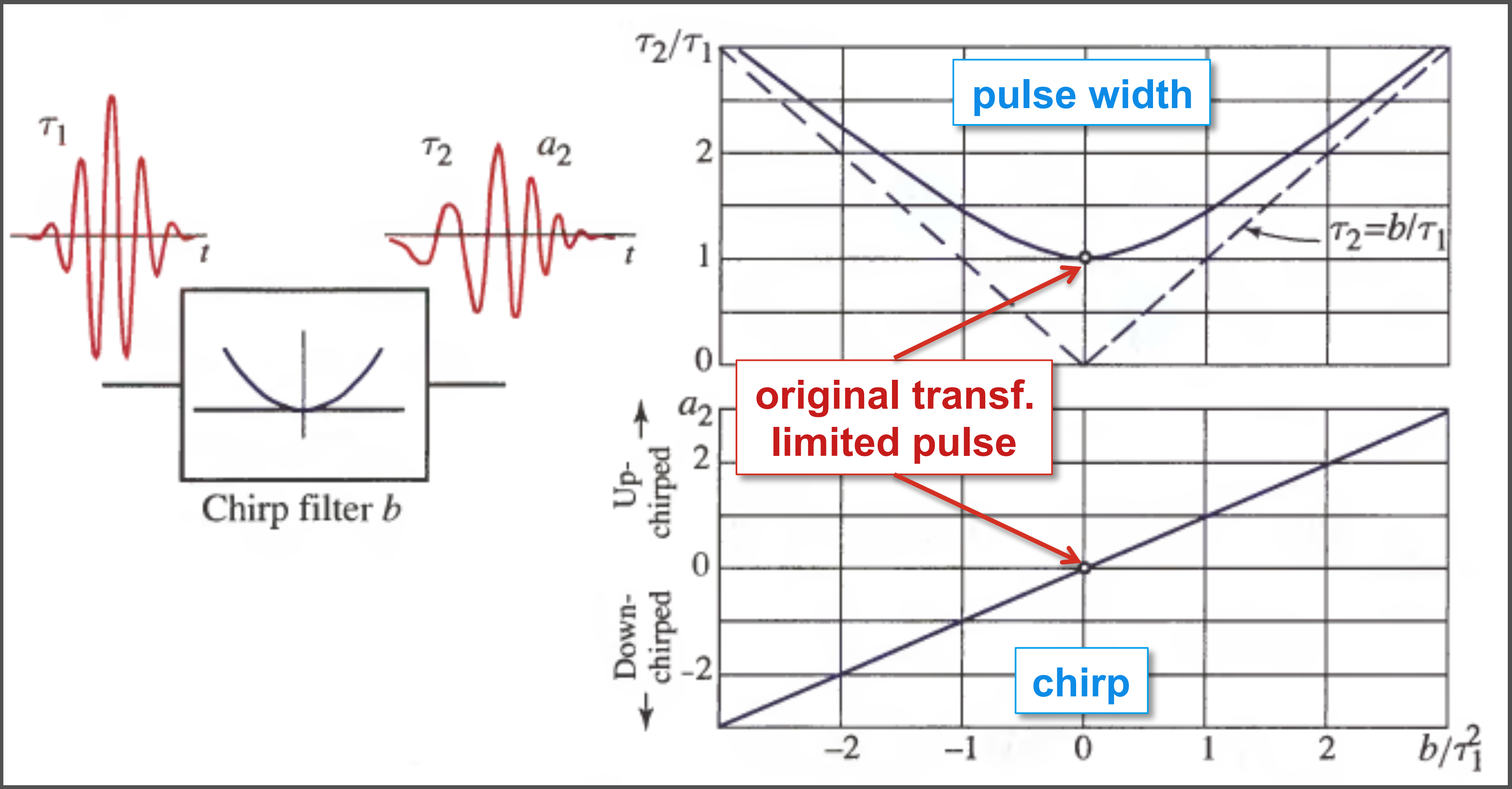
- The **temporal width increases** by a factor depending on *b*.
- The **initial TL pulse becomes chirped** with chirp parameter proportional to *b*.
- The spectral width remains the same.

