Introduction to Commodity Derivatives
MF702, BU, Fall 2020

Roza Galeeva: groza@bu.edu

Outline

Introduction to Commodity Derivatives

Most Popular Options

- Spread Options
- Swaptions

Time Dependent Volatility

- Total Realized Variance
- Instantaneous Volatility
- Black-Scholes formula under time dependent volatility

Samuelson Effect

- Samuelson effect
- Different forms of Samuelson parameterization

Spread Options

We start with the most popular option on two assets F_1 and F_2 which is a spread option with payoff

$$\max(F_1(T) - F_2(T) - K, 0) \tag{1}$$

where are T is the option expiry and K is the strike. Spread options are very important in energy markets, as many energy assets have a spread options embedded in them.

- *Power Plants*: Evaluation of power plants can be represented as a spark spread option: spread between power and fuel prices. The payoff is

  $$\max(P - HR * G - V, 0) \tag{2}$$

  where P is power price, G is the fuel price, HR is the heat rate and V is the variable cost of running plant.
- *Gas storage*: The value of the storage can be replicated through calendar spread options: buy and inject natural gas in months with low gas prices (October), extract and sell in the months with high gas prices (January).
- *Transmission lines, pipelines* : Transmission contracts can be viewed as a right to move power or gas from a liquid point A to a liquid point B.

Swaptions

- A swaption is an option to enter a swap at expiry. The payoff of a swaption can be written as

$$\max\left(\frac{\sum_i df_i * N_i * F_i(T)}{\sum_i df_i * N_i} - K, 0\right) \tag{3}$$

  where $F_i(T)$ are the future prices in the underlying swap price at swaption expiry T, df_i are discount factors from swaption expiry T until futures expiries T_i and N_i are the volumes and K is the strike.
- Swaptions is one of the most popular derivative products traded, and is a hedging tool used in oil, gas, and power markets.
- Daily evaluation of a swaption must take into account a non-trivial dynamics of the volatilities of the underlying futures (Samuelson effect), as well as the dynamics of correlations
- The inputs to a swaption evaluation include future prices F_i, implied volatilities σ_i, interest rates, correlations matrix between futures.

Total Realized Variance and Instantaneous volatility

- We will denote future price by $F(t, T)$, where we are looking at future price at time T for a contract with expiration T. [1]
- The instantaneous volatility $\sigma(t, T)$ is the volatility of future contract T measured on a short time interval $[t, t_1]$ with length of the interval $t_1 - t \to 0$.
- For example, we can define a piecewise constant volatility $\sigma(t, T) = \sigma_1$ for $t \in [t_0, t_1]$, $\sigma(t, T) = \sigma_2$ for $t \in [t_1, t_2]$,.... with $t_0 < t_1 < t_2 < ..t_n = T$.
- The total variance realized on the interval $[t_0, T]$ will be given by

$$Var_{t_0,T} = \sum_{i=1}^{n} \sigma_i^2 (t_i - t_{i-1}) \tag{4}$$

- When $\max_i (t_i - t_{i-1}) \to 0$ the sum 4 becomes an integral:

$$Var_{t_0,T} = \int_{t_0}^{T} \sigma^2(s, T) ds \tag{5}$$

[1] We will be referring to a future contract using the expiration of the contract T, which is unique

Average volatility and the Volatility in BS

(i) $\sigma(t, T)$ in 5 is called the *instantaneous* volatility of a future contract T.

(ii) From the total realized variance we can calculate the average volatility $\sigma^{av}_{[t_0, T]}$ on the interval $[t_0, T]$ from:

$$(\sigma^{av}_{[t_0, T]})^2 (T - t_0) = Var_{[t_0, T]}$$

(iii) This is the volatility which will be used in the BS formula as an input, since the expressions for d_1 , d_2 depend on the total variance.

$$d_1 = \frac{\ln(F/K) + \sigma^2/2(T - t_0)}{\sigma\sqrt{T - t_0}}, \quad d_2 = d_1 - \sigma\sqrt{T - t_0}$$

(iv) Therefore, in the case of time dependent volatility, we need to calculate the average volatility and plug it in the BS formula.

