Counterparty Credit Risk

MF 731 Corporate Risk Management

Outline

Counterparty credit risk.

Value adjustments for Over the Counter (OTC) derivatives.

- CVA, DVA for an "OTC" European option.
- CVA, DVA for CDS in a constant hazard rate model.

Counterparty Credit Risk

Counterparty credit risk:

"The risk associated to counterparty default, in an OTC transaction."

E.g.: you are the protection buyer in a 5 year CDS, and Goldman Sachs is the protection seller. What is the risk that Goldman Sachs might default within the next 5 years?

Counterparty Credit Risk

Counterparty risk management poses a number of challenges. We need to

- Asses the risk to a given counterparty across all derivative positions.
  - In particular, for different product types.
- Adjust prices to account for the risk.
- Mitigate the risk, typically through collateral postings.

Counterparty Credit Risk

We will not cover

- Aggregation of counterparty risk across positions.
- The details of collateral postings.

We will cover

- Adjusting prices for counterparty risk.

We seek to understand the <u>V</u>alue <u>A</u>djustments

- BCVA: <u>B</u>ilateral <u>C</u>redit.
- CVA: <u>C</u>redit.
- DVA: <u>D</u>ebt.

The Value Adjustments

Where we are headed:

Computing BCVA, CVA, DVA for a CDS.

To gain intuition:

We first treat a hypothetical situation where we have bought a European option through an OTC transaction.

Though contrived (typically, one may buy the option on an exchange), this will help us understand the calculations.

European Option

Black-Scholes model:

Risky asset: $\frac{dX_t}{X_t} = \mu dt + \sigma dW_t$.

Money market account: $\frac{dB_t}{B_t} = rdt$.

Option payoff: $V(T) = V(X_T)$ at T.

Non-negative payoff: $V(x) \geq x$ for all $x > 0$.

Examples. Call: $V(T) = (X_T - K)^+$. Put: $V(T) = (K - X_T)^+$.

Option value at $t \leq T$:

$$V(t) = E^{\mathbb{Q}}\left[e^{-r(T-t)}V(X_T) \mid \mathcal{F}_t\right].$$

$\mathbb{Q}$: risk neutral measure. $\mathbb{F} = \{\mathcal{F}_t\}_{t \leq T}$: underlying filtration.

Value at a stopping time $\tau \leq T$:

$$V(\tau) = E^{\mathbb{Q}}\left[e^{-r(T-\tau)}V(X_T) \mid \mathcal{F}_\tau\right].$$

European Option Example

Label the buyer "B", and the seller "S".

How does the option value change taking into account default of B or S?

τ^B: buyer default time. τ^S: seller default time.

τ^B, τ^S: stopping times.

$\tau_B < \tau_S \Rightarrow \xi = B$

$\tau_S < \tau_B \Rightarrow \xi = S$

$\tau = \min\{\tau^B, \tau^S\}$: first default time. $\xi \in \{B, S\}$: name of first to default.

We take the buyer's perspective on payoffs.

The Payoffs

If $\tau > T$ (no default by maturity):

Payoff of $V(T) = V(X_T)$ at T.

(time 0) Discounted payoff: $V(T)e^{-rT}$.

If $\tau \leq T$ (either B or S defaults):

A close-out fee is paid at τ.

Industry convention: recovery at market value.

Taking into account loss given default.

The Payoff if S Defaults First

If $\tau \le T$, $\xi = S$, to compute the close out fee we

Identify the theoretical option value at τ:

$$V(\tau) = E^{\mathbb{Q}}\left[e^{-r(T-\tau)}V(X_T) \mid \mathcal{F}_\tau\right] \ge 0.$$

S owes B $V(\tau)$, but cannot pay the full amount. B instead receives $(1-\delta^S)V(\tau)$: δ^S loss given default for S.

Payoff at τ: $1_{\tau \le T} 1_{\xi = S}(1-\delta^S)V(\tau)$.

Discounted payoff: $1_{\tau \le T} 1_{\xi = S}(1-\delta^S)V(\tau)e^{-r\tau}$.

