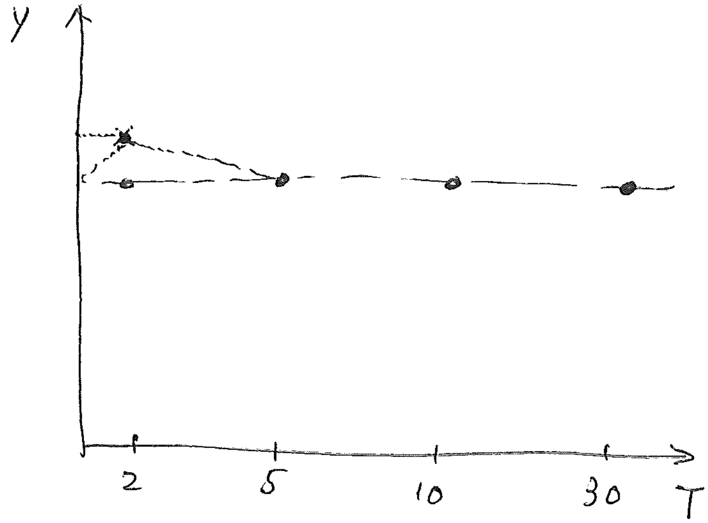

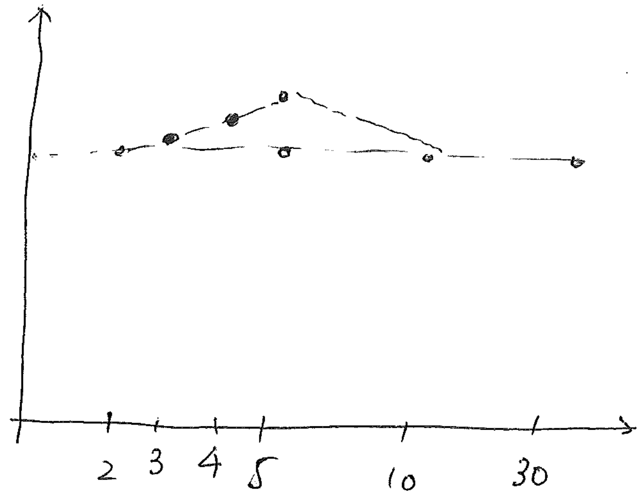

Example:

Reference yield curve. flat, at 5%.

Annuity (30 yr). with $A = \$3250$ every 6 months.

$$\text{Ref } Px = A \sum_{i=1}^{60} \frac{1}{(1+y/2)^i} = 100,453.13$$

perturbations: 1 bp shocks to 2, 5, 10, 30 yr yields.

$y_i^* = 5\%$. $\quad i = 1, 2, 3, 4$.

$y_1 = 5.01\%$, $\quad y_i^* = 5\%$ $\quad i = 2, 3, 4$.

$\Rightarrow d(t) \qquad t = \frac{1}{2}, 1, \cdots, 30$.

$$Px = A \sum_{t=\frac{1}{2}}^{30} d(t)$$

$$\Rightarrow \quad f(5.01,\ 5,\ 5,\ 5) = 100,452.15$$

$$f(5,\ 5.01,\ 5,\ 5) = 100,449.36$$

$$f(5,\ 5,\ 5.01,\ 5) = 100,410.77$$

$$f(5,\ 5,\ 5,\ 5.01) = 100,385.88$$

$$\frac{\Delta P_x}{\Delta y_1} = \frac{100,452.15 - 100,453.13}{0.0001}$$

$$DV01_1 = -\frac{\Delta P_x}{10,000\,\Delta y_1} = 0.98$$

$$D_1 = -\frac{1}{P_x}\frac{\Delta P_x}{\Delta y_1} = 0.098$$

Rate	DV01	Duration
2yr	0.98	0.098
5yr	3.77	0.375
10yr	42.36	4.217
30yr	67.25	6.695

