Supplementary Materials for "Prompting multi-scale adaptive sparsifying frame network with provably Lipschitz continuity for all-in-one CT reconstruction"

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This document is organized as follows. We first provide the detailed proofs in Sec. I, and then report the experimental results in Sec. II.

I. The proofs

A. Proof of Lemma 1

According to the definition of a single tight frame network, for $\forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, we have

$$\begin{aligned} & \left\|\boldsymbol{W}^{\mathrm{T}} \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\boldsymbol{W}^{\mathrm{T}} \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \left\|\boldsymbol{W}^{\mathrm{T}}\left[\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right]\right\|_2^2 \\ \leq & \left\|\boldsymbol{W}^{\mathrm{T}}\right\|_2^2\left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \lambda_{\max }\left(\boldsymbol{W} \boldsymbol{W}^{\mathrm{T}}\right)\left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \end{aligned} \tag{1}$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue.

Let $(\boldsymbol{W}\boldsymbol{x})_i$ represents the i-th element of $\boldsymbol{W}\boldsymbol{x}$. The soft thresholding operator $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})$ is defined as

$$\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})=\begin{cases}(\boldsymbol{W} \boldsymbol{x})_i+\lambda_i, & (\boldsymbol{W} \boldsymbol{x})_i<-\lambda_i \\ 0, & \left|(\boldsymbol{W} \boldsymbol{x})_i\right| \leq \lambda_i \\ (\boldsymbol{W} \boldsymbol{x})_i-\lambda_i, & (\boldsymbol{W} \boldsymbol{x})_i > \lambda_i .\end{cases} \tag{2}$$

Let $(\boldsymbol{W}\boldsymbol{y})_i$ represents the i-th element of $\boldsymbol{W}\boldsymbol{y}$. The soft thresholding operator $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y})$ is defined as

$$\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})=\begin{cases}(\boldsymbol{W} \boldsymbol{y})_i+\lambda_i, & (\boldsymbol{W} \boldsymbol{y})_i<-\lambda_i \\ 0, & \left|(\boldsymbol{W} \boldsymbol{y})_i\right| \leq \lambda_i \\ (\boldsymbol{W} \boldsymbol{y})_i-\lambda_i, & (\boldsymbol{W} \boldsymbol{y})_i > \lambda_i .\end{cases} \tag{3}$$

Consider the following scenarios:

C1: $(\boldsymbol{W}\boldsymbol{x})_i < (\boldsymbol{W}\boldsymbol{y})_i$

- If $(\boldsymbol{W}\boldsymbol{x})_i < (\boldsymbol{W}\boldsymbol{y})_i < -\lambda_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}$.

$$\begin{aligned} & \left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \|(\boldsymbol{W} \boldsymbol{y}+\boldsymbol{\Lambda})-(\boldsymbol{W} \boldsymbol{x}+\boldsymbol{\Lambda})\|_2^2 \\ = & \|\boldsymbol{W} \boldsymbol{y}-\boldsymbol{W} \boldsymbol{x}\|_2^2 \\ \leq & \|\boldsymbol{W}\|_2^2\|\boldsymbol{y}-\boldsymbol{x}\|_2^2 \\ = & \|\boldsymbol{y}-\boldsymbol{x}\|_2^2 . \end{aligned} \tag{4}$$

- If $(\boldsymbol{W}\boldsymbol{x})_i < -\lambda_i < (\boldsymbol{W}\boldsymbol{y})_i < \lambda_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \mathbf{0}$.

$$\begin{aligned} & \left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \|\mathbf{0}-(\boldsymbol{W} \boldsymbol{x}+\boldsymbol{\Lambda})\|_2^2 \\ = & \|-\boldsymbol{W} \boldsymbol{x}-\boldsymbol{\Lambda}\|_2^2 \\ \leq & \|\boldsymbol{W} \boldsymbol{y}-\boldsymbol{W} \boldsymbol{x}\|_2^2 \\ \leq & \|\boldsymbol{W}\|_2^2\|\boldsymbol{y}-\boldsymbol{x}\|_2^2 \\ = & \|\boldsymbol{y}-\boldsymbol{x}\|_2^2 . \end{aligned} \tag{5}$$

- If $(\boldsymbol{W}\boldsymbol{x})_i < -\lambda_i < \lambda_i < (\boldsymbol{W}\boldsymbol{y})_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} - \boldsymbol{\Lambda}$.