$$b = \tau_1^2 \rightarrow \tau_2 = \sqrt{2}\tau_1$$

$$b >> \tau_1^2 \rightarrow \tau_2 \approx |b|/\tau_1$$



Chirp filtering of a transform-limited Gaussian pulse: results



Chirp filtering of a chirped Gaussian pulse

In this case the chirped filter is applied to an already chirped Gaussian pulse with chirp a_1

$$\tilde{H}_e(\delta\omega) = \exp\left(-i\, b\delta\omega^2/4\right)$$

$$A_1(t) = A_{10}\exp\left(-(1-ia_1)t^2/\tau_1^2\right)$$

Width

$$\tau_2 = \tau_1\sqrt{1+2a_1 b/\tau_1^2 + \left(1+a_1^2\right)b^2/\tau_1^4}$$

Chirp parameter

$$a_2 = a_1 + \left(1+a_1^2\right)b/\tau_1^2$$

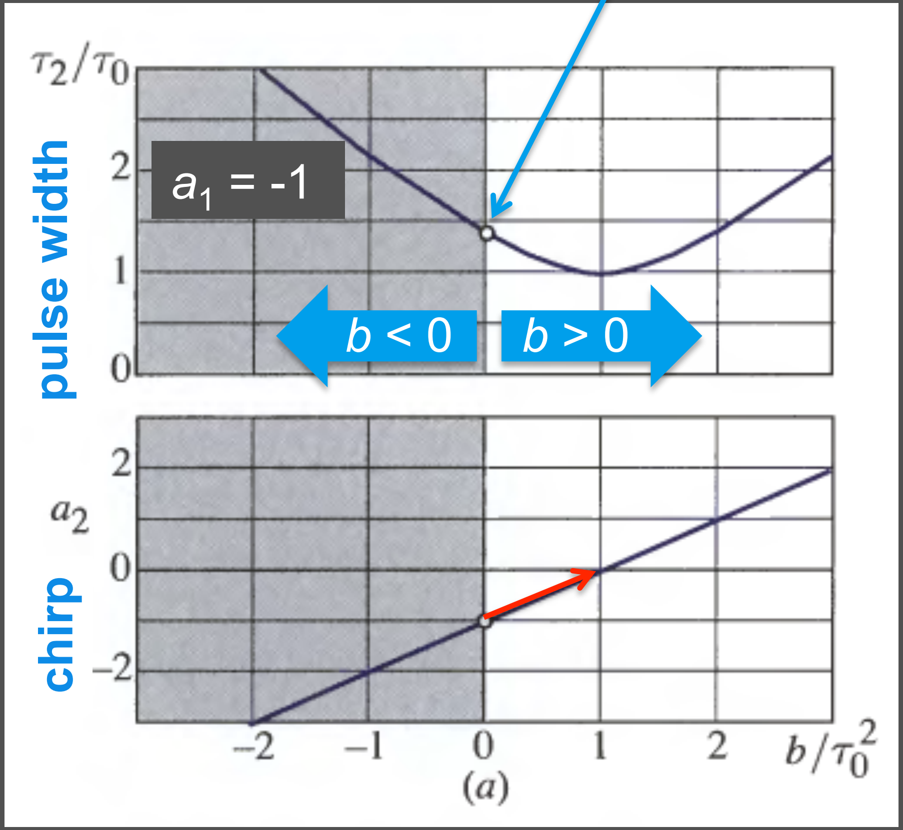
The great difference now is that the output pulse can be **shorter** than the original one. The minimum width τ_0 and required chirp coeff. b are:

$$\tau_0 = \frac{\tau_1}{\sqrt{1+a_1^2}}$$

$$b_{\min} = -a_1\tau_0^2 = -\frac{a_1}{1+a_1^2}\tau_1^2$$

Example: Compression and expansion of laser pulses (1)

