1.

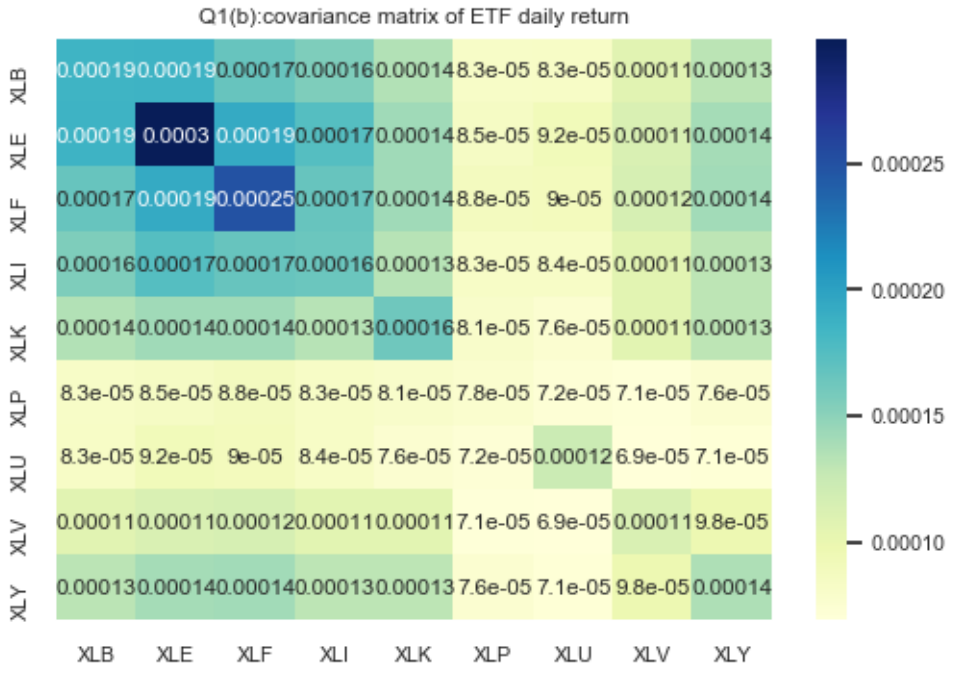
(a) Load data

```
yf.pdr_override()
etf_name = ['XLB','XLE','XLF','XLI','XLK','XLP','XLU','XLV','XLY']
etf_df = []
for etf in etf_name:
    etf_df.append(pdr.get_data_yahoo(etf, start="2010-01-01", end="2020-12-04"))
```

