Back-testing & Quant Trading

Goals:

- Discuss back-testing quantitative investment strategies.
- Describe some common algorithmic trading strategies.
- Discuss the practicalities of implementing quant strategies, and the challenges of building strategies on options & futures.

Quantitative Investment Strategies: Overview

- Quantitative investment strategies rely on mathematical methods in order to deduce patterns in underlying asset prices.
- Quantiative investment strategies may be based on only price data, or may be based on price data in conjunction with fundamental data.
- In general, Quant investment methods try to take advantage of the central limit theorem and law of large numbers in a context where the odds are marginally skewed in your favor.
- The canonical example of this is the flip of a biased coin (say with 51% probability of heads).

Quantitative Investment Strategies: Overview

- On any individual observation, we have no idea what will happen.
- As time progresses, however, and the number of repeated experiments accumulates, other factors tend to cancel out and these strategies are able to take advantage of the bias.
- The degree to which the odds are skewed in your favor is ultimately determined by your **underlying signal**.
- The process of uncovering these signals is referred to as **alpha research**.
- One such signal might be momentum or mean-reversion of an asset, or exposure to a particular factor of interest (i.e. Size, or Book to Market)

Quantitative Investment Strategies: Stages of an Alpha Research Project

- An alpha research project should begin with some piece of theory or market observation that is turned into a hypothesis to prove or disprove during the research process.
- In any alpha research project our null hypothesis should be that the signal does not work and we should require the burden of proof to show us that the signal is valid.
- After the initial hypothesis, the next step is to identify and store the appropriate data sets. These data sets must then be cleaned.
- The data should then be split into in and out of sample periods.

Quantitative Investment Strategies: Stages of an Alpha Research Project

- The next step would be to build out the signal and whatever parameter estimation is associated with the signal.
- Next, we would take that signal and analyze it's correlation with market movements in the in-sample period.
- If these results are compelling, we would then subject our signal to a full-blown back-test on the in-sample period, and would tweak the model as necessary.
- Finally, we would use our out-of-sample data to validate any positive results that we got in-sample.
- If the results are still compelling, then we would continue with a truly live out-of-sample test, usually initially without any capital.

Back-testing Strategies: Overview

- When we back-test an investment strategy our goal is to simulate as closely as possible what would have happened if we had been engaging in the strategy during the period.
- In a back-test, we build and adjust our hypothetical portfolio according to some set of quantitative rules.
- When we build a back-test we need to be cognizent of what information is available to us at the time we need to make trading decisions.
- There are certain obvious examples, i.e. we should never look ahead in the data, but sometimes they are more subtle.

Components of a Back-testing Engine

- **Data Collection** (asset prices, forward / futures curves, volatility surfaces)
- **Data Cleaning** (checking for splits, arbitrage checks on futures / options)
- **Signal Generation** (i.e. calculating asset expected returns)
- **Position Calculator** for Strategies (using signals generate re-balances / trades)
- Strategy Level **Portfolio Construction** Algorithm
- **Simulated Trading** Algorithm
- **Portfolio Valuation**

Back-testing Strategies: Parameter Estimation

- In order to generate our signals, for example by calculating our expected return model, we almost always need to estimate a set of parameters.
- This is usually accomplished via regression, machine learning or other econometric techniques.
- Earlier in the course, we discussed momentum or mean-reversion in a single time series. In this case, our calibrated coefficient of momentum / mean-reversion would be a model parameter.
- This parameter estimation will likely need to be repeated periodically in the back-test as we evolve time forward to take into account the dynamic nature of the underlying relationships.

Back-testing Strategies: Parameter Estimation

- In order for our signals to work we need to have signals that are not only statistically significant on the data that we use but also to have a good fit to data that is out-of-sample and in some cases not yet known.
- This presents a natural tradeoff between in-sample fit and minimizing the probability of overfitting.
- In practice I try to make my trading models parsimonious and only use parameterizations that have some economic justification.
- That said, this process is a bit of an art and others have different approaches.

