Model Risk

MF 731 Corporate Risk Management

Outline

Qualitative discussion on model risk.

Examples in quantifying model risk.

- Uncertainty in VaR.
- Derivative prices in calibrated models.

Model Risk

"The risk associated with using a mis-specified or inappropriate model."

Either for asset valuation or risk measuring.

Examples

Using Black-Scholes when volatility is clearly stochastic.

Calibrating a model to European option prices, but using it to price path dependent options.

Model Risk

Models are approximations to reality.

Frameworks which enable us to produce outputs (e.g. asset prices, hedging strategies, risk measures) based upon hypotheses about inputs (e.g. log returns, volatilities).

Models should not be considered as “reality”.

Emmanuel Derman: “A model is just a toy, though occasionally a very good one, in which case people call it a theory.”

Model Risk

By definition, a model is a simplified structure.

We should not expect it to give perfect answers.

In fact, we want the “best - simplest” model.

Not all uncertainty in output is model risk.

Simulations produce a small degree of variation in the answers, simply by sampling random variables.

Thus, measuring model risk can be subtle.

If our model produces a strange answer, is the model wrong, or did we sample a rare event?

Model Risk

Further complicating things, sometimes "incorrect" models give good answers.

E.g.: Many assumptions underlying the Black-Scholes option pricing formula are clearly not true.

I.i.d. normal log returns, no bid-ask spread or liquidity constraints.

But, the resulting hedging strategies work well in practice.

So, how should we assess model risk?

Understanding the Model

Before using a model, Derman suggests we should:

- Understand the securities involved and their markets.
- Isolate the most important variables. Separate causal variables (exogenous) from caused variables (endogenous).
- Determine which variables are deterministic and which are stochastic.
  - E.g: Black-Scholes: volatility deterministic, log return stochastic.
- Determine how the exogenous variables affect the endogenous ones.

Understanding the Model

Derman's suggestions continued:

Consider how the model can be solved, and look for the simplest solution.

Program the model, taking into account programming considerations.

Test and back-test the model.

Periodically evaluate the model's performance.

Sources of Model Risk

What are the main sources of model risk?

overlap with each other

- Mis-specification (wrong model).
- Mis-application (good model but wrong situation).
- Mis-implementation (good model but execution errors).

Other sources:

- Incorrect calibration.
- Programming problems.
- Data problems.

Warning: there is overlap between these sources!

Mis-Specification: Types and Examples

Stochastic processes: we model stock prices via geometric Brownian motion when log returns have fat tails.

Missing risk factors: we ignore stochastic volatility; we ignore relevant points on the yield curve.

Asset relationships: we ignore, or incorrectly identify, correlations between log returns.

Market Frictions: we ignore transactions costs, liquidity constraints.

Mis-Application: Examples

Using Black-Scholes to price interest rate options, rather than, e.g., Heath - Jarrow - Morton.

Good equity model $\neq$ Good fixed income model.

Letting a model become out-of-date (I): not updating parameters.

Letting a model become out-of-date (II): not switching to a new, superior, model.

Good model in 1995 $\neq$ Good model in 2019.

Mis-Implementation

A model cannot completely specify when it should be used!

- Too many variables/market instruments to satisfy all the assumptions.

Model mis-implementation may arise because of

- Problems in valuation: i.e. using mark to market prices, mark to model prices, mid-prices, improperly cleaned P&L data.

Model Risk and VaR: Case Studies

Beder (1995): Estimated VaR for 3 hypothetical portfolio types using 8 common VaR methodologies. Estimates varied by up to a factor of 14!

Berkowitz, Obrien (2002): Examined VaR models used by 6 financial institutions. Found models are *a*) too complicated, *b*) too conservative, and yet *c*) miss extreme losses.

Simple models to estimate VaR are superior.

Case Studies

Marshall, Seigal (1997): compared how different software systems applied the Risk Metrics approach.

Common framework: one-day VaR, $\alpha = .95$, first order losses for derivatives.

Found that VaR estimates could vary by up to 30%. Variations positively correlated with model complexity.

“Models behave stochastically, treating models as black-boxes is unwise.”

Other Sources of Model Risk

Incorrect calibration: parameter estimation error; un-realistic sampling periods, etc.

