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Homework Assignment 3
703 Programming for mathematical Finance
Volatility Experiments

2020-10-22

1 Data Downloading

Download historical data of SPY and VIX from 2010-01-01 to 2020-06-30 and show head of data:

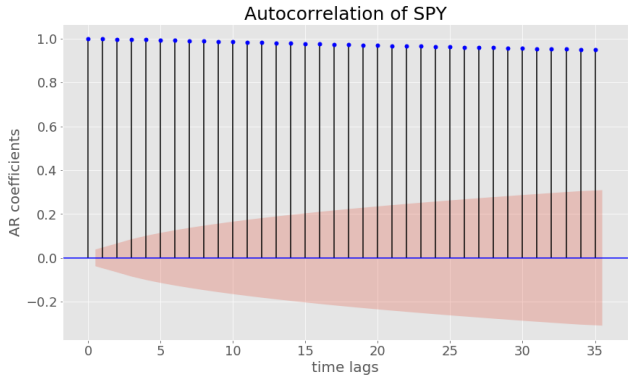
```
In [155]: SPY_data
Out[155]:
                Adj Close
Date
2010-01-04    91.475693
2010-01-05    91.717857
2010-01-06    91.782425
2010-01-07    92.169884
2010-01-08    92.476562
```

```
In [156]: VIX_data
Out[156]:
                Adj Close
Date
2010-01-04    20.040001
2010-01-05    19.350000
2010-01-06    19.160000
2010-01-07    19.059999
2010-01-08    18.129999
```

Figure 1: Daily and monthly return correlation and co-variance matrix

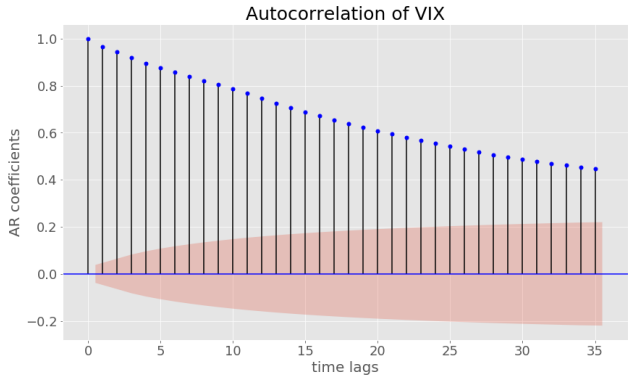
2 Autocorrelation Test

The results of AR-test as follows:




```
AR_test for SPY index:
    AR_coeff  p_value
0   1.000000      0.0
1   0.998449      0.0
2   0.997152      0.0
3   0.995634      0.0
4   0.994145      0.0
```

Figure 2: SPY AR-test




```
AR_test for VIX index:
    AR_coeff  p_value
0   1.000000      0.0
1   0.966323      0.0
2   0.944198      0.0
3   0.919572      0.0
4   0.895132      0.0
```

Figure 3: VIX AR-test

From the figures above, we can clearly see that SPY and VIX are highly auto-correlated, SP has higher autocorrelation and it is not a stationary data. The reason may be that VIX is not a trade-able asset so it is more mean-reverting. However, SPY is trade-able and so we are less likely to find mean reversion due to market efficiency.

3 Correlation of SPY and implied volatility

daily correlation between SPY and VIX is -0.18675842857383018
monthly correlation between SPY and VIX is -0.18620058421058086
A significant negative correlation can be found between SPY and VIX index on daily and monthly basis, which implies that the implied volatiliy is not constant cross stock prices. But one of the assumptions of Black–Scholes-Merton model is that volatility should be constant. Then this means that investors are willing to pay an extra ”premium” when markets have downside risk.

4 Rolling 90-day Correlation of SPY and implied volatility

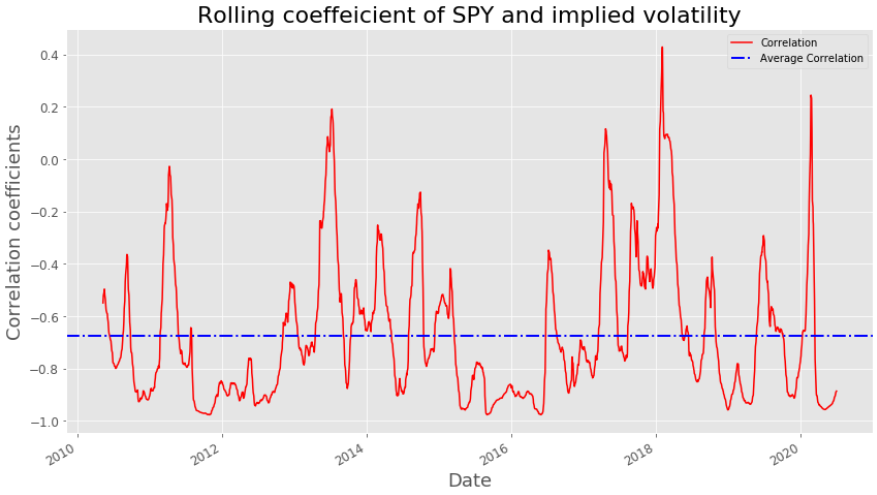


Figure 4: 90-days Correlation of SPY and implied volatility

Apparently, the rolling correlation coefficients has a very huge volatility and its long term average is approximately -0.7. It deviated from the long run average periodically, especially, in 2008, 2013 and 2018.

