Credit & Interest Rate Markets

Goals:

- Describe the most common linear products that trade in credit and interest rate markets.
- Discuss curve construction process for interest rate and credit term structures.

Interest Rates: Traded Products

- **Libor**: Libor is the instantaneous rate at which banks offer to lend to each other.
- **OIS**: Fed funds effective rate. The average rate at which banks lend to each other over a given day.
- **FRA's** / **Eurodollar Futures**: agreements to pay or receive Libor at a future date in exchange for a fixed rate set at trade initiation.
- **Swaps**: A swap is an agreement to exchange cash-flows at a set of pre-determined dates. The most common type of swap is a fixed for floating rate swap.
- **Basis Swaps**: A basis swap is a floating for floating rate swap where Libor is exchanged for Fed Funds / OIS.

Cash vs. Synthetic Instruments

- Bonds, including treasuries and other sovereign bonds, as well as corporate bonds, are cash instruments.
- Derivatives, such as swaps, futures and credit default swaps are synthetic instruments.
- In a cash instrument, a principal amount is received at maturity and there is an upfront cost to purchase the bond.
- In a synthetic instrument, no cashflow changes hands at trade initiation and there is no principal payment at maturity. The trade instead references a set notional value for calculation of each of its cashflows.

Forward Rate Agreements & Eurodollar Futures

- Forward Rate Agreements (FRA's) are agreements to exchange Libor for some fixed rate at some specified future date.
- FRA's are quoted in terms of the annualized rate. The actual payment will be multiplied by the trade **notional** as well as the **year fraction** of the rate exchanged.
- For example, if 3-month Libor is exchanged, a multiplier of 0.25 will be applied to the annualized rate.
- Eurodollar futures are futures contracts linked to 3 month Libor.
- They are exchange traded and have standardized terms.
- Eurodollar futures are quoted in price: $P = 100(1 - L)$

Difference between FRAs and Eurodollar Futures

- The difference between a FRA and a Eurodollar future is that a FRA is an OTC contract and a Eurodollar future is exchange traded.
- Exchange traded products require adjustments to their posted collateral daily, whereas this is not the case for OTC instruments.
- Because of the negative correlation between the P&L of a Eurodollar future and discount rates, a convexity correction emerges in Eurodollar futures.
- Specifically, if we are required to post collateral when rates are higher, we are posting collateral when the value we could earn on our deposits is highest.

Swaps

- A swap is an agreement to exchange cashflows periodically at predetermined future dates.
- A vanilla swap is a *fixed* for *floating rate* swap.
- Swaps can either be spot starting or forward starting.
- Valuing a swap requires the present value of the two legs:
  - Fixed Leg:

$$\text{PV(fixed)} = \sum_{i=1}^{N} \delta_{t_i} C D(0, t_i) \tag{1}$$

    Where C is the set fixed coupon and $D(0, t_i)$ is the discount factor from today until time t_i and δ_{t_i} is the time between payments.

– Floating Leg

$$\text{PV(float)} = \sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0, t_i) \tag{2}$$

Where L_{t_i} is the set fixed coupon and $D(0, t_i)$ is the discount factor from today until time t_i and δ_{t_i} is the time between payments.

- A break-even, or fair swap rate is the rate that equates the present values of the two legs.
- Equating these legs and solving for the unknown, the fixed coupon rate, we are left with:

$$\hat{C} = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(0, t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0, t_i)} \tag{3}$$

Basis Swaps

- A Basis swap is a floating for floating rate swap where the Libor rate is exchanged for the OIS rate.
- In a multi-curve paradigm, we must create a term structure of both Libor and OIS rates.
- Basis swaps provide us the link between the two, and enable us to extract the Libor-OIS basis as a function of time.
- The breakeven basis swap rate can be written as:

$$\hat{B} = \frac{\sum_{i=1}^{N} \delta_{t_i}(L_{t_i} - F_{t_i})D(0, t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(0, t_i)} \tag{4}$$

Where L_{t_i} is the libor rate and F_{t_i} is the fed funds, or OIS rate.

Term Structure of Interest Rates

- Our goal is to extract a consistent, coherent term structure, or yield curve from the set of quoted products, including swaps, FRA's and Eurodollar futures.
- To do this correctly, we really need two term structures, one for Libor and one for the basis between Libor and OIS.
- The term structure is a representation of the interest rate surface that enables us to compute any instantaneous, forward or swap rate.
- The standard representation would a set of underlying **forward rates**.

Components of a Term Structure

- Using our calibrated term structure, we should be able to easily compute the following quantities at any point on our yield curve:
  - Zero Coupon Bonds (Discount Factors)
  - Spot Rates
  - Forward Rates
  - Swap Rates
- A spot rate is the rate that we can instantaneously borrow or lend at for a specified time.
- A forward rate is the rate that we can lock in today to borrow or lend at some point in the future for a specified time.
- A discount factor is the price of a zero-coupon bond starting today with some future maturity date that pays $1 at maturity.