Input pulse: $A_1(t) = A_{10} \exp\left(-(1 - ia_1)t^2/\tau_1^2\right)$



$$a_1 = -1$$

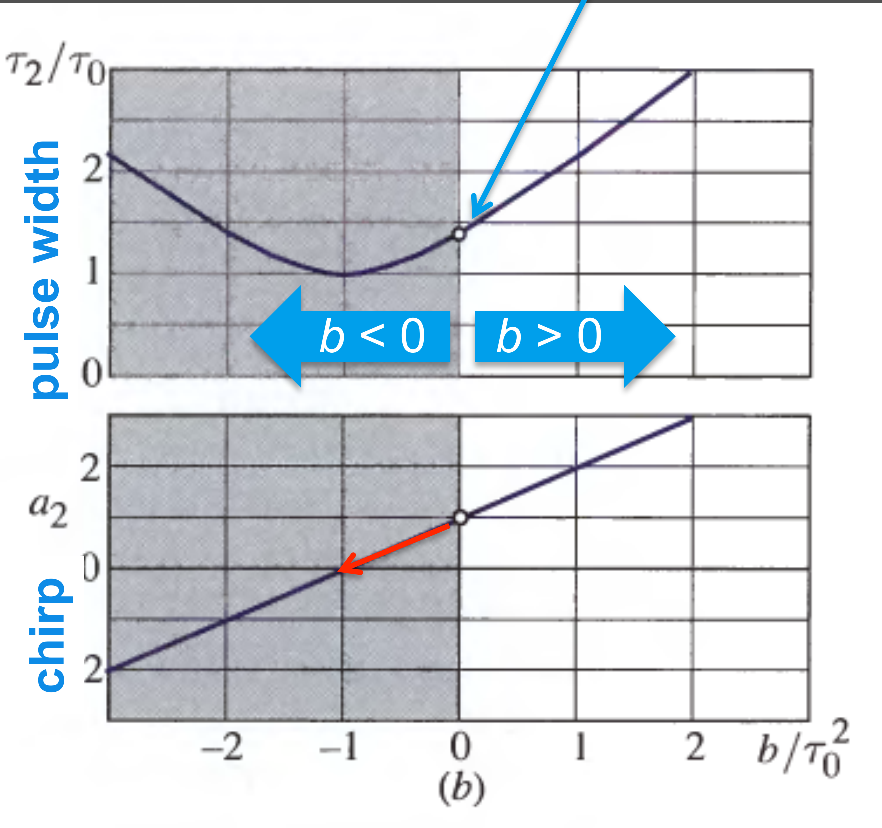
$$b_{min} = -\frac{a_1}{1 + a_1^2}\tau_1^2 = \frac{1}{2}\tau_1^2$$

$$\tau_0 = \frac{\tau_1}{\sqrt{1 + a_1^2}} = \frac{\tau_1}{\sqrt{2}}$$

$$\therefore b_{min} = \tau_0^2$$

Example: Compression and expansion of laser pulses (2)

