Q Learning

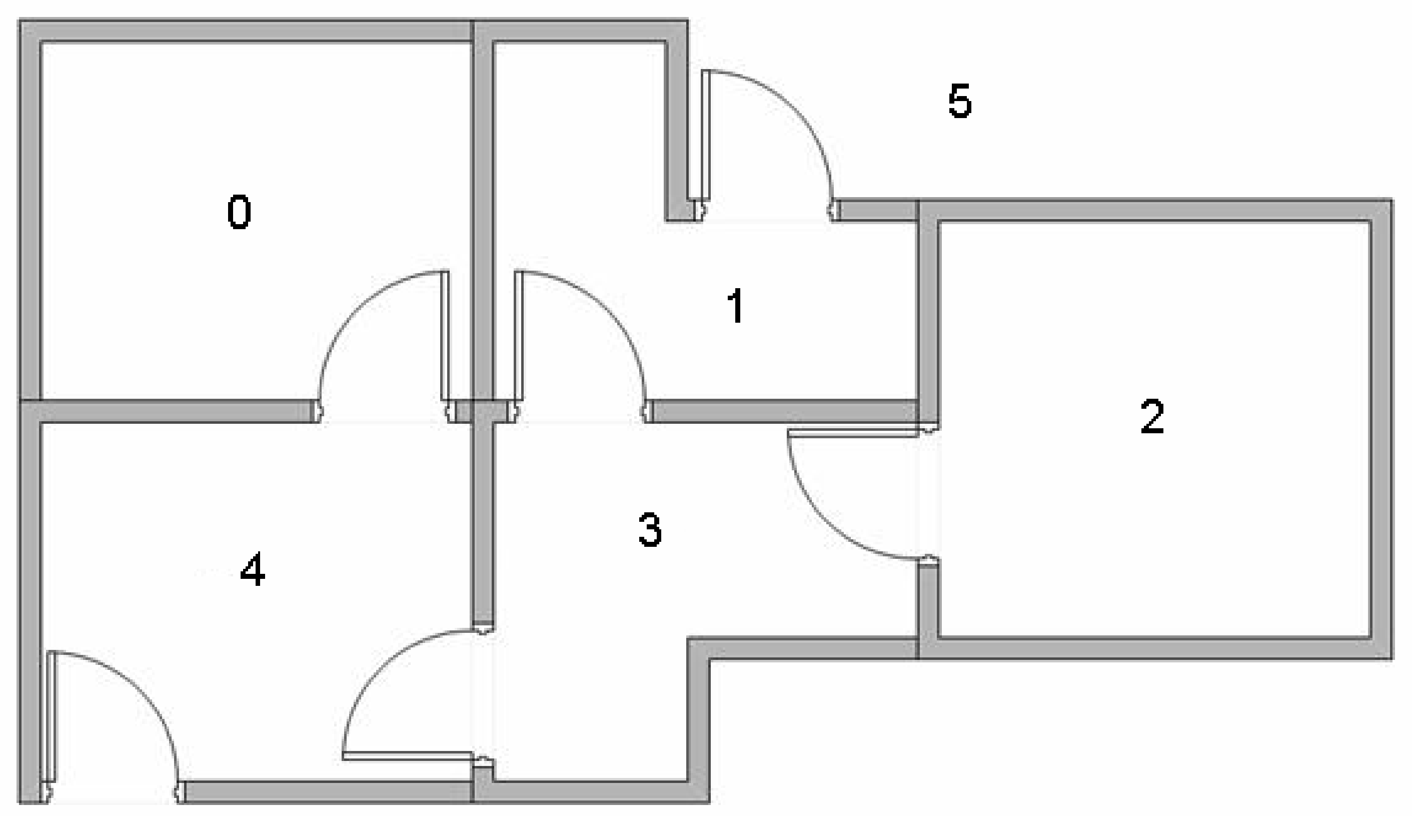
Agenda

- What is Q Learning
- Example
- Q Learning - Algorithm
- Taxi solving using Q Learning