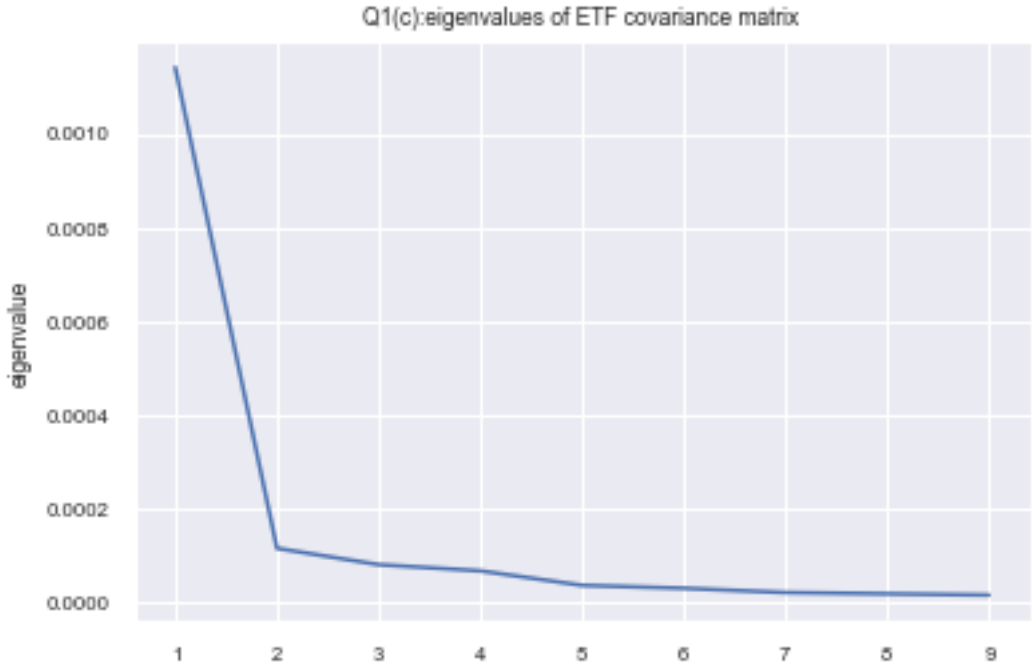
```
[2750 rows x 6 columns],
                 Open        High         Low       Close   Adj Close   Volume
Date
2010-01-04   29.900000   30.110001   29.900000   30.000000   25.727346  5443900
2010-01-05   30.010000   30.139999   29.820000   30.110001   25.821678  6162200
2010-01-06   30.090000   30.219999   30.020000   30.150000   25.855986  4246900
2010-01-07   30.340000   30.410000   30.139999   30.400000   26.070374  5736700
2010-01-08   30.290001   30.410000   30.120001   30.389999   26.061798  6438000
...                ...         ...         ...         ...         ...      ...
2020-11-27  158.600006  158.789993  157.740005  158.220001  158.220001  1108300
2020-11-30  157.979996  158.059998  155.210007  157.259995  157.259995  3288000
2020-12-01  158.490005  159.160004  157.490005  158.429993  158.429993  4369100
2020-12-02  157.809998  157.809998  156.770004  157.440002  157.440002  2154400
2020-12-03  157.479996  158.750000  157.479996  157.880005  157.880005  3005700

[2750 rows x 6 columns]]
```

(b)

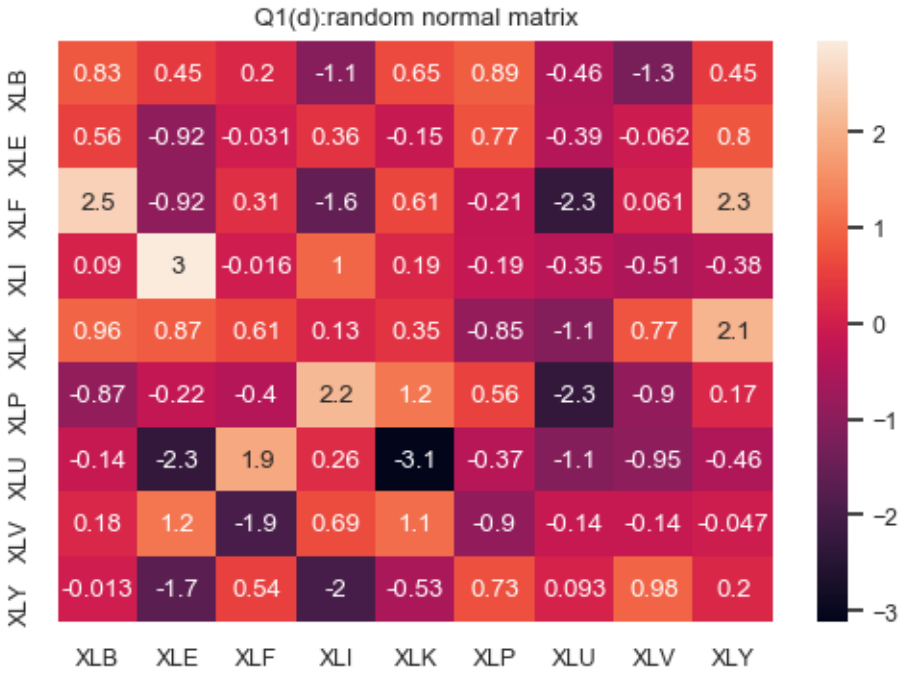


(c)

