MF796 Assignment 2

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Problem 1

(1)

The price of the call calculated by BS formula is: 0.01567390695679638.

(2) The BSM formula has the form:

$$C_0 = S_0 N(d_1) - K e^{-rT} N(d_2)$$

where

$$N(d_1) = \int_{-\infty}^{d_1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

I used the Gaussian-Legendre as my method and choose -5 as lower bond which is sufficient to precision to calculated N(d1) and N(d2).

$$\Phi(d) = \int_{-1}^{1} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{((d+5)x+d-5)^2}{8}\right) \frac{d+5}{2} dx$$

Price calculated by different quadrature method are:

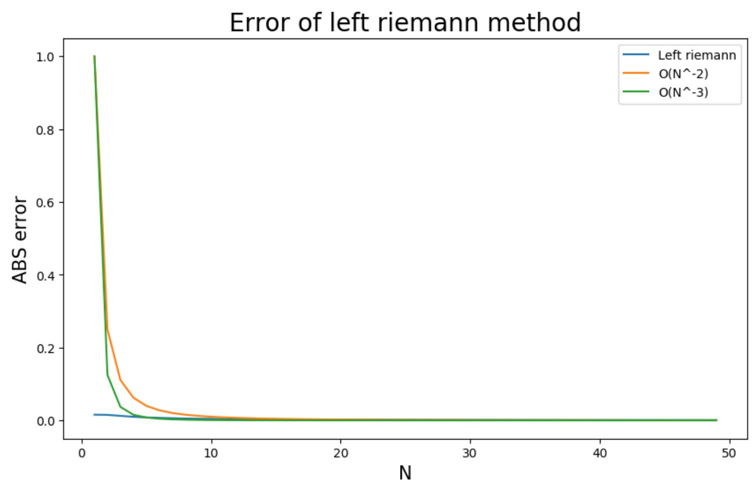
N	Left	Mid	Gaussian
5	0.0075	0.0029	0.01566
10	0.0114	0.0079	0.01567
50	0.0148	0.0139	0.01567

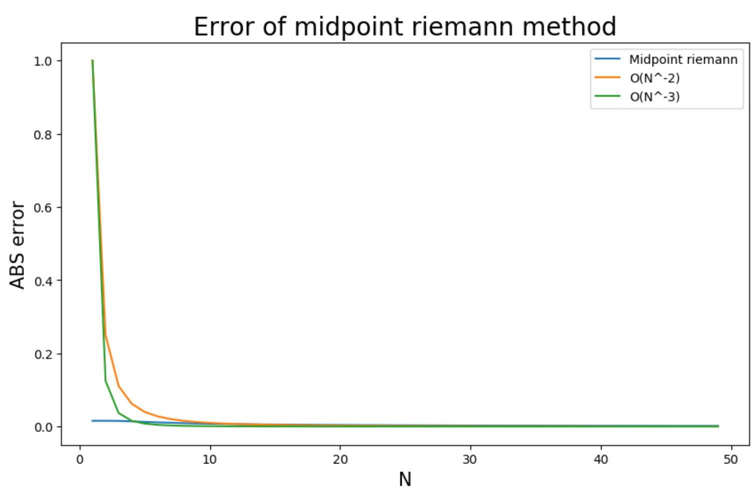
Errors are as follows:

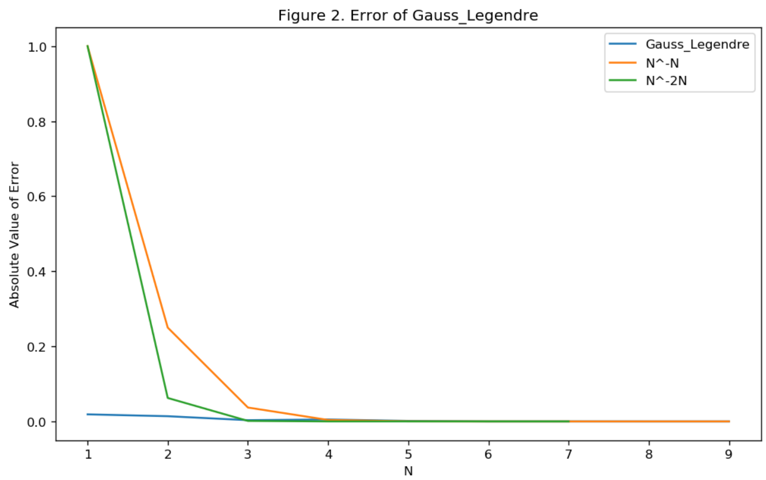
N	Left	Mid	Gaussian
5	0.0081	0.0127	0.0000
10	0.0042	0.0077	0.0000
50	0.0008	0.0017	0.0000

(3)

The figure below shows that the absolute error of Left-Riemann and Midpoint methods, with the plot of N^−i , where i = 2,3. As could be seen the error of Midpoint method is of order between O(N^−2) to O(N^−3), and the Gauss method is smaller than O(N^-2N)







(4)

The Gauss-Legendre method is my choice since it is efficient and easy to implement when N is large in general. However, to use Gaussian nodes I need to do carefully proper transformation and in some cases it would be quite complicated. Furthermore, I need to change variables so that $(-\infty, d1]$ becomes $[-1,1]$ or rescale so that the integrand is zero near the ends of [-1,1] and the function is supposed to be smooth over the integration domain.

Problem 2

(a)

For the current stock price, I used the close price of SPY500 on January 29th 2020 as S0, which is 326.61. Since r is 0, we could assume the mean of St is still S0. The price of Vanilla Option is 0.023445. (I chose midpoint rule here.)

(b)

For Contingent option, the price should be:

$$
\begin{aligned}
C &= \mathbb{E}^{Q}\left[e^{-rT}\left(S_{1T}-K_1\right)^{+} 1_{\{S_{2T}<375\}}\right] \\
&= \int_{-\infty}^{375} \int_{380}^{\infty} e^{-rT}\left(s_1-K_1\right) \psi\left(s_1, s_2\right) ds_1 ds_2
\end{aligned}
$$

Where

$$\psi(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \times$$
$$\exp\left\{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_1)^2}{\sigma_1^2} + \frac{(y-\mu_2)^2}{\sigma_2^2} - \frac{2\rho(x-\mu_1)(y-\mu_2)}{\sigma_1\sigma_2}\right)\right\}$$

Plugging in the value of the parameters in the above formula, we see that the price of the contingent call is 0.01528.

(3)

ρ	**Value of the contingent option**
0.8	0.01944
0.5	0.02262
0.2	0.02334

(4)

Yes. For a contingent call, when the positively higher the correlation, the contingent option value is lower. Intuitively, when correlation is high, there is a higher chance that both the SPY and SPY 6 month are lower than 375, and then the contingent option expires out of money.

(5)

6 month SPY	**Value of the contingent option**
370	0.00851
360	0.000481

(6)

Yes. When the threshold increases, the contingent condition is easier to achieve, there is a higher chance that the option exists until maturity.

(7)

To make contingent equal to vanilla, we need to make the K2 be infinity rather than 375. In addition, making $\rho = 0$ will make the contingency independent from vanilla, and would make the contingent option equal to vanilla times the probability of contingency occurring.