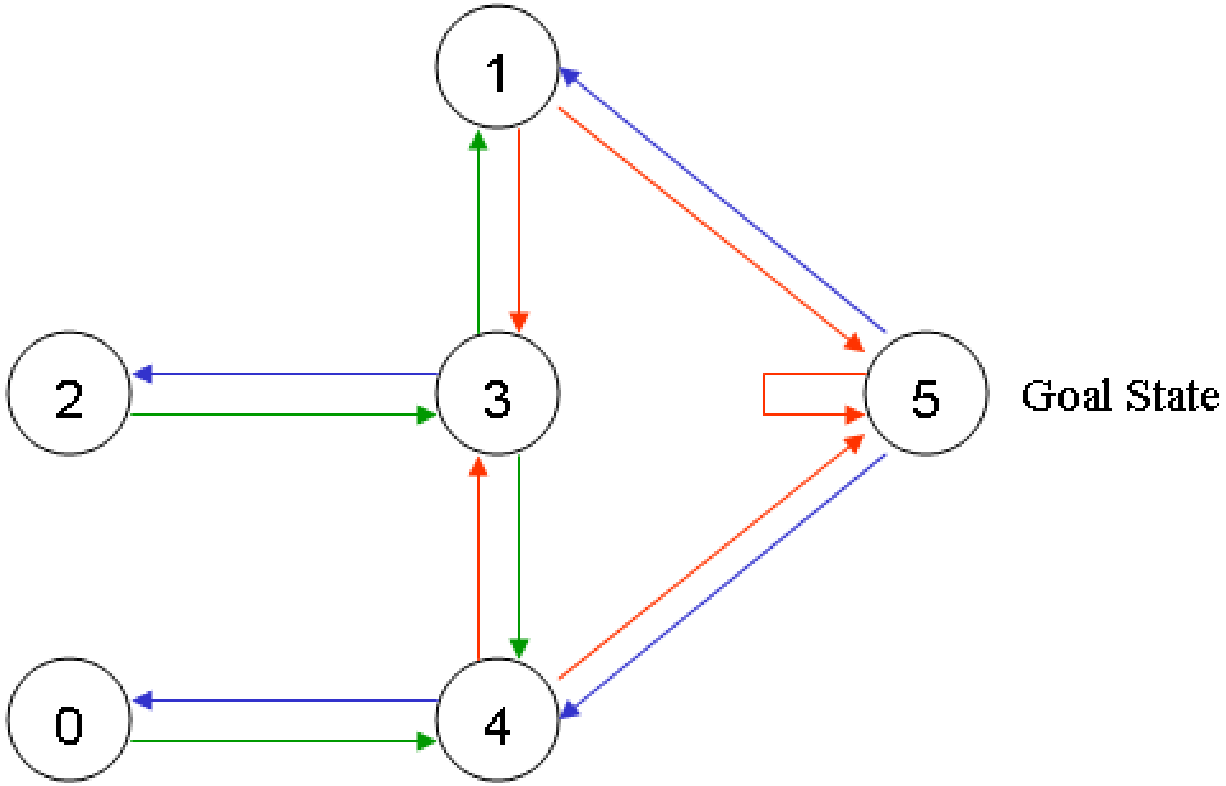
Q-learning

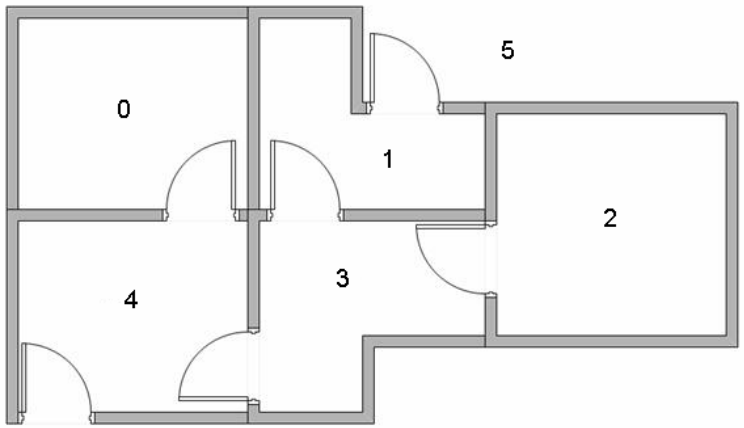
- Q-learning is a reinforcement learning technique used in machine learning.
- The goal of Q-Learning is to learn a policy, which tells an agent which action to take under which circumstances.