Input pulse: $A_1(t) = A_{10}\exp\left(-(1-ia_1)t^2/\tau_1^2\right)$



$$a_1 = 1$$

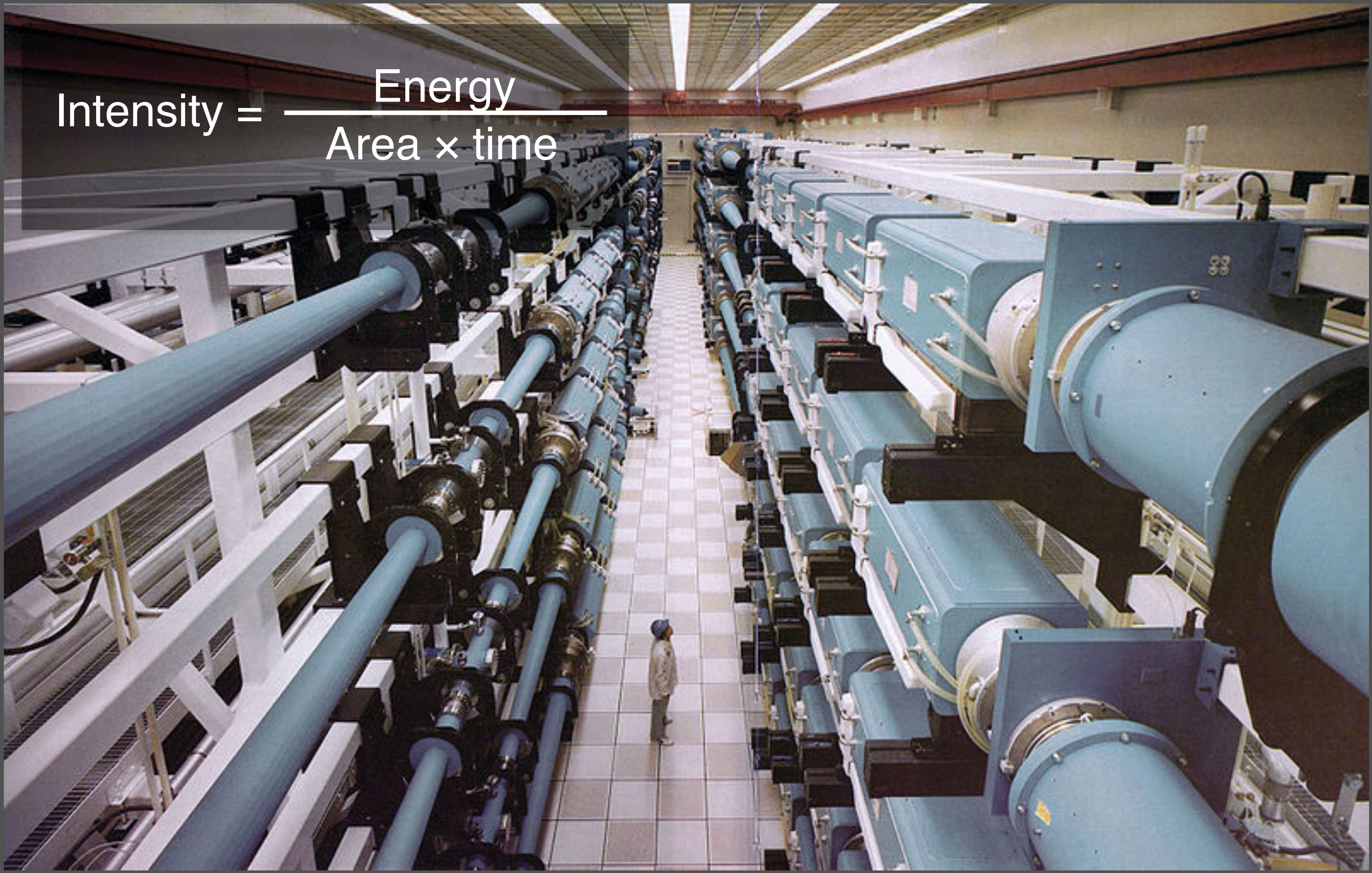
$$b_{\min} = -\frac{a_1}{1+a_1^2}\tau_1^2 = -\frac{1}{2}\tau_1^2$$

$$\tau_0 = \frac{\tau_1}{\sqrt{1+a_1^2}} = \frac{\tau_1}{\sqrt{2}}$$

$$\therefore b_{\min} = -\tau_0^2$$

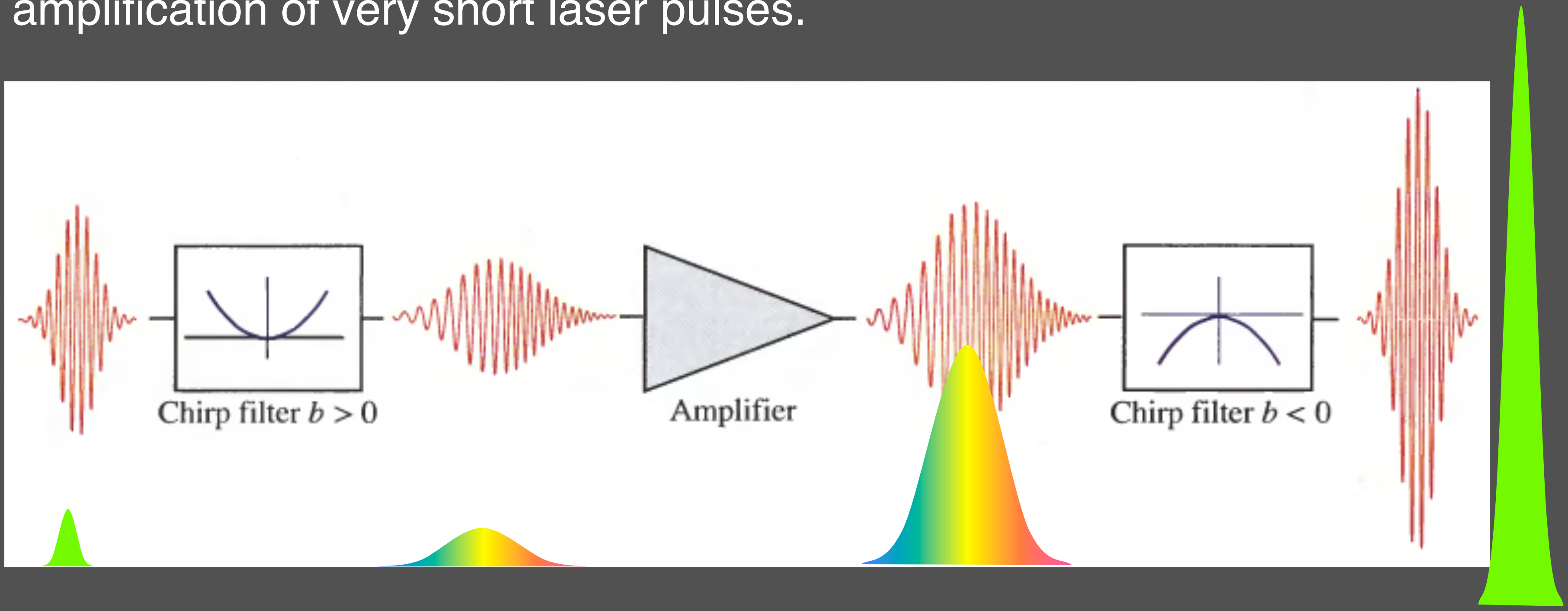
In the mid-1980's high power (~TW) lasers were very large