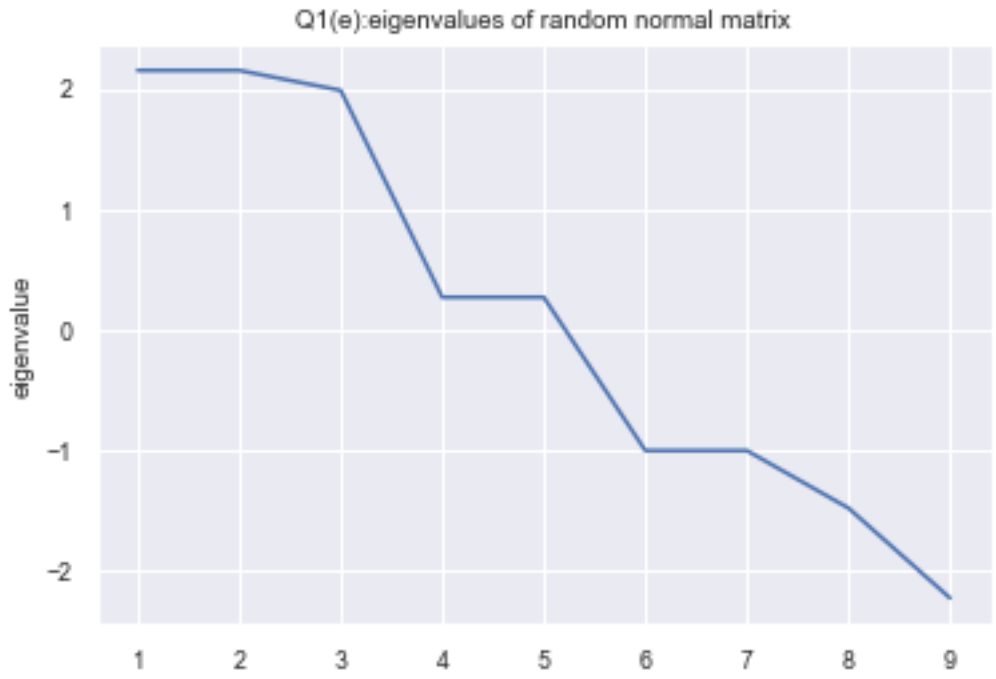


All of eigenvalues are positive. But, 8 out of 9 eigenvalues are smaller that 0.0001, which are very closed to 0 and not statistically significant. Only the largest eigenvalue is significant.

(d)



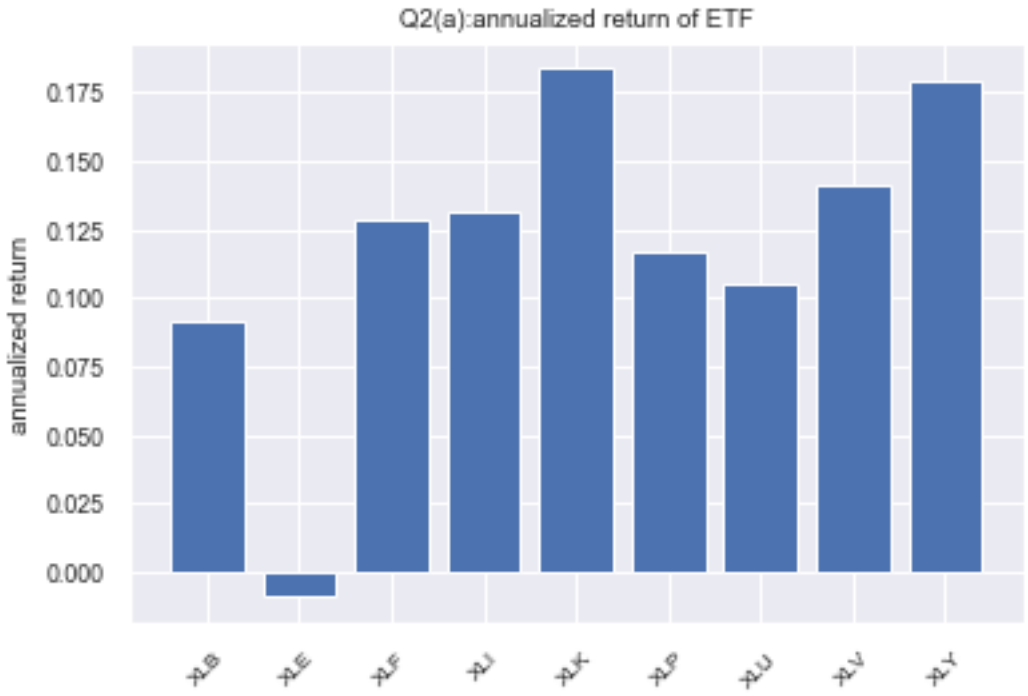
(e)