Example - 5 rooms in a building connected by doors

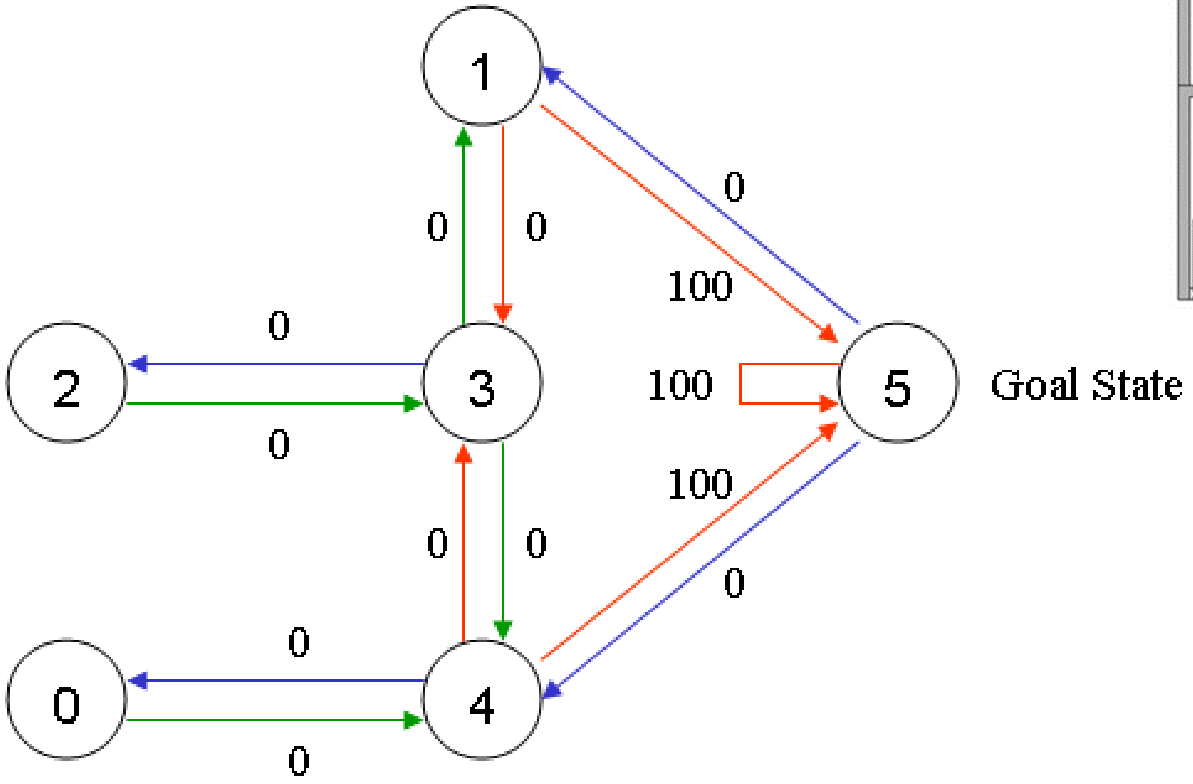


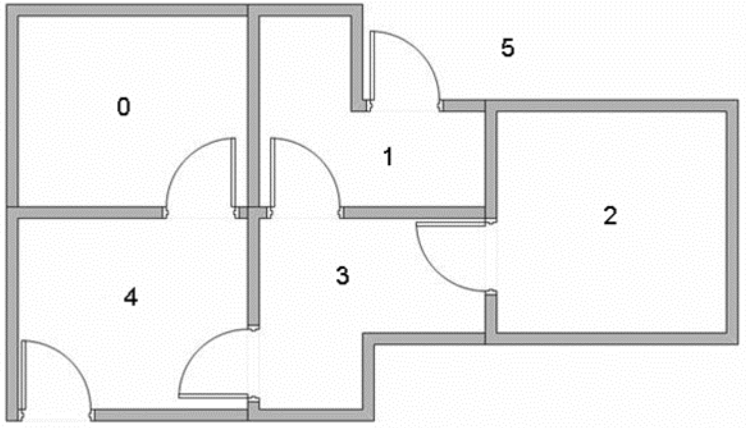
Rooms as a graph



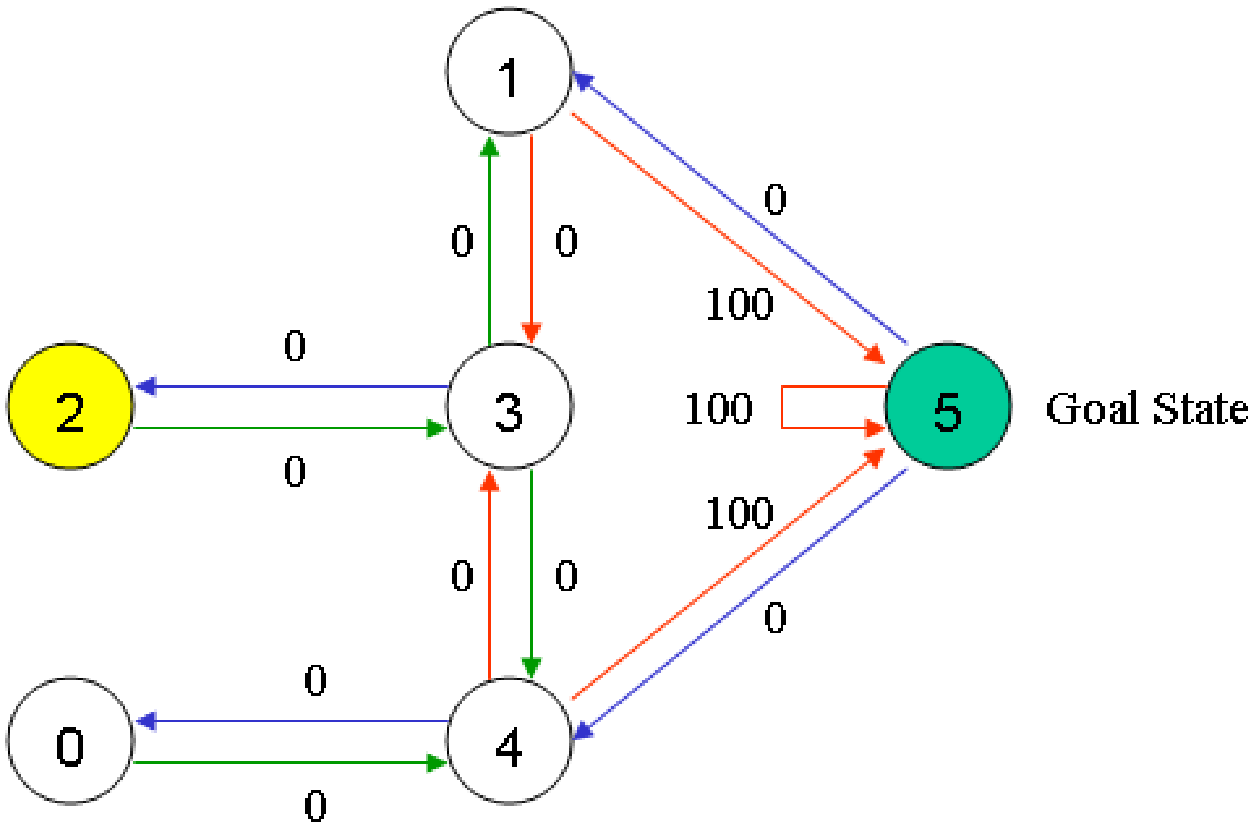


Graph Representation with reward





Graph Representation with reward - Goal



Reward table - "matrix R"

$$
\begin{array}{ccc}
 & & \textbf{Action} \\
 & \textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
R = & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix}
-1 & -1 & -1 & -1 & 0 & -1 \\
-1 & -1 & -1 & 0 & -1 & 100 \\
-1 & -1 & -1 & 0 & -1 & -1 \\
-1 & 0 & 0 & -1 & 0 & -1 \\
0 & -1 & -1 & 0 & -1 & 100 \\
-1 & 0 & -1 & -1 & 0 & 100
\end{bmatrix}
\end{array}
$$

“Q Matrix"

- A similar matrix, "Q",
- brain of our agent, representing the memory of what the agent has learned through experience.
- The rows of matrix Q represent the current state of the agent, and the columns represent the possible actions leading to the next state (the links between the nodes).

Transition rule of Q learning

Q(state, action) = R(state, action) + Gamma * Max[Q(next state, all actions)]

Q-Learning algorithm

1. Set the gamma parameter, and environment rewards in matrix R.
2. Initialize matrix Q to zero.
    - For each episode:
        - Select a random initial state.
        - Do While the goal state hasn't been reached.
            - Select one among all possible actions for the current state.
            - Using this possible action, consider going to the next state.
            - Get maximum Q value for this next state based on all possible actions.
            - Compute: Q(state, action) = R(state, action) + Gamma * Max[Q(next state, all actions)]
            - Set the next state as the current state.
    - End Do
3. End For

$$Q_{t+1}(s_t,a_t) = \underbrace{Q_t(s_t,a_t)}_{\text{old value}} + \underbrace{\alpha_t(s_t,a_t)}_{\text{learning rate}} \times \left[\overbrace{\underbrace{R_{t+1}}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \underbrace{\max_a Q_t(s_{t+1},a_t)}_{\text{estimate of optimal future value}}}^{\text{learned value}} - \underbrace{Q_t(s_t,a_t)}_{\text{old value}} \right]$$

Algorithm to utilize the Q matrix

1. Set current state = initial state.
2. From current state, find the action with the highest Q value.
3. Set current state = next state.
4. Repeat Steps 2 and 3 until current state = goal state.

