MF796 Assignment 4

Shi Bo

U56082126

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1-- Implementation of Breeden-Litzenberger

#######################(a)#######################

According to BSM model, we can have that:

$$K_C = S_0 e^{-\sigma\sqrt{T}\Phi^{-1}(\Delta_C)+T(r+\frac{\sigma^2}{2})}$$

$$K_P = S_0 e^{\sigma\sqrt{T}\Phi^{-1}(-\Delta_P)+T(r+\frac{\sigma^2}{2})}$$

The table for extracted strike is:

	1M	3M
10DP	89.138758	84.225674
25DP	95.542103	93.470685
40DP	98.700642	98.127960
50D	100.138720	100.338826
40DC	101.262273	102.141761
25DC	102.783751	104.532713
10DC	104.395838	107.422225

######################(b)######################

By using ***np. ployfit*** we get volatility function for strikes:

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The interpolation fitted 1M coef and intercept are:  [-0.01378835  1.55778814]
The interpolation fitted 3M coef and intercept are:  [-0.00767355  0.93223457]
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and:

$$\sigma_{1M}(K) = -0.0138K + 1.5578$$

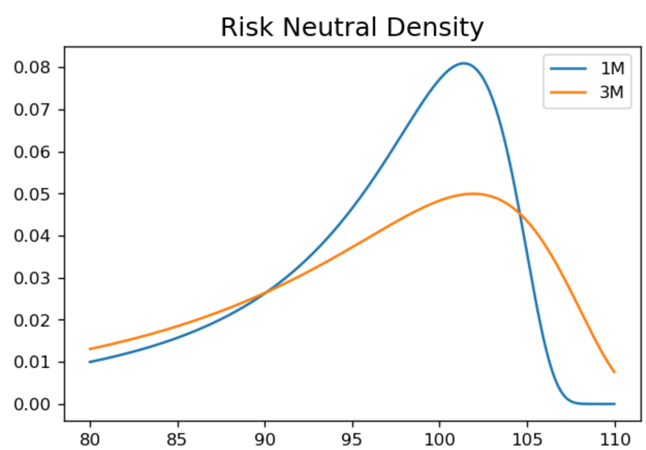
$$\sigma_{3M}(K) = -0.0077K + 0.9322$$

######################(c)######################

We could compute the risk neutral density (from slides) like:

$$\phi(K) = e^{rT}\frac{c(K-h) - 2c(K) + c(K+h)}{h^2}$$

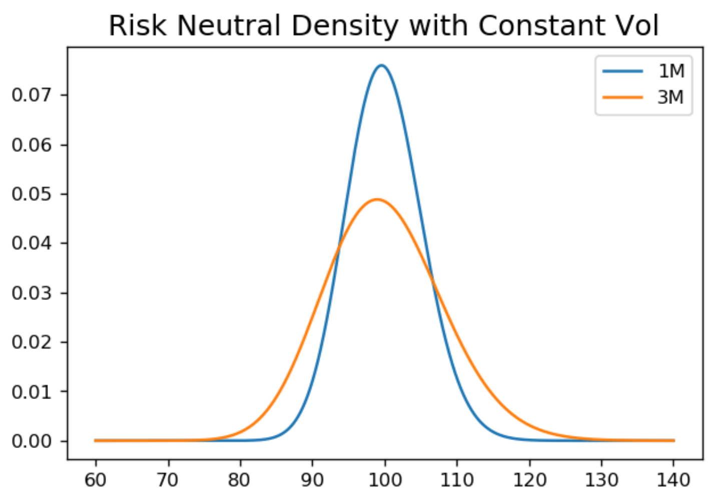
The plot of 1M and 3M risk neutral density are:



Both distributions are skewed. 3M has fatter tails than 1M. The probability for larger K is higher when expiry is 3M.

######################(d)######################

The plot risk neutral density calculated by constant volatility is:



######################(e)######################

Price of 1M European Digital Put Option with Strike 110: **0.98**

Price of 3M European Digital Call Option with Strike 105: **0.32**

Price of 2M European Call Option with Strike 100: **2.74**

2-- Calibration of Heston Model

######################(a)######################

In order to prevent arbitrage in our data, we must check:

(1) Call (put) prices that are monotonically decreasing (increasing) in strike.

(2) Call (put) prices whose rate of change relative to strike is greater than -1 (0) and smaller than 0 (1).

(3) Call and put prices that are convex with respect to changes in strike.

After these three checking process, there are no opportunities in the given data set using mid price.

######################(b)(c)######################

We can use ”L-BFGS-B” method to include bounds for the optimization. The initial values that I chose are (1.5, 0.1, 0.25, -0.5, 0.1) and tried several different upper and lower bound for guessing.