Back-testing Strategies: In-sample and Out-of-sample Periods

- When back-testing quant strategies, we almost always try many sets of parameters before finally choosing a calibrated set.
- This inflates our results, as we won't have this luxury when we are live trading!
- Additionally, we often calibrate our parameters over some fraction of the data and then run the back-test on that same period (so for this period we have looked ahead).
- The more the data is mangled beforehand, the less likely your results are to persist.

Back-testing Strategies: In-sample and Out-of-sample Periods

- One general rule of thumb is that you should maintain an out-of-sample period of at least one-third of the data.
- That out-of-sample period should then only be used after the parameters have been fully optimized in the in-sample period.
- The degradation of results in the out-of-sample period vs the in-sample period is then quite informative.
- Of course, it is somewhat likely that the first test we run on the out-of-sample period will not produce terrific results, and this will lead us to go back and tweak our parameters/model. Remember that once we've done this the out-of-sample data is no longer really out-of-sample!

Back-testing Strategies: Transactions Costs

- Handling of transaction costs is a critical component of a robust back-testing algorithm.
- Many strategies appear to be appealling until the proper transaction costs are added...
- This phenomenon is especially true for higher frequency strategies which are by nature more sensitive to transaction costs.
- Modeling transaction costs requires us to estimate the following costs:
  - Commissions
  - Bid-Offer
  - Market Impact / Slippage

Back-testing Strategies: Transactions Costs

- In my back-testing algorithms, I think of transactions costs as the expected difference between where the model thinks I can execute and where I will actually be able to execute.
- Some of these costs are fixed, and do not depend on the size of your trade relative to the overall.
- Others will depend greatly on your size, especially the market impact component.

Back-testing Strategies: Transactions Costs

- The simplest approach to modeling transaction costs is to assume that each time you trade a certain percentage is paid in transaction costs.
- In practice I have found that 5bps is a fairly accurate, if a little conservative assumption.
- There is a great deal of literature on modeling market impact and transaction costs for those interested, but the details are beyond the scope of this course.
- In practice, I embed within my back-tests (and trading algorithms) a cap on the shares that I'm allowed to trade, and I set this based on the asset's adjusted daily volume.

Back-testing Strategies: Judging Performance

- Once you have finished your rigorous research process, beginning with a piece of theory that you validated experimentally via historical data, the next step is to decide how exciting the strategy is, and whether it should be allocated capital.
- There are some helpful metrics that can guide us here (i.e. Sharpe Ratio), but in a lot of cases this part of the quant investment process is somewhat subjective.
- In particular, you need to decide how likely these results are to persist out-of-sample and how reasonable your assumptions were.
- And you need to be sure you didn't fall for any common quant traps...

Back-testing Strategies: Common Quant Traps

- When back-testing quant strategies we are forced to overcome many potential pitfalls, including, just to name a few:
  - Looking Ahead
  - Overfitting
  - Unrealistic execution assumptions
  - Survivorship Bias
- Some of these are easy to spot but others are more pernicious.
- The only way to really avoid these traps is to follow a robust research process.
- Only the researcher will know how legitimate your results are, and I highly recommend a cautious and honest approach.

Back-testing Strategies: Common Performance Metrics

- Annualized Return / Annualized Volatility
- Market Beta
- Sharpe Ratio

$$\text{SR} = \frac{\mathbb{E}[R] - r}{\sigma}$$

  where r is the risk-free rate and σ is the strategies expected volatility.
- Max (peak to trough) Drawdown
- VaR / CVaR
- Book Size / Leverage
- Turnover

Back-testing Strategies: Common Performance Metrics

- Mean Absolute Deviation
- Downside risk metrics (i.e. Sortino Ratio)
- Skewness / Kurtosis
- It is common for a back-testing engine to output a set of statistics, including those above for each strategy that is run through the back-test.

Back-testing Strategies: Sharpe / Information Ratios

- The gold standard for measuring the success or failure of a quant strategy is it's sharpe ratio, or information ratio.
- Sharpe ratio is not without flaws. For example it does not include skewness and kurtosis.
- Nonetheless is does provide a consistent way to compare strategies and accounts for differences in volatility and expected returns.
- A portfolio with a high sharpe ratio but with a large amount of negative skewness and kurtosis is not as attractive as its sharpe ratio would indicate.