The Payoff if B Defaults First

If $\tau \leq T$, $\xi = B$, to compute the close-out fee we

Identify the theoretical option value at τ:

$$V(\tau) = E^{\mathbb{Q}}\left[e^{-r(T-\tau)}V(X_T) \mid \mathcal{F}_\tau\right] \geq 0.$$

S owes B $V(\tau)$. Since S has not defaulted, S pays B full amount $V(\tau)$.

Payoff at τ: $1_{\tau \leq T} 1_{\xi = B} V(\tau)$.

Discounted payoff: $1_{\tau \leq T} 1_{\xi = B} V(\tau) e^{-r\tau}$.

BCVA

This yields the "real" option value

$$V^r(0) = E^{\mathbb{Q}}\left[\underbrace{1_{\tau>T}V(T)e^{-rT}}_{\text{No default}}\right] + E^{\mathbb{Q}}\left[\underbrace{1_{\tau\le T}1_{\xi=S}(1-\delta^s)V(\tau)e^{-r\tau}}_{S\text{ defaults first}}\right]$$

$$+ E^{\mathbb{Q}}\left[\underbrace{1_{\tau\le T}1_{\xi=B}V(\tau)e^{-r\tau}}_{B\text{ defaults first}}\right];$$

$$= E^{\mathbb{Q}}\left[1_{\tau>T}V(T)e^{-rT} + 1_{\tau\le T}V(\tau)e^{-r\tau}\right]$$

$$- \delta^S E^{\mathbb{Q}}\left[1_{\tau\le T}1_{\xi=S}V(\tau)e^{-r\tau}\right].$$

$1_{\xi=B} + 1_{\xi=S} = 1$

$= E^{Q}[e^{-r(T\wedge\tau)} V_{T\wedge\tau}] - E^{Q}[1_{\tau\le T} 1_{\xi=S} V(\tau)e^{-r\tau}]$

$V(0) = E^{Q}[e^{-rT} V_T]$

The bilateral credit value adjustment is:

$$\mathrm{BCVA}(0) \triangleq V(0) - V^r(0).$$

$$\mathrm{BCVA}(0) = V(0) - V^r(0)$$

"Bilateral": the adjustment takes into account defaults for <u>both</u> the buyer and the seller.

Note: $V^r(0) = V(0) - \mathrm{BCVA}(0)$.

"Real" price is theoretical price less bilateral credit adjustment.

Warning: BCVA need not always be positive.

Especially for CDS, this value may be negative.

CVA

CVA: the change in option value due to S's default.

Basic idea (note: $V(t) \geq 0$ in our example):

Assume $\tau \leq T$ and $\xi = S$. Two options:

Buyer receives $V(\tau)$: as if there were no loss.

Buyer receives $(1-\delta^S)V(\tau)$: incorporating the loss.

The difference is $\delta^S V(\tau)$. ← "loss" (hypothetical) of $\delta^S \cdot V_\tau$ at τ

(hypothetically) Realized at τ.

The time 0 value is the CVA:

$$\mathrm{CVA}(0) = E^{\mathbb{Q}}\left[1_{\tau \leq T} 1_{\xi = S} \delta^S V(\tau) e^{-r\tau}\right].$$

DVA

DVA: the change in option value due to B's default.

Basic idea (note: $V(t) \geq 0$ in our example)

Assume $\tau \leq T$ and $\xi = B$. Since S has not defaulted, the two options are:

Buyer receives $V(\tau)$: as if there were no loss.

Buyer receives $V(\tau)$: S has not defaulted.

Difference is 0 at τ so the time 0 value is $\mathrm{DVA}(0) = 0$.

$\mathbb{E}^{\mathbb{Q}}[1_{\tau\leq T} 1_{\xi=B}\, 0] = 0$

WARNING: DVA is typically not 0. That it is 0 is specific to this example. For CDS it will not be 0.

A process M is a mart. if $\mathbb{E}^{\mathbb{Q}}[M_t | \mathcal{F}_s] = M_s$, $s<t$.

Risk neutral pricing theory:

$$t \mapsto e^{-rt} V_t \quad \text{is a} \quad \mathbb{Q}\text{-mart.}$$

Optional Sampling thm:

- I can plug stopping times into the mart. equation.