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{time}}$$



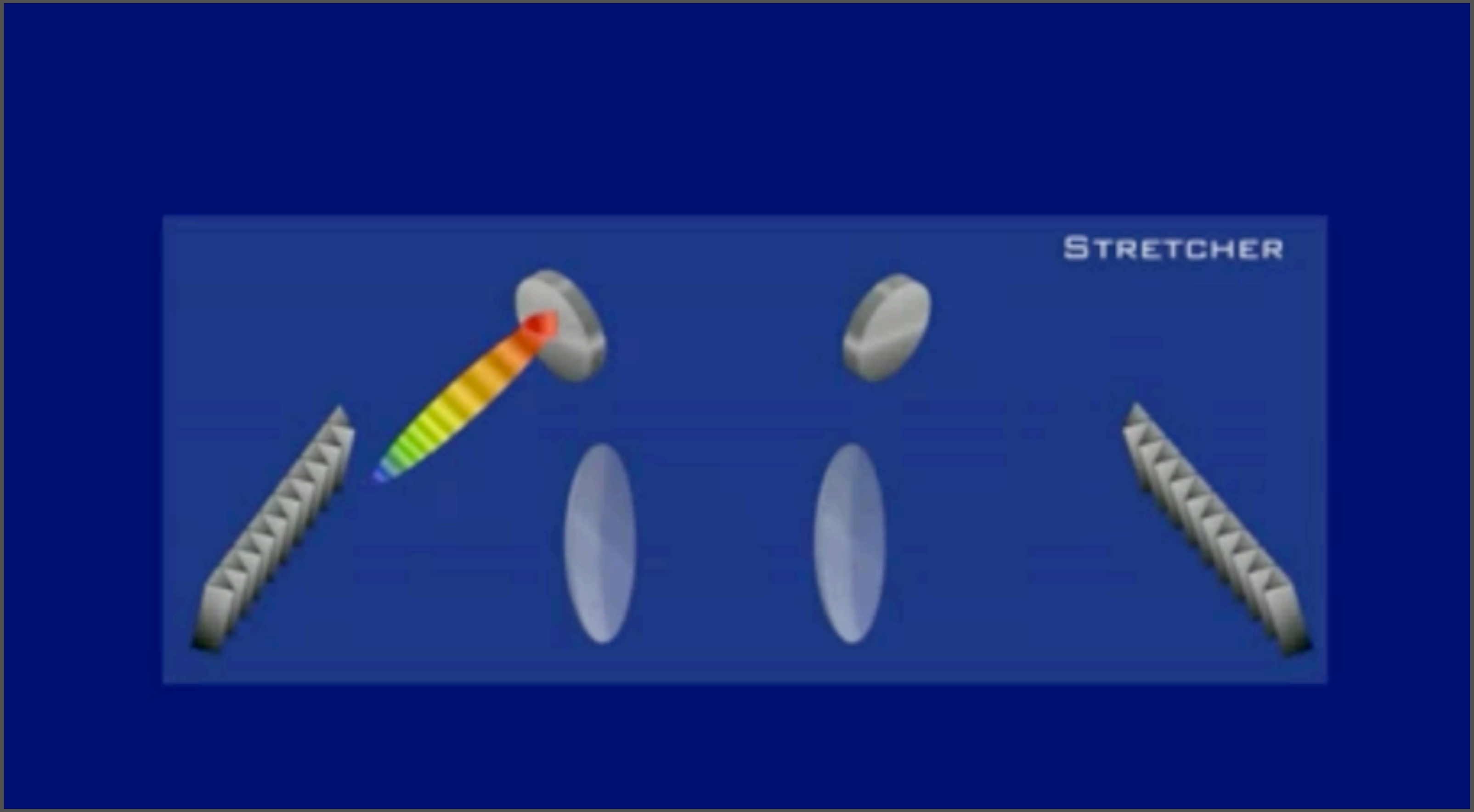
Application: Chirped pulse amplification