Back-testing Strategies: Sharpe / Information Ratios

- It also doesn't factor in latent risk factors, such as leverage, that may lead to higher than expected vol. and large drawdowns in certain market conditions.
- An alternative metric is to use the strategy's maximum drawdown instead of volatility in the denominator of the calculation. This will help to incorporate the strategy's tail behavior.
- In practice, for quant strategies, legitimate sharpe ratios above 2 are considered excellent and these types of strategies are highly sought after. Alas, they are also hard to find.

Back-testing Strategies: Interpreting Sharpe / Information Ratios

- If we assume that the returns of our underlying strategy are known with certainty and normally distributed, then our strategies Sharpe Ratio has some intuitive rules of thumb.
- If we have a Sharpe Ratio of X, the it will take an X standard deviation move for the strategy to break even.
- If we have a Sharpe Ratio of X, then the probability that the strategy will lose money on a given period is the CDF evaluated at $-X$.

Back-testing Strategies: Interpreting Sharpe / Information Ratios

- So if our Sharpe Ratio is 1...
  - Then it takes a one standard deviation move for the strategy.
  - The probability that we will lose money is 35%
- A higher Sharpe leads to a lower loss probability.
- A sharpe is 2, has only a 5% chance of losing money.
- Keep these rules of thumb in mind when judging your sharpe ratios and doing sanity checks on your results.
- For example, given this, is a sharpe ratio of 5 is realistic?
- This approximation will be less accurate for strategies with high levels of skewness and kurtosis.

Out-of-sample Back-test Performance

- Once we have finished developing a strategy and are convinced that the risk/return profile is compelling, the next step is to conduct a true out of sample test.
- In this out-of-sample test we generally perform simulated or paper trading. This stage is designed to catch any look ahead or other biases or inconsistencies, and make sure that the model is implemented correctly.
- Note that it is recommended that we do this prior to allocating capital to a strategy even if we had an out-of-sample period in our research process. (Why?)

Slippage: Comparing Back-tested & Live Performance

- Assuming that the paper trading period goes smoothly, the next step will be to begin live trading (generally with a small amount of capital at first).
- Once we begin live trading, there will be many factors that will determine success or failure of the strategy, including:
  - Slippage
  - Unanticipated Transactions Costs
  - Regime Changes
  - Small Sample Sizes
  - Capital Constraints, Stop Losses, etc.

Slippage: Comparing Back-tested & Live Performance

- Some of these factors are beyond your control, however, slippage is a tractable factor for us to monitor.
- In particular, every day that the strategy is live we should compare the strategies actual profit or loss to the profit or loss obtained in the back-test for the same day.
- If these are in line, it is a sign that you have modeled t-costs and market impact accurately, and should give you confidence.
- If these diverge, it is a serious warning signal and you should try to explain it immediately.
- If it is likely to be a recurring divergence, it should give you much less confidence.

Live vs. Back-test Sharpe Ratios

- Generally speaking, once you start trading a strategy you will observe a sharpe ratio that is lower than your back-tested sharpe.
- The degree that it is lower will of course depend on the individual strategy, how robust the research process was, how many parameters were estimated, etc.
- Some rules of thumb that are common within the industry are to **subtract half** from your back-tested sharpe ratio or to **divide it by 2**.

Common Stat-Arb Strategies

- Single Stock Autocorrelation Momentum / Mean-Reversion
- Cross Asset Autocorrelation (Lead / Lag Relationships)
- Cointegration / Pairs Trading
- Factor Modeling
- Statistical Factor Decomposition (PCA)

Back-testing Options Strategies: Overview

- Back-testing options strategies requires an additional level of complexity because we need additional info to value an option.
- Options also have fixed expiries so monitoring them historically is more challenging because there is no single ticker with a time series for the entire historical period.
- In practice, this means that in addition to asset prices, we need to incorporate changes to forward / future curves and volatility surfaces.

Back-testing Options Strategies: Overview

- It is important to remember that we are trying to simulate the behavior of engaging in this strategy historically, so we need to simulate buying the option at the price that it would have traded at and selling the option at the price that it would have traded at the time of the sale.
- We should not, however, lock today's option pricing and forward curve and only simulate changes to the asset price. While this may be useful analysis, it is not a back-test.
- Volatility strategy back-tests are less precise because there is less certainty about the true market implied volatility surface.