  - bounded stopping time

$$\mathbb{E}[M_\tau | \mathcal{F}_\sigma] = M_\sigma$$

$$= \sigma \equiv 0, \quad \mathbb{E}^{\mathbb{Q}}[M_\tau] = M_0$$

$$V(0) = \mathbb{E}^{\mathbb{Q}}[e^{-rT} V_T] = M_0 = V(0)$$

$$M_t = e^{-rt} V_t$$

optional sampling

$$V^r(0) = \mathbb{E}^{\mathbb{Q}}[e^{-r(\tau \wedge T)} V_{\tau \wedge T}] + DVA(0) - CVA(0) = V(0) + DVA(0) - CVA(0)$$

$$BCVA(0) = V(0) - V^r(0) = CVA(0) - DVA(0)$$

Notes on CVA and DVA

Since $\{V(t)e^{-rt}\}_{t \le T}$ is a $\mathbb{Q}$ Martingale

$$\begin{aligned} V^r(0) &= E^{\mathbb{Q}}\left[1_{\tau > T} V(T)e^{-rT} + 1_{\tau \le T} V(\tau)e^{-r\tau} - \delta^S 1_{\tau \le T} 1_{\xi = S} V(\tau)e^{-r\tau}\right]; \\ &= E^{\mathbb{Q}}\left[V(T \wedge \tau)e^{-r(T\wedge\tau)} - \underbrace{\delta^S 1_{\tau \le T} 1_{\xi = S} V(\tau)e^{-r\tau}}_{\text{CVA}(0)}\right]; \quad \text{DVA}(0) = 0 \\ &= V(0) - \text{CVA}(0) + \text{DVA}(0) \end{aligned}$$

Thus, $\text{BCVA}(0) = V(0) - V^r(0) = \text{CVA}(0) - \text{DVA}(0)$.

Assume we are the buyer:

CVA: price adjustment due to seller default.

Downward: the value of our position decreases.

DVA: price adjustment due to our default.

Upward: we owe less if we default because we cannot pay!
Can our default really increase our value?

Application: Call Option

Payoff: $V(T) = (X_T - K)^+$, $V(t) = C^{BS}(t, X_t)$.

C^{BS}: Black-Scholes price for a call option.

Assume $\tau^B \perp\!\!\!\perp \tau^S$, with constant intensities γ^B, γ^S.

Explicit formula for the CVA!

$$\mathrm{CVA}(0) = C^{BS}(0, X_0)\frac{\gamma^S \delta^S}{\gamma^S + \gamma^B}\left(1 - e^{-(\gamma^S + \gamma^B)T}\right).$$

Parameters:

$r = 2\%$, $\sigma = 30\%$, $T = 1$, $K = 55$, $X_0 = 52$.
$\gamma^B = 3\%$, $\gamma^S = 5\%$. $\delta^S = 50\%$.

Numeric values:

$\mathrm{CVA}(0) = 0.1298$. 2.40% of theoretical option value.

The Value Adjustments for CDS

The idea behind the calculations is the same.

Compute theoretical CDS value $V(t)$ for all $t \leq T$.

Compute actual value $V^r(0)$, taking into default of B, S.

$\mathrm{BCVA}(0) = V(0) - V^r(0)$.

$\mathrm{CVA}(0)$: adjustment due to default of S.

$\mathrm{DVA}(0)$: adjustment due to default of B.

$\mathrm{BCVA}(0) = \mathrm{CVA}(0) - \mathrm{DVA}(0)$.

CDS

However, the calculations are much trickier.

- CDS contract involves multiple payments over time.
- Payments are from B to S and vice versa.
- There is another default we have to worry about:
  - That of the underlying reference entity R.

Despite all this, we can still compute the value adjustments.

CDS: Theoretical Value

Recall the (theoretical) CDS price derivation:

Payment dates $0 = t_0 < t_1 < t_2 < ... < t_{T\Delta} = T$.

$t_n = n/\Delta$ for $n = 1, ..., T\Delta$.

Δ: payment frequency. E.g. 4 (quarterly).

B pays S a fixed premium x/Δ at each payment date.

x: annualized CDS spread.

τ^R: default time of R. If $\tau^R \in (t_{n-1}, t_n]$:

LGD

S pays B δ^R. B makes no further premium payments to x.

We ignore the accrued interest B would have to pay S at τ^R.

CDS: Theoretical Value

We need the theoretical value $V(t)$ for all $t \leq T$.