CPA was introduced in 1986 and represented a revolution in the amplification of very short laser pulses.



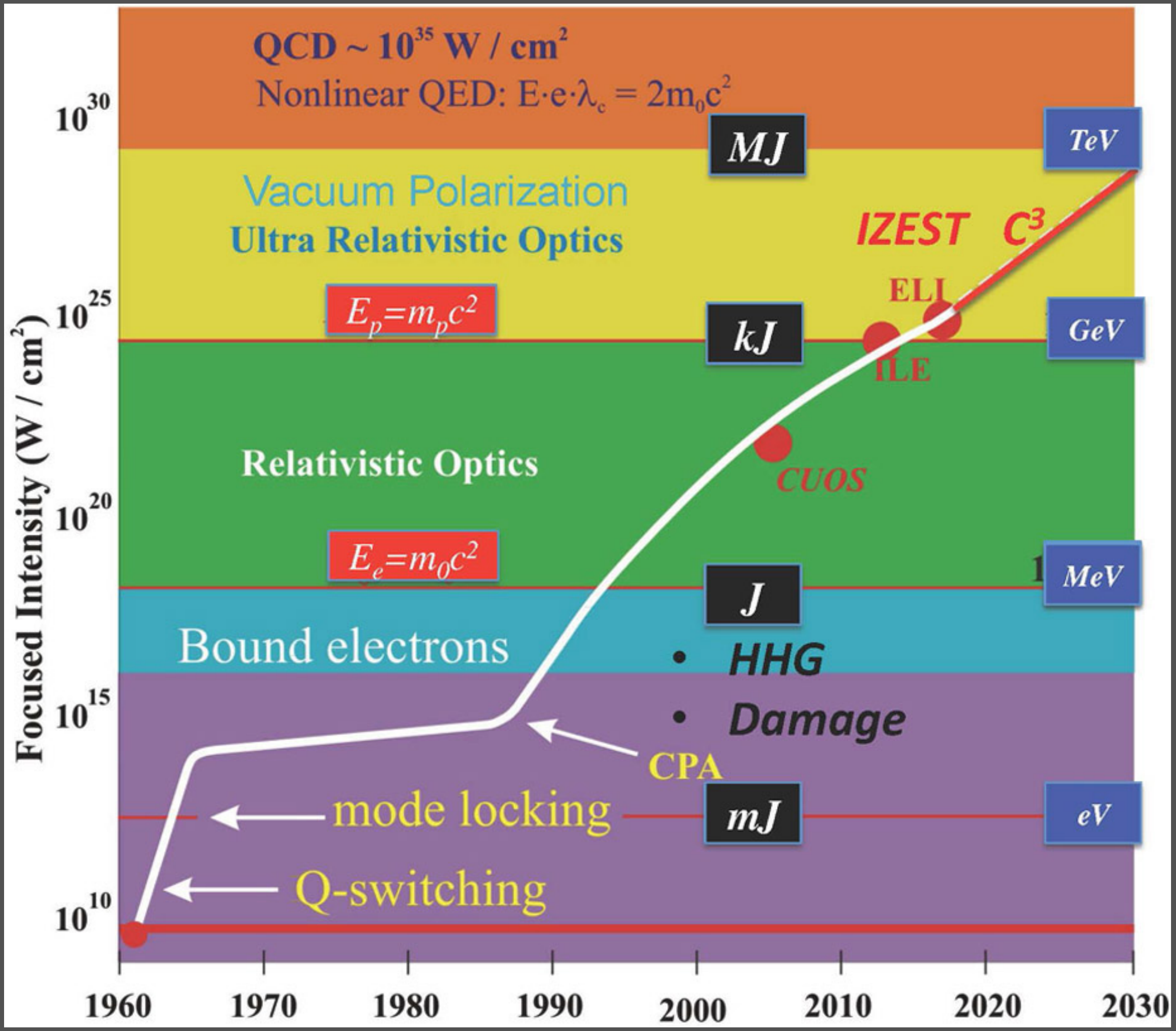
TL pulse → Stretching → Amplification → Compression