Case 1: $\quad y(t) = 5.01\,\%. \quad \forall t.$

$$\Delta P_x = \sum \frac{\Delta P_x}{\Delta y_i^*} \Delta y_i \qquad \text{first order}$$

$$= -\,(0.98 + 3.77 + 42.36 + 67.25)\ 1\,bp.$$

$$= -\,114.36$$

$y(t) = 5.01\%$

$$P_X = A \sum_{i=1}^{60} \frac{1}{\left(1+\frac{5.01\%}{2}\right)^i} = 100,338.81$$

$$\Delta P_X = P_X - P_X^* = 100,338.81 - 100,453.13$$

$$= -114.32$$

$$\approx -114.36$$

$$\begin{array}{r} 453.13 \\ -338.81 \\ \hline 114.32 \end{array}$$

Case 2: $\Delta y_1 = 5\text{bp}$ $\Delta y_2 = 3\text{bp}$. $\Delta y_3 = 2\text{bp}$ $\Delta y_4 = -1\text{bp}$

$$\Delta P_X = -(0.98 \times 5 + 3.77 \times 3 + 42.36 \times 2 + 67.25 \times (-1))$$

first order

$$= -33.68$$

Hedging using key rates.

Hold ZCB with ~~maturi~~ maturities 2, 5, 10, 30 yr

so that $DV01_i = 0$ for $i = 1, 2, 3, 4$.

DV01 for ZCB.

ZCB	DV01 (100F)	Duration.
2yr	0.018810	1.8810
5yr	0.04376	4.376
10yr	0.077946	7.7946
30yr	0.154543	15.4543

To match DV01.

$$\frac{F_1}{100} \cdot 0.018810 = 0.98 \Rightarrow F_1 = 5,209.99$$

$$\frac{F_2}{100} \cdot 0.04376 = 3.77 \Rightarrow F_2 = 8,632.93$$

$$\frac{F_3}{100} \cdot 0.077946 = 42.36 \Rightarrow F_3 = 54,345.32$$

$$\frac{F_4}{100} \cdot 0.154543 = 67.25 \Rightarrow F_4 = 43,515.40$$

$\square$

X N-dim.

$$\mu = \mathbb{E}[X] = \begin{pmatrix} \mathbb{E}[X_1] \\ \mathbb{E}[X_2] \\ \vdots \\ \mathbb{E}[X_N] \end{pmatrix} \qquad \underset{\substack{\uparrow \\ N \times N}}{\Sigma} = \mathbb{E}\left[(X - \mathbb{E}[X]) (X - \mathbb{E}[X])^T \right]$$

$$= \begin{pmatrix} Var(X_1) & \cdots & Cov(X_1, X_N) \\ \vdots & \ddots & \vdots \\ Cov(X_1, X_N) & \cdots & Var(X_N) \end{pmatrix}$$

$X \qquad \Sigma = C D C^T$

$$Y = C^T (X - \mu)$$

$$\mathbb{E}[Y] = \mathbb{E}[C^T (X - \mu)] = C^T \mathbb{E}[X - \mu] = C^T 0 = 0$$

$$Cov(Y) = \mathbb{E}[Y Y^T] = \mathbb{E}[C^T (X - \mu)(X - \mu)^T C]$$

$$= C^T \mathbb{E}[(X - \mu)(X - \mu)^T] C$$

$$= C^T \underbrace{\Sigma}_{\parallel} C$$

$$= \underbrace{C^T C}_{1_N} \overbrace{D}\, \underbrace{C^T C}_{1_N}$$

$$= D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_N \end{pmatrix}$$

$\mathbb{E}[Y_i] = 0. \quad \forall i = 1, \cdots N$

$\mathrm{Var}(Y_i) = \lambda_i$

$\mathrm{cov}(Y_i, Y_j) = 0. \quad \forall i \neq j$

$$Y = C^T (X - \mu) \qquad C^T C = C C^T = 1_N$$

$$C Y = \underbrace{C C^T}_{1_N} (X - \mu)$$

$$X = \mu + C Y$$

$$X_1 = \mu_1 + \sum_{n=1}^{N} C_{1n} Y_n \approx \mu_1 + C_{11} Y_1 + C_{12} Y_2 + \overbrace{\sum_{n=3}^{N} C_{1n} Y_n}^{\varepsilon_1}$$