$$\begin{aligned} & \left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \|(\boldsymbol{W} \boldsymbol{y}-\boldsymbol{\Lambda})-(\boldsymbol{W} \boldsymbol{x}+\boldsymbol{\Lambda})\|_2^2 \\ = & \|\boldsymbol{W} \boldsymbol{y}-\boldsymbol{W} \boldsymbol{x}-2\boldsymbol{\Lambda}\|_2^2 \\ \leq & \|\boldsymbol{W} \boldsymbol{y}-\boldsymbol{W} \boldsymbol{x}\|_2^2 \\ \leq & \|\boldsymbol{W}\|_2^2\|\boldsymbol{y}-\boldsymbol{x}\|_2^2 \\ = & \|\boldsymbol{y}-\boldsymbol{x}\|_2^2 . \end{aligned} \tag{6}$$

- If $-\lambda_i \leq (\boldsymbol{W}\boldsymbol{x})_i < (\boldsymbol{W}\boldsymbol{y})_i \leq \lambda_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \mathbf{0}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \mathbf{0}$.

$$\begin{aligned} & \left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \|\mathbf{0}-\mathbf{0}\|_2^2 \\ = & 0 . \end{aligned} \tag{7}$$

- If $-\lambda_i \leq (\boldsymbol{W}\boldsymbol{x})_i \leq \lambda_i < (\boldsymbol{W}\boldsymbol{y})_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \mathbf{0}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} - \boldsymbol{\Lambda}$.

$$\begin{aligned} & \left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \|(\boldsymbol{W} \boldsymbol{y}-\boldsymbol{\Lambda})-\mathbf{0}\|_2^2 \\ \leq & \|\boldsymbol{W} \boldsymbol{y}-\boldsymbol{W} \boldsymbol{x}\|_2^2 \\ \leq & \|\boldsymbol{W}\|_2^2\|\boldsymbol{y}-\boldsymbol{x}\|_2^2 \\ = & \|\boldsymbol{y}-\boldsymbol{x}\|_2^2 . \end{aligned} \tag{8}$$

- If $\lambda_i < (\boldsymbol{W}\boldsymbol{x})_i < (\boldsymbol{W}\boldsymbol{y})_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} - \boldsymbol{\Lambda}$.

$$\begin{aligned} & \left\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{y})-\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W} \boldsymbol{x})\right\|_2^2 \\ = & \|(\boldsymbol{W} \boldsymbol{y}-\boldsymbol{\Lambda})-(\boldsymbol{W} \boldsymbol{x}-\boldsymbol{\Lambda})\|_2^2 \\ \leq & \|\boldsymbol{W} \boldsymbol{y}-\boldsymbol{W} \boldsymbol{x}\|_2^2 \\ \leq & \|\boldsymbol{W}\|_2^2\|\boldsymbol{y}-\boldsymbol{x}\|_2^2 \\ = & \|\boldsymbol{y}-\boldsymbol{x}\|_2^2 . \end{aligned} \tag{9}$$

C2: $(\boldsymbol{W}\boldsymbol{y})_i < (\boldsymbol{W}\boldsymbol{x})_i$

- If $(\boldsymbol{W}\boldsymbol{y})_i < (\boldsymbol{W}\boldsymbol{x})_i < -\lambda_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}$.

$$\begin{aligned} &\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\|_2^2 \\ =& \|(\boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}) - (\boldsymbol{W}\boldsymbol{x} + \boldsymbol{\Lambda})\|_2^2 \\ =& \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{x}\|_2^2 \\ \leq& \|\boldsymbol{W}\|_2^2 \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 \\ =& \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 . \end{aligned} \tag{10}$$

- If $(\boldsymbol{W}\boldsymbol{y})_i < -\lambda_i < (\boldsymbol{W}\boldsymbol{x})_i < \lambda_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{0}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}$.

$$\begin{aligned} &\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\|_2^2 \\ =& \|(\boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}) - \boldsymbol{0}\|_2^2 \\ =& \|\boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}\|_2^2 \\ \leq& \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{x}\|_2^2 \\ \leq& \|\boldsymbol{W}\|_2^2 \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 \\ =& \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 . \end{aligned} \tag{11}$$

- If $(\boldsymbol{W}\boldsymbol{y})_i < -\lambda_i < \lambda_i < (\boldsymbol{W}\boldsymbol{x})_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}$.

$$\begin{aligned} &\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\|_2^2 \\ =& \|(\boldsymbol{W}\boldsymbol{y} + \boldsymbol{\Lambda}) - (\boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda})\|_2^2 \\ =& \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{x} + 2\boldsymbol{\Lambda}\|_2^2 \\ \leq& \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{x}\|_2^2 \\ \leq& \|\boldsymbol{W}\|_2^2 \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 \\ =& \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 . \end{aligned} \tag{12}$$

