I. Supplementary Material

A. Proof of Lemma 1

According to the paradigm compatibility, we can construct the following inequality:

$$\begin{aligned}
&||\boldsymbol{W}^{\mathrm{T}}soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})-\boldsymbol{W}^{\mathrm{T}}soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})||_2^2\\
&=||\boldsymbol{W}^{\mathrm{T}}[soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})]||_2^2\\
&\le||\boldsymbol{W}^{\mathrm{T}}||_2^2||soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})||_2^2\\
&=\lambda_{max}(\boldsymbol{W}\boldsymbol{W}^{\mathrm{T}})||soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})||_2^2\\
&=\lambda_{max}(\boldsymbol{W}\boldsymbol{W}^{\mathrm{T}})||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2
\end{aligned} \tag{1}$$

Let $(\boldsymbol{W}\boldsymbol{x})_i$ represent the i-th element of $\boldsymbol{W}\boldsymbol{x}$. The soft thresholding operator $soft(\boldsymbol{W}\boldsymbol{x},\boldsymbol{\varepsilon})$ is defined as:

$$soft(\boldsymbol{W}\boldsymbol{x},\boldsymbol{\varepsilon})=\begin{cases}(\boldsymbol{W}\boldsymbol{x})_i+\varepsilon_i, & (\boldsymbol{W}\boldsymbol{x})_i<-\varepsilon_i\\ 0, & |(\boldsymbol{W}\boldsymbol{x})_i|\le\varepsilon_i\\ (\boldsymbol{W}\boldsymbol{x})_i-\varepsilon_i, & (\boldsymbol{W}\boldsymbol{x})_i>\varepsilon_i\end{cases} \tag{2}$$

We divide our discussion of Eqn. (1) into the following scenarios:

$(i)(\boldsymbol{W}\boldsymbol{x}_1)_i<(\boldsymbol{W}\boldsymbol{x}_2)_i$

$a.$ if $(\boldsymbol{W}\boldsymbol{x}_1)_i<(\boldsymbol{W}\boldsymbol{x}_2)_i<-\varepsilon_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon})-(\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon})||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{3}$$

$b.$ if $(\boldsymbol{W}\boldsymbol{x}_1)_i<-\varepsilon_i\le(\boldsymbol{W}\boldsymbol{x}_2)_i\le\varepsilon_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=0$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||0-(\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon})||_2^2\\
&=||-\boldsymbol{W}\boldsymbol{x}_1-\boldsymbol{\varepsilon}||_2^2\\
&=||-\boldsymbol{\varepsilon}-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{4}$$

$c.$ if $(\boldsymbol{W}\boldsymbol{x}_1)_i<-\varepsilon_i<\varepsilon_i<(\boldsymbol{W}\boldsymbol{x}_2)_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{\varepsilon})-(\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon})||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1-2\boldsymbol{\varepsilon}||_2^2\\
&\le||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{5}$$

$d.$ if $-\varepsilon_i\le(\boldsymbol{W}\boldsymbol{x}_1)_i<(\boldsymbol{W}\boldsymbol{x}_2)_i\le\varepsilon_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=0, soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=0$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||0-0||_2^2\\
&=0
\end{aligned} \tag{6}$$

$e.$ if $-\varepsilon_i\le(\boldsymbol{W}\boldsymbol{x}_1)_i\le\varepsilon_i<(\boldsymbol{W}\boldsymbol{x}_2)_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=0, soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{\varepsilon})-0||_2^2\\
&\le||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{7}$$

$f.$ if $\varepsilon_i<(\boldsymbol{W}\boldsymbol{x}_1)_i<(\boldsymbol{W}\boldsymbol{x}_2)_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_1-\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{\varepsilon})-(\boldsymbol{W}\boldsymbol{x}_1-\boldsymbol{\varepsilon})||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{8}$$

$(ii)(\boldsymbol{W}\boldsymbol{x}_2)_i<(\boldsymbol{W}\boldsymbol{x}_1)_i$

$a.$ if $(\boldsymbol{W}\boldsymbol{x}_2)_i<(\boldsymbol{W}\boldsymbol{x}_1)_i<-\varepsilon_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon})-(\boldsymbol{W}\boldsymbol{x}_1+\boldsymbol{\varepsilon})||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{9}$$

$b.$ if $(\boldsymbol{W}\boldsymbol{x}_2)_i<-\varepsilon_i\le(\boldsymbol{W}\boldsymbol{x}_1)_i\le\varepsilon_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=0$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon})-0||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon}||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2-(-\boldsymbol{\varepsilon})||_2^2\\
&\le||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{10}$$

