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21:01

$T = 10 \quad F = 10{,}000, \qquad q = 4\%, \qquad P_x^1 = 9{,}060$

$T = 10 \quad F = 5{,}000 \qquad q = 8\% \qquad P_x^2 = 6{,}146.50$

discounting factors, on the same maturity must be the same.

$$\frac{9{,}060}{10{,}000} = \frac{0.04}{2} \boxed{\sum_{i=1}^{20} d(i/2)} + \boxed{d(10)}$$

$$\frac{6{,}146.50}{5{,}000} = \frac{0.08}{2} \sum_{i=1}^{20} d(i/2) + d(10)$$

$$d(10) = 0.5827 \qquad \sum_{i=1}^{20} d(i/2) = \frac{\frac{9{,}060}{10{,}000} - d(10)}{\frac{0.04}{2}}$$

ZCB $F = 20.000 \qquad T = 10$ yr

$$P_x^3 = 20{,}000\, d(10) = 20{,}000 \times 0.5827$$

(b) $A = 500$ paid twice per year.

$$P_x^4 = 500 \sum_{i=1}^{20} d(i/2)$$

(c) Coupon bond with $F = 7,500$, $q = 6\%$, $T = 10$

$$P_x^5 = F\left(\frac{6\%}{2}\sum_{i=1}^{20} d(i/2) + d(10)\right)$$

3. $\hat{R}(T)$, $R^{for}_{0,\eta,T}$

1 borrowed at η will become

$(1 + R^{for}_{0,\eta,T})^{T-\eta}$ at time T.

$$(1+\hat{R}(T))^T = (1+\hat{R}(\eta))^\eta (1+R^{for}_{0,\eta,T})^{T-\eta}$$

$$(1+R^{for}_{0,\eta,T})^{T-\eta} = \frac{(1+\hat{R}(T))^T}{(1+\hat{R}(\eta))^\eta} = \frac{(1+\hat{R}(T))^\eta}{(1+\hat{R}(\eta))^\eta}(1+\hat{R}(T))^{T-\eta}$$

$\hat{R}(T) > \hat{R}(\eta)$ for $\eta \leq T$. $\geq (1+\hat{R}(T))^{T-\eta}$

$\Rightarrow R^{for}_{0,\eta,T} \geq \hat{R}(T)$.

4. $\hat{r}(t) = 6\%$ for all t, $y = \hat{r}(t) = 6\%$

ZCB, $F = 1{,}000{,}000$ $T = 2, 5, 10$

(a) DV01, duration.

$$DV01 = \sum_i F^i \underbrace{DV01^i}_{\text{with } F=1} = \sum_i \underbrace{DV01^i}_{F=F^i}$$

ZCB y, T

$$P_X = F\left(1+\frac{y}{2}\right)^{-2T} = f(y)$$

$$DV01 = -\frac{\dot{f}(y)}{10{,}000} = \frac{F}{10{,}000}\left(\frac{T}{1+y/2}\right)\left(1+\frac{y}{2}\right)^{-2T}$$

$$DV01(B_1) = \frac{1{,}000{,}000}{10{,}000}\left(\frac{2}{1+\frac{0.06}{2}}\right)\left(1+\frac{0.06}{2}\right)^{-4}$$

$DV01(B_2)$ . . . .

$$DV01(X) = DV01(B_1) + DV01(B_2) + DV01(B_3)$$

ZCB, $$D = \frac{T}{1+y/2}$$

$D(B_1)$, $D(B_2)$, $D(B_3)$

$$D(X) = \frac{P_X(B_1)}{P_X(X)} D(B_1) + \frac{P_X(B_2)}{P_X(X)} D(B_2) + \frac{P_X(B_3)}{P_X(X)} D(B_3)$$

$$P_X(X) = P_X(B_1) + P_X(B_2) + P_X(B_3)$$

$$D(X) = 4.96007.$$

(b). par-coupon $T = 7$. $F = ?$

$y = 6\%$. $P_X = F \Leftrightarrow q = y$

$q = 6\%$.

DV01 for general coupon bond, set $q = y$

$$DV01 = \frac{F}{10{,}000} \frac{1}{y} \left(1 - \frac{1}{(1+y/2)^{2T}}\right) = DV01 \text{ from (a)}$$

where $y = 6\%$.

$F = 1,896,729.9$.

(c) $\hat{r}(2) = 0.065$, $\hat{r}(5) = 0.0642$, $\hat{r}(10) = 0.0638$

$$P_X(B_1) = 1,000,000 \left(1 + \frac{0.065}{2}\right)^{-4}$$

$P_X(B_2)$

+) $P_X(B_3)$

$P_X(X)$

$\Delta P_X(X) = -43,612$.