Q-Learning Example By Hand

- We'll start by setting the value of the learning parameter Gamma = 0.8, and the initial state as Room 1.
- Initialize matrix Q as a zero matrix:

Initialize matrix Q as a zero matrix

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

initial state as Room 1

- There are two possible actions for the current state 1:
  - go to state 3, or
  - go to state 5.
- By random selection, we select to go to 5 as our action.

$$
\begin{array}{cc}
 & \textbf{Action} \\
\textbf{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
R = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix}
\end{array}
$$

Reinforcement learning

- Q(state, action) = R(state, action) + Gamma * Max[Q(next state, all actions)]

- Q(1, 5) = R(1, 5) + 0.8 * Max[Q(5, 1), Q(5, 4), Q(5, 5)] = 100 + 0.8 * 0 = 100

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & \text{Action} \\ \text{State} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \end{array}$$

$$R = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix}$$

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Next episode - Randomly chosen initial state

- State 3 is the initial state
- 3 possible actions: go to state 1, 2 or 4. By random selection, we select to go to state 1 as our action.
- Now if in state 1. It has 2 possible actions: go to state 3 or state 5. Then, we compute the Q value
- Q(state, action) = R(state, action) + Gamma * Max[Q(next state, all actions)]
- Q(3, 1) = R(3, 1) + 0.8 * Max[Q(1, 3), Q(1, 5)] = 0 + 0.8 * Max(0, 100) = 80

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Action

$$R = \begin{array}{c} \text{State} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} -1 & -1 & -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 & -1 & 100 \\ -1 & -1 & -1 & 0 & -1 & -1 \\ -1 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & -1 & 0 & -1 & 100 \\ -1 & 0 & -1 & -1 & 0 & 100 \end{bmatrix} \end{array}$$

The matrix Q

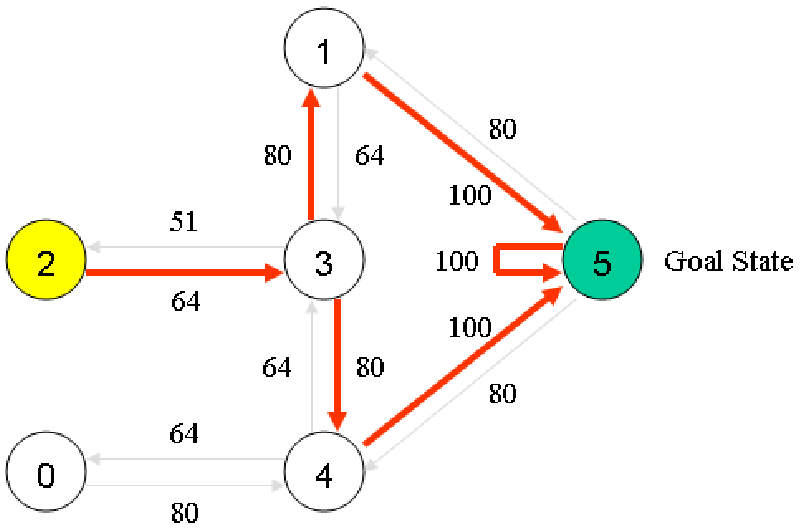
$$
Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 80 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