Components of a Term Structure

- Zero Coupon Bonds (Discount Factors):

$$D(S,T) = exp\left(-\int_S^T f(s)ds\right) \tag{5}$$

- Forward Rates

$$F(S,T) = \frac{1}{\delta}\left[exp\left(\int_S^T f(s)ds\right) - 1\right] \tag{6}$$

- Spot Rates

$$\text{Spot}(T) = F(0,T) \tag{7}$$

Components of a Term Structure

- Swap Rates

$$S(S,T) = \frac{\sum_{i=1}^{N} \delta_{t_i} L_{t_i} D(S,t_i)}{\sum_{i=1}^{N} \delta_{t_i} D(S,t_i)} \tag{8}$$

- NOTE: These formulas assume a single rate curve. To incorporate today's multi-curve paradigm we'd need to update the formulas to include Libor-OIS basis.

Extracting a Yield Curve from Market Data

There are two main methods for extracting yield curve construction:

- **Bootstrapping**: Bootstrapping works iteratively beginning with the nearest expiry instrument. For this instrument, we find the constant rate that enables us to match the market observed price. The algorithm then moves onto the next instrument, fixed the part of the term structure that was used to match the previous set of instruments, and solving for the rate over the remaining period that matches the next market price.
- **Optimization**: Alternatively, we can use optimization to minimize the squared distance from our market data.
- For more information, see: Curve Fitting Lecture Notes

Yield Curve: Bootstrapping Procedure

- The exact details of bootstrapping a yield curve in the presence of Libor & OIS rates and a basis between them is beyond the scope of this course.
- Having said that, here's a summary of a simple bootstrapping procedure to give you the idea:
  - Order the securities that you have market prices for by maturity.
  - Begin with the closest maturity security. Invert the formula for ithe FRA / Eurodollar / Swap rate to find the **constant forward rate** that allows you to match the market traded FRA / Eurodollar / Swap Rate.
  - Fix this **constant forward rate** for this time interval.

  - Move to the next traded instrument. Holding the forward rate constant for the cashflows expiring prior to the last instrument, find the **constant forward rate** of the remaining dates that enable us to match market price.
  - Continue to do this until all market instruments have been matched.

- You will have a chance to work through a simple example of this on your next homework.

Credit Markets: Overview

- Credits markets consists of a number of a wide array of products, including:
  - Corporate / Sovereign Bonds
  - Single Name Credit / Sovereign Credit Default Swaps
  - Index Credit Default Swaps
  - Index Credit Default Swaptions
  - Index Credit Default Tranches (CDO's)
  - Credit ETF's
  - Bonds or other derivatives linked to mortgages
- Bonds and CDS are **linear** instruments and are therefore the simplest to model.

Bond Pricing

- In the absence of default, the price of a coupon paying bond can be written as:

$$B(t,T) = \sum_{i=1}^{N} \delta_{t_i} C D(0,t_i) + P D(0,T) \tag{9}$$

Where P is the bond's principal, C is the bond's coupon, T is the bond's maturity and $D(0,t_i)$ is the discount factor until time t_i.

- This bond is sensitive to changes in interest rates, but assumes that the cashflows are certain.

- If we introduce default risk, some or all of these coupons in addition to the principal might not be received.

Pricing a Defaultable Bond

- When pricing a defaultable bond, we need to account for the possibility that the issuer will be unable or unwilling to re-pay a coupon or principal amount.
- Should this happen, we may get some recovery amount for the bond, R, less than the principal value.
- This makes the distribution of the underlying bond price bi-modal and we must handle:
  - Default Case
  - No-Default Case

Modeling Defaults: Hazard Rate Model

- Pricing defaultable bonds and CDS requires modeling of default events for the underlying assets.
- The standard approach to pricing and risk management of defaultable bonds and CDS is a **reduced form model**.
- In this model, we use a hazard rate approach, and structure it similarly to how we structured discount factor and a forward rates in the context of a yield curve.

Modeling Defaults: Hazard Rate Model

- Recall that a discount factor can be written as:

$$D(0,T) = exp(-\int_0^T f_\tau d\tau) \tag{10}$$

- Similarly, a survival probability can be written as:

$$Q(0,T) = exp(-\int_0^T \lambda_\tau d\tau) \tag{11}$$

- The instantaneous forward rate, f_τ above is analogous to the instantaneous default intensity, or hazard rate, λ_τ
- Mathematically speaking, we treat defaults as Poisson processes. The time between defaults then follows an exponential distribution, which will give us probability of default in some period equal to: $\lambda_\tau Q(0,T)$.