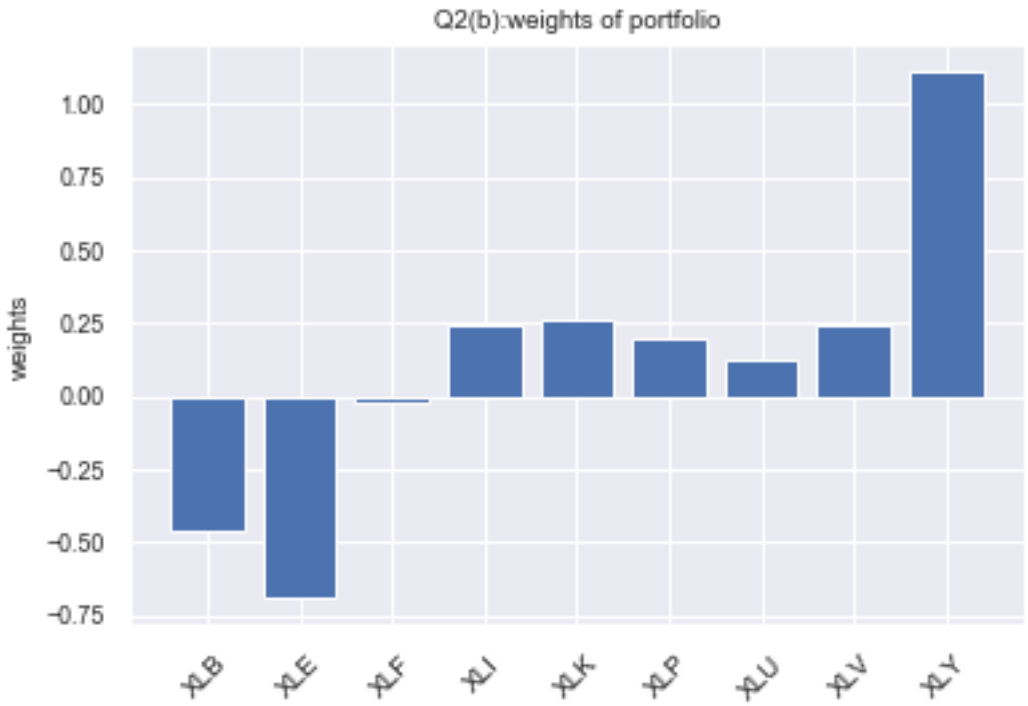
For eigenvalues of random matrix, 5 out of 9 are positive and 4 out of 9 are negative. Compared with eigenvalues from historical covariance matrix, eigenvalues from random matrix distribute more smoothly.

2.