Examples:

NatWest Bank: incorrect (too high) volatility estimates used to price interest rate options. Large theoretical values led to significant trading losses.

Bank of Tokyo-Mitsubishi: calibrated swaption model to at the money swaptions, but used it to price out of the money swaptions. Led to large losses.

Other Sources

Programming problems:

Some overlap here with operational risk.

Examples:

Bugs in code, discretization errors, rounding errors.

Poorly commented code: programmers cannot tell what was previously done.

A major problem, as commenting is now mandated by regulation.

Other sources

Data problems.

Garbage in - garbage out.

This is particularly bad for valuating mortgage backed securities. Many relevant data items (borrower income, employment, etc) were either not in the loan-application or falsely entered.

Mis-handling of time.

This is particularly bad for fixed income instruments. Many different coupon/accrual conventions.

Other Sources

Endogenous model risk.

Subtle: arises because of how traders react to models.

Example: VaR-based trading limits:

- Traders face VaR-based limits to their activities.
- If traders know that a model under-estimates the risk of certain positions/instruments, they will gravitate towards these instruments to loosen their restrictions.
- Ju, Pearson (1999): bias is large when the number of assets is large, and the data used to estimate covariances is sparse. Firms should be very careful when adopting VaR based trading restrictions.

Quantifying Model Risk

How do we quantify model risk?

Important note:

- Any attempt to quantify model risk is actually an attempt to quantify a *particular form* of model risk.
  - We cannot specify all the risks or model may be subject to, but we can specify certain risks and/or uncertainties.
- As such, any estimate of model risk is inherently too low!
  - If our estimate is low, the true model risk may be high due to factors we have not estimated.
  - If our estimate is high, then our model risk is also high!

Quantifying Model Risk

To keep things concrete, we will focus on estimating two types of model risk.

- Uncertainty in risk measures (i.e. VaR) due to unknown parameters.
- The variation in model prices for options not calibrated to the model.
  - We calibrate a model to fit call/put option prices.
  - But, this calibration may not be unique.
  - For other options, we may see a wide range of prices across the calibrated models.

VaR and Unknown Parameters

The setting:

We own λ_t shares of S_t over $[t, t+\Delta]$.

Loss: $L_{t+\Delta} = -\lambda_t S_t \left(e^{X_{t+\Delta}} - 1\right)$. $X_{t+\Delta}$: log return.

Distribution: $X_{t+\Delta} \overset{\mathcal{F}_t}{\sim} N\left(\mu_{t+\Delta}, \sigma^2_{t+\Delta}\right)$.

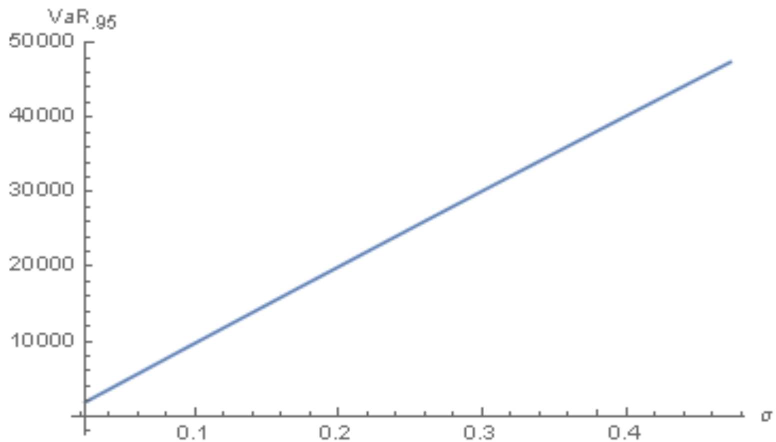
$\mathrm{VaR}_\alpha = \theta_t \left(1 - e^{\mu_{t+\Delta} + \sigma_{t+\Delta} N^{-1}(1-\alpha)}\right)$, $\theta_t = \lambda_t S_t$.

