MF 790 : Stochastic Calculus

Fall 2021

Instructor: Scott Robertson

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Office Hours: 8 - 9:30 AM on Tuesdays (virtual) and 11 AM - 12:30 PM on Wednesdays (in person).

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Office : HAR 519
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Office Hours : 7-9 PM on Wednesdays (virtual).

Course Website: On Questrom Tools : *questromtools.bu.edu*

Lecture Schedule: The class meets Thursdays from 8 - 10:45 AM in HAR 322.

Adjustments due to COVID. According to the Learn from Anywhere (LfA) model, lectures will be broadcast live and recorded over Zoom. Similarly, office hours (for both myself and the TAs) will be held both in person and over Zoom. All Zoom meeting links will be available over Questrom tools. However, international students must conform to what is required in order to obtain their visa. In particular, if classroom attendance is mandated, then students are expected to attend class.

Course Recording Policy As mentioned above, lectures will be recorded and made available on Questrom Tools. Students may not share the recordings with anyone not registered in the course, and may not post them in a public platform. Students have the right to opt-out of being part of the class recording. Please contact me or the TA to discuss options for attending the course in such cases.

Important Scheduling Notes:

(1) The mid-term exam will take place the week of October 11 - 15. Further details regarding the exam will be given as the date approachers.
(2) The week of November 23 - 27 (Thanksgiving week), there will no class..
(3) The final exam will take place during finals week, with the specific date being given later.

Prerequisites : There are no official pre-requisites for the class. However, students are expected to have a working knowledge of calculus, linear-algebra and probability theory.

Primary Textbook:

Stochastic Calculus for Finance II : Continuous-Time Models
Steven E. Shreve
Springer, ISBN-13: 978-1141923110

Supplemental Textbook

Stochastic Calculus for Finance I : The Binomial Asset Pricing Model
Steven E. Shreve
Springer, ISBN-13: 978-0387249681

Grading: The course grade is determined as follows:

Midterm Exam : 40% Final Exam : 40% Homework : 20%

Exams: Due to Covid-19, students will have the option to take the exam in person, or online. The exam will be the same for all students : only the location where students complete the exam will differ. Precise technical details on how the exam will be administered will be given later. As always, during the exam students may not give or receive assistance, or communicate with other students in any fashion. Violation of this policy will be treated seriously, and it is expected students abide by the rules set forth in the Academic Code of Conduct.

Homework: Homework problems will be posted each week. Assignments will be collected every two weeks, for a total of 5-6 assignments. For example, week one problems will be posted by Thursday, September 2nd and week two problems posted by Thursday, September 9th, with both sets of problems due on Thursday, September 16th. Homework solutions will be posted by 6 PM on the Friday following the due date. There will be no problems posted for the weeks of October 11-15 (mid-term) and November 22-26 (Thanksgiving).

Homework submissions may be uploaded through Questrom Tools (preferred method), or turned in at the beginning of class the week it is due. Late homework will not be accepted. However, we will drop your lowest homework score when computing the homework average.

You are encouraged to collaborate on the homework. However, you should not submit any work which you do not fully understand. In the case where a group of students have suspiciously similar homework submissions, each member of the group may be asked to produce a solution without notes. You may not use course material from previous years to help with the homework: this includes asking students who previously took the course for help.

Course Objectives: This is a first course in stochastic calculus for finance, aiming to give a general introduction to students in the Asset Management, Financial Technology and Risk Management concentrations. Students will learn the necessary tools from stochastic calculus to price and hedge both assets and derivatives in a continuous time setting. The stochastic calculus content of the course is also used for fixed income, advanced derivatives, credit risk models, foreign exchanges and commodities.

Course Content: The course will cover the following topics. Each numbered topic will be covered over one, or possibly two, lectures. The last topic (jump processes) is tentative. All references below are to the class textbook.