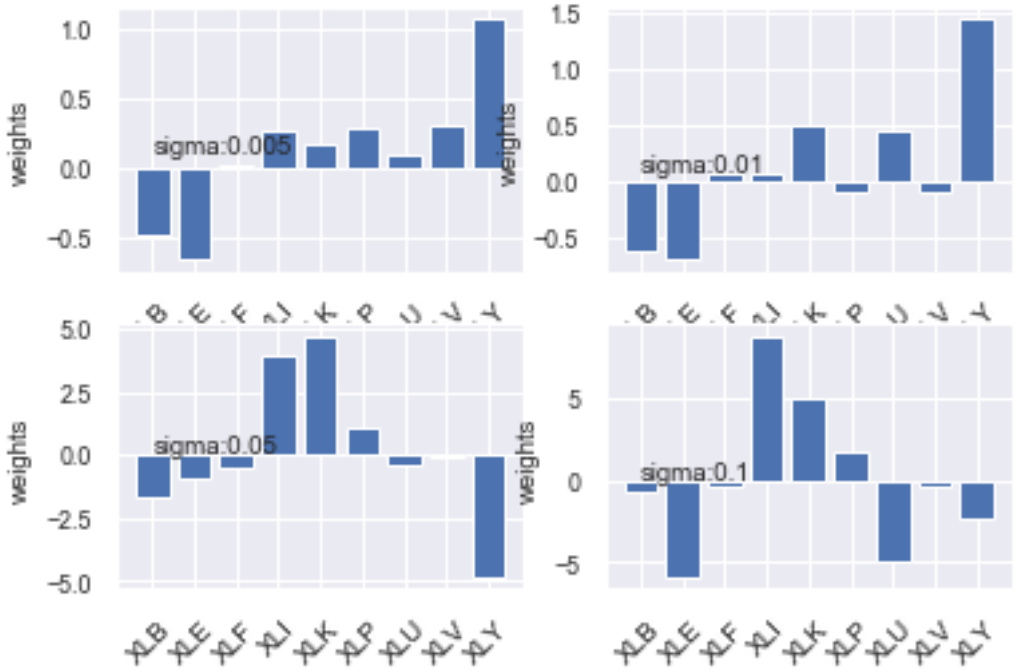
(a)



(b)

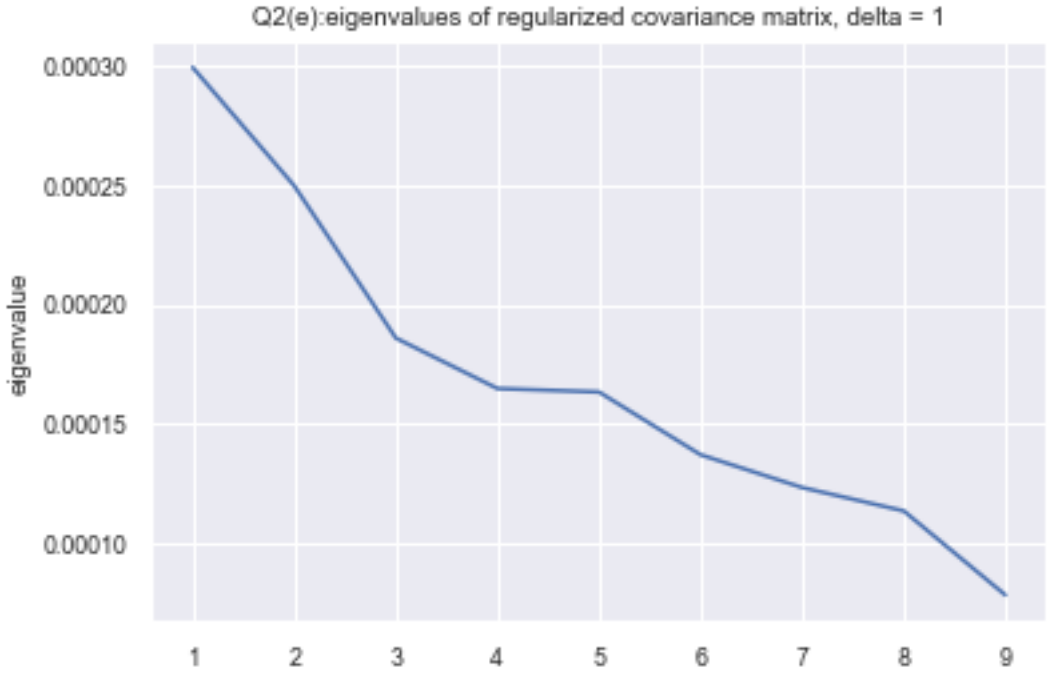


(c)



