

MF796 Assignment 6

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February 22, 2021

1: Numerical PDEs

(Q1)

The reason is that we can exercise two American calls at different time, but we have no right to terminate short call. Oppositely, it is feasible for us to early exercise a call spread which is equivalent to early executing both calls simultaneously.

The closing price of SPY on March 3 is 382.62.

(Q2)

I get it from U.S. **DEPARTMENT OF THE TREASURY** (<https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield>) which is 0.04%.

(Q3)

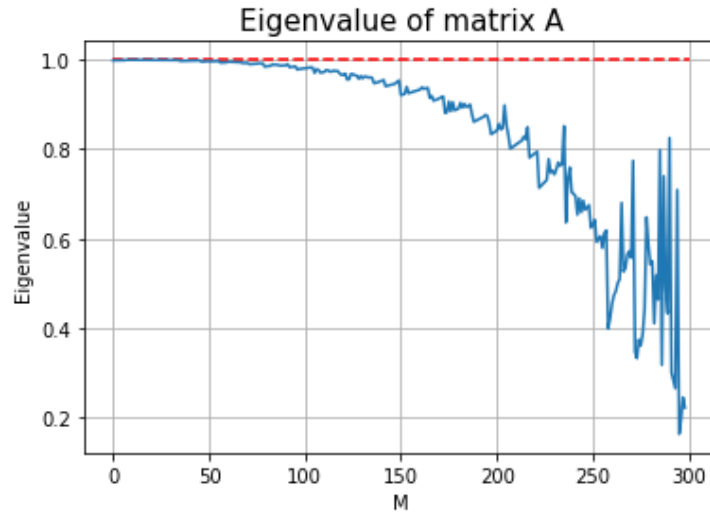
First, I searched the updated (March 5) implied volatility corresponding to the option price on SPY on Yahoo Finance, which are 21.9% and 19.09% respectively. Then, I averaged them $(21.9\% + 19.09\%) / 2 = 20.49\%$ to make it more reasonable.

(Q4)

I chose $S_{max} = 600$ since SPY is hard to reach at 600, and $N = 5000$ since I want more precise result and $M = 300$, $T = 7/12$ (approximately). Then, use these parameters to calculate h_s and h_t which are 2 and $7/12/5000$. The choice of these parameters must satisfy following condition to ensure stability:

$$h_t < \left(\frac{1}{\sigma^2 S_{max}^2} \right) h_s^2 = O(h_s^2)$$

(Q5)



All the eigenvalues of A have absolute value less than 1, which is reasonable.

(Q6)

$$\begin{bmatrix} a_1 & u_1 & & & \\ l_2 & a_2 & u_2 & & \\ & \ddots & \ddots & \ddots & \\ & & l_{M-2} & a_{M-2} & u_{M-2} \\ & & & l_{M-1} & a_{M-1} \end{bmatrix} \begin{bmatrix} C(s_1, t_j) \\ C(s_2, t_j) \\ \vdots \\ C(s_{M-2}, t_j) \\ C(s_{M-1}, t_j) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_j \end{bmatrix}$$

where

$$b_j = u_{M-1} \left(s_M - K e^{-r(t_N - t_j)} \right)$$

The call price without early exercise is: 2.3868606818134657 by using numerical PDE.

(Q7)

The call price with early exercise is: 4.46811942679873.

(Q8)

The premium that the difference between the American and European call spreads is: 2.0812587449852646 which is reasonable, since the early exercise should give the option a positive premium, which covers the benefit of early exercise.