Homework 1

Shi Bo

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1 Task1: Historical Analysis of Sector ETFs

(a) Download historical price data from January 1st 2010 for the following ETFs on yahoo finance or another site of your choice. Clean/check the data for splits and other anomalies.

Let us first download the historical price data by using yfinance package, Head of price data is shown as follows:

	A	B	C	D	E	F	G	H	I	J	K
1	Date	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
2	2010-01-04	91.8419	26.9230	43.2547	7.6123	22.8315	19.6175	20.0680	21.1976	26.2979	25.7822
3	2010-01-05	92.0850	27.0101	43.6077	7.7522	22.9122	19.5922	20.0756	20.9452	26.0400	25.8767
4	2010-01-06	92.1498	27.4691	44.1299	7.7678	22.9605	19.3730	20.0605	21.0680	26.3062	25.9111
5	2010-01-07	92.5388	27.2554	44.0637	7.9336	23.2104	19.2971	20.0605	20.9725	26.3978	26.1259
6	2010-01-08	92.8468	27.6353	44.3506	7.8870	23.5813	19.4236	19.9928	20.9521	26.4393	26.1173
7	2010-01-09	92.97643	27.48492	44.29175	7.892157	23.83929	19.3477	20.04547	21.17031	26.5891	26.06578

Figure 1: Head of price data

and check whether there are anomalies, results as follows:

	A	B	C
1		price_data_ano	return_data_ano
2	SPY	0	0
3	XLB	0	0
4	XLE	0	0
5	XLF	0	0
6	XLI	0	0
7	XLK	0	0
8	XLP	0	0
9	XLU	0	0
10	XLV	0	0
11	XLY	0	0

Figure 2: checking anomalies

It is obvious that there are no anomalies.

(b) Calculate the annualized return and standard deviation of each ETF.

First, let us define the formula for annualized return: $r_{annual} + 1 = (r_{daily} + 1)^{252}$

	Ann_Return	Standard_Deviation
SPY	0.195725701	0.172437486
XLB	0.125817978	0.215697478
XLE	-0.028835667	0.266611814
XLF	0.177075833	0.250046858
XLI	0.181218495	0.203021191
XLK	0.276061968	0.200980881
XLP	0.174344699	0.140314253
XLU	0.150100113	0.176249539
XLV	0.207917382	0.168656183
XLY	0.266835169	0.185543688

Figure 3: annualized return and std

(c) Calculate the co-variance matrix of daily and monthly returns. Comment on the differences in correlations at different frequencies.

