MF796 Assignment 3

Shi Bo

U56082126

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1 Option Pricing via FFT Techniques

(a) Exploring FFT Technique Parameters

i.

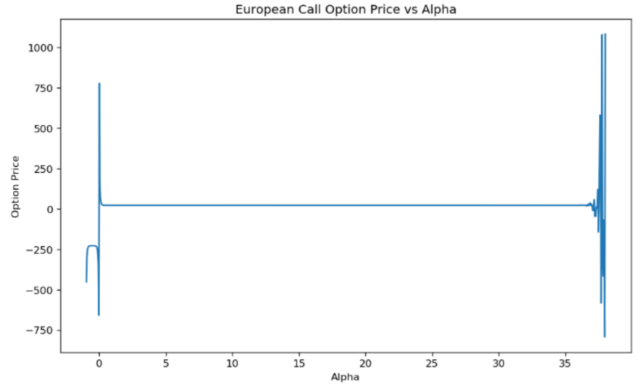
We need to build the characteristic function and x_j first which has the formula:

$$x_j = \frac{[(2-\delta_{j-1})\Delta\nu]e^{-\int_0^T r_u du}}{2(\alpha+i\nu_j)(\alpha+i\nu_j+1)} e^{-i(\ln S_0 - \frac{\Delta k N}{2})\nu_j} \Phi(\nu_j - (\alpha+1)i)$$

Then, we implemented FFT technique to transform vector x into vector y. By doing this, we can use y_j to get $C_T(k_j)$:

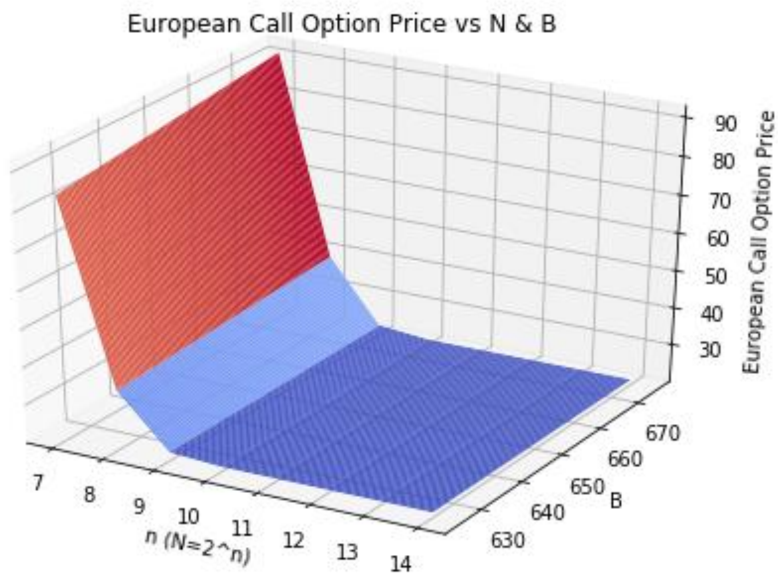
$$C_T(k_j) = \frac{e^{-\alpha[\ln S_0 - \Delta k(\frac{N}{2}-(j-1))]}}{\pi} Re(y_j)$$

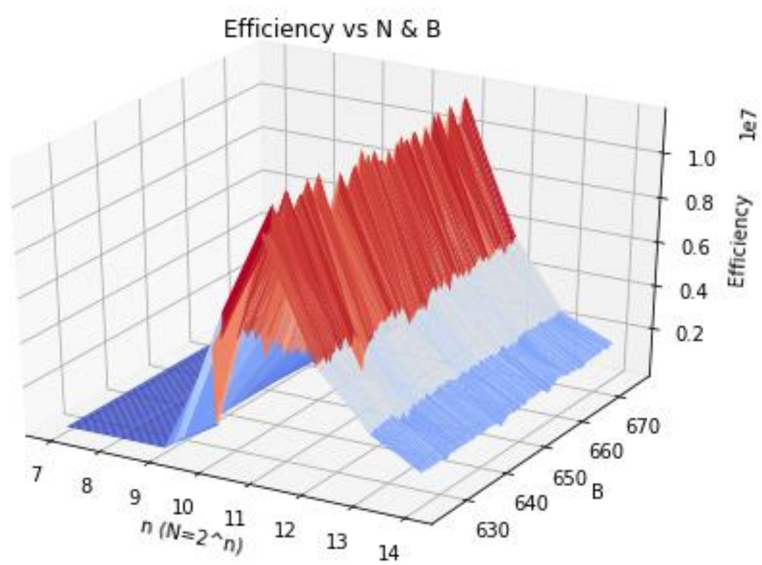
Eventually, we use C_T to interpolate and fit a curve of call price VS K. By plugging in the corresponding K, we will get the call option price. I set n = 11 and B = 675 after the efficiency comparison and chose different alpha ranging from -1 to 40 to find the most stable price (see chart below).