Breakout Room 1 Questions

1. Assume we are looking at a contract with expiration in 5 years. Assume its volatility is constant in each year (so is piecewise constant) and is given by

$$\sigma(t) = \begin{cases} 0.2 & \text{for } t \in [0,1] \\ 0.25 & \text{for } t \in [1,2] \\ 0.3 & \text{for } t \in [2,3] \\ 0.4 & \text{for } t \in [3,4] \\ 0.5 & \text{for } t \in [4,5] \end{cases}$$

Question: If we want to price an option on this contract with maturity of 5 years, what is the average volatility we need to plug into the BS formula? What about an option with expiration of 4 years?

The answer is in the spreadsheet.

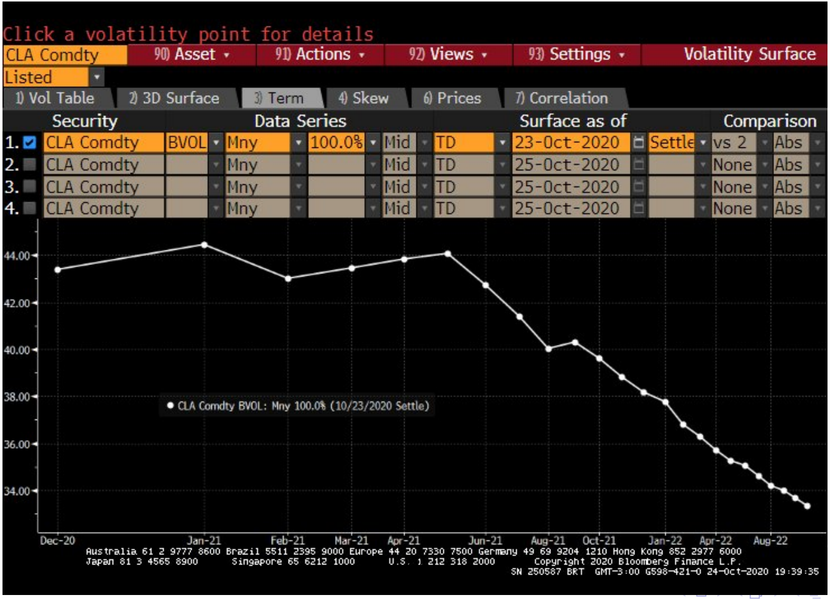
Samuelson effect in commodities

- ★ The increase of forward price volatility , both historical and implied, as they get closer to their maturity is called Samuelson effect, see Samuelson [1965]. This can explain by the fact that more information is available and there is more trading; uncertainty in the weather , etc.
- ★ This backwardation of volatility of forward contracts arises across most markets but this effect is especially pronounced in energy markets, as a particular steep increase in volatility occurs in the last six (or less) months of the life of the contract.
- ★ There are different ways to parameterize the Samuelson effect. For example, in Swindle [2014] the author suggests to model the decay of volatility by choosing a model for implied variance with two exponential decays.

$$\Sigma^2(t,T)(T-t) = \sigma_0^2\left(e^{-2B(T-t)} + \lambda e^{-2\beta(T-t)}\right) \tag{6}$$

where $\Sigma(t,T)$ is the implied volatility quoted in the market for contract with expiration T, B is the global decay and β is a long term decay (much smaller), σ_0 is the normalization factor to match the implied market volatility.

WTI Implied Vols



Modeling Instantaneous Volatility

- ⋆ In some cases, instead of modeling the total variance as in 6, it is more convenient to model the instantaneous volatility. We can always integrate the instantaneous vol as in 5. Besides, it would be very difficult to use 6 to calculate the total realized covariance between two contracts , therefore a correlation between them.
- ⋆ We can use the representation as in 6 but now for instantaneous volatility;

$$\sigma(t, T) = \sigma_0 \sqrt{\left(e^{-2B(T-t)} + \lambda e^{-2\beta(T-t)}\right)}$$

- ⋆ The simplest case is $\lambda = 0$ so a simple exponential decay of instantaneous volatility. This form was used in Galeeva, Haversang [2020]

$$\sigma(t, T) = \sigma_0 e^{-B(T-t)} \tag{7}$$

- ⋆ Another case is $\beta = 0$. For all considered parameterizations, the volatility only depends on the time to the expiration of the future, $T - t$.

Breakout Room 2

1. We choose a simple exponential decay model for instantaneous volatility 7. Let's assume:
   (i) Consider a future contract which expires in two years $T = 2$, $t_0 = 0$
   (ii) The implied ATM vol for this contract is quoted at $\Sigma = 50\%$.
   (iii) Based on calibration of the data, the best decay parameter $B = 0.2$ (per year).