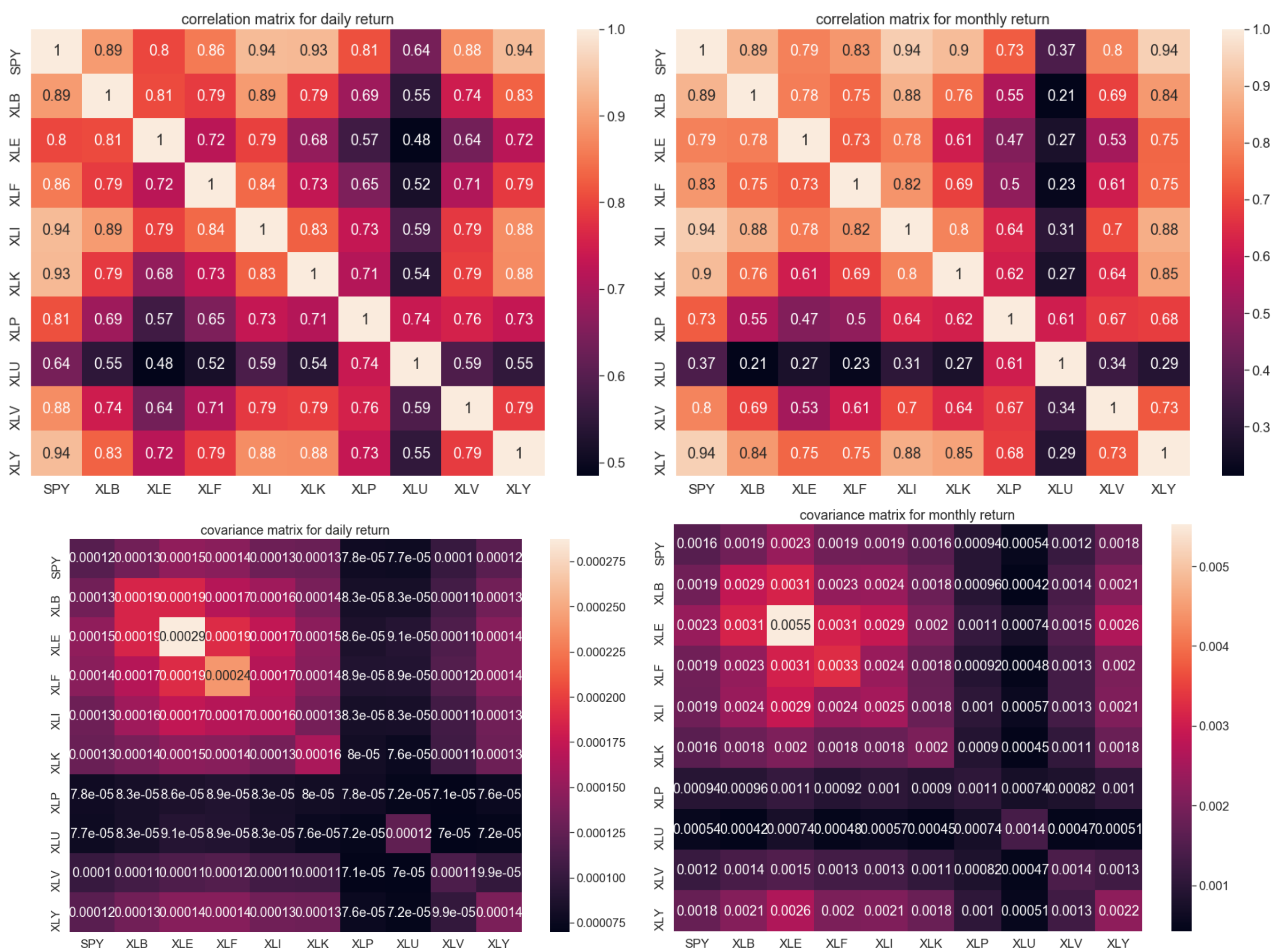

Figure 4: Daily and monthly return correlation and co-variance matrix

From the figures above, we can clearly see that correlation of daily returns are slightly higher than that of monthly returns, but the difference is tiny. Oppositely, situations can not be found on co-variance matrix. One of the reason is probably that these co-variances is not adjusted by the standard deviation of the returns themselves.And the difference these two values is represented as below:

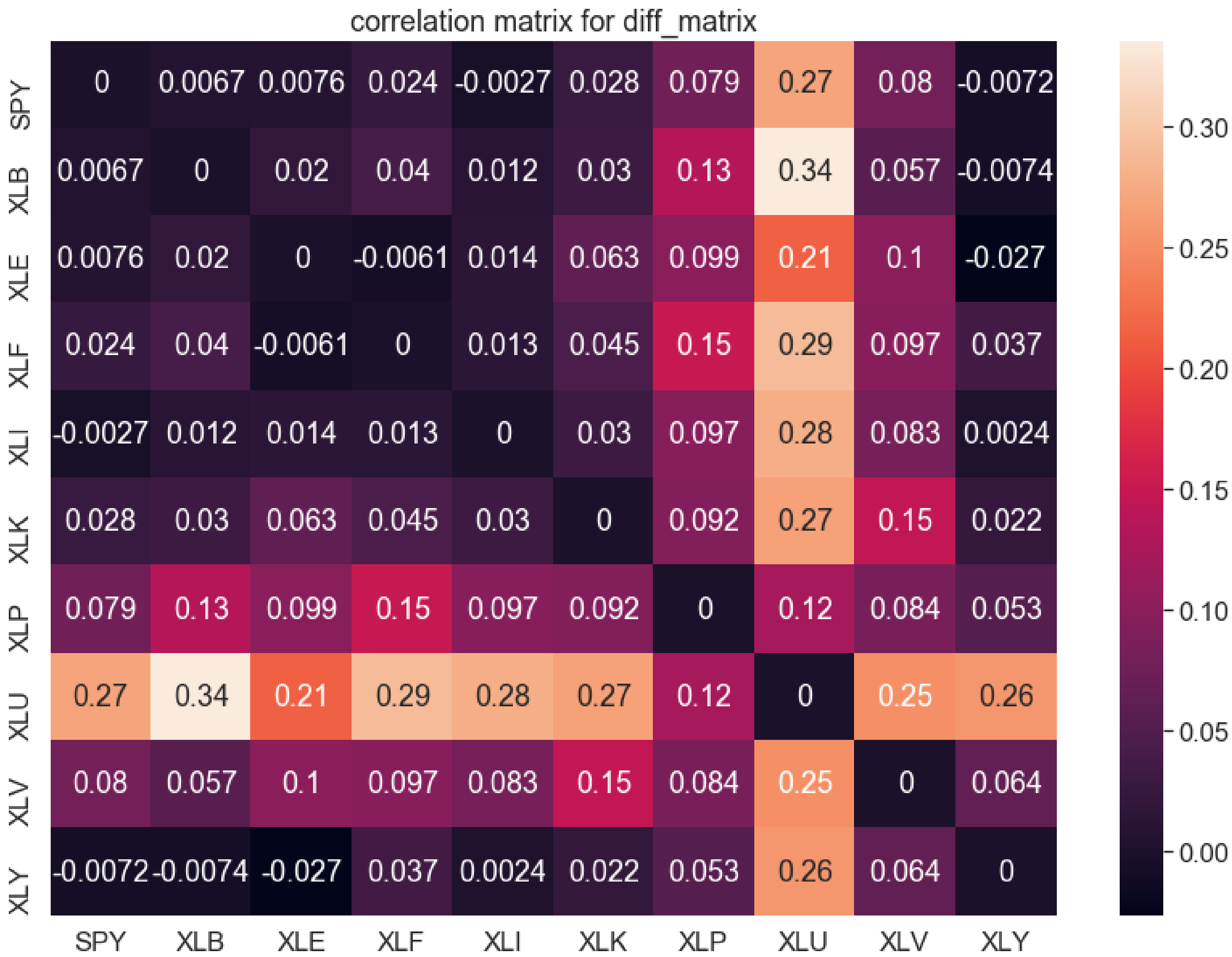

Figure 5: annualized return and std

(d) Calculate a rolling 90-day correlation of each sector ETF with the SP index. Do the correlations appear to be stable over time? What seems to cause them to vary the most?

The curve for rolling 90-day correlation of each sectors ETF with the SP index is presented as below:

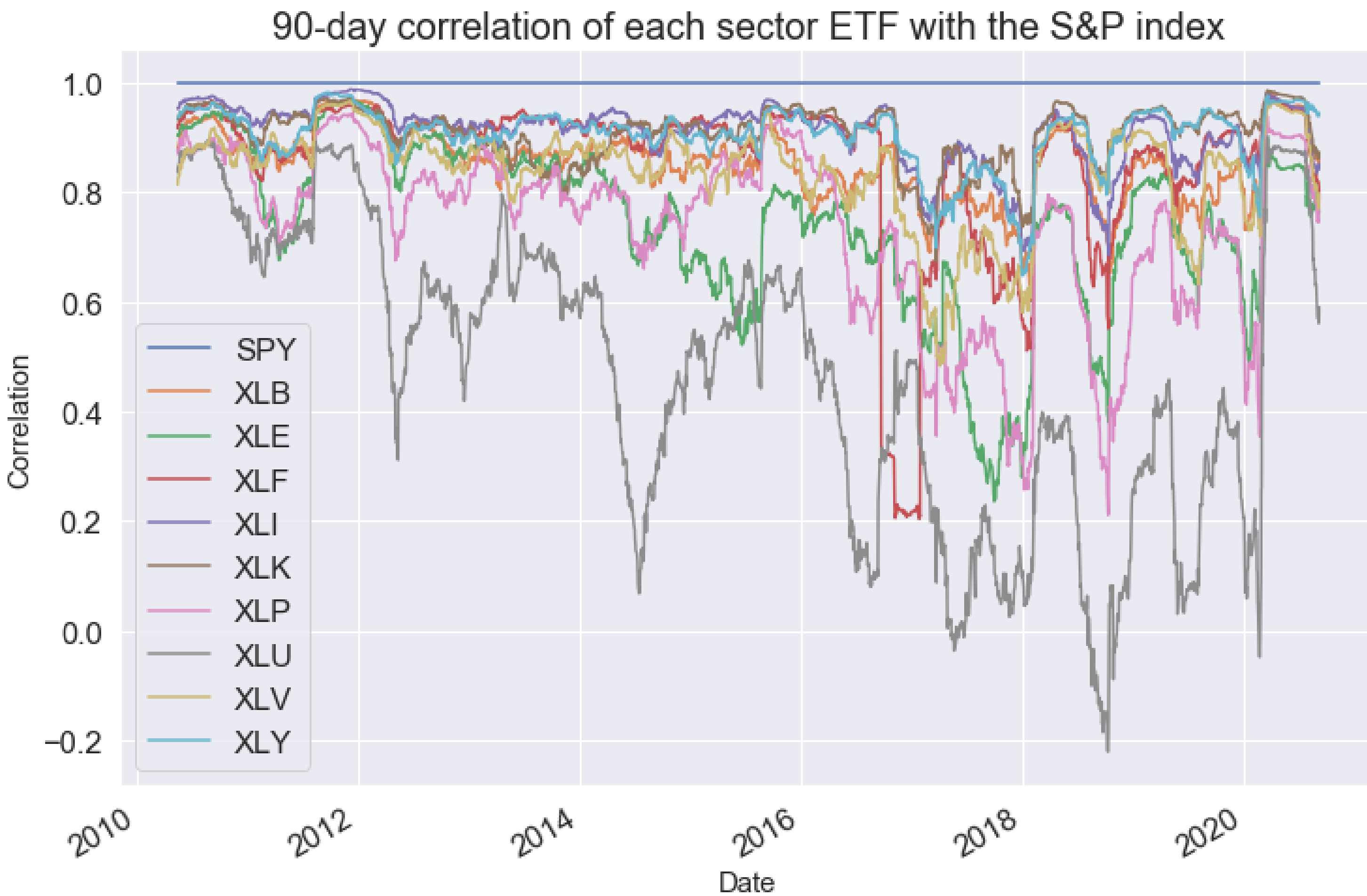

Figure 6: rolling correlation

Clearly, 90-days correlation of each sector ETF with the SP index is not stable over time and it fluctuates smoothly from 2010 to the beginning of 2013 and soon drastically in the rest of period.This may owing to the divergent performances of different ETF with respect to SP index at different time.

(e) Consider a single factor CAPM model where returns are composed of the market return and an idiosyncratic component:

$$r_{i,t} = \beta r_{\mathrm{mkt},t} + \epsilon_{i,t}$$

For each sector ETF, **compute it's to the market using the above model. This will require computing a linear regression on stock returns. Compute the for the entire historical period and also compute rolling 90-day 's. Are the 's that you calculated consistent over the entire period? How do they compare to the rolling correlations?**

	SPY	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
beta	1	0.716061	0.515315	0.60103	0.795031	0.80047	0.999341	0.628332	0.899155	0.868913

Figure 7: beta(whole period)