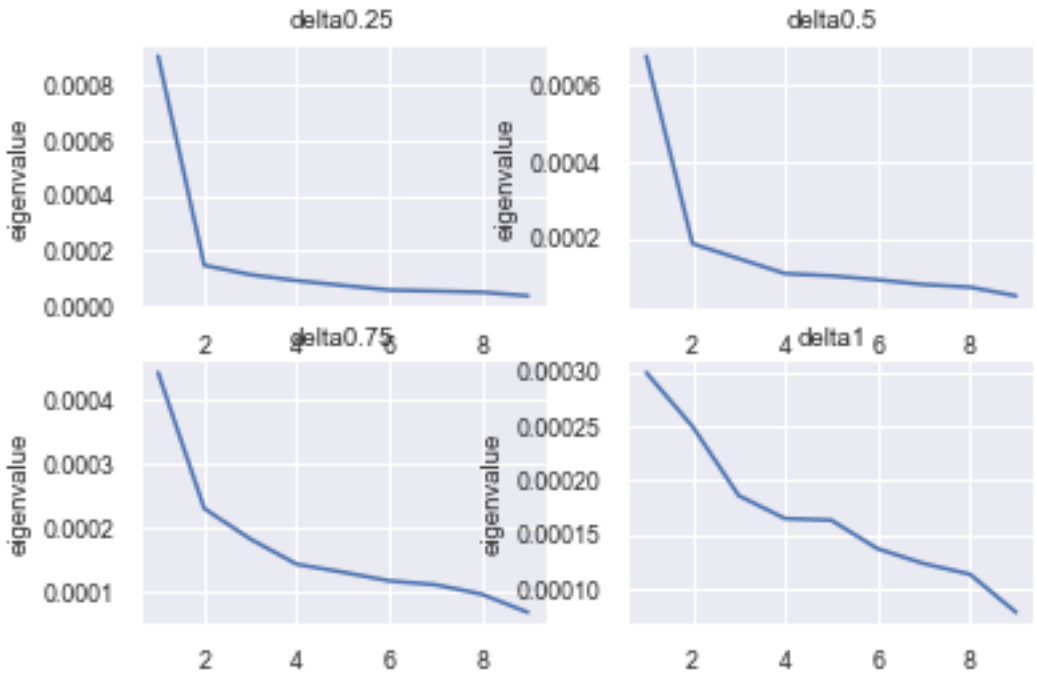
When sigma increase, weights change significantly, which is unstable.

(e)