- If $-\lambda_i \leq (\boldsymbol{W}\boldsymbol{y})_i < (\boldsymbol{W}\boldsymbol{x})_i \leq \lambda_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{0}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{0}$.

$$\begin{aligned} &\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\|_2^2 \\ =& \|\boldsymbol{0} - \boldsymbol{0}\|_2^2 \\ =& 0. \end{aligned} \tag{13}$$

- If $-\lambda_i \leq (\boldsymbol{W}\boldsymbol{y})_i \leq \lambda_i < (\boldsymbol{W}\boldsymbol{x})_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{0}$.

$$\begin{aligned} &\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\|_2^2 \\ =& \|\boldsymbol{0} - (\boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda})\|_2^2 \\ =& \|\boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda}\|_2^2 \\ \leq& \|\boldsymbol{W}\boldsymbol{x} - \boldsymbol{W}\boldsymbol{y}\|_2^2 \\ \leq& \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{x}\|_2^2 \\ \leq& \|\boldsymbol{W}\|_2^2 \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 \\ =& \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 . \end{aligned} \tag{14}$$

- If $\lambda_i < (\boldsymbol{W}\boldsymbol{y})_i < (\boldsymbol{W}\boldsymbol{x})_i$, then $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x}) = \boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda}$, $\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) = \boldsymbol{W}\boldsymbol{y} - \boldsymbol{\Lambda}$.

$$\begin{aligned} &\|\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\|_2^2 \\ =& \|(\boldsymbol{W}\boldsymbol{y} - \boldsymbol{\Lambda}) - (\boldsymbol{W}\boldsymbol{x} - \boldsymbol{\Lambda})\|_2^2 \\ =& \|\boldsymbol{W}\boldsymbol{y} - \boldsymbol{W}\boldsymbol{x}\|_2^2 \\ \leq& \|\boldsymbol{W}\|_2^2 \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 \\ =& \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 . \end{aligned} \tag{15}$$

From the above inequalities, it can be concluded that

$$\left\|\boldsymbol{W}^{\mathrm{T}}\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{y}) - \boldsymbol{W}^{\mathrm{T}}\mathcal{S}_{\boldsymbol{\Lambda}}(\boldsymbol{W}\boldsymbol{x})\right\|_2^2 \leq \lambda_{\max}\left(\boldsymbol{W}\boldsymbol{W}^{\mathrm{T}}\right) \|\boldsymbol{y} - \boldsymbol{x}\|_2^2 .$$

Therefore, the single tight frame N is ε-Lipschitzian, where $\varepsilon = \sqrt{\lambda_{\max}(\boldsymbol{W}\boldsymbol{W}^{\mathrm{T}})}$. ■

B. Proof of Lemma 2

In our optimization problem, we set a convex smooth ℓ_2 data-fidelity term $f(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{2}}\|\mathbf{y} - \mathbf{P}\mathbf{x}\|_{\mathbf{2}}^{\mathbf{2}}$. A smooth function refers to a function that is infinitely differentiable. Thus, there exists a constant $L_f > 0$ such that the gradient ∇f of $f(\cdot)$ satisfies

$$\|\nabla f(\mathbf{x}_1) - \nabla f(\mathbf{x}_2)\|_2 \leq L_f \|\mathbf{x}_1 - \mathbf{x}_2\|_2, \forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n.$$

The gradient of our objective function, i.e., ∇F can be calculated as follows

$$\nabla F(\mathbf{x}) = -\mathbf{P}^{\mathrm{T}}(\mathbf{y} - \mathbf{P}\mathbf{x}) + \lambda M(\mathbf{x}),$$

where $M(\cdot)$ is ε-Lipschitzian based on Lemma 1. Then, for $\forall \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$