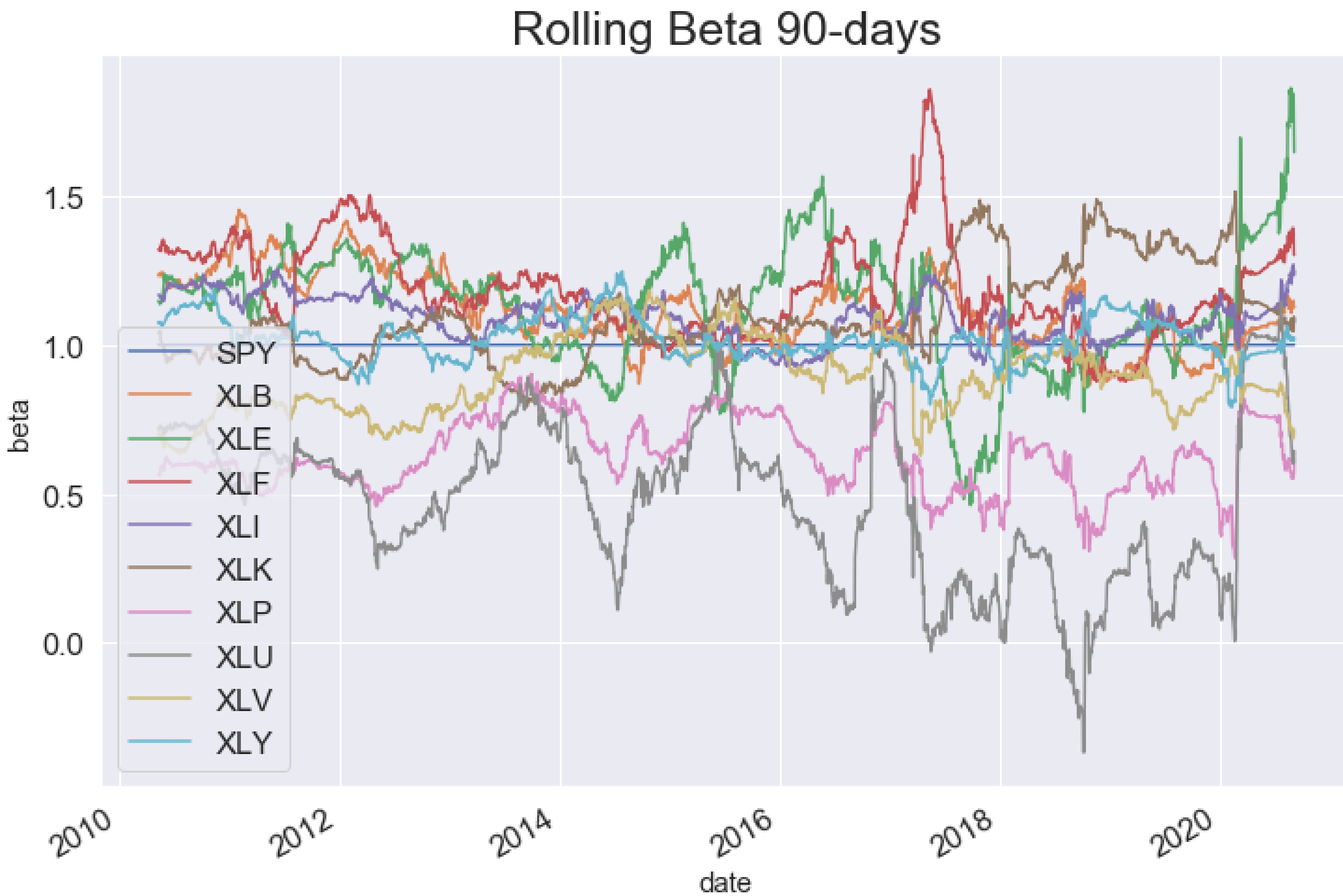

Figure 8: rolling beta 90-days

From the charts above, we could summarize that 90-days rolling beta of these sectors is almost stable. However, the sector of Consumer Staples and Utilities is not quite consistent with others. Moreover, for the whole period beta, Energy sector is not that closely related to market with the lowest beta of 0.515315 and it is not tough to imagine that the Consumer Staple sector has highest beta with corresponding to market.

2 Task2: Exotic Option Pricing via Simulation

Consider a one-year fixed strike lookback option which enables the buyer to choose the point of exercise for the option at its expiry. Recall that the dynamics of the Black-Scholes model can be written as:

$$S_t = rS_t dt + \sigma S_t dW_t$$

And that each Brownian motion increment dW t is normally distributed with mean zero and variance t. Assume that r = 0, S0 = 100 and sigma = 0.25.