VaR and Unknown Parameters

Dropping sub-scripts.

$$\mathrm{VaR}_\alpha = \theta\left(1 - e^{\mu+\sigma N^{-1}(1-\alpha)}\right) = \theta\left(1 - e^{\mu-\sigma N^{-1}(\alpha)}\right).$$

Clearly, VaR depends upon σ:

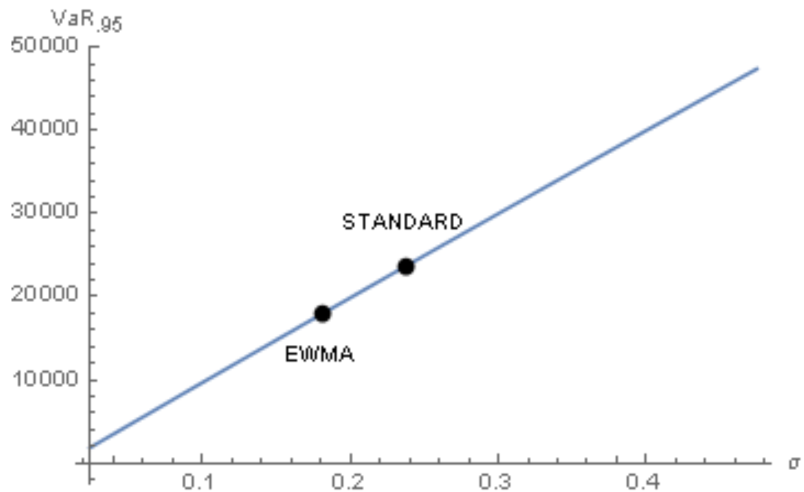


Problem: σ is unknown. We have to estimate it.

Uncertainty in VaR due to uncertainty in σ.

Our Estimate of σ Matters

Example: MSFT stock. 5 yrs historical data. \$1M portfolio value. Empirical and EWMA σ.



Difference of almost \$6, 000.

Accounting for Uncertainty in σ

How do we account for our estimate of σ in our estimate of VaR?

Abstract theory:

Assume $\{X_i\}_{i=1}^{\infty}$ are i.i.d. $N(\mu, \sigma^2)$ r.v.

For $n = 1, 2, \ldots$ set

$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$: sample mean.

$S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}_n\right)^2$: sample variance.

Then $\frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$.

χ^2_k: chi-square r.v. with k degrees of freedom.

$\chi^2_k \sim \sum_{i=1}^{k} Z_i^2$ where $Z_i \sim N(0,1)$ are i.i.d..

Wonderful result : $\frac{(n-1)S_n^2}{\sigma^2} \perp\!\!\!\perp \left(\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}}\right)^2$

= not obvi

- not obvious
- particular for normal dist

$\Rightarrow \frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$

loss 1 degree of freedom

Uncertainty in σ

$$\frac{(n-1)S_n^2}{\sigma^2} = \frac{1}{\sigma^2}\sum_{j=1}^{n}(X_j-\bar{X}_n)^2$$
$$= \frac{1}{\sigma^2}\sum_{j=1}^{n}(X_j-\mu+\mu-\bar{X}_n)^2$$

To see why $\frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$:

$$\underbrace{\sum_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2}_{\chi^2_n} = \frac{(n-1)S_n^2}{\sigma^2} + \underbrace{\left(\frac{\bar{X}_n-\mu}{\sigma/\sqrt{n}}\right)^2}_{\chi^2_1}.$$

$$= \sum_{j=1}^{n}\left(\frac{X_j-\mu}{\sigma}\right)^2 + \frac{2}{\sigma^2}(\mu-\bar{X}_n)\sum_{j=1}^{n}(X_j-\mu) + \frac{n}{\sigma^2}(\mu-\bar{X}_n)^2$$

Follows by a direct calculation.

$$\left(\frac{2}{\sigma^2}(\mu-\bar{X}_n)n(\bar{X}_n-\mu) = \frac{2}{\sigma^2}(\mu-\bar{X}_n)\sum_{j=1}^{n}(X_j-\mu)\right)$$

$$= \sum_{j=1}^{n}\left(\frac{X_j-\mu}{\sigma}\right)^2 - \left(\frac{\mu-\bar{X}_n}{\sigma/\sqrt{n}}\right)^2$$

Left hand side above is χ^2_n. Right most term is χ^2_1.

Two terms on right hand side are independent.

This is the hard step.