Interest rate process $r = \{r(t)\}_{t \geq 0}$.

Discounted cash flows at $t \leq T$: $\tau^R > t$

buyer's prospective

$$\Pi(t) = 1_{t < \tau^R \leq T} \underset{\text{pay back}}{\delta^R} e^{-\int_t^{\tau^R} r(u)du} - \frac{X}{\Delta} \sum_{n=1}^{T\Delta} 1_{t \leq t_n < \tau^R} e^{-\int_t^{t_n} r(u)du}.$$

Time t theoretical value is thus $V(t) = E^{\mathbb{Q}}\left[\Pi(t) \mid \mathcal{F}_t\right]$.

CDS: Actual Value

We now account for counterparty default.

Set

$\tau = \min\{\tau^R, \tau^S, \tau^B\}$: first default time.

$\xi \in \{R, S, B\}$: identity of first to default.

$\Pi^r(t)$: discounted actual flows at $t \leq T$, accounting for default.

If $\tau > T$ (no one defaults) or $\tau \leq T, \xi = R$ (R first to default): – no adjustment needed

$\Pi_1^r(0) = \Pi(0)$: no change in flows.

CDS: Actual Value

If $\tau \leq T$, $\xi = S$ (S defaults first)

1) B already paid x/Δ at t_n for all $t_n < \tau$.

2) Close out payment at τ ($V(\tau)$: theoretical value).

If $V(\tau) > 0$, S owes $V(\tau)$, but can only pay $(1-\delta^S)V(\tau)$.

If $V(\tau) < 0$, B owes $-V(\tau)$, and can pay full amount.

Net payment: $(1-\delta^S)V(\tau)^+ - V(\tau)^-$. B's prospective

Recall: $x^+ = \max[x, 0]$, $x^- = -\min[x, 0]$, $x = x^+ - x^-$.

Present value of payments:

$$\Pi_2^r(0) = \left((1-\delta^S)V(\tau)^+ - V(\tau)^-\right) e^{-\int_0^\tau r(u)du}$$
$$- \frac{x}{\Delta} \sum_{n=1}^{T\Delta} 1_{t_n<\tau} e^{-\int_0^{t_n} r(u)du}.$$

CDS: Actual Value

If $\tau \le T$, $\xi = B$ (B defaults first)

- B already paid x/Δ at t_n for all $t_n < \tau$.
- Close out payment at τ ($V(\tau)$: theoretical value)
  - If $V(\tau) > 0$, S owes B, $V(\tau)^+$, and can pay full amount. B gets $V(\tau)$
  - If $V(\tau) < 0$, B owes S, $V(\tau)^-$, but can only pay $(1-\delta^B)V(\tau)^-$.
  - Net payment: $V(\tau)^+ - (1-\delta^B)V(\tau)^-$. at τ.
- Present value of payments:

$$\Pi_3^r(0) = \left(V(\tau)^+ - (1-\delta^B)V(\tau)^-\right) e^{-\int_0^\tau r(u)du} - \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t_n<\tau} e^{-\int_0^{t_n} r(u)du}.$$

CDS: Actual Value

We computed the discounted value in all scenarios

- No default or R defaults first.
- S defaults first.
- B defaults first.

The actual time 0 discounted flows are

$$\boxed{\begin{aligned}\Pi^r(0) = {} & \left(1_{\tau > T} + 1_{\tau \leq T, \xi = R}\right) \Pi_1^r(0) + 1_{\tau \leq T, \xi = S} \Pi_2^r(0) \\ & + 1_{\tau \leq T, \xi = B} \Pi_3^r(0).\end{aligned}}$$

The actual time 0 value is $V^r(0) = E^{\mathbb{Q}}\left[\Pi^r(0)\right]$.

CDS: BCVA

BCVA is the same as before:

$$\mathrm{BCVA}(0) = V(0) - V^r(0).$$

The difference between the theoretical and actual values, taking defaults of B and S into account.

Alternatively, $V^r(0) = V(0) - \mathrm{BCVA}(0)$.

CDS: CVA

To compute CVA we see what happens if S defaults.

If $\tau \leq T$, $\xi = S$ the game stops at τ:

B actually receives $(1 - \delta^S)V(\tau)^+ - V(\tau)^-$.