(a) Generate a series of normally distributed random numbers and use these to generate simulated paths for the underlying asset. What is the mean and variance of the terminal value of these paths? Does it appear to be consistent with the underlying dynamics?

First, let us generate a random walk process with simulated time as 1000, step as 250. The plot of the simulated paths is as follows:

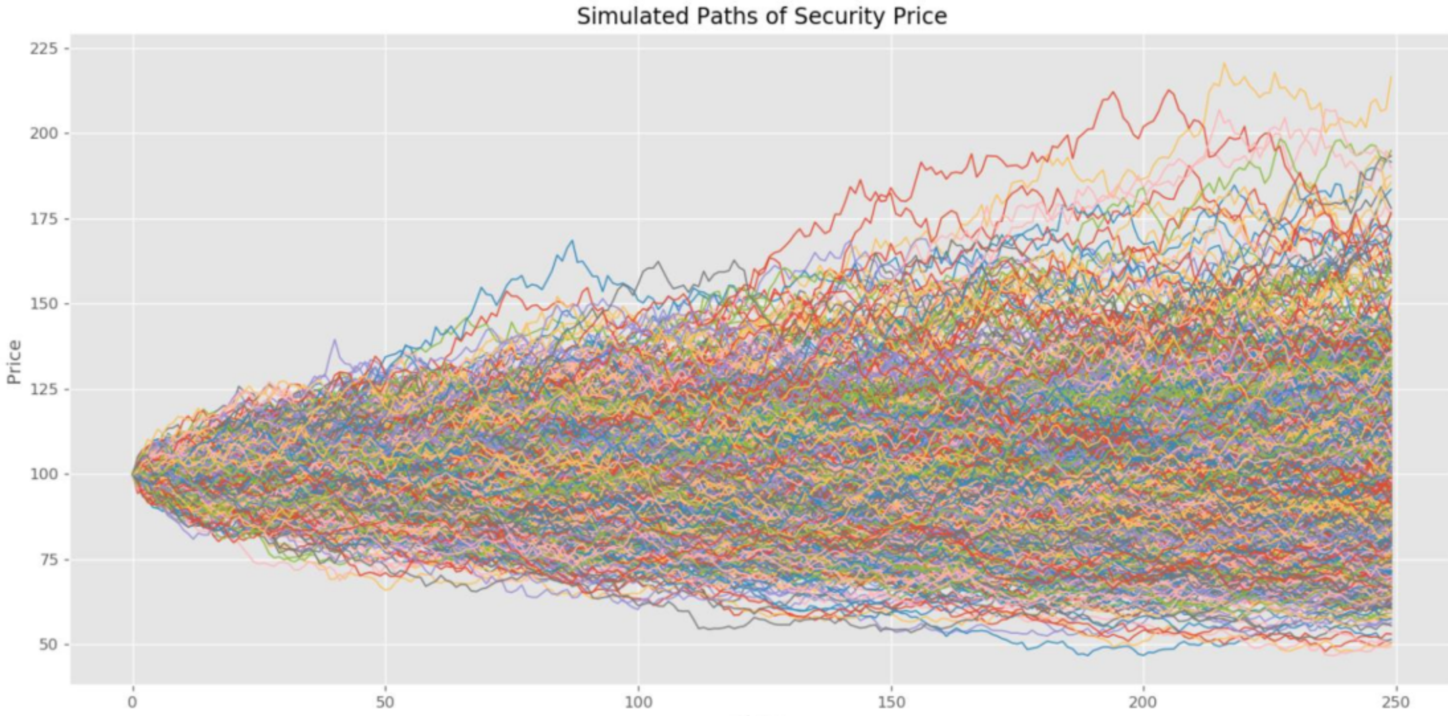

Figure 9: Random walk
The mean of the terminal value of these paths is: *100.82462161210616*
The std of the terminal value of these paths is: *626.9332522796601*
The paths of the simulations are consistent with the underlying dynamics, because they all have a clear tendency that the price of the second day depends on the price of the last day, plus a random walk.

(b) Calculate the payoff of a European put option with strike 100 along all simulated paths. Make a histogram of the payoffs for the European option. What is the mean and standard deviation of the payoffs?