We can see that from approximately 2 to 35 the option price is most stable then we chose alpha equals to 1.5. The price calculated as 21.27.

ii.

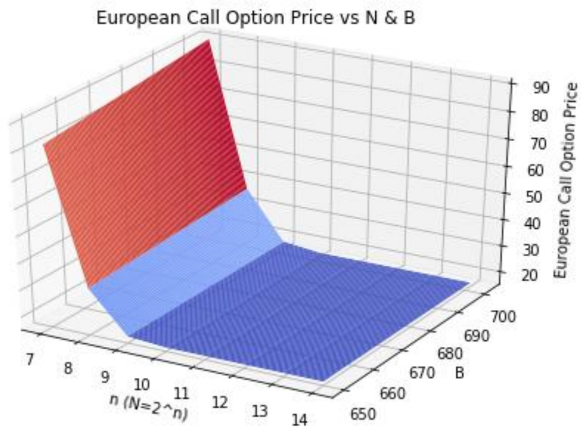


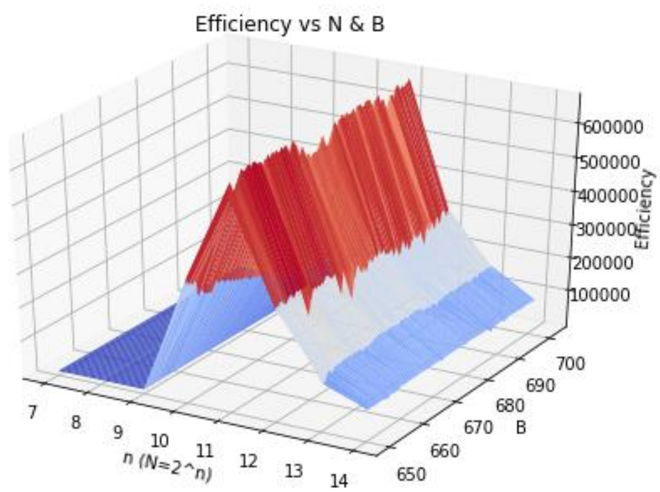


When n = 11 and B = 630 we have the highest computational efficiency.

iii.

We selected n = 11 and B = 675 to calculate the option price which is 16.73. The highest computational efficiency was achieved at n = 11 and B = 690.

