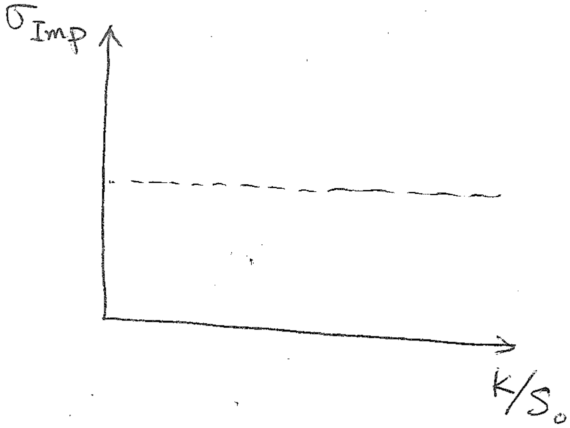
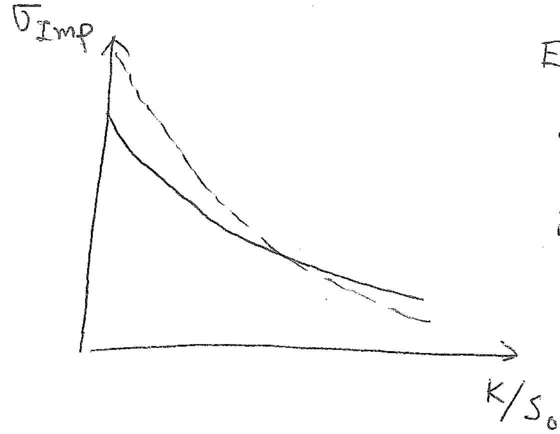


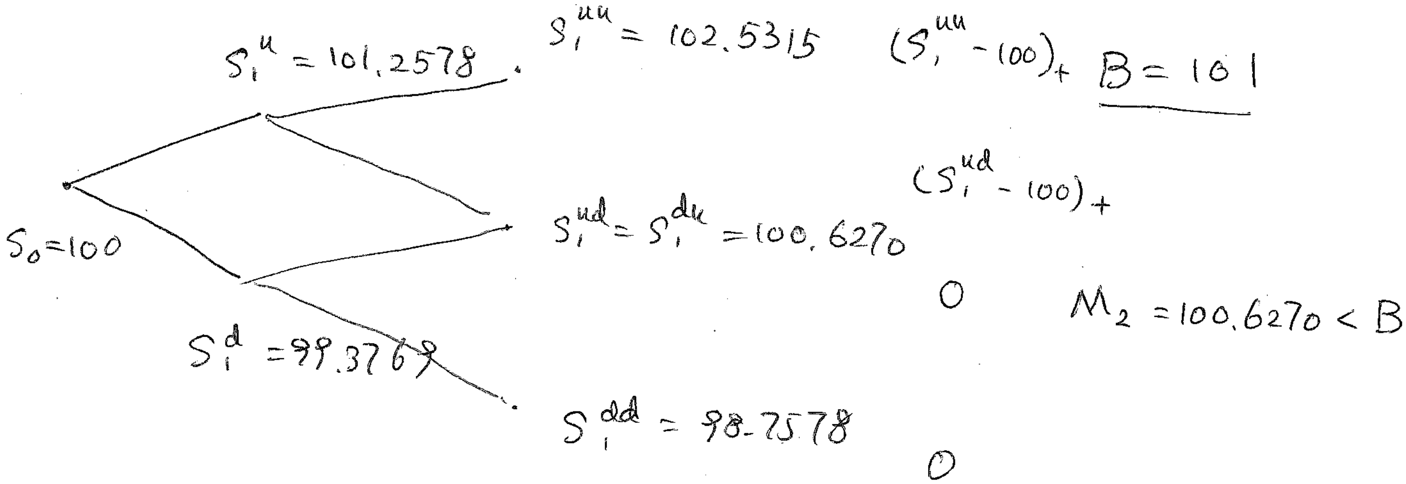


Excess demand of put option

increase put option value in small moneyness.