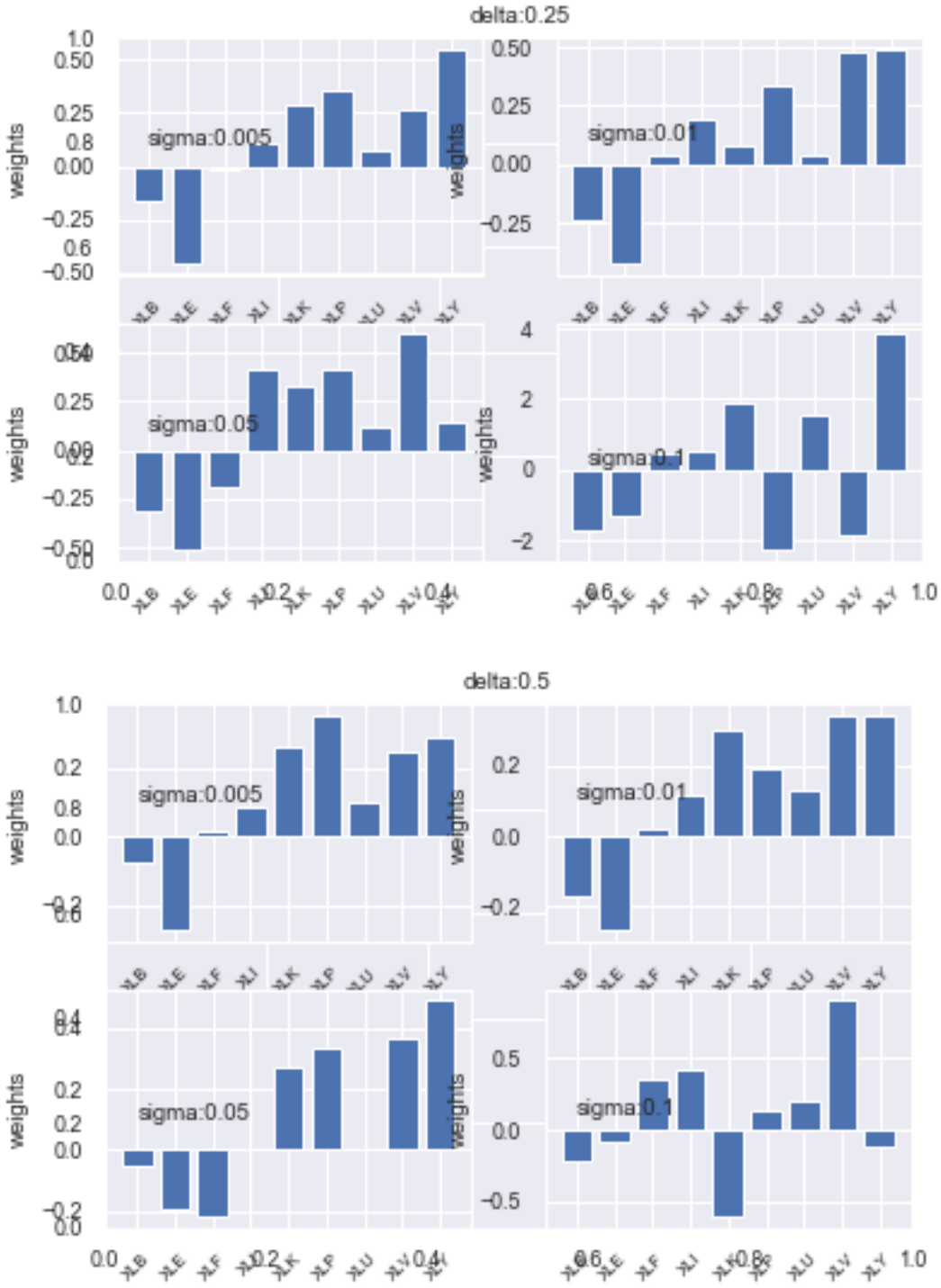
Since regularized covariance matrix is diagonal matrix, its rank is 9.