Back-testing Options Strategies: Details

- To back-test an options strategy, we begin by collecting data for the asset, its **implied forward curve**, and its **vol. surface**.
- We should ensure that forward curve and the volatility surface pass **arbitrage checks**.
- In some cases, we may use either calibration of a stochastic model or interpolation to simplify storage of the volatility surface.
- We then need to iterate through history and make our buy or sell decisions on the options. This will be how we compute our portfolio weights.
- We also need to know how to value each option position each day. This will involve finding the market traded volatility and pricing with the correct time to maturity remaining.

Back-testing Options Strategies: Example

- Consider an example of an investor who everyday buys a 1M expiry ATM call option on the S&P and holds it to expiry.
- To back-test this strategy we would do the following:
  - Collect historical S&P prices
  - Collect option prices for all S&P options
  - Iterate through history, and on each day compute the price of buying an at-the-money call option (requires option prices)
  - For each day mark the value of the position using the option prices and the forwards.
  - Compute the daily profit and loss for each individual trade, and aggregative over all of the daily trades put on.
  - Compute the relevant aggregate statistics including Sharpe.

Common Options Strategies

- Short Volatility Strategies
- Implied vs. Realized Volatility Strategies
- Mean-Reversion Strategies
- Skew or Calendar Strategies
- Dispersion Strategies
- Relative Value Strategies

Asset Allocation: Overview

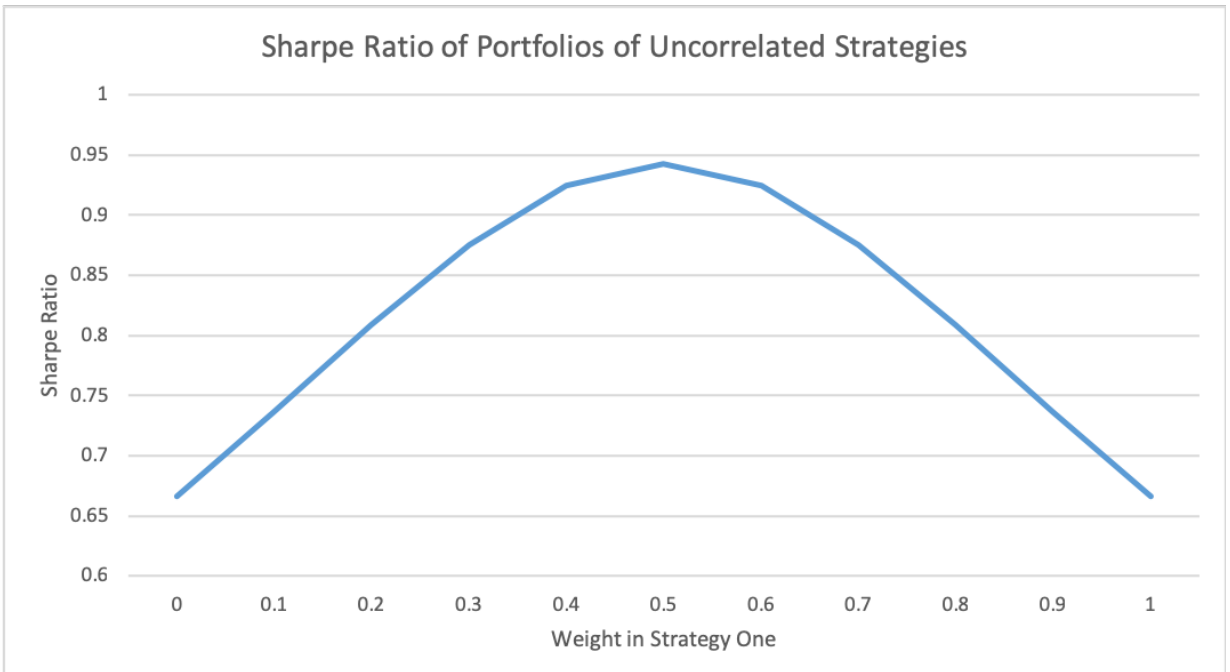
- Once we have completed the signal generation / strategy design portion of our project, the final step is choosing how to allocate between the one or more strategies that you've created and cash.
- Generally speaking this will rely on using portfolio optimization techniques which we will discuss in the next few lectures.
- Clearly, optimal allocation between the strategies will depend on:
  - Strategy Expected Returns
  - Strategy Volatilites
  - Correlation between the strategies