(put option deep out-of-money)

→ increase of σ_{Imp}.



$$\sum_{i=1}^{N} \frac{1}{(1+y/2)^i} = \frac{1}{1+y/2} \underbrace{\sum_{i=0}^{N-1} \frac{1}{(1+y/2)^i}}_{\displaystyle = \frac{1-\frac{1}{(1+y/2)^N}}{1-\frac{1}{1+y/2}}} = \frac{1}{\cancel{1+y/2}} \frac{1}{\frac{y/2}{\cancel{1+y/2}}} \left(1-\frac{1}{(1+y/2)^N}\right)$$

$$= \frac{2}{y}\left(1 - \frac{1}{(1+y/2)^N}\right)$$

Example: 3 securities ZCB 2, 5, 10 yr

S^1, S^2, S^3

portfolio A: 1M face value in S^2.

B: F^1 face in S^1, F^3 face in S^3

$\hat{r}(2) = 5.78\%$. $\hat{r}(5) = 6.02\%$. $\hat{r}(10) = 6.26\%$.

Value: $X_A = 1M\ P^2$

$X_B = F^1 P^1 + F^3 P^3$

(1) $X_A = X_B \Rightarrow 1MP^2 = F^1 P^1 + F^3 P^3$

$$P^1 = \frac{1}{\left(1+\frac{\hat{r}(2)}{2}\right)^4}$$

$$D = \frac{T}{1+y/2} \qquad y = \hat{r}$$

$$D_A = \frac{T_2}{1+\frac{\hat{r}(5)}{2}} = \frac{2}{1+\frac{\hat{r}(5)}{2}}$$

$$D_B = \frac{F^1 P^1}{1m\ P^2} D^1 + \frac{F^3 P^3}{1m\ P^2} D^3$$

(2) $D_A = D_B \Rightarrow \dfrac{T_2}{1+\frac{\hat{r}(5)}{2}} = \dfrac{F^1 P^1}{1m\ P^2} \dfrac{T_1}{1+\frac{\hat{r}(2)}{2}} + \dfrac{F^3 P^3}{1m\ P^2} \dfrac{T_3}{1+\frac{\hat{r}(10)}{2}}$

solve (1) and (2) to find F_1, F_2.

$F^1 = 520,386.$ $F^3 = 516,843$

Case 1 "parallel" shift in the spot curve by 35 bp.

$\hat{r}(2) = 5.78\% + 0.35\%$ $\hat{r}(5) = 6.02\% + 0.35\%$

$\hat{r}(10) = 6.26\% + 0.35\%$

$$\Delta X_A = X_A^{new} - X_A^{old}$$

$$= 1m\,(P^{2,new} - P^{2,old})$$

$$P^{2,new} = \frac{1}{\left(1+\frac{\hat{r}(5,new)}{2}\right)^{10}} \qquad P^{2,old} = \frac{1}{\left(1+\frac{\hat{r}(5,old)}{2}\right)^{10}}$$

$$= -12,511.7 \quad (\text{exact})$$

$$\Delta X_B = F^1(P^{1,new} - P^{1,old}) + F^3(P^{3,new} - P^{3,old})$$

$$= -12,448.8 \quad (\text{exact})$$

Linear approximation.

1st order

$$\Delta X_A \approx - X_A D_A \Delta y$$

$$\Delta X_B \approx - X_B D_B \Delta y$$

($X_A = X_B$, $D_A = D_B$, $\Delta y = \Delta y$)

Case 2. $\Delta \hat{r}(T) = -35bp.$

$$\Delta X_A = 12,747.7 \quad (\text{exact})$$

$$\Delta X_B = 12,813.2 \quad (\text{exact})$$

short A , long B

Case 1. $\Delta X_B - \Delta X_A = -12{,}488.8 - (-12{,}511.7)$

$= 62.9$

Case 2. $\Delta X_B - \Delta X_A = 12{,}813.2 - 12{,}747.7$

$= 65.5$

Convexity

$$\left.\begin{aligned} C_A &= 25.916 \\ C_B &= 40.007 \end{aligned}\right\}$$ hedged portfolio is long in convexity!