calibrated	starting points	lower bounds	upper bounds	squared error
$(\hat{\kappa},\hat{\theta},\hat{\sigma},\hat{\rho},\hat{\nu}_0)$	$(\kappa^s,\theta^s,\sigma^s,\rho^s,\nu_0^s)$	$(\underline{\kappa},\underline{\theta},\underline{\sigma},\underline{\rho},\underline{\nu}_0)$	$(\bar{\kappa},\bar{\theta},\bar{\sigma},\bar{\rho},\bar{\nu}_0)$	$\sum_{\tau,K}(\tilde{c}(\tau,K,\vec{p})-c_{\tau,K})^2$
(2.50,0.062,1.14,-0.83,0.035)	(0.05,0.2,0.2,-0.6,0.03)	(0.001,0,0,-1,0)	(2.5,1.0,2.0,0.2,0.5)	28.33
(2.73,0.061,1.18,-0.84,0.035)	(0.05,0.2,0.2,-0.6,0.03)	(0.001,0,0,-1,0)	(5.0,2.0,2.0,1.0,1.0)	28.26
(2.50,0.063,1.21,-0.82,0.036)	(0.25,0.06,0.8,-0.25,0.09)	(0.001,0,0,-1,0)	(2.5,1.0,2.0,0.2,0.5)	28.22
(4.72,0.054,1.65,-0.82,0.040)	(0.25,0.06,0.8,-0.25,0.09)	(0.001,0,0,-1,0)	(5.0,2.0,2.0,1.0,1.0)	27.77
(2.50,0.063,1.20,-0.82,0.036)	(1.5,0.1,0.25,-0.5,0.1)	(0.001,0,0,-1,0)	(2.5,1.0,2.0,0.2,0.5)	28.22
(4.64,0.054,1.63,-0.82,0.040)	(1.5,0.1,0.25,-0.5,0.1)	(0.001,0,0,-1,0)	(5.0,2.0,2.0,1.0,1.0)	27.77

The calibrated values do not change much as we change the starting points of the parameters. So, we have relatively robust calibration algorithm.

#######################(d)#######################

By minimizing the sum of weighted squared errors, we have:

calibrated	starting points	lower bounds	upper bounds	weighted squared error
$(\hat{\kappa},\hat{\theta},\hat{\sigma},\hat{\rho},\hat{\nu}_0)$	$(\kappa^s,\theta^s,\sigma^s,\rho^s,\nu_0^s)$	$(\underline{\kappa},\underline{\theta},\underline{\sigma},\underline{\rho},\underline{\nu}_0)$	$(\bar{\kappa},\bar{\theta},\bar{\sigma},\bar{\rho},\bar{\nu}_0)$	$\sum_{\tau,K}w_{\tau,K}(\tilde{c}(\tau,K,\vec{p})-c_{\tau,K})^2$
(1.35,0.072,0.86,-0.76,0.032)	(0.05,0.2,0.2,-0.6,0.03)	(0.001,0,0,-1,0)	(2.5,1.0,2.0,0.2,0.5)	234.94
(0.81,0.102,0.85,-0.75,0.032)	(0.05,0.2,0.2,-0.6,0.03)	(0.001,0,0,-1,0)	(5.0,2.0,2.0,1.0,1.0)	244.04
(2.50,0.057,1.03,-0.77,0.033)	(0.25,0.06,0.8,-0.25,0.09)	(0.001,0,0,-1,0)	(2.5,1.0,2.0,0.2,0.5)	223.64
(3.52,0.052,1.18,-0.77,0.034)	(0.25,0.06,0.8,-0.25,0.09)	(0.001,0,0,-1,0)	(5.0,2.0,2.0,1.0,1.0)	221.70
(2.50,0.057,1.03,-0.77,0.033)	(1.5,0.1,0.25,-0.5,0.1)	(0.001,0,0,-1,0)	(2.5,1.0,2.0,0.2,0.5)	236.15
(3.51,0.052,1.74,-0.77,0.034)	(1.5,0.1,0.25,-0.5,0.1)	(0.001,0,0,-1,0)	(5.0,2.0,2.0,1.0,1.0)	221.71

From the table we can see the new optimized parameters are slightly different with (c), but the difference is not very large. Since the weights are inversely proportional to the bid-ask spread, the options with greater liquidity are assigned with larger weights.

3-- Hedging Under Heston Model

#######################(a: i)#######################

The BSM delta can be calculated as $\Delta_{BS} = e^{-qT}N(d_1)$ and Heston delta is $\Delta_H = \frac{c(S+d)-c(S-d)}{2d}$ by finite difference method. We selected optimal parameters obtained from last question, which are (κ, θ, σ, ρ, ν0) = [3.52, 0.052, 1.18, -0.77, 0.034]. the delta in the Heston model computed using central difference method is Δ H = 0.48 and the delta in the BS model is given by Δ BS = 0.34. They are different due to the existence of stochastic volatility and Heston delta, from my prospective, is better than BSM delta, since it takes the volatility effect into account.

#######################(a: ii)#######################

According to the property of delta, if we hold a long position of call, we need to short delta position of underlying asset to build a delta neutral portfolio.

#######################(b: i)######################

Using (κ, θ, σ, ρ, ν0) = (3.52, 0.052, 1.18, −0.77, 0.034) again to calculate Heston vega. The Heston vega is 133.12170274010364 via finite difference method (changed two parameters) and BSM vega by calculation is 48.83836381057.

$$\mathrm{Vega}_H = \frac{c(\theta+d, \nu_0+d) - c(\theta-d, \nu_0-d)}{2d}$$

$$\mathrm{Vega}_{BS} = S_0 e^{-qT}\sqrt{T}\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}$$

######################(b: ii)######################

The negative calibrated value of the parameter ρ in the Heston model implies that there is a negative relationship between the underlying asset and volatility. Therefore, one unit increase in the volatility will lead to a smaller increase in the option price in the Heston model compared to that in the BS model.