1) $\sum_{j=1}^{n}\left(\frac{X_j-\mu}{\sigma}\right)^2 \sim \chi^2_n$ – by def

2) $\bar{X}_n = \frac{1}{n}\sum_{j=1}^{n}X_j \sim N(\mu, \frac{\sigma^2}{n})$

$\Rightarrow \frac{\bar{X}_n-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$\Rightarrow \left(\frac{\bar{X}_n-\mu}{\sigma/\sqrt{n}}\right) \sim \chi^2_1$

Thus, the result follows.

Assume we know μ but not σ.

<u>Method</u>

$$q_\beta : \beta \text{ quartile of } \frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$P\left(q_{\beta/2} \leq \frac{(n-1)S_n^2}{\sigma^2} \leq q_{1-\beta/2}\right)$$

$$= F_{\chi^2_{n-1}}(q_{1-\beta/2}) - F_{\chi^2_{n-1}}(q_{\beta/2})$$

$$= 1 - \beta/2 - \beta/2 = 1-\beta$$

With $1-\beta$ likelihood

$$q_{\beta/2} \leq \frac{(n-1)S_n^2}{\sigma^2} \leq q_{1-\beta/2}$$

$$\Rightarrow \underbrace{S_n\sqrt{\frac{n-1}{q_{1-\beta/2}}}}_{\underline{\sigma}_\beta} \leq \sigma \leq \underbrace{S_n\sqrt{\frac{n-1}{q_{\beta/2}}}}_{\bar{\sigma}_\beta}$$

Idea: data gives us S_n and hence $\bar{\sigma}_\beta$ and $\underline{\sigma}_\beta$.

$\Rightarrow$ $1-\beta$ confidence that $\underline{\sigma}_\beta \leq \sigma \leq \bar{\sigma}_\beta$

$\Rightarrow$ $1-\beta$ confidence that $\theta(1-e^{\mu-\underline{\sigma}_\beta N^{-1}(\alpha)}) \leq VaR_\alpha \leq \theta(1-e^{\mu-\bar{\sigma}_\beta N^{-1}(\alpha)})$

To produce CI's,

1) take data $\{X_i\}_{i=1}^n$ — n most recent log-returns

$\Rightarrow$ compute $S_n = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X}_n)$

$\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$

$\Rightarrow$ compute $\underline{\sigma}_\beta = S_n\sqrt{\frac{n-1}{q_{1-\beta/2}}}$, $\bar{\sigma}_\beta = \sqrt{\frac{n-1}{q_{\beta/2}}}$

2) We say $\widehat{VaR}_\alpha = \theta(1-e^{\mu-S_n N^{-1}(\alpha)})$ with $1-\beta$ likelihood

$$\theta(1-e^{\mu-\underline{\sigma}_\beta N^{-1}(\alpha)}) \leq VaR_\alpha \leq \theta(1-e^{\mu-\bar{\sigma}_\beta N^{-1}(\alpha)})$$

Uncertainty in σ

Write q_β as the β quantile of $\frac{(n-1)S_n^2}{\sigma^2}$.

With $100(1-\beta)\%$ certainty

$$q_{\beta/2} \leq \frac{(n-1)S_n^2}{\sigma^2} \leq q_{1-\beta/2}.$$

Equivalently: $\sqrt{\frac{n-1}{q_{1-\beta/2}}}S_n =: \underline{\sigma}_\beta \leq \sigma \leq \overline{\sigma}_\beta := \sqrt{\frac{n-1}{q_{\beta/2}}}S_n.$

Recall: $\mathrm{VaR} = \theta\left(1 - e^{\mu - \sigma N^{-1}(\alpha)}\right).$

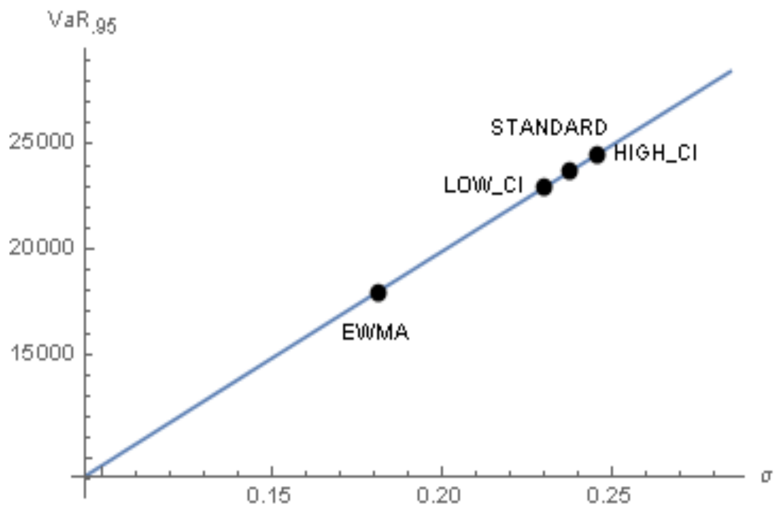
With $100(1-\beta)\%$ confidence we know

