MF 790 HW 6, Part 2

This assignment is due on Thursday, December 2nd at 8 AM. Problem 1 is worth 30 points, for a total of 30 points.

1. Forwards and Futures Prices in the Hull-White Model. Recall in the Hull-White model with constant coefficients, we assume under $\mathbb{Q}$ that the money market rate process has dynamics

$$dR_t = \kappa(\theta - R_t)dt + adW_t^{\mathbb{Q}},$$

where $\kappa, a > 0$ and $\theta \in \mathbb{R}$. Note that have written a for the volatility. Next, let us assume that the risky asset S has $\mathbb{Q}$ dynamics

$$\frac{dS_t}{S_t} = R_t dt + \sigma dW_t^{\mathbb{Q}}.$$

where $\sigma > 0$.

(a) Using the bond-pricing result we derived in class, for a fixed $T > 0$ and for each $t \leq T$ identify the forward price For_t for a forward contract on S maturing at t. Be as explicit as possible in your answer.

(b) For $t \leq T$, identify the futures price Fut_t for a futures contract on S maturing at T. To do so, use the following steps

(i) Prove the identity $\text{Fut}_t = S_t \times \mathbb{E}^{\widetilde{\mathbb{P}}}\left[e^{\int_t^T R_u du}\middle|\mathcal{F}_t\right]$ where

$$\frac{d\widehat{Z}_t}{\widehat{Z}_t} = \sigma dW_t^{\mathbb{Q}}, \widehat{Z}_0 = 1, \qquad \frac{d\widetilde{\mathbb{P}}}{d\mathbb{Q}} = \widehat{Z}_T.$$

(ii) By *slightly* adjusting the bond pricing result we did in class for the Hull-White model (e.g. we have R instead of $-R$ and the coefficients κ, θ, a might change), explicitly identify

$$\mathbb{E}^{\widetilde{\mathbb{P}}}\left[e^{\int_0^T R_u du}\middle|\mathcal{F}_t\right]$$

and hence the futures price.

(c) How do the forward and futures prices compare? Is one always bigger than the other?