Typical CPA setup



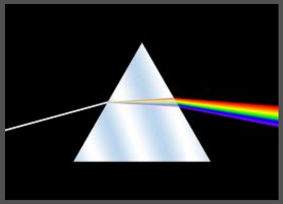
https://youtu.be/jkyEURnb9A4

Evolution of peak laser intensity

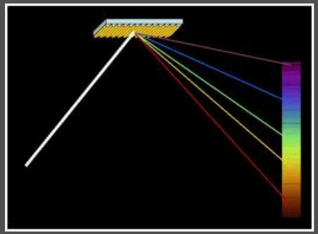


Examples of implementation of chirp filters

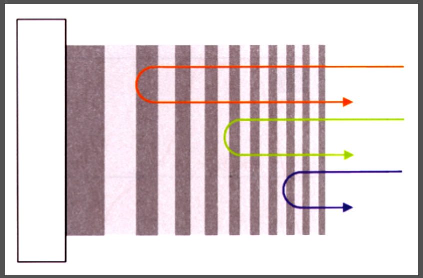
- Material dispersion




- Angular dispersion

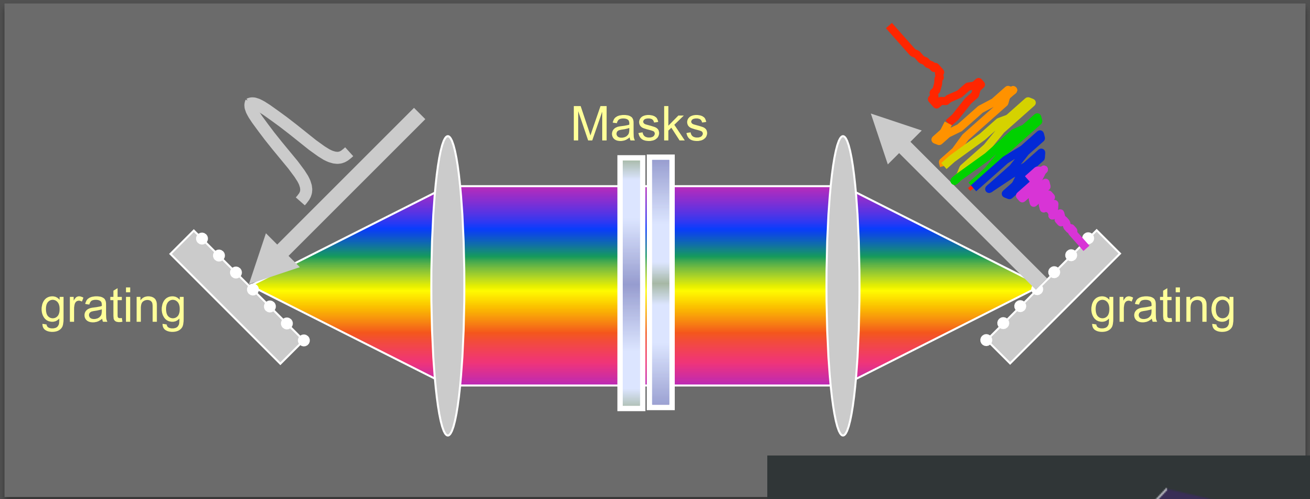



- Interferometric dispersion




- Others: polarization dispersion, nonlinear dispersion…

Pulse shaping



- To generate pulses that control chemical reactions or other phenomena
- To generate trains of pulses for telecommunications
- To precompensate for distortions that occur in dispersive media