The histogram of payoff of European put option is shown as follows:

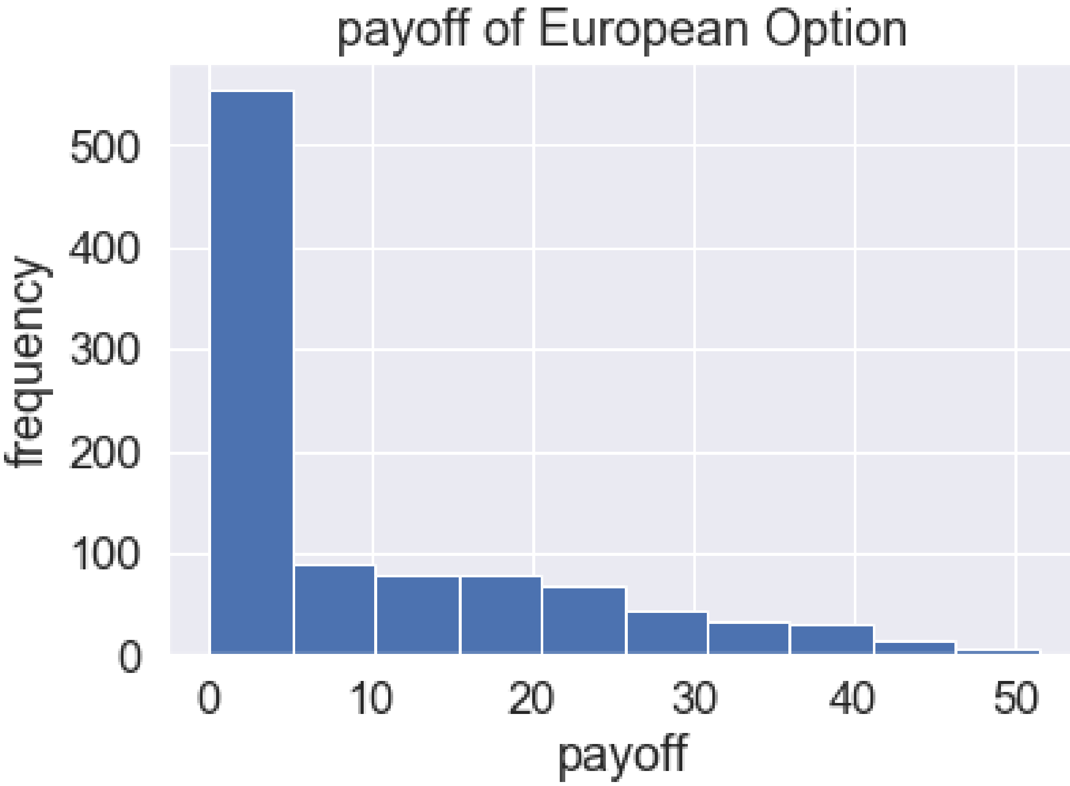

Figure 10: payoff of European option
the mean of the payoffs: *9.35*
the standard deviation of the payoffs: *12.28*

(c) Calculate a simulation approximation to the price of a European put option by taking the average discounted payoff across all paths.

Since r = 0, the discounted value of the mean of payoffs is the same as the original mean of payoffs. Therefore, the approximate price of the option is the mean of payoffs, which is <u>9.73368954774781</u>.

(d) Compare the price of the European put option obtained via simulation to the price you obtain using the Black-Scholes formula. Comment on any difference between the two prices.
By using the black-scholes formula,

$$p_0 = \Phi(-d_2) K e^{-rT} - \Phi(-d_1) S_0$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left(\log\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right)$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

we have the price of European Option calculated by BSM is *9.94764496602258* and the difference between these two methods is *-0.2139554182747698З*. As we notice here, the difference between these two values are very tiny and in fact, we could imagine that as the times of simulation increases the difference will be smaller and smaller.

(e) Calculate the payoff of a fixed strike lookback put option with stike 100 along all simulated path. (HINT: The option holder should exercise at the minimum price along each simulated

path). Calculate the simulation price for the lookback option by averaging the discounted payoffs.

