Sensitivity Analysis and Hedging I

- DV01, hedging with DV01

Motivating Remark

- the price for a q-coupon bond with N remaining payments (at $1/2, 1, \ldots, N/2$)

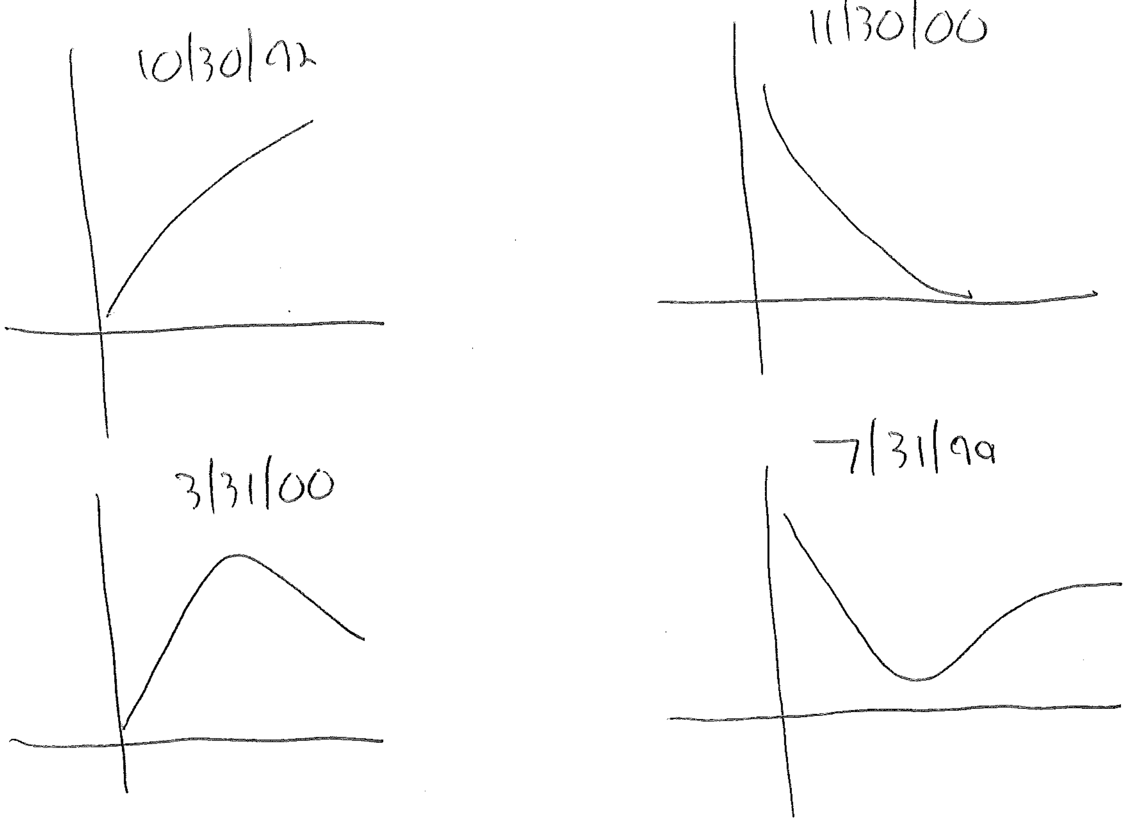
$$\frac{Px}{F} = q/2 \sum_{i=1}^{N} \frac{1}{(1+\hat{r}(i/2)/2)^i} + \frac{1}{(1+\hat{r}(N/2)/2)^N}$$

- so, even though the cash flows are fixed and will happen with certainty, since the price depends on (all of) the spot rates, the price displays interest rate risk
    - roughly: if rates go up, prices go down.

- we need a way to measure the sensitivity of a bond's price to interest rates.

In fact, in its entirety, the spot curve can move in surprising/complex ways

e.g. (continuously compounded) spot curves



- measuring price sensitivity to the whole curve can be hard.

- sometimes it is possible to measure interest-rate-risk or sensitivity by selecting one, or a few, key interest rate factors

  i.e. spot curve changes driven

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by a short, medium, or long term rate.

- this will not capture all movements, but it can provide a good approximation.

- we will start by measuring sensitivities to a <u>single</u> interest rate factor

One Factor Models

Here, we assume there is a single interest rate factor y s.t. we can think of prices as a function of y

obstractly: $P_X = f(y)$

WARNING

y is <u>not</u> necessarily the bond's YTM

- using y to be consistent with Tuckman.