Bullet: long a med range horizon product T_2

Barbell: long a short, long horizon products. T_1, T_3

YTM for all products are the same, y. $T_1 < T_2 < T_3$

$$\text{price}^{\text{bullet}} = \frac{1}{(1+y/2)^{2T_2}}$$

$$D^{\text{bullet}} = \frac{T_2}{1+y/2}$$

$$C^{\text{bullet}} = \frac{T_2^2 + T_2/2}{(1+y/2)^2}$$

barbell, T_1. T_3

$$D^{barbell} = \partial \cdot \frac{T_1}{1+y/2} + (1-\partial)\frac{T_3}{1+y/2} = \frac{T_2}{1+y/2}$$

$$\partial = \frac{F_1 P(T_1)}{F_1 P(T_1) + F_3 P(T_3)}$$

$$Price^{barbell} = F_1 P(T_1) + F_3 P(T_3) = P(T_2)$$

$$\partial = \frac{T_3 - T_2}{T_3 - T_1}$$

$$F_1 = \frac{T_3 - T_2}{T_3 - T_1} \frac{P(T_2)}{P(T_1)}$$

$$F_3 = \frac{T_2 - T_1}{T_3 - T_1} \frac{P(T_2)}{P(T_3)}$$

Convexity:

$$C = \frac{T^2 + T/2}{(1+y/2)^2}$$

$$C^{bullet} = \frac{T_2^2 + T_2/2}{(1+y/2)^2}$$

$$C^{barbell} = \partial \cdot \frac{T_1^2 + T_1/2}{(1+y/2)^2} + (1-\partial)\frac{T_3^2 + T_3/2}{(1+y/2)^2}$$

$$g(T) = \frac{T^2 + T/2}{(1+y/2)^2} \quad \leftarrow \text{ convex in } T.$$

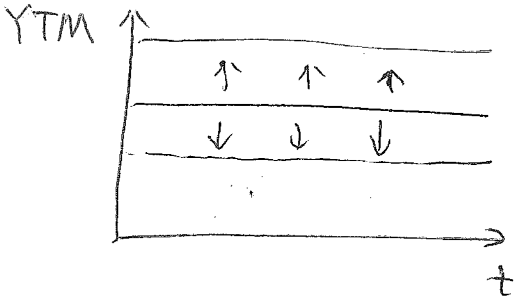
$$C^{bullet} = g(T_2) \qquad T_2 = \partial T_1 + (1-\partial) T_3$$

$$= g(\partial T_1 + (1-\partial) T_3)$$

$$\wedge|$$

$$C^{barbell} = \partial g(T_1) + (1-\partial) g(T_3)$$

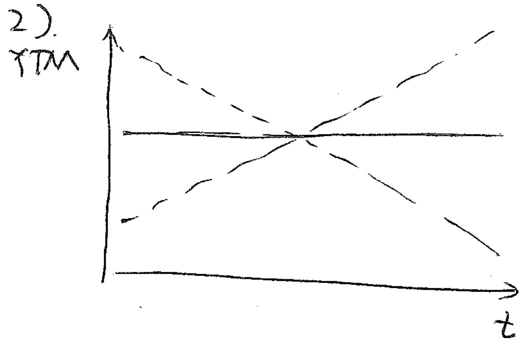
1) parallel shift



make money

long barbell short bullet.

2).

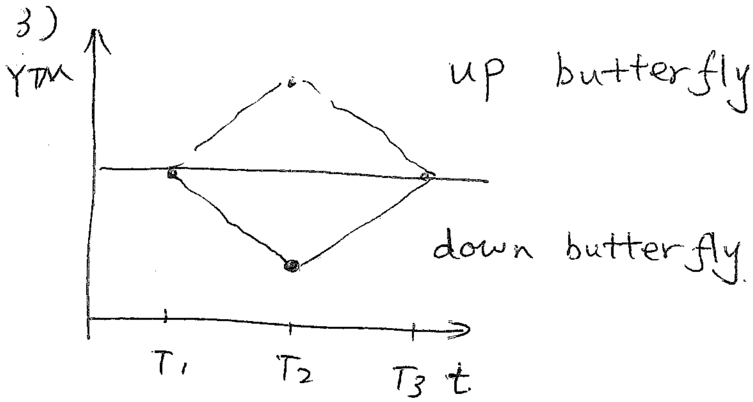


lose money

if short yield ↑.

long yield ⇓

3)



down butterfly : loss money.

value of barbell does n't change

value of bullet becomes larger.