$$\theta\left(1 - e^{\mu - \underline{\sigma}_\beta N^{-1}(\alpha)}\right) \leq \mathrm{VaR}_\alpha \leq \theta\left(1 - e^{\mu - \overline{\sigma}_\beta N^{-1}(\alpha)}\right).$$

Uncertainty in σ

$$\underbrace{\theta\left(1-e^{\mu-\underline{\sigma}_\beta N^{-1}(\alpha)}\right)}_{\text{Low CI}} \leq \mathrm{VaR}_\alpha \leq \underbrace{\theta\left(1-e^{\mu-\overline{\sigma}_\beta N^{-1}(\alpha)}\right)}_{\text{High CI}}.$$

EWMA estimated VaR typically will not lie in the confidence interval.



- It is expected that the EWMA based VaR estimate will not lie within the CI.

A methodology for EWMA

- not completely rigorous
- Assume $\mu = 0$

$\{X_j\}_{j=1}^{n}$ i.i.d $N(0, \sigma^2)$

Set $\sigma^2_{n,EWMA} = (1-\lambda)\sum_{j=1}^{n}\lambda^{n-j}X_j^2$

$$\sigma^2_{new} = \lambda\sigma^2_{old} + (1-\lambda)X^2_{new}$$

$$= \lambda((1-\lambda)X^2_{n-1} + \lambda\sigma^2_{n-1}) + (1-\lambda)X_n^2$$

$$= (1-\lambda)X^2_{n-0} + \lambda(1-\lambda)X^2_{n-1} + \lambda^2(1-\lambda)\cdots$$

$$\sigma^2_{n,EWMA} = (1-\lambda)\sum_{j=1}^{n}\lambda^{n-j}X_j^2 + \cancel{\lambda^n\sigma^2_{n-n}}$$ very small

$$Var(\sigma^2_{n,EWMA}) = \sum_{j=1}^{n}(1-\lambda)^2\lambda^{2(n-j)}Var(X_j^2), \quad Var(X_j^2) = E[X_j^4] - E[X_j^2]^2 = 3\sigma^4 - \sigma^4 = 2\sigma^4$$

$$= 2\sigma^4(1-\lambda)^2\sum_{j=1}^{n}\lambda^{2(n-j)} = 2\sigma^2(1-\lambda)^2\frac{(1-\lambda^{2n})}{1-\lambda^2 = (1-\lambda)(1+\lambda)}$$

$$= \frac{2\sigma^4(1-\lambda)}{1+\lambda}(1-\lambda^{2n})$$

$$\approx \frac{2\sigma^4_{n,EWMA}(1-\lambda)}{1-\lambda} \quad \text{Assume } n \text{ large}$$

$$Stdev(\sigma^2_{n,EWMA}) \approx \sigma^2_{n,EWMA}\sqrt{\frac{2(1-\lambda)}{1+\lambda}}$$

$$\overline{\sigma}^2_{n,EWMA} = \sigma^2_{n,EWMA} + K\sigma^2_{n,EWMA}\sqrt{\frac{2(1-\lambda)}{1+\lambda}}, \quad \underline{\sigma}^2_{n,EWMA} = \sigma^2_{n,EWMA} - K\sigma^2_{n,EWMA}\sqrt{\frac{2(1-\lambda)}{1+\lambda}}$$

- VaR bound based on this

Uncertainty in σ

We can also produce confidence intervals for EWMA.

Methodology is not as rigorous.

Idea: assume $\mu = 0$, $\{X_j\}_{j=1}^{\infty}$ i.i.d. $N(0, \sigma^2)$ r.v..

Set $\sigma^2_{n,EWMA} := (1-\lambda)\sum_{j=1}^{n} \lambda^{n-j} X_j^2$.