$$\begin{aligned} &\|\nabla F(\mathbf{x}_1) - \nabla F(\mathbf{x}_2)\|_2 \\ =& \left\|-\mathbf{P}^{\mathrm{T}}(\mathbf{y} - \mathbf{P}\mathbf{x}_1) + \lambda M(\mathbf{x}_1) - \left(-\mathbf{P}^{\mathrm{T}}(\mathbf{y} - \mathbf{P}\mathbf{x}_2) + \lambda M(\mathbf{x}_2)\right)\right\|_2 \\ =& \left\|\mathbf{P}^{\mathrm{T}}\mathbf{P}(\mathbf{x}_1 - \mathbf{x}_2) + \lambda\left(M(\mathbf{x}_1) - M(\mathbf{x}_2)\right)\right\|_2 \\ \leq& \left\|\mathbf{P}^{\mathrm{T}}\mathbf{P}(\mathbf{x}_1 - \mathbf{x}_2)\right\|_2 + \lambda\|M(\mathbf{x}_1) - M(\mathbf{x}_2)\|_2 \\ \leq& \left\|\mathbf{P}^{\mathrm{T}}\mathbf{P}\right\|_2 \cdot \|\mathbf{x}_1 - \mathbf{x}_2\|_2 + \varepsilon\lambda\|\mathbf{x}_1 - \mathbf{x}_2\|_2 \\ \leq& (L' + \varepsilon\lambda)\|\mathbf{x}_1 - \mathbf{x}_2\|_2 , \end{aligned} \tag{16}$$

where $L' = \lambda_{\max}\left(\mathbf{P}^{\mathrm{T}}\mathbf{P}\right)$.

Therefore, ∇F is Lipschitz continuous with constant $L = L' + \varepsilon\lambda$. ■

C. Proof of Lemma 3

Let $g(t) = F(t(\mathbf{x} - \mathbf{y}) + \mathbf{y})$, we have

$$\begin{aligned} &F(\mathbf{x}) = g(1), F(\mathbf{y}) = g(0), \\ &g'(t) = \nabla F(t(\mathbf{x} - \mathbf{y}) + \mathbf{y})^{\mathrm{T}}(\mathbf{x} - \mathbf{y}). \end{aligned}$$

Hence, we can derive that

$$\begin{aligned} &F(\mathbf{x}) - F(\mathbf{y}) - \langle\nabla F(\mathbf{y}), \mathbf{x} - \mathbf{y}\rangle \\ =& g(1) - g(0) - \nabla F(\mathbf{y})^{\mathrm{T}}(\mathbf{x} - \mathbf{y}) \\ =& \int_0^1 g'(t)\,dt - \nabla F(\mathbf{y})^{\mathrm{T}}(\mathbf{x} - \mathbf{y}) \\ =& \int_0^1 \left[\nabla F(t(\mathbf{x} - \mathbf{y}) + \mathbf{y}) - \nabla F(\mathbf{y})\right]^{\mathrm{T}}(\mathbf{x} - \mathbf{y})\,dt \\ \leq& \int_0^1 \|\nabla F(t(\mathbf{x} - \mathbf{y}) + \mathbf{y}) - \nabla F(\mathbf{y})\|_2 \cdot ||\mathbf{x} - \mathbf{y}||_2\,dt \\ \leq& \int_0^1 L\,\|t(\mathbf{x} - \mathbf{y})\|_2 \cdot \|\mathbf{x} - \mathbf{y}\|_2\ dt \\ =& \frac{L}{2}||\mathbf{x} - \mathbf{y}||_2^2, \end{aligned} \tag{17}$$

which completes the proof. ■

II. Experimental results

To further evaluate the effectiveness of our constructed algorithm, we supplement the comparison results of different algorithms on SVCT task and LACT task, as shown in Fig. 1 and Fig. 2.

The reconstruction results of various SVCT reconstruction methods from 120 views on the AAPM dataset are presented in Fig. 1. The FBP method yields images of inferior quality, rendering crucial details and structures indistinct. While RED-CNN and FBPConvNet mitigate some artifacts, they lead to a loss of vital details. Moreover, CNNMAR and MSANet fall short of accurately recovering intricate details. Besides, DuDoTrans and FreeSeed result exhibit blurred edges. Remarkably, our method generates visual results replete with the most texture details and the least noise. Fig. 2 displays the reconstruction results of different LACT reconstruction methods from 150 views on the AAPM dataset. As we can observe, our method consistently outperforms other methods. It yields enhanced organ boundary recovery, small structure recovery, and boundary artifact elimination.