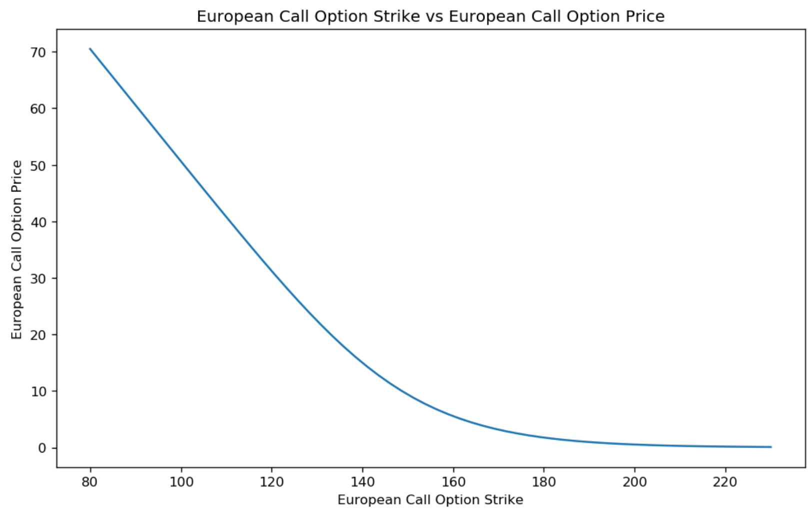


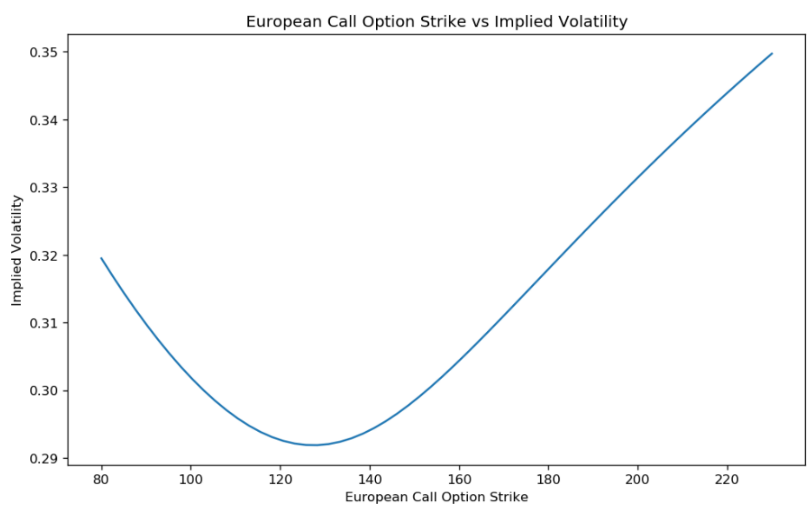


(b) Exploring FFT Technique Parameters

i.

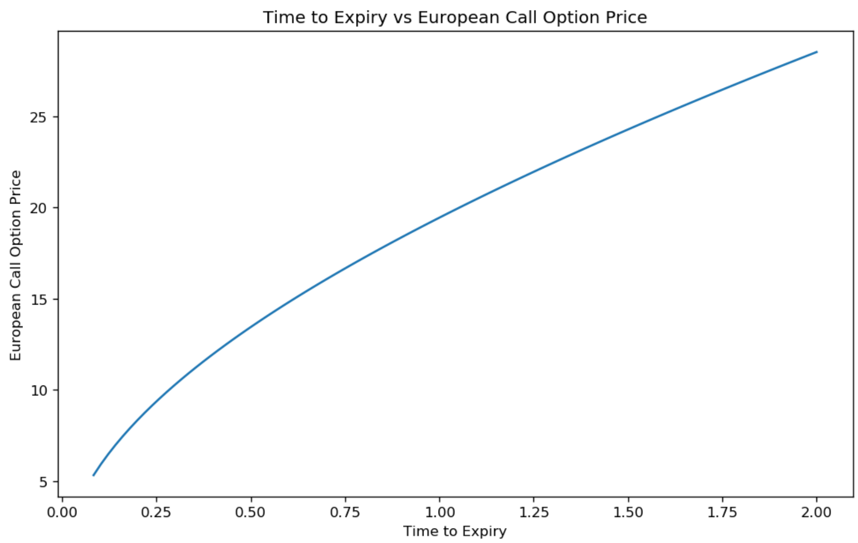
We calculate the option price by choosing different strike price via Heston Model of which the parameters are $\alpha = 1.5$, n = 11 and B = 700, then using the option price to reversely calculate the implied volatility by using Black-Scholes model. I use root function in Python to help me solve the equation.

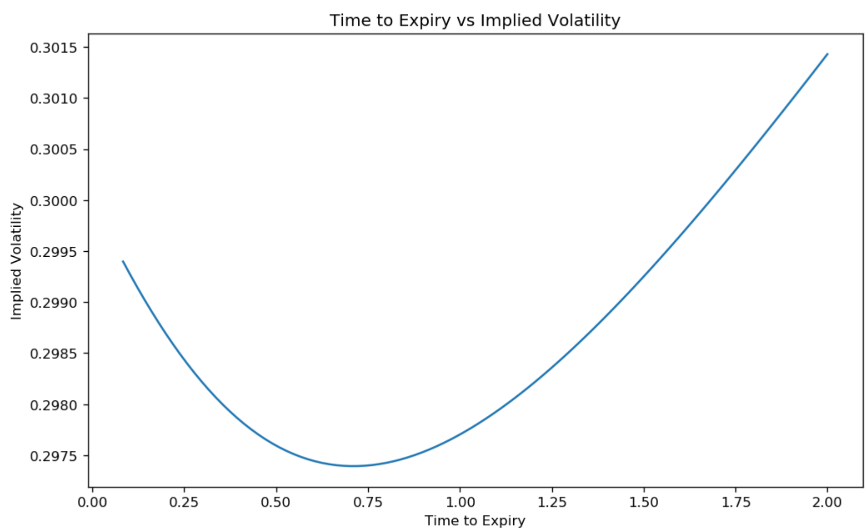




ii.