(d) Find $\underline{y}$ so that $P_X(X)$ from (c)

$$P_X(X) = 1,000,000\left(\left(1+\frac{y}{2}\right)^{-4} + \left(1+\frac{y}{2}\right)^{-10} + \left(1+\frac{y}{2}\right)^{-20}\right)$$

try and error.

$y = 0.06$ RHS < LHS

$\Rightarrow$ $y = 0.065$ RHS > LHS

1) y_D v.s. y_A

$$\frac{P_D}{F_D} = \frac{q}{2}\sum_{i=1}^{20} \frac{1}{\left(1+\frac{y_D}{2}\right)^i} + \frac{1}{\left(1+\frac{y_D}{2}\right)^{20}}$$

$$= \frac{q}{2}\sum_{i=1}^{20} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i} + \frac{1}{\left(1+\frac{\hat{r}(10)}{2}\right)^{20}}$$

$\hat{r}(10)$ largest

$$\gtrsim \frac{q}{2}\sum_{i=1}^{20} \frac{1}{\left(1+\frac{\hat{r}(10)}{2}\right)^i} + \frac{1}{\left(1+\frac{\hat{r}(10)}{2}\right)^{20}}$$

$$y_D < \hat{r}(10) = y_A.$$

2) y_c v.s. y_A. $T = 10$.

$$\frac{P_c}{A_c} = \sum_{i=1}^{20} \frac{1}{\left(1+\frac{y_c}{2}\right)^i} = \sum_{i=1}^{20} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i}$$

$\hat{r}(10)$ is the largest $\rightarrow$

$$> \sum_{i=1}^{20} \frac{1}{\left(1+\frac{\hat{r}(10)}{2}\right)^i}$$

$$y_C < \hat{r}(10) = y_A, \qquad y_B < y_A.$$

3). y_B, v.s. y_C $T_B = 5$, $T_C = 10$,

$$\frac{P_C}{A_C} = \sum_{i=1}^{20} \frac{1}{\left(1+\frac{y_C}{2}\right)^i} = \sum_{i=1}^{20} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i}$$

$$= \sum_{i=1}^{10} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i} + \sum_{i=11}^{20} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i}$$

$$\overset{\|}{\frac{P_B}{A_B}} = \sum_{i=1}^{10} \frac{1}{\left(1+\frac{y_B}{2}\right)^i}$$

$$y_B \le \hat{r}(5) \le \hat{r}(i/2) \quad i = 11, \cdots, 20,$$

$$\sum_{i=1}^{20} \frac{1}{\left(1+\frac{y_C}{2}\right)^i} = \sum_{i=1}^{10} \frac{1}{\left(1+\frac{y_B}{2}\right)^i} + \sum_{i=11}^{20} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i}$$

$$< \sum_{i=1}^{10} \frac{1}{\left(1+\frac{y_B}{2}\right)^i} + \sum_{i=11}^{20} \frac{1}{\left(1+\frac{y_B}{2}\right)^i}$$

$$= \sum_{i=1}^{20} \frac{1}{\left(1+\frac{y_B}{2}\right)^i}$$

$y_B < y_C$

In summary, $y_B < y_C < y_A$, $y_D < y_A$

3) y_C v.s. y_D

$$\frac{P_D}{F_D} = \frac{q}{2}\sum_{i=1}^{20} \frac{1}{\left(1+\frac{\hat{r}(i/2)}{2}\right)^i} + \frac{1}{\left(1+\frac{\hat{r}(10)}{2}\right)^{20}}$$

$$= \frac{q}{2}\sum_{i=1}^{20} \frac{1}{\left(1+\frac{y_D}{2}\right)^i} + \frac{1}{\left(1+\frac{y_D}{2}\right)^{20}}$$

$$y_D < \hat{r}(10) \Rightarrow \frac{1}{(1+\frac{\hat{r}(10)}{2})^{20}} < \frac{1}{(1+\frac{y_D}{2})^{20}}$$

$$\Rightarrow \sum_{i=1}^{20} \frac{1}{(1+\frac{\hat{r}(i/2)}{2})^i} > \sum_{i=1}^{20} \frac{1}{(1+\frac{y_D}{2})^i}$$

$$\parallel$$

$$\sum_{i=1}^{20} \frac{1}{(1+\frac{y_C}{2})^i}$$

$$\Rightarrow y_D > y_C$$

In conclusion, $y_B < y_C < y_D < y_A$.

7. A. 10 premium bond

B. ZCB 10.

C. ZCB η = D_{mac} = 10 yar par-coupon bond

$D_{mac}(B) = 10$

D_{mac} coupon bond $\leq$ maturity

$\Rightarrow \eta \leq 10$

A 10 yr premium Bond. has a higher coupon than 10 yr par coupon bond.

$P_X > F \Leftrightarrow q > y$

$P_X = F \Leftrightarrow q = y$

$D_{mac} \downarrow$ when $q \uparrow$

$D_{mac}(A) < D_{mac}(\text{par coupon}) = \eta$