(1) Probability Theory Review
  Probability spaces, filtrations and random variables (Ch 1.1, 2.1, 1.2).
  Expectation and conditional expectations (Ch 1.3-1.5, 2.3).
  Stochastic process: martingales and Markov processes (Ch 2.3).
(2) Brownian Motion
  Random walks and scaled random walks (Ch 3.2).
  Brownian motion definition (Ch 3.3).
  Quadratic variation of Brownian motion (Ch 3.4).
  Martingale property of Brownian motion (Ch 3.3).
  Markov property of Brownian motion (Ch 3.5).
(3) Itô Integrals, Itô 's Formula and the Black-Scholes equation
  Itô Integral for simple integrands (Ch 4.2).
  Itô Integral for general integrands (Ch 4.3).
  Itô -Doeblin formula (Ch 4.4).
  Black-Scholes-Merton formula (Ch 4.5).
(4) Risk Neutral Pricing
  Lévy characterization of Brownian motion (Ch 4.6).
  Change of measure and Girsanov's theorem (Ch 5.2).

Risk-neutral measure (Ch 5.2).
The risk-neutral price for a contingent claim (Ch 5.2)

(5) Multi-stock models and the Fundamental Theorems
Multivariate stochastic calculus (Ch 4.6).
Multi-stock models, market price of risk equations (Ch 5.4).
Martingale representation theorem (Ch 5.3).
Fundamental theorems of asset pricing (Ch 5.4).

(6) Dividend Paying Stocks, Forwards and Futures
Dividend paying stocks (Ch 5.5).
Forwards and Futures (Ch 5.6).

(7) Stochastic Differential Equations and Partial Differential Equations
Stochastic differential equations (SDE): introduction and examples (Ch 6.2).
Partial differential equations (PDE): introduction and examples (Ch 6.4).
Connections between SDE and PDE: Itô (revisited) and Feynman-Kač (Ch 6.6).

(8) Exotic Options
Reflection principle for Brownian motion (Ch 3.6).
Maximum of a Brownian motion with drift (Ch 7.2).
Path-dependent options I: knockout, barrier, look-back and Asian (Ch 7.3-7.5).
Path-dependent options II: American (Ch 8).
Options replication using calls and puts.
Variance options.

(9) Change of Numéraire
Numéraire (Ch 9.2).
Forward measure (Ch 9.4).
Foreign and domestic risk-neutral measure (Ch 9.3).

(10) Jump Processes (time-permitting)
The Poisson and compound Poisson processes (Ch 11.2-11.3).
Stochastic calculus for jump processes (Ch 11.4-11.5).
Change of measure for jump processes (Ch 11.6).
Risk-neutral pricing for jump processes (Ch 11.7).

COVID and a Statement of Questrom Community Norms

In addition to the expectations outlined in the COVID-19 Health Commitments & Expectations for Boston University Students, we expect all members of our Questrom community to adhere to the following (selected) classroom norms:

- Wear an appropriate personal protection equipment (PPE) face covering when in the Hariri building, including in the Hariri classrooms and offices. Students not wearing a face covering will be asked to leave and are expected to comply with the request.
- Be willing to display your "green screen" compliance app upon request (e.g., in class or for a meeting).

Diversity and Inclusion Statement This course has been developed with the knowledge that identity and culture may impact students' reactions to the course content. If there are topics or presentation styles that you feel would benefit from incorporation of a differing perspective, or from the support of our Questrom's Office of Diversity & Inclusion, please inform me and I will explore opportunities to engage a wide variety of perspectives. If you have concerns or ideas about diversity and inclusion at Questrom you can also reach the Questrom Diversity & Inclusion office at myQuestrom@bu.edu.

Academic Accommodations for Students with Special Needs In keeping with University policy, any student with a disability who needs or thinks they need academic accommodations must call the Office of Disability Services at 617-353-3658 or stop by 19 Deerfield Street to arrange a

confidential appointment with a Disability Services staff member. Accommodation letters must be delivered to me in a timely fashion (not later than two weeks before any major examination). Please note that accommodations will not be delivered absent an official letter of accommodation.