The matrix Q

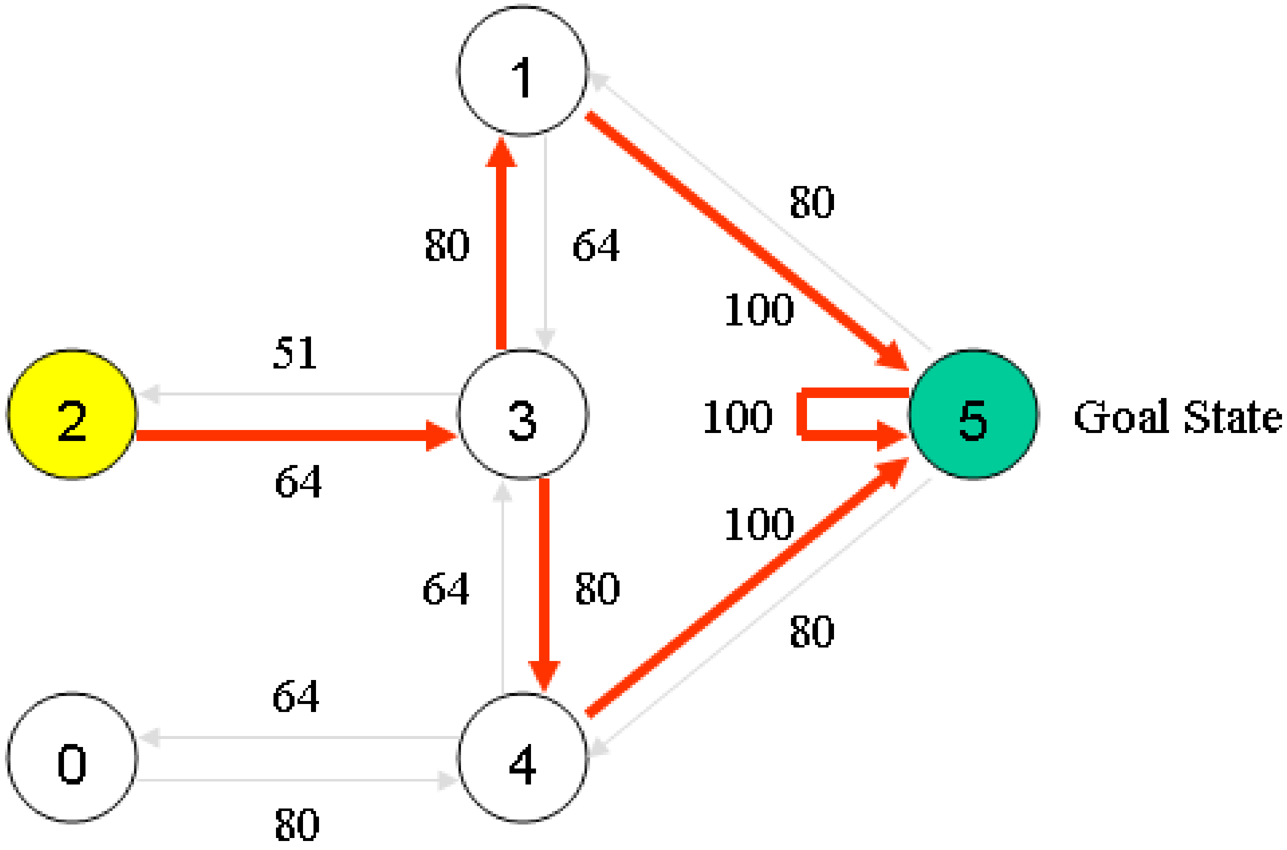
$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 400 & 0 \\ 0 & 0 & 0 & 320 & 0 & 500 \\ 0 & 0 & 0 & 320 & 0 & 0 \\ 0 & 400 & 256 & 0 & 400 & 0 \\ 320 & 0 & 0 & 320 & 0 & 500 \\ 0 & 400 & 0 & 0 & 400 & 500 \end{bmatrix} \end{array}$$

The matrix Q

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 80 & 0 \\ 0 & 0 & 0 & 64 & 0 & 100 \\ 0 & 0 & 0 & 64 & 0 & 0 \\ 0 & 80 & 51 & 0 & 80 & 0 \\ 64 & 0 & 0 & 64 & 0 & 100 \\ 0 & 80 & 0 & 0 & 80 & 100 \end{bmatrix} \end{array}$$



The Solved Graph



The Solved Graph

```
for episode in range(1,1001):
    done = False
    G, reward = 0,0
    state = env.reset()
    while done != True:
            action = np.argmax(Q[state])
            state2, reward, done, info = env.step(action)
            Q[state,action] += alpha * (reward + np.max(Q[state2]) - Q[state,action])
            G += reward
            state = state2
    if episode % 50 == 0:
        print('Episode {} Total Reward: {}'.format(episode,G))
```

State

```
Q[state,action] += alpha * (reward + np.max(Q[state2]) - Q[state,action])
```