Absent default, B receives $V(\tau) = V(\tau)^+ - V(\tau)^-$.

The loss is thus $\delta^S V(\tau)^+$ which is "realized" at τ.

$$\mathrm{CVA}(0) = E^{\mathbb{Q}}\left[1_{\tau \leq T, \xi = S}\delta^S V(\tau)^+ e^{-\int_0^\tau r(u)du}\right].$$

CDS: DVA

For DVA we see what happens if B defaults.

If $\tau \leq T$, $\xi = B$ the game stops at τ:

B actually receives $V(\tau)^+ - (1 - \delta^B) V(\tau)^-$.

Absent its own default B receives $V(\tau) = V(\tau)^+ - V(\tau)^-$.

The gain(!) due to B's default is $\delta^B V(\tau)^-$, "realized" at τ.

$$\mathrm{DVA}(0) = E^{\mathbb{Q}}\left[1_{\tau \leq T, \xi = B} \delta^B V(\tau)^- e^{-\int_0^\tau r(u)du}\right].$$

time 0 price of gain

Theoretical problem (revisited): how can B gain from its own default??

CDS: Formula Recap

$$\Pi(t) = 1_{t<\tau^R\le T}\delta^R e^{-\int_t^{\tau^R} r(u)du} - \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t\le t_n<\tau^R} e^{-\int_t^{t_n} r(u)du};$$

$$V(\tau) = E^{\mathbb{Q}}\left[\Pi(\tau) \;\middle|\; \mathcal{F}_\tau\right]; \quad V(0) = E^{\mathbb{Q}}\left[\Pi(0)\right];$$

$$\Pi_1^r(0) = \Pi(0);$$

$$\Pi_2^r(0) = \left((1-\delta^S)V(\tau)^+ - V(\tau)^-\right) e^{-\int_0^\tau r(u)du} - \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t_n<\tau} e^{-\int_0^{t_n} r(u)du};$$

$$\Pi_3^r(0) = \left(V(\tau)^+ - (1-\delta^B)V(\tau)^-\right) e^{-\int_0^\tau r(u)du} - \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t_n<\tau} e^{-\int_0^{t_n} r(u)du};$$

$$\Pi^r(0) = (1_{\tau>T} + 1_{\tau\le T,\xi=R})\,\Pi_1^r(0) + 1_{\tau\le T,\xi=S}\Pi_2^r(0) + 1_{\tau\le T,\xi=B}\Pi_3^r(0);$$

$$V^r(0) = E^{\mathbb{Q}}\left[\Pi^r(0)\right].$$

CDS: Value Adjustment Recap

$$\begin{aligned}
\mathrm{BCVA}(0) &= V(0) - V^r(0); \\
\mathrm{CVA}(0) &= E^{\mathbb{Q}}\left[\mathbb{1}_{\tau \le T, \xi = S}\, \delta^S V(\tau)^+ e^{-\int_0^\tau r(u)du}\right]; \\
\mathrm{DVA}(0) &= E^{\mathbb{Q}}\left[\mathbb{1}_{\tau \le T, \xi = B}\, \delta^B V(\tau)^- e^{-\int_0^\tau r(u)du}\right].
\end{aligned}$$

Similar to before, a long calculation shows:

$$\mathrm{BCVA}(0) = \mathrm{CVA}(0) - \mathrm{DVA}(0).$$

The latter formula gives us an "easier" way to compute V^r, as well as $\mathrm{BCVA}(0)$.

CDS: Computational Considerations

How do we actually compute these adjustments?

Focus on CVA:

$$\mathrm{CVA}(0) = E^{\mathbb{Q}}\left[1_{\tau \le T, \xi = S}\delta^S V(\tau)^+ e^{-\int_0^\tau r(u)du}\right].$$

To compute this we have to evaluate

$$V(t) = E^{\mathbb{Q}}\left[1_{t<\tau^R\le T}\delta^R e^{-\int_t^{\tau^R} r(u)du} - \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t\le t_n<\tau} e^{-\int_t^{t_n} r(u)du} \mid \mathcal{F}_t\right].$$

$\tau = \min\left\{\tau^R, \tau^S, \tau^B\right\}.$

ξ: name of first to default.