So, if $P_x = f(y)$ then

1) first order approx.

$$\Delta P_x \approx \dot{f}(y)\Delta y$$

2) second order approx

$$\Delta P_x \approx \dot{f}(y)\Delta y + \frac{1}{2}\ddot{f}(y)(\Delta y)^2$$

example

Zero with maturity T, $y = YTM$ $(= \hat{r}(T))$

$$P_x = f(y) = F(1+y/2)^{-2T}$$

$\Rightarrow$

$$\dot{f}(y) = -2TF(1+y/2)^{-2T-1}\cdot\frac{1}{2}$$

$$= -\left(\frac{T}{1+y/2}\right)F(1+y/2)^{-2T}$$

$$= -\left(\frac{T}{1+y/2}\right)f(y) \quad \text{(note: it is negative)}$$

$\Rightarrow$

$$\ddot{f}(y) = \frac{T^2 + \frac{1}{2}T}{(1+y/2)^2}f(y) \quad \text{(note: it is positive)}$$

so

$$\Delta P_x \approx -\left(\frac{T}{1+y/2}\right)P_x\cdot\Delta y \quad (1^{st}\text{ order})$$

$$\approx -\left(\frac{T}{1+y/2}\right)P_x\,\Delta y + \frac{T^2+\frac{1}{2}T}{(1+y/2)^2}P_x(\Delta y)^2 \quad (2^{nd}\text{ order})$$

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Relative Changes

$$\frac{\Delta Px}{Px} \approx -\left(\frac{T}{1+y/2}\right)\Delta y$$

$$\approx -\left(\frac{T}{1+y/2}\right)\Delta y + \frac{3}{4}\left(\frac{T}{1+y/2}\right)^2(\Delta y)^2$$

In general:

- bond not necessarily a zero.
- y not necessarily YTM.

We define

$$DV01 = -\frac{\Delta Px}{10{,}000\,\Delta y}$$

- "Dollar Value of a Basis Point"

  or

  "Dollar Value of an '01"

  or

  "Price Value of a Basis Point"

- Basis Point (bp)

  $100bp = 1\%$ or $1bp = 0.0001$

  - changes in rates often quoted in bp.

Notes

1) why 10,000 ?

Δy : absolute change in y

e.g. $y_0 = 5\%$, $y_1 = 5.02\%$

$\Rightarrow \Delta y = .02\% = 0.0002$.

so

$10{,}000\,\Delta y$: bp change in y

$\Rightarrow 10{,}000\,\Delta y = 2\,bp$ (above example)

so DV01 tells us how much the price will change if y moves 1bp.

2) Why the minus sign?

- in general, prices will go down if rates go up
  - warning: for some fixed income products the opposite is true.
- use minus sign to think of DV01 as a positive #.

3) If $Px = f(y)$ and f is known, smooth

$$DV01 = - \frac{f'(y)}{10{,}000} \quad \text{if } \Delta y \text{ is small}$$

4) By definition, DV01 changes with the face

e.g. zero: $Px = F \cdot d(T) = F \frac{1}{(1+y/2)^{2T}}$

$$\Rightarrow DV01 = F \cdot \frac{d}{dy}\left(\frac{1}{(1+y/2)^{2T}}\right) \quad \Delta y \text{ small.}$$

typically, we may quote DV01 per 100 Face, but be careful to take F into accaunt.

example

Px^1 (per 100 face) $= 100$

Px^2 (" " ") $= 99.973$

$y^{(1)} = 5.00\%$

$y^{(2)} = 5.02\%$

then, for a face F

$$DV01 = \frac{F(100 - 99.973)/100}{10{,}000 \cdot .02}$$

$$= \frac{F}{100} \frac{.027}{10{,}000(.0002)}$$

$$= \frac{F}{100} \times .0135$$

Yield Based DV01

when $y = YTM$, DV01 is called

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the "yield based" DV01

- sometimes people say "DV01" for yield based DV01
  - make sure to ask if you are not sure.

Hedging with DV01.

- say we have a bond B whose yield is 4%.
- say we also have an option on a bond (e.g. call, put, etc) with price P.
- assume, somehow, we know that (100 face of option)

$P = 8.0866$ if the bonds yield y is 4.01%

$P = 8.2148$ if $y = 3.99\%$

$\Rightarrow$ DV01 (option, 100 face)

$$= - \frac{8.0866 - 8.2148}{10000(.0401 - .0399)} = .0641.$$

(10)

Next, suppose we can estimate/~~calculation~~ calculate the bond's DV01 at $y = 4\%$ to be .0857. (100 face)

Q.

If we own \$100 million face of the option, how can we hedge against small interest moves with the bond.

I.e. how much of B should we buy/sell so ~~at~~ our portfolio DV01 is 0?

– for F face of B our DV01 is

$$\frac{F}{100} \cdot .0857.$$

(11)

- for \$100 million face of the option our DV01 is

$$\frac{100{,}000{,}000}{100} \cdot .0641$$

- we thus want

$$0 = \frac{F}{100} \times .0857 + \frac{100{,}000{,}000}{100} \times .0641$$

$\Rightarrow F < 0$

why?

- if $y \nearrow$ we lose money on the option ($DV01 > 0$)
- for the band, the same fact holds, since its DV01 is positive, if we are long the band. Thus we want to shart the band.

Solving

$$F = \frac{-100{,}000{,}000}{100} \cdot \frac{.0641}{.0857} \approx -74{,}795{,}799$$

(12)

Note: If securities A, B both have positive DV01, then to hedge a long position in A we must short B

- A, B opposite DV01 sign

$\Rightarrow$ Long B to hedge long A.