As $n \uparrow \infty$:

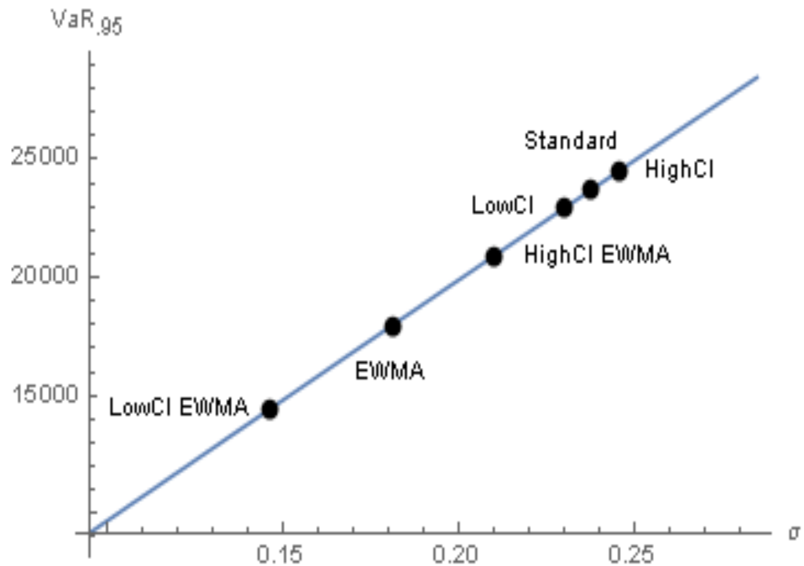
$$\mathrm{Var}\left[\sigma^2_{n,EWMA}\right] = \frac{2\sigma^4(1-\lambda)}{1+\lambda}(1-\lambda^{2n}) \approx \frac{2\sigma^4_{n,EWMA}(1-\lambda)}{1+\lambda}.$$

So for a given multiple k we set

$$\overline{\sigma}^2_{n,EWMA}, \underline{\sigma}^2_{n,EWMA} = \sigma^2_{n,EWMA}\left(1 \pm k\sqrt{\frac{2(1-\lambda)}{1+\lambda}}\right).$$

Uncertainty in σ

We then produce low,high VaR estimates using $\overline{\sigma}^2_{n,EWMA}, \underline{\sigma}^2_{n,EWMA}$.



μ not know. (simulation)

Method

1) Take our data $\{X_j\}_{j=1}^{n}$ and estimate $\bar{X}_n$, S_n^2

2) for $m, \cdots, M$

① We sample σ^m using that $\frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$

$\Rightarrow \sigma^m = S_n \sqrt{\frac{n-1}{(\chi^2_{n-1})^m}}$

② CLT $\Rightarrow \frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$\mu \stackrel{d}{=} \bar{X}_n - \sigma/\sqrt{n}\, Z$, $Z \sim N(0,1)$

$\mu^m = \bar{X}_n - \sigma^m/\sqrt{n}\, Z^m$, $Z^m \sim N(0,1)$

③ $VaR_\alpha^m = \theta(1 - e^{\mu^m - \sigma^m N^{-1}(\alpha)})$

Output VaRα estimate. $\widehat{VaR_\alpha} = \frac{1}{M}\sum_{m=1}^{M} VaR_\alpha^m$ and get standard CI

Uncertainty in μ

In practice, we do not know μ.

Here, we can do something similar, but must use simulation.

- We take our sample data (n samples) and estimate $\bar{X}_n, S_n^2$.
- For $m = 1, \ldots, M$ trials:
  - We sample σ^m using that $\frac{(n-1)S_n^2}{\sigma^2} \sim \chi^2_{n-1}$.
  - We then sample $\mu^m \sim N\left(\bar{X}_n, \frac{(\sigma^m)^2}{n}\right)$ since by the CLT we should have $\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$.
  - We use μ^m, σ^m to estimate VaR_α^m.
- We then output the average of $\{\mathrm{VaR}_\alpha^m\}_{m=1}^M$, along with confidence intervals.

Derivative Prices in Calibrated Models

We now turn to the second example of model risk.

Idea: suppose we have a model for the price S.