Pricing a Defaultable Bond

- The No-default case is the same as we saw before, except that now we need an additional survival probability term.
- The default case is new. Conditional on default, we will receive the discounted recovery value.
- The price of a defaultable bond can be written as:

$$B(t,T) = \sum_{i=1}^{N} \delta_{t_i} C D(0,t_i) Q(0,t_i) + P D(0,T) Q(0,T) + \int_0^T D(0,\tau) Q(0,\tau) R \lambda d\tau$$

Here, R is the recovery value, $Q(0,t_i)$ is the survival probability until time t_i and λ is the default intensity, or hazard rate.

Credit Default Swaps

- A credit default swap is analogous to buying life insurance on a corporation or government.
- In a CDS, we exchange a periodic fixed coupon payment in exchange for a payment conditional on default of the underlying issuer.
- There is a precise definition of what exactly constitutes a default, which may impact the CDS's valuation
- Like an interest rate swap, a CDS consists of two legs:
  - Premium or Fee Leg
  - Default Leg

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Credit Default Swaps

- The value of the premium and default legs can be written as:

$$\text{PV(premium)} = \sum_{i=1}^{N} \delta_{t_i} S D(0, t_i) Q(0, t_i)$$

$$\text{PV(default)} = \int_0^T D(0, \tau) Q(0, \tau) R \lambda d\tau$$

- Notice that there is no exchange of a principal payment.
- The break-even, or fair CDS spread is the spread that equates the values of the two legs:

$$\hat{S} = \frac{\int_0^T D(0, \tau) Q(0, \tau) R \lambda d\tau}{\sum_{i=1}^{N} \delta_{t_i} D(0, t_i) Q(0, t_i)} \tag{12}$$

Credit Default Swaps: Risks

- In order to implement a CDS pricing model, we need to employ a quadrature technique, that is, we need to numerically approximate the value of the integral in the default leg present value formula.
- CDS contracts have exposure to many risk factors:
  - Jump to Default Risk
  - Credit Spread / Hazard Rate Risk
  - Interest Rate Risk
  - Changes to the Recovery Rate
- CDS contracts have a number of conventions and standard features that are beyond the scope of this course. For more information see CDS Snac Docs

Term Structure of Hazard Rates

- CDS for a given corporation of government entity may trade for multiple maturities, just as a corporation of sovereignty may issue bonds with different maturities.
- In practice, the most liquid CDS contract has a five year maturity.
- To connect multiple CDS for a given entity, we need to take a credit spread curve and extract a term structure of hazard rates.
- This process is a **calibration** of our hazard rate model to market traded credit spreads.

CDS Curve: Bootstrapping Procedure

- The bootstrapping procedure for a CDS curve is analogous to the bootstrapping procedure we used to construct a yield curve.
- Specifically, we begin with the shorted maturity CDS and find the hazard rate that enables us to match the traded market spread.
- We then fix this hazard rate for the time period, move to the next maturing CDS and find the hazard rate that matches this CDS's traded spread.
- When we do this, we need to solve the CDS breakeven, or fair spread formula in (12) for λ.

CDS Curve: Bootstrapping Procedure

- Notice that to do this, we need to make λ a function of time to match more than one CDS spread. The standard specification is to assume $\lambda(t)$ is piecewise constant.
- Also note that when we do this all other parameters, such as recovery rate and interest rates should be fixed and set as inputs to our bootstrapping procedure.

Corporate Bonds vs. CDS

- You may notice that a **long position in a defaultable bond** and a **short protection position in a CDS** have similar risk profiles.
- In particular, they are both exposed to **default risk** of the underlying issuer.
- They are also both have sensitivity to changes in interest rates.
- One might think that this position is riskless and therefore the bond and the CDS should trade at the same spread.
- The main difference between the two is that a defaultable bond is a **cash** instrument that requires us to post collateral and a CDS is a **synthetic** instrument that requires no cashflows at origination.
- This funding difference causes bonds and CDS to trade at different spread levels.

Corporate Bonds with Credit Risk

- Because of this difference in spreads on the bond and the CDS that arise due to funding, when we model a defaultable cash bond we need to include what is referred to as a basis between the two spreads.
- Investors may analyze this basis and try to spot anomalies.

Derivatives on Rates & Credit

- Both the credit and rates markets have liquid OTC derivatives markets.
- In both markets, swaptions are the most liquid derivatives.
  - Credit Default Swaptions on Indices
  - Interest Rate Swaptions
- Other derivatives also exist and trade frequently, such as credit tranches and exotics in rates.
- These products may seem esoteric but they actually make up a significant part of the market.
- We will discuss dive into modeling of these derivatives in the next lecture...