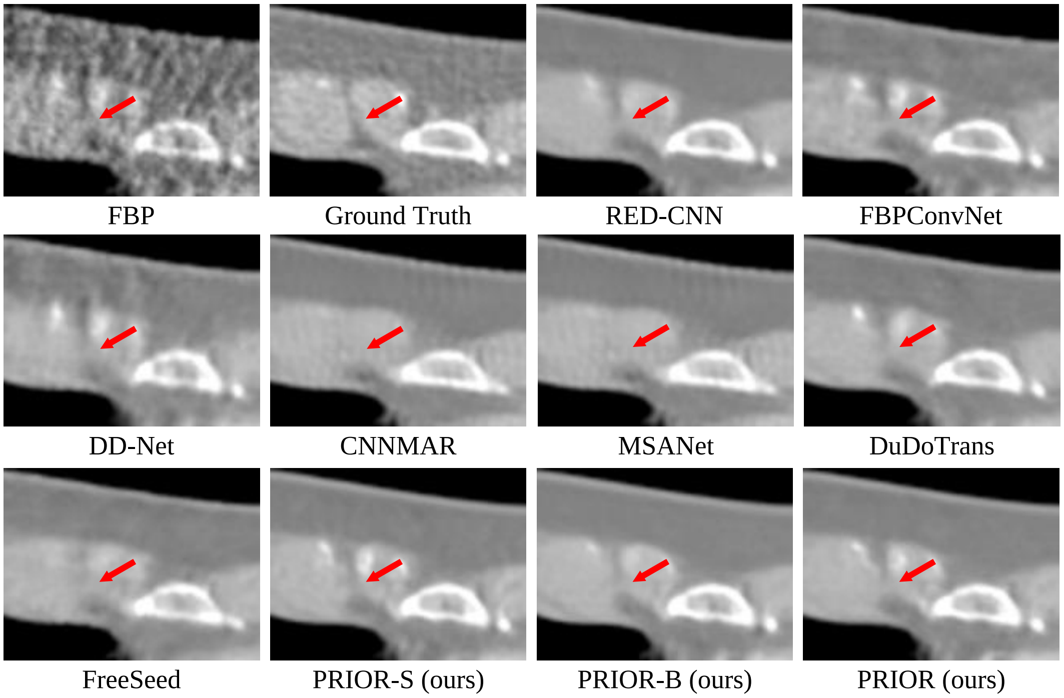

Fig. 1. SVCT reconstruction results from 120 views using different methods. Regions of interest are zoomed in for better viewing. The display window of reconstructed results is [-600, 600] HU for a better observation of small details.

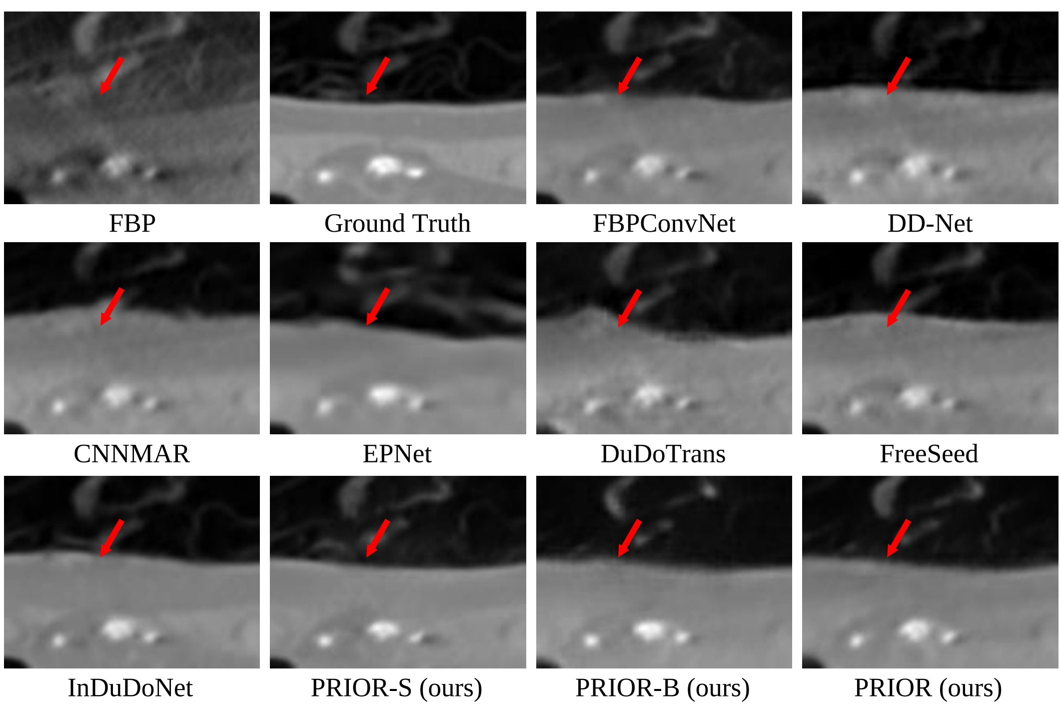

Fig. 2. LACT reconstruction results from 150 views using different methods. Regions of interest are zoomed in for better viewing. The display window of reconstructed results is [-1000, 1000] HU for a better observation of small details.