5 Premium of Volatility

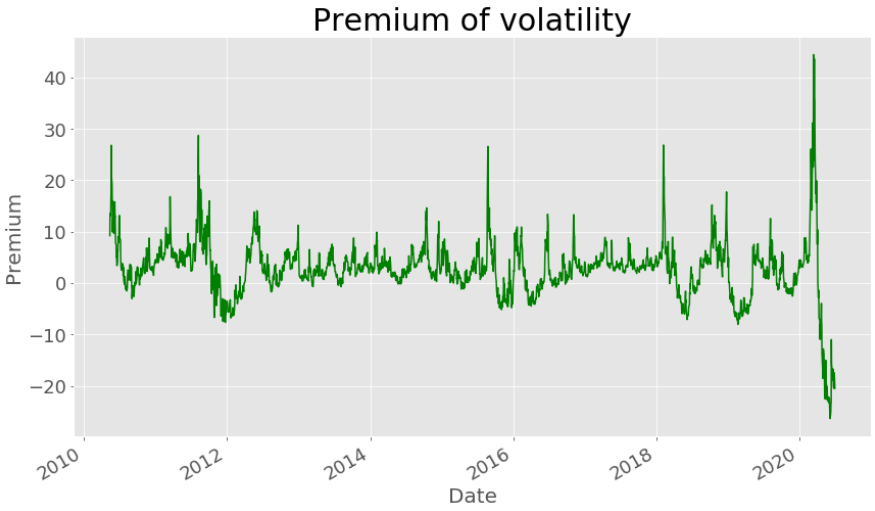


Figure 5: Premium of Volatility

$$\mathrm{R_t} = Ln\frac{\mathrm{P_t}}{\mathrm{P_{t-1}}}$$

Where:
Ln = natural logarithm
Pt = Underlying Reference Price ("closing price") at day t
Pt–1 = Underlying Reference Price at day immediately preceding day t

$$\mathrm{Vol} = 100 \cdot \sqrt{\frac{252}{\mathrm{n}} \sum_{\mathrm{t=1}}^{\mathrm{n}} \mathrm{R_t^2}}$$

Where:
Vol = Realized volatility
252 = a constant representing the approximate number of trading days in a year
t = a counter representing each trading day
n = number of trading days in the measurement time frame
Rt = continuously compounded daily returns as calculated by the formula above

The premium is positive except for a few of periods including 2008 crisis. The premium is high when market is doing well and is low when shocks hit for which the result also can be found in recent period. Before Covid-19, the premium increased significantly and when the virus broke out, the premium is falling sharply.

6 Straddle portfolio construction

prices of straddle:

Date	SPY	VIX	Call Price	Put Price
2010-01-04	91.475693	0.2004	2.110874	2.110874
2010-01-05	91.717857	0.1935	2.043609	2.043609
2010-01-06	91.782425	0.1916	2.024972	2.024972
2010-01-07	92.169884	0.1906	2.022910	2.022910
2010-01-08	92.476562	0.1813	1.930631	1.930631

Figure 6: Straddle Price

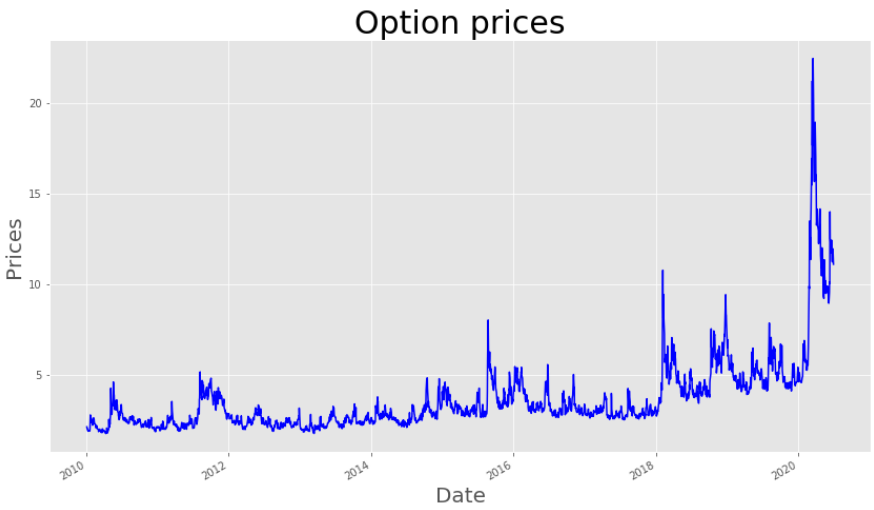


Figure 7: Option prices

7 Payoff of Straddles

straddle info with profit and loss and payoff:

Date	SPY	VIX	Call Price	Put Price	at expiry 1M later
2010-01-04	91.475693	0.2004	2.110874	2.110874	88.650642
2010-01-05	91.717857	0.1935	2.043609	2.043609	85.914337
2010-01-06	91.782425	0.1916	2.024972	2.024972	86.091965
2010-01-07	92.169884	0.1906	2.022910	2.022910	85.470390
2010-01-08	92.476562	0.1813	1.930631	1.930631	86.543930

Date	payoff	profit
2010-01-04	2.825050	-1.396698
2010-01-05	5.803520	1.716302
2010-01-06	5.690460	1.640515
2010-01-07	6.699493	2.653673
2010-01-08	5.932632	2.071370

Figure 8: P and L table

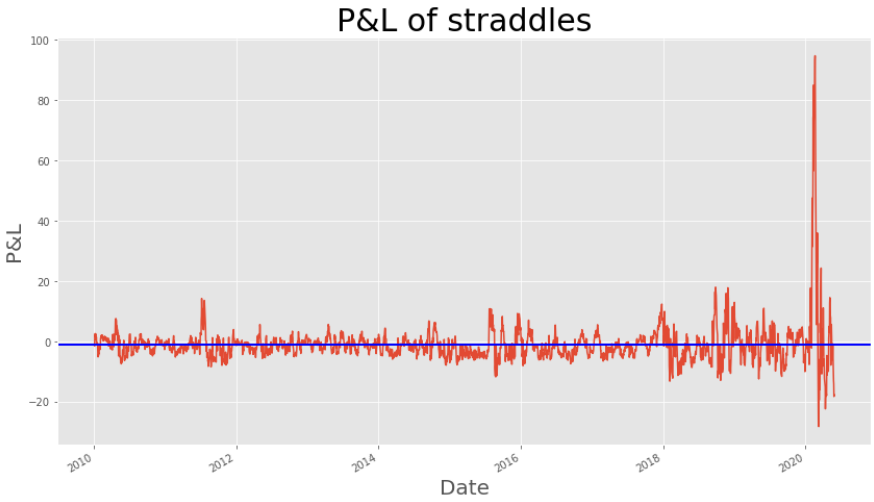


Figure 9: P and L plot

The payoff of 1-month straddle is the absolute difference between SP 500 today and SP 500 one month later and the average P/L of these straddles is: -1.06946. The volatility of this payoff appears extreme abnormality due to Covid-19.

8 P/L of straddles volatility premium

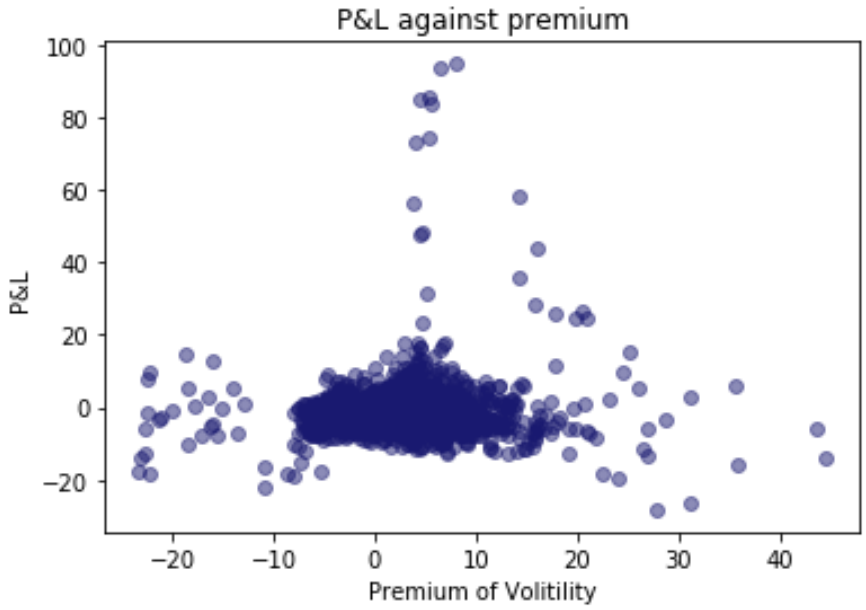


Figure 10: scatter plot of P and L against Premium

OLS Regression Results

Dep. Variable:	Premium of volitility	R-squared (uncentered):	0.001
Model:	OLS	Adj. R-squared (uncentered):	0.000
Method:	Least Squares	F-statistic:	1.897
Date:	Thu, 08 Oct 2020	Prob (F-statistic):	0.169
Time:	23:05:26	Log-Likelihood:	-8107.6
No. Observations:	2529	AIC:	1.622e+04
Df Residuals:	2528	BIC:	1.622e+04
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
profit	-0.0240	0.017	-1.377	0.169	-0.058	0.010

Omnibus:	525.807	Durbin-Watson:	0.089
Prob(Omnibus):	0.000	Jarque-Bera (JB):	7915.521
Skew:	0.550	Prob(JB):	0.00
Kurtosis:	11.597	Cond. No.	1.00

Figure 11: OLS results

The weakly negative relationship between them could be found as the above results represent.The intuition probably is that the payoff of this straddle is positively related to the realized volatility but the option prices that we need to pay is positively related to the implied volatility. So the PL of this straddle is negatively related to volatility premium.