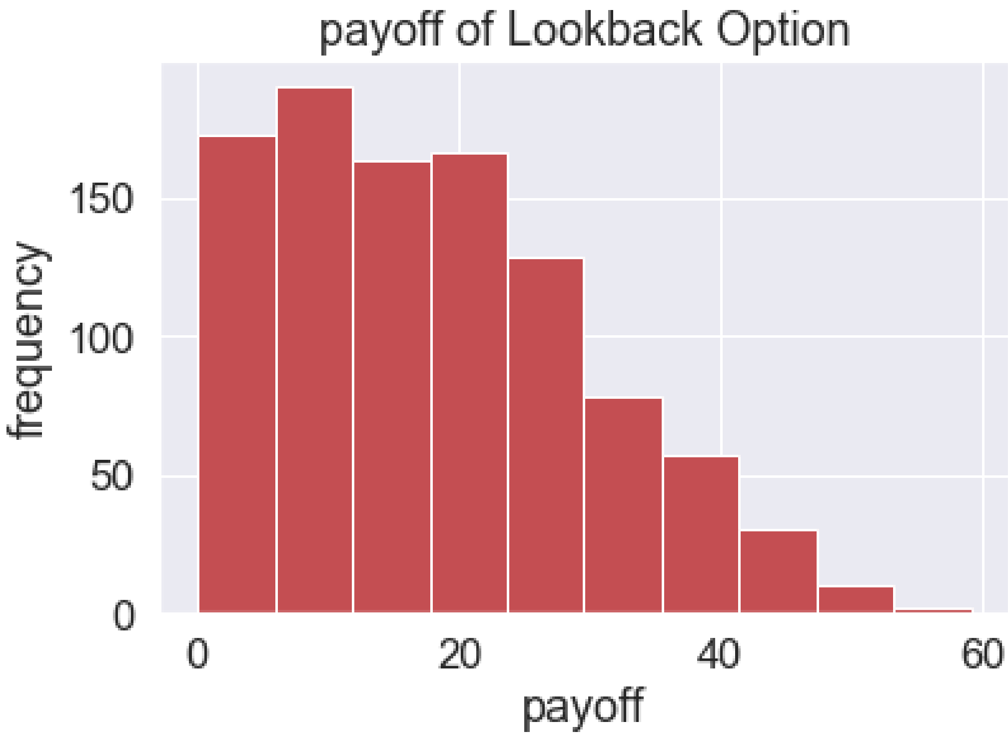

Figure 11: lookback payoff

For the lookback option, we find the minimum price along the path and use it to calculate the payoff. Since r = 0, the discounted value of the mean of lookback payoffs is the same as the original mean of lookback payoffs. Therefore, the simulation price of the lookback option is 17.660706649379307.

(f) Calculate the premium that the buyer is charged for the extra optionality embedded in the lookback. When would this premium be highest? Lowest? Can it ever be negative?

The premium is calculated by the simulation lookback option price minus the simulation European option price, which is 7.987083047954233.
The premium is highest when the option begins and lowest when the option expires. This is because when the minimum is at the beginning, the discounted payoff is the same as the original payoff, hence the premium is highest. It is lowest at the end of the option because the discounted payoff is lowest.
The premium cannot be negative. Suppose the execution price for lookback option is S_l , and execution price for European option is S_e . The payoff for European option is $(\max(K - S_e, 0)) \times e^{-rT}$. Since $S_l \leq S_e$, and the discount period of lookback option is less than T, the payoff of lookback option cannot be smaller than European option. Hence, the premium cannot be negative.

(g) Try a few different values of and comment on what happens to the price of the European, the Lookback option and the spread/premium between the two.
Choosing sigma = 0.25,0.5,0.75,1.00, the price of European, lookback option and premium are:

sigma	European	lookback	premium
0.25	9.947644966	18.0504093	7.67189305
0.5	19.74126514	31.99288582	12.71692295
0.75	29.23395333	47.32568818	16.17701501
1	38.29249225	45.28351876	16.2987625

Figure 12: different sigma

If increasing the sigma, the price of options would increase, the premium would increase,is decreasing the sigma, the price of options would decrease, the premium would decrease.