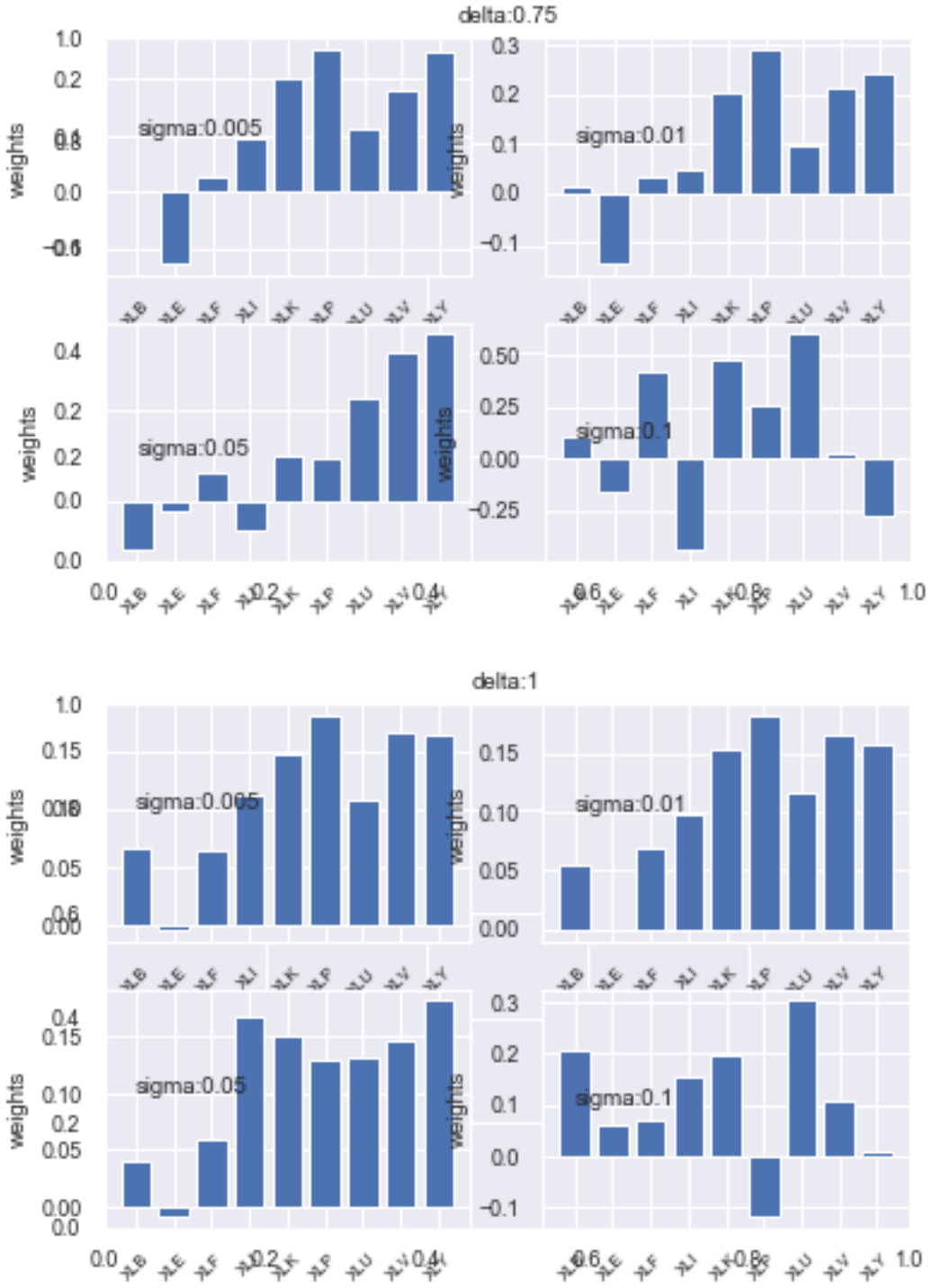
(f)



When delta change from 0.25 to 1, all eigenvalues are still positive for each delta. But, when delta increase, eigenvalues distribute more smoothly, like eigenvalues from random normal matrix. And, more eigenvalues are higher than 0.0001 (more significant), compared with eigenvalues from low delta regularized covariance matrix.

(g)


delta:0.25
sigma:0.005
sigma:0.01
sigma:0.05
sigma:0.1
weights
delta:0.5
sigma:0.005
sigma:0.01
sigma:0.05
sigma:0.1
weights



When delta increase, weights are more stable if sigma change. Especially, when delta is 0 in part c, regularized matrix is just historical covariance matrix and weights are the most unstable for sigma's change. When delta is 1, regularized matrix is diagonal matrix and weights are the most stable for sigma's change.