Question 2 What σ_0 should we choose to match the quoted implied vol?

Answer Breakout 2

- In order to calibrate σ_0 we calculate the total variance based on the model for instantaneous volatility and match it with the variance based on the implied vol quote:

$$Var = \sigma_0^2 \int_0^2 \exp\left(-2B(T-s)\right) ds = \Sigma^2 * 2 = 0.5^2 * 2 = 0.5$$

- Let $\tau = T - s$, $ds = -d\tau$. Calculating the integral we will get

$$Var = \frac{\sigma_0^2}{2B}(1 - exp(-4B)) = 0.5$$

- Therefore,

$$\sigma_0 = 0.602655$$

Measuring Samuelson strength

No matter which Samuelson parameterization is chosen, the main parameters capturing the Samuelson effect in our opinion are

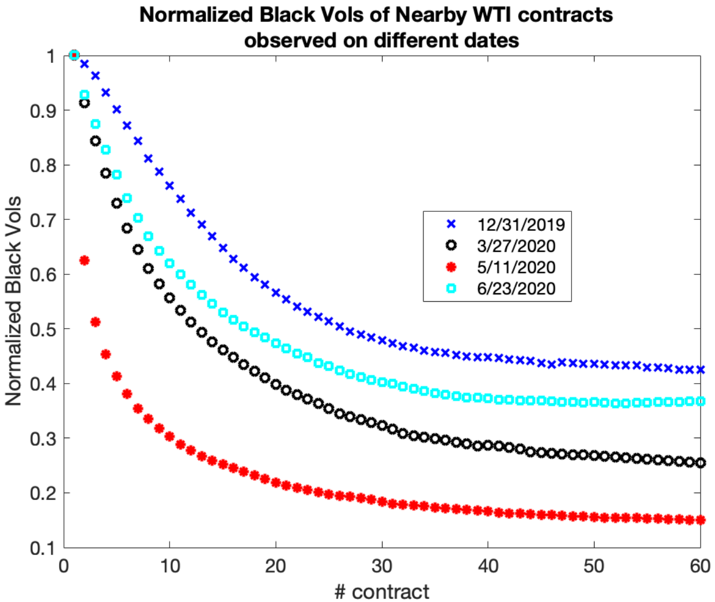
1. The rate of exponential decay B of volatility with time to expiration $T - t$. Larger values of the rate B increase the strength of Samuelson.
2. The long term level of volatility as $T - t \to \infty$. Its larger values make Samuelson weaker. The long term volatility could be a constant or a slowly decaying function of $T - t$ as in 6
3. We do not even need a particular parameterization, and can only show the normalized vols observed from the history.

Analysis on the recent data

- We can compare Samuelson strength in different historical periods.
- We expect to see much stronger Samuelson in turbulent times, such as during the crash in April 2020.
- The first graph on next slide shows normalized volatilities, calculated over different 30 days time intervals [2]
- We start with May 2019, when we see moderate levels of volatility and a moderate decay.
- Starting February 2020 Samuelson becomes stronger and volatilities increase.
- A particularly strong Samuelson is observed in May 2020, the period which involves the April crash. We see a very sharp decay of volatilities , and a lower long term level
- Finally in June 2020, we can see some kind of return to normality.

[2] we give the dates t_2

Results



Normalized Black Vols of Nearby WTI contracts
observed on different dates
Normalized Black Vols
contract
12/31/2019
3/27/2020
5/11/2020
6/23/2020
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60

References

1. Black, Fisher and M. Scholes, “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy*, 81, 1973, 163 – 175.
2. Black, Fischer, “The Pricing of Commodity Contracts.” *Journal of Financial Economics* 3 (March 1976), pp. 167-179.
3. Eydeland, "Energy and Power Risk Management", Wiley 2003.
4. Galeeva, Haversang, "Parameterized Calendar Correlations: Decoding Oil and Beyond", *Journal of Derivatives*, Spring 2020.
5. Samuelson, "Proof that Properly Anticipated Prices Fluctuate Randomly", *Industrial Management Review*", 6, 1965.
6. Swindle, "Valuation and Risk Management in Energy Markets", *Cambridge University press*, 2014.