Stochastic volatility in continuous time:

$\frac{dS_t}{S_t} = rdt + \sigma_t dW_t^{\mathbb{Q}}$, $\mathbb{Q}$: risk neutral measure.

Stochastic volatility in discrete time:

$\frac{\Delta S_n}{S_n} = (e^r - 1) + \sigma_n Z^{\mathbb{Q}}$, $Z^{\mathbb{Q}} = \pm 1$ with prob. $1/2$.

Note: σ_n known over $[n, n+1]$.

Goal: find the volatility process $\{\sigma_t\}_t$ (or $\{\sigma_n\}_n$) which fit to observed option prices.

A Discrete Time Picture

$N = 2,\ r = 0$:

$$S_0 = S_0$$

$$S_1^u = S_0(1 + \sigma_0)$$

$$S_1^d = S_0(1 - \sigma_0)$$

$$S_2^{uu} = S_1^u(1 + \sigma_1^u)$$

$$S_2^{ud} = S_1^u(1 - \sigma_1^u)$$

$$S_2^{du} = S_1^d(1 + \sigma_1^d)$$

$$S_2^{dd} = S_1^d(1 - \sigma_1^d)$$

3 free params $\sigma_0, \sigma^u, \sigma^d$

Fitting the Model

We observe market prices for call options with maturities $\{T_i\}_{i=1}^{I}$ and strikes $\{K_j\}_{j=1}^{J}$:

Write the price as $C^{m}(T_i, K_j)$. market

$\frac{\Delta S_n}{S_n} = (\sigma r - 1) + \sigma_n Z_n^{\mathbb{Q}}$

The theoretical prices are

We can produce model price as well.

$$C^{\mathbb{Q}}(T_i, K_j) = E^{\mathbb{Q}}\left[e^{-rT_i}(S_{T_i} - K_j)^{+}\right].$$

We seek a volatility process σ so that for all i, j:

matches market and model price

$$\boxed{C^{m}(T_i, K_j) = C^{\mathbb{Q}}(T_i, K_j).}$$

This is called "fitting the model".

Too Many Models?

It is easy to fit the model.

Typically: many processes σ can fit market prices.

Example: discrete time $N = 1, 2, \ldots$.

$N = 1$: we can fit one option price (σ_0 free parameter).

$N = 2$: we can fit three option prices ($\sigma_0, \sigma_1^u, \sigma_1^d$ free parameters).

$N = N$: we can fit $2^N - 1$ option prices.

In continuous time we can fit to a continuum of option prices.

A Family of Models

Therefore, given $\{C^m(T_i, K_j)\}$, we expect to have a *family* of models $\{M_a\}_{a\in\mathcal{A}}$ which fit the observed call option prices.

collection of models which callibrate.

每一个σ_a对应M_a

I.e.: one model for each fitted volatility function σ_a.

Now, let X be the payoff for a derivative which is not in the set of calibrated call options.

Each model $M_a, a \in \mathcal{A}$ yields a price for X:

$$P_a(X) = E^{\mathbb{Q}^a}\left[e^{-rT}X\right], \; a \in \mathcal{A}.$$

In the calibration set, every model have same prices, 外面的不知道.

Write $\mathbb{Q}^a$ to stress dependence upon M_a.

X is not in the callibration set. No reason to expect $P_a(X)$ is the same for all a.

(P_a call (with T_j and K_j) $= C^m(T_j, K_j)$ $\forall a$)

Model Risk as a Price Range

Since each M_a is calibrated, we do not a-priori know which model gives the "correct" price for X.

This is our model risk when evaluating X.

We define a measure of the model risk as:

$$\mu_{\mathcal{A}}(X) := \max_{a \in \mathcal{A}} \{P_a(X)\} - \min_{a \in \mathcal{A}} \{P_a(X)\}.$$ Size of the variation.

$\mu_{\mathcal{A}}(X)$ is the variation in prices across models consistent with observed option prices.

An Example of the Range

Typically, models are calibrated using at, or near at, the money options.

The range of prices $\mu_{\mathcal{A}}(X)$ can be large.