Asset Allocation: Combining Strategies

- Finding the proper allocation of our sub-strategies is a critical part of building a successful portfolio.
- We could have a set of good signals but if they aren't properly allocated to we may still see large and unncessary drawdowns.
- To gain some intuition, let's start with the case of two strategies and see how our portfolio varies as we change the sub-strategy weights...

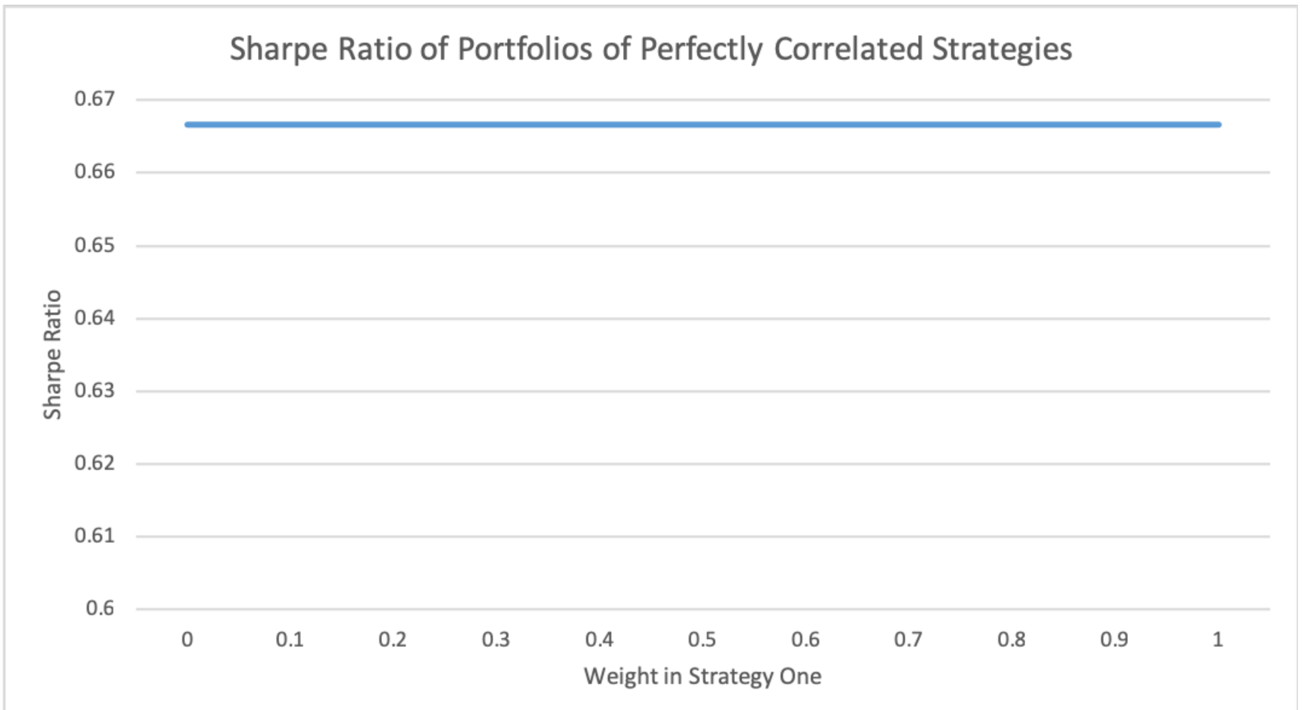
Sharpe Ratios of Portfolio of Two Uncorrelated Strategies

Lets consider two strategies, each of which has an expected return of 10% and an annualized volatility of 15%. How will the sharpe change as we vary the weight in the first strategy?



