MF 703 Homework 4

Friday, October 23th

Shi Bo
shibo@bu.edu

a) Zero coupon bonds' prices

$$\frac{100}{(1+y)^n}$$

Maturity	YTM	Price
1	0.025	97.561
2	0.026	94.996
3	0.027	92.3185
5	0.03	86.2609
10	0.035	70.8919
30	0.04	30.8319

The zero coupon bond with the shortest maturity has the highest price. This is reasonable because the bond with the shortest maturity has the smallest risk.

b) Duration of zero coupon bonds

Maturity	YTM	Duration
1	0.025	0.9756
2	0.026	1.94927
3	0.027	2.92107
5	0.03	4.85434
10	0.035	9.66186
30	0.04	28.8501

The higher the yield is, the lower the bond price is. The price is more sensitive to higher yield. Besides, the duration of zero coupon bonds equal with its maturity.

c) Prices of coupon bonds

Maturity	YTM	Price
1	0.025	100.488
2	0.026	100.77
3	0.027	100.854
5	0.03	100
10	0.035	95.8417
30	0.04	82.708

The bonds with YTM greater than 0.03 are below 100 dollars. The others are above 100 dollars. This is because, when the yields of the bonds are smaller than coupon rate, the price would be greater than face value. When the yields of the bonds are greater than coupon rate, the price would be smaller than face value.

d) Duration of coupon bonds

Maturity	YTM	Duration
1	0.025	0.975618
2	0.026	1.92098
3	0.027	2.83719
5	0.03	4.57966
10	0.035	8.45877
30	0.04	18.3707

Zero coupon bonds have higher duration than coupon bonds. Since there is no cash flow during the whole period except at maturity for zero coupon bonds, it requires longer time to give all its cash flow than coupon bonds.

e) Convexity

Maturity	YTM	Zero-coupon	Coupon
1	0.025	1.87683	1.82217
2	0.026	5.94315	5.90546
3	0.027	11.074	10.5908
5	0.03	27.595	25.6348
10	0.035	101.593	85.0172
30	0.04	856.429	458.642

The convexities of the bonds are all positive. This is because yield curve is convex in nature, due to quadratic nature of equations we use in compounding and discounting

f) Initial value of the portfolio

The initial value of the portfolio is: -0.112503

g) Duration and convexity of the portfolio

The duration of the portfolio is following: 48.8606
The convexity of the portfolio is following: -678.149
Duration is greater. However, in terms of absolute quantity, convexity is much bigger.

h) The number of units required to realize duration neutral is: -1.97

i) Initial value of portfolio when rates sell off by 100 basis points

In this case, the initial value of portfolio is -0.0500793

j) Initial value of portfolio when rates rally by 100 basis points

In this case, the initial value of portfolio is -0.159592
Considering that the decrease of portfolio value due to rise in interest rate is greater than the increase of portfolio value due to fall in interest rate, convexity contribute to hedging the risk of interest rate going up. Thus, this portfolio may be good to hold.

k) Cash flow of amortizing bond

Year	Cash flow
1	23
2	23
3	23
4	23
5	23

l) Price and Duration of amortizing bond

The 5-year amortizing bond price is following: 105.916
The 5-year amortizing duration is following: 2.85857
The price of this amortized bond is greater than its zero coupon and coupon equivalents. That is because more cash flow is prepaid in advance, which can also be reflected by its smaller duration.