$c.$ if $(\boldsymbol{W}\boldsymbol{x}_2)_i<-\varepsilon_i<\varepsilon_i<(\boldsymbol{W}\boldsymbol{x}_1)_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=\boldsymbol{W}\boldsymbol{x}_1-\boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||(\boldsymbol{W}\boldsymbol{x}_2+\boldsymbol{\varepsilon})-(\boldsymbol{W}\boldsymbol{x}_1-\boldsymbol{\varepsilon})||_2^2\\
&=||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1+2\boldsymbol{\varepsilon}||_2^2\\
&\le||\boldsymbol{W}\boldsymbol{x}_2-\boldsymbol{W}\boldsymbol{x}_1||_2^2\\
&\le||\boldsymbol{W}||_2^2||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2\\
&=||\boldsymbol{x}_2-\boldsymbol{x}_1||_2^2
\end{aligned} \tag{11}$$

$d.$ if $-\varepsilon_i\le(\boldsymbol{W}\boldsymbol{x}_2)_i<(\boldsymbol{W}\boldsymbol{x}_1)_i\le\varepsilon_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})=0, soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})=0$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2,\boldsymbol{\varepsilon})-soft(\boldsymbol{W}\boldsymbol{x}_1,\boldsymbol{\varepsilon})||_2^2\\
&=||0-0||_2^2\\
&=0
\end{aligned} \tag{12}$$

e. if $-\varepsilon_i \leq (\boldsymbol{W}\boldsymbol{x}_2)_i \leq \varepsilon_i < (\boldsymbol{W}\boldsymbol{x}_1)_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_2, \boldsymbol{\varepsilon}) = 0, soft(\boldsymbol{W}\boldsymbol{x}_1, \boldsymbol{\varepsilon}) = \boldsymbol{W}\boldsymbol{x}_1 - \boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2, \boldsymbol{\varepsilon}) - soft(\boldsymbol{W}\boldsymbol{x}_1, \boldsymbol{\varepsilon})||_2^2 \\
&= ||0 - (\boldsymbol{W}\boldsymbol{x}_1 - \boldsymbol{\varepsilon})||_2^2 \\
&= ||\boldsymbol{\varepsilon} - \boldsymbol{W}\boldsymbol{x}_1||_2^2 \\
&\leq ||\boldsymbol{W}\boldsymbol{x}_2 - \boldsymbol{W}\boldsymbol{x}_1||_2^2 \\
&\leq ||\boldsymbol{W}||_2^2 ||\boldsymbol{x}_2 - \boldsymbol{x}_1||_2^2 \\
&= ||\boldsymbol{x}_2 - \boldsymbol{x}_1||_2^2
\end{aligned} \tag{13}$$

f. if $\varepsilon_i < (\boldsymbol{W}\boldsymbol{x}_2)_i < (\boldsymbol{W}\boldsymbol{x}_1)_i$, then $soft(\boldsymbol{W}\boldsymbol{x}_2, \boldsymbol{\varepsilon}) = \boldsymbol{W}\boldsymbol{x}_2 - \boldsymbol{\varepsilon}, soft(\boldsymbol{W}\boldsymbol{x}_1, \boldsymbol{\varepsilon}) = \boldsymbol{W}\boldsymbol{x}_1 - \boldsymbol{\varepsilon}$,:

$$\begin{aligned}
&||soft(\boldsymbol{W}\boldsymbol{x}_2, \boldsymbol{\varepsilon}) - soft(\boldsymbol{W}\boldsymbol{x}_1, \boldsymbol{\varepsilon})||_2^2 \\
&= ||(\boldsymbol{W}\boldsymbol{x}_2 - \boldsymbol{\varepsilon}) - (\boldsymbol{W}\boldsymbol{x}_1 - \boldsymbol{\varepsilon})||_2^2 \\
&= ||\boldsymbol{W}\boldsymbol{x}_2 - \boldsymbol{W}\boldsymbol{x}_1||_2^2 \\
&\leq ||\boldsymbol{W}||_2^2 ||\boldsymbol{x}_2 - \boldsymbol{x}_1||_2^2 \\
&= ||\boldsymbol{x}_2 - \boldsymbol{x}_1||_2^2
\end{aligned} \tag{14}$$

From the above inequalities, it can be seen that the upper bound of $||soft(\boldsymbol{W}\boldsymbol{x}_2, \boldsymbol{\varepsilon}) - soft(\boldsymbol{W}\boldsymbol{x}_1, \boldsymbol{\varepsilon}))||_2^2$ is either 0 or $||\boldsymbol{x}_2 - \boldsymbol{x}_1||_2^2$. The $\boldsymbol{W}^{\mathrm{T}} soft(\boldsymbol{W}\boldsymbol{x}, \boldsymbol{\varepsilon})$ is $\lambda_{max}(\boldsymbol{W}\boldsymbol{W})^{\mathrm{T}}$-Lipschitz.