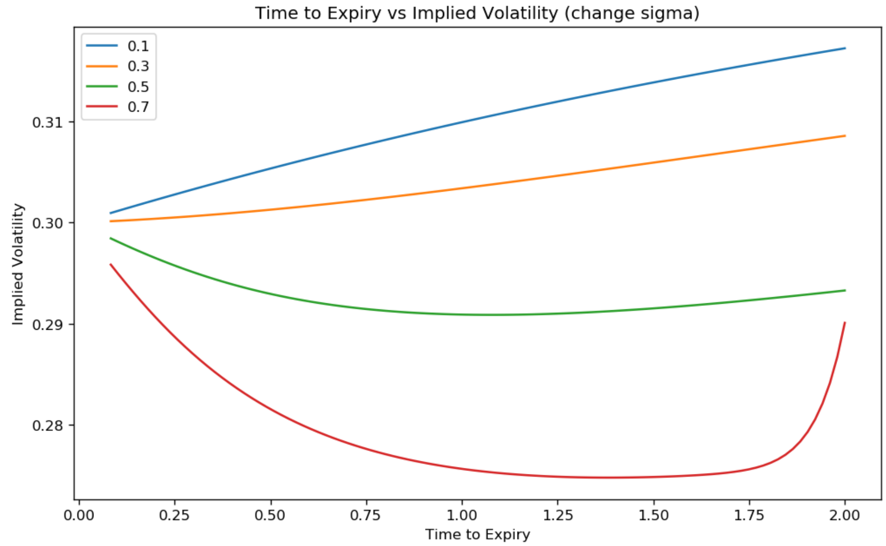
See charts below.

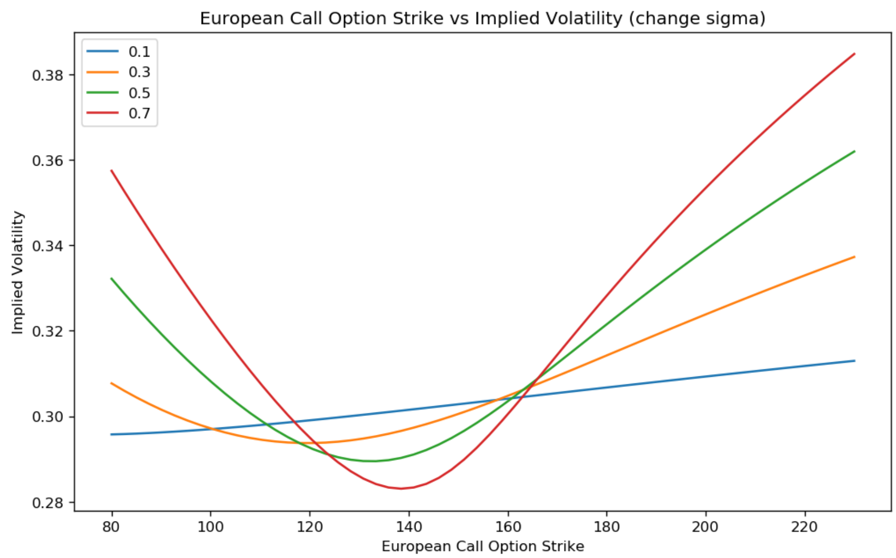




iii.