Sharpe Ratios of Portfolio of Two Perfectly Positively Correlated Strategies

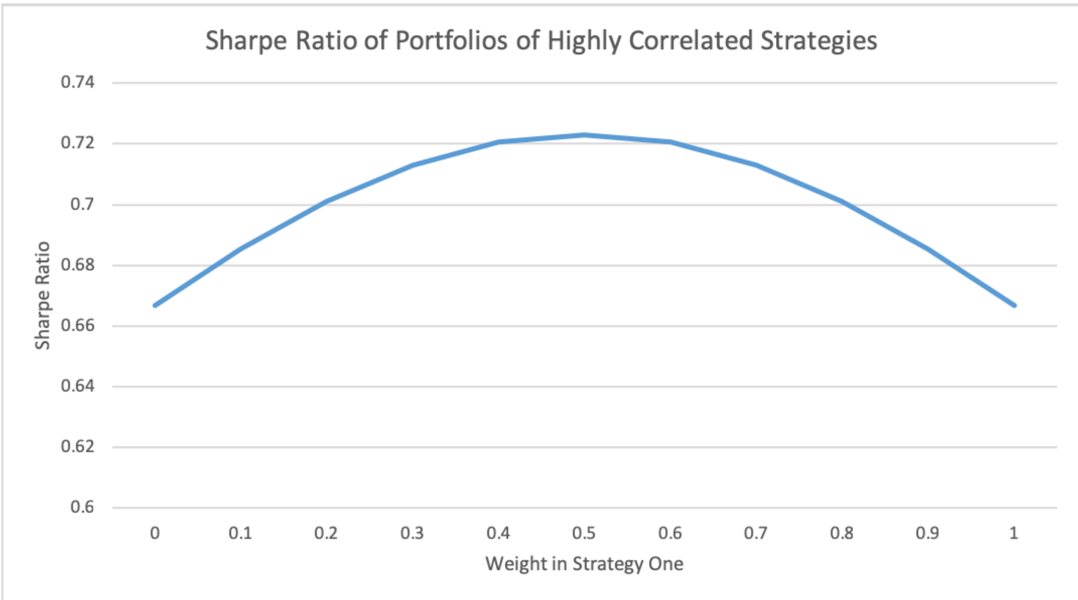
What happens if the assets are perfectly positively correlated?



Then there is no diversification benefit, unsurprisingly.

Sharpe Ratios of Portfolio of Two Highly Positively Correlated Strategies

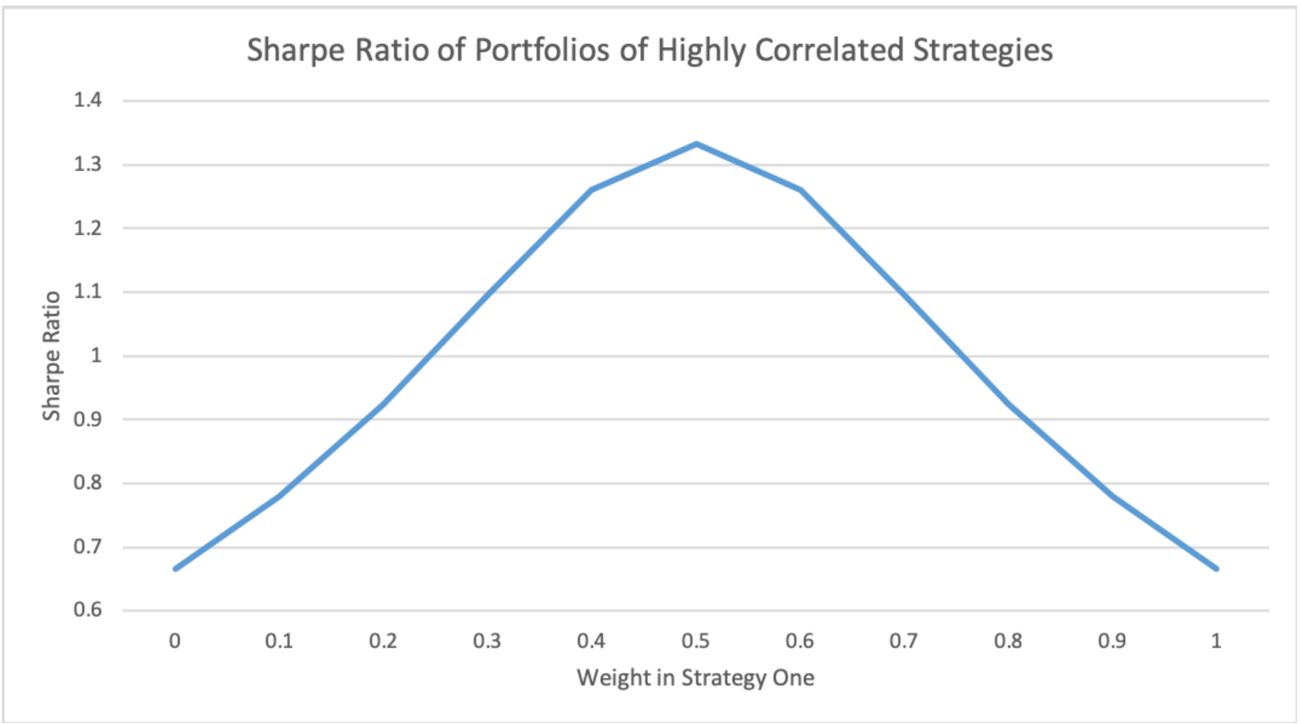
And if the assets are highly, but not perfectly, positively correlated, we have:



We can see that there is some marginal improvement to the sharpe ratio, but it is not as much as in the case of uncorrelated assets.

Sharpe Ratios of Portfolio of Two Highly Negatively Correlated Strategies

And we can see that if the two strategies are negatively correlated then we get the maximal benefit from diversification:



Asset Allocation: Comments

- The previous examples illustrate one of the most powerful and important concepts in quantitative investing and portfolio management.
- In particular, as we know from mean-variance analysis, diversification enables us to eliminate (idiosyncratic) volatility without sacrificing expected return.
- We see the same phenomenon play out here. That is, we can improve our portfolio sharpe ratio by adding a strategy that is not perfectly correlated with our existing strategy.
- Notice that the benefit of adding a strategy is highest when the new strategy is negatively correlated.

Asset Allocation: Comments

- In practice this principal is critical and is a key tool for building a successful portfolio.
- In many cases, we can develop an attractive portfolio by building a set of uncorrelated sub-strategies with relative low sharpe ratios.
- If enough uncorrelated sub-strategies can be created, then the portfolio itself may have a compelling sharpe.
- **Question:** What is the Sharpe of a portfolio that has N equally weighted uncorrelated sub-strategies each of which are uncorrelated and each with individual sharpe ratios equal to 1?

Asset Allocation: Portfolio Optimization of Several Strategies

- In the case of many sub-strategies, we might consider the following different types of strategy weightings:
  - Equally Weighted
  - Mean Variance Optimal Portfolio Weights
  - Risk Parity Weights
- Unless we choose equally weighted, allocating between them becomes a non-trivial exercise in portfolio optimization.
- If we do choose equal weights, then we are not taking advantage of the correlation structure of the sub-strategies.
- We are also implicitly weighting the higher volatility strategies the most.

Asset Allocation: Portfolio Optimization of Several Strategies

- When using a risk parity or mean-variance approach to portfolio construction, we need to keep in mind the following caveats:
  - Estimation Error
  - Sensitivity of Weights to Expected Returns
  - Stability of the Covariance Matrix
  - Non-stationarity of the underlying returns, variances and covariances.

Asset Allocation: Comments on Portfolio Optimization of Several Strategies

- When we are optimizing on a set of sub-strategies it is likely that they will be less correlated than the original asset universe.
- Remember that there is a great deal of uncertainty in the expected returns that we compute.
- Two strategies with back-tested returns of 10 and 20 percent respectively in reality may have very similar true expected returns.
- This is true for variance and covariances as well.
- Some strategies may show a volatility of 5% if you look post-crisis but 20% if you include the crisis. Which estimate should you use in this situation?

Asset Allocation: Comments on Portfolio Optimization of Several Strategies

- Because of this, I tend to assume that my strategies all have similar sharpe ratios when conducting this portfolio optimization procedure.
- In general, I highly recommend using some sort of regularization process on the input returns and volatilities to make sure that you get the most robust results.