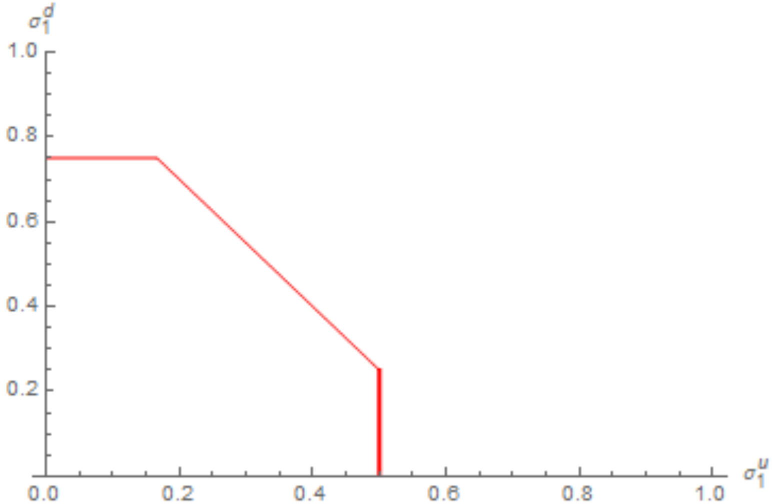
Example: discrete time, $N = 2, r = 0$.

Calibration to at the money calls $C^m(1, S_0), C^m(2, S_0)$.

An Example of the Range

Calculation shows

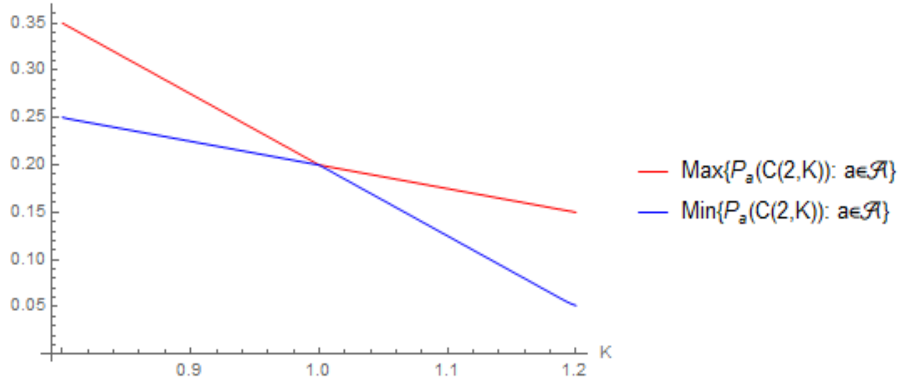
$\sigma_0 = 2\frac{C^m(1,S_0)}{S_0}$, but σ_1^u, σ_1^d must lie on the curve



and we need $C^m(1, S_0) < C^m(2, S_0) < \frac{S_0}{4}$.

An Example of the Range

$\mu_{\mathcal{A}}(X)$ for $X = C(2, K)$. $\overline{P}$: average price.



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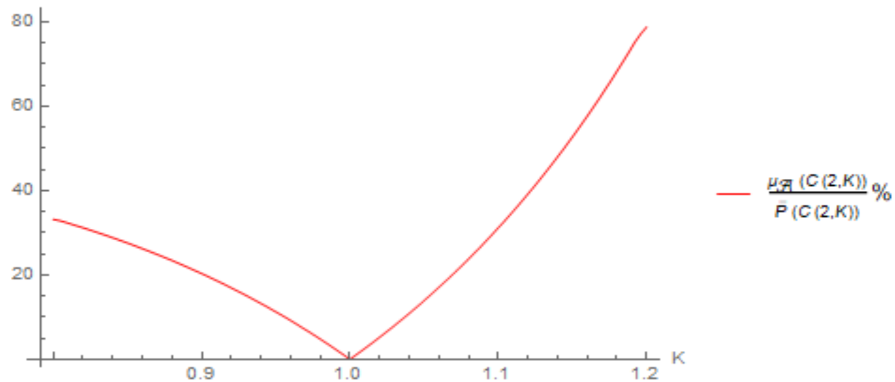

Example Conclusions

For out of the money calls the range is really large.

$\mu_{\mathcal{A}}(C(2, 1.2))$ is 79% of the average model price!

In continuous time:

This problem can be even more severe, especially for out of the money options.

Models consistent with options prices can yield wildly different path behavior.

E.g: we can find continuous models and models with jumps which are calibrated.