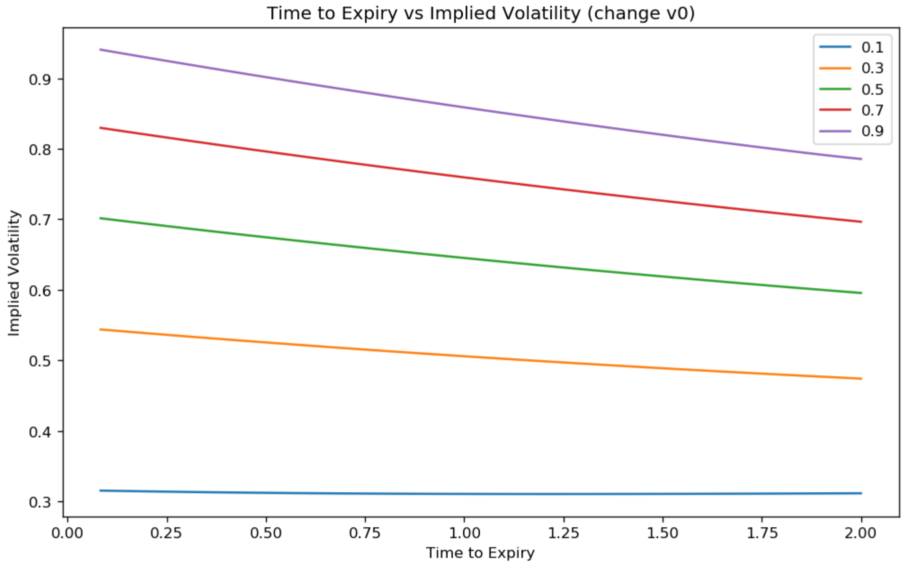
By changing σ from 0.1 to 0.7, we will have:

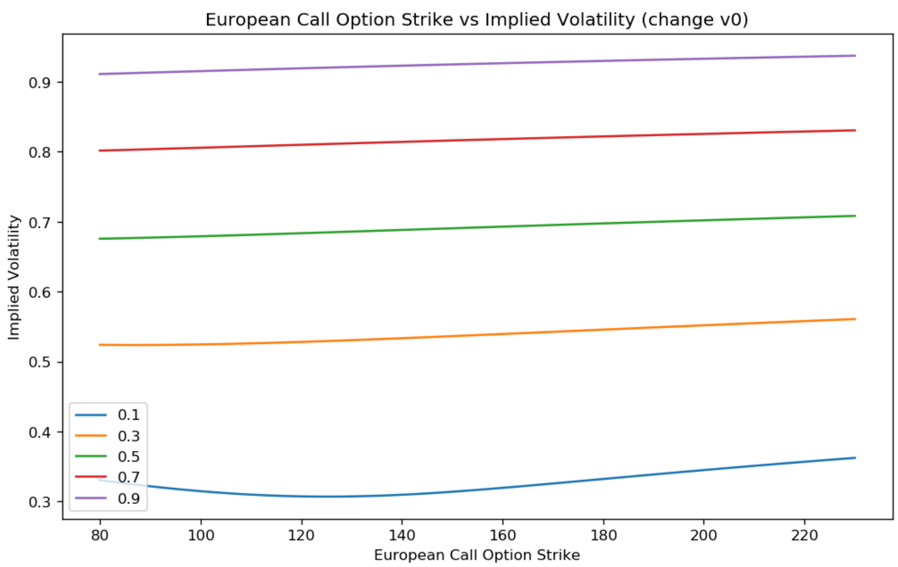




When σ is smaller, volatility skew shifts down and left and we can see that the smaller the sigma the flatter the curve. In addition that, term structure shifts up and becomes increasing.

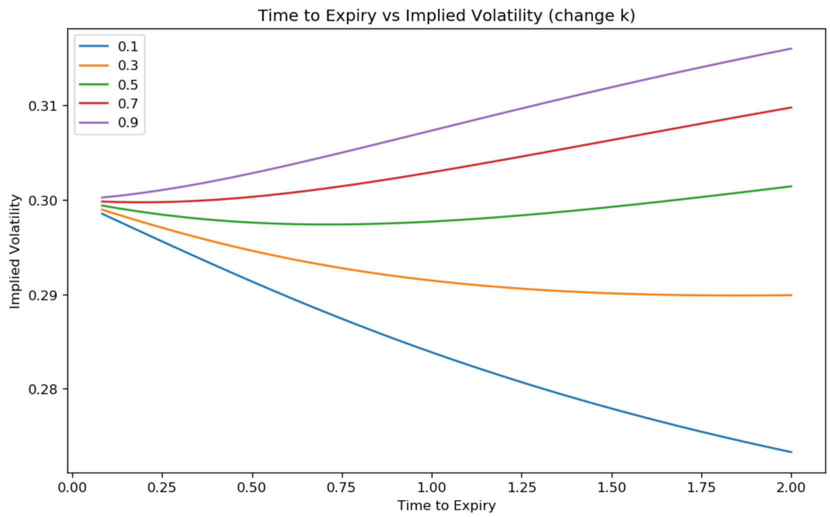
By changing v0 from 0.1 to 0.9, we will have:

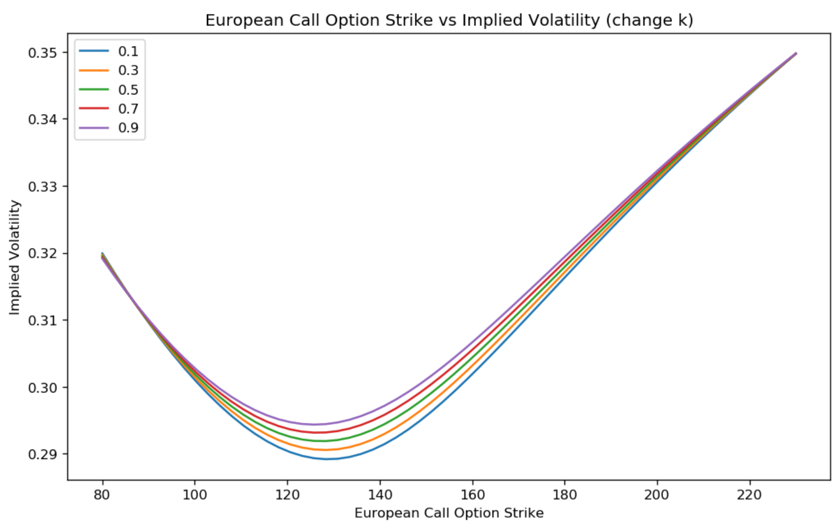




As v0 increases, volatility surface goes up to higher level. In other words, the extent of volatility smile remains basically the same. And volatility term structure goes up and becomes decreasing.

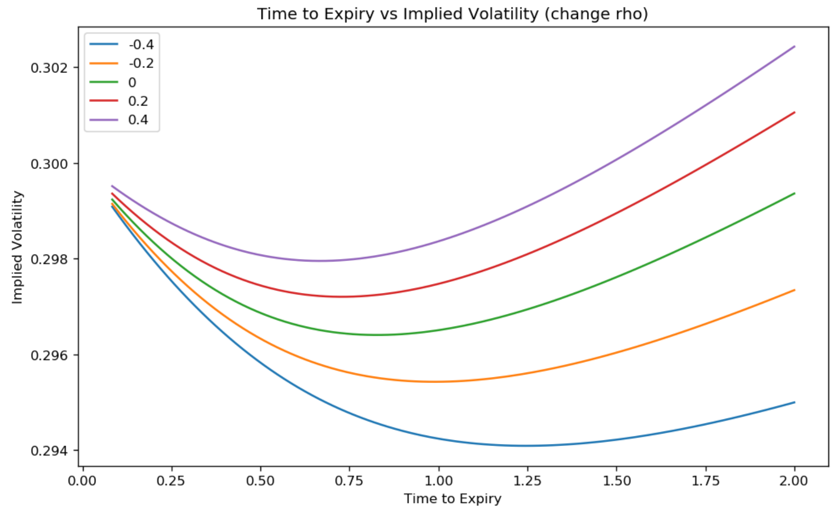
By changing kappa from 0.1 to 0.9, we will have:

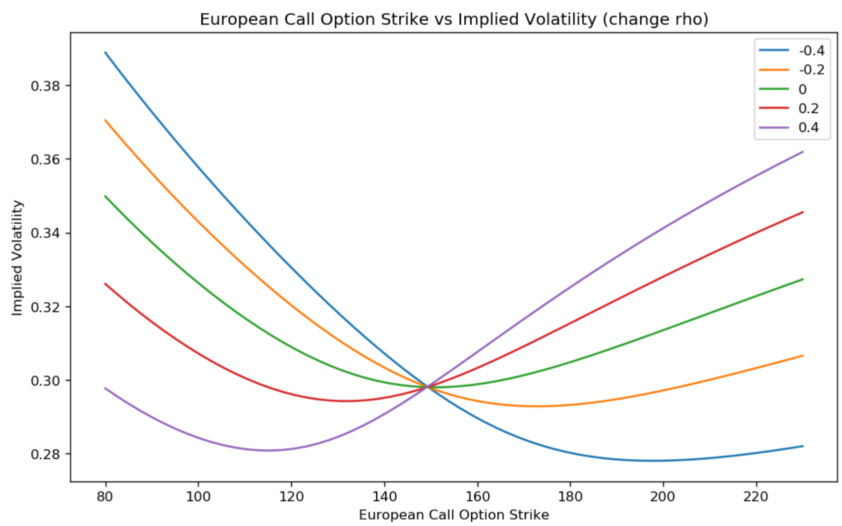




As kappa increases, volatility surface remains basically the same. But volatility term structure goes up and becomes increasing.

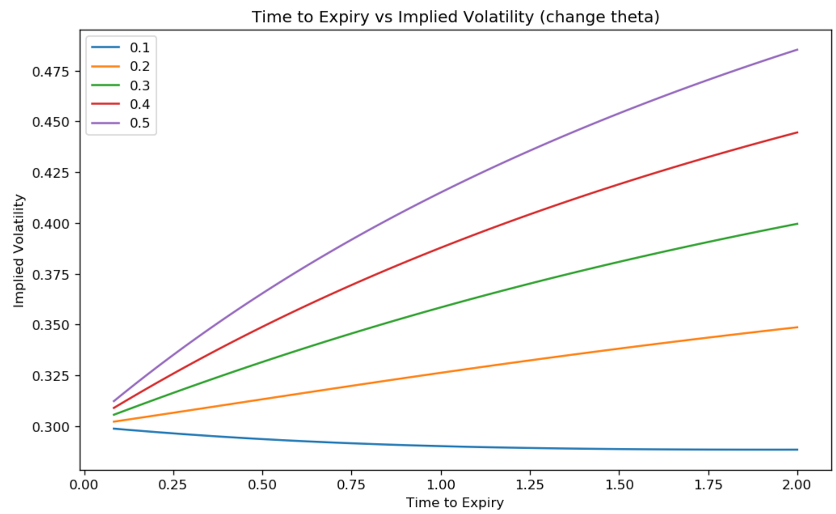
By changing rho from -0.4 to 0.4, we will have:

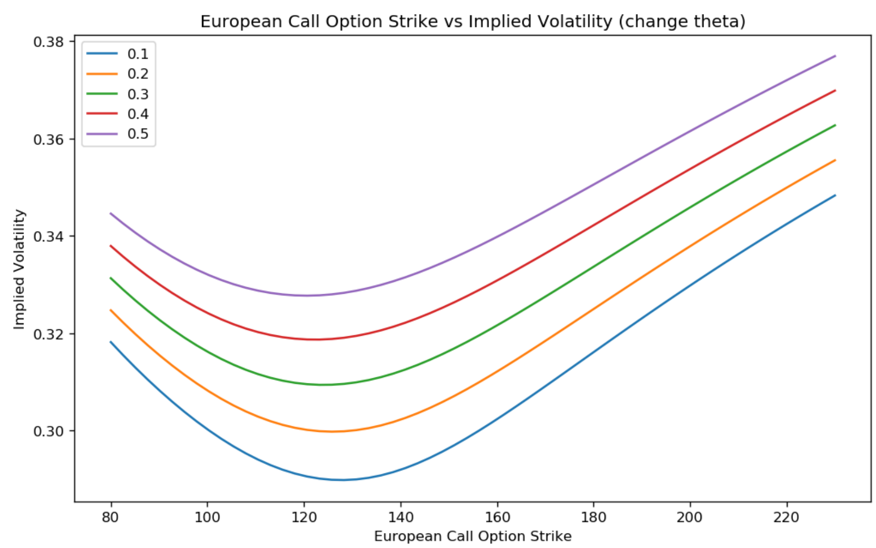




As rho increases, volatility surface shifts left and down. And volatility term structure goes up to higher level.

By changing theta from 0.1 to 0.5, we will have:





As theta increases, volatility surface goes up but not quite much. And volatility term structure goes up and becomes increasing.