$r = \{r(t)\}_{t\ge 0}$:

CDS: Value Adjustments for Constant Intensities

We consider a simplified setting where

Constant spot rate r.

$\{\tau^i\}_{i \in \{R,B,S\}}$ are independent with constant intensities $\gamma^i, i \in \{R, B, S\}$.

$$\mathbb{Q}\left[\tau^i \leq t\right] = 1 - e^{-\gamma^i t} \text{ for } i \in \{R, B, S\}.$$

Easy to simulate:

Sample $\tau^i \sim \mathrm{Exp}\left(\gamma^i\right)$, $i \in \{R, B, S\}$ independently.

Set $\tau = \min\left\{\tau^R, \tau^B, \tau^S\right\}$, write ξ accordingly.

Easy to introduce dependence via copulas.

CDS: Information Flow

Consequences of independence:

$$\tau = \min\left\{\tau^R, \tau^S, \tau^B\right\} \text{ has intensity } \gamma = \gamma^R + \gamma^S + \gamma^B.$$

$\perp\!\!\!\perp$, exponential r.v.

$$\begin{aligned}\mathbb{Q}[\tau \geq t] &= \mathbb{Q}\left[\tau^R \geq t, \tau^S \geq t, \tau^R \geq t\right];\\ &= \mathbb{Q}\left[\tau^R \geq t\right]\mathbb{Q}\left[\tau^S \geq t\right]\mathbb{Q}\left[\tau^B \geq t\right];\\ &= e^{-\gamma^R t}e^{-\gamma^S t}e^{-\gamma^B t} = e^{-\gamma t}.\end{aligned}$$

$\tau \overset{\mathbb{Q}}{\sim} \exp(\gamma = \gamma^R + \gamma^S + \gamma^B)$

$$\mathbb{Q}\left[\xi = i \middle| \tau\right] = \frac{\gamma^i}{\gamma}; \quad i \in \{R, B, S\}.$$

Proof follows similar arguments.

CDS: CVA

Formulas simplify considerably. First,

$$V(t) = E^{\mathbb{Q}}\left[\overbrace{1_{t<\tau^R\leq T}\delta^R e^{-r(\tau^R-t)}}^{\text{protection side}} - \overbrace{\frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t\leq t_n<\tau^R} e^{-r(t_n-t)}}^{\text{premium side}} \;\middle|\; \mathcal{F}_t\right];$$

$$= 1_{\tau^R>t}\tilde{V}(t);$$

$$\underbrace{\tilde{V}(t)}_{\text{deterministic}} = \delta^R \int_t^T e^{-r(s-t)} \overbrace{\gamma^R e^{-\gamma^R(s-t)}}^{\text{conditional CDF}} ds \qquad t < \tau_R < T$$

$$- \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t\leq t_n} e^{-r(t_n-t)} \int_{t_n}^{\infty} \gamma^R e^{-\gamma^R(s-t)} ds;$$

$$= \frac{\delta^R\gamma^R}{r+\gamma^R}\left(1 - e^{-(r+\gamma^R)(T-t)}\right) - \frac{x}{\Delta}\sum_{n=1}^{T\Delta} 1_{t\leq t_n} e^{-(r+\gamma^R)(t_n-t)}$$

CDS: CVA

With this expression ($1_{\xi=S}1_{\tau^R>\tau} = 1_{\xi=S}$):

$$
\begin{aligned}
\mathrm{CVA}(0) &= E^{\mathbb{Q}}\left[1_{\tau\le T}1_{\xi=S}\delta^S V(\tau)^+ e^{-r\tau}\right];\\
&= \delta^S E^{\mathbb{Q}}\left[1_{\tau\le T}1_{\xi=S}1_{\tau^R>\tau}\tilde{V}(\tau)^+ e^{-r\tau}\right];\\
&= \delta^S E^{\mathbb{Q}}\left[1_{\tau\le T}1_{\xi=S}\tilde{V}(\tau)^+ e^{-r\tau}\right];\\
&= \delta^S \int_0^T \gamma e^{-\gamma t}\frac{\gamma^S}{\gamma}\tilde{V}(t)^+ e^{-rt}dt;\\
&= \delta^S\gamma^S \int_0^T \tilde{V}(t)^+ e^{-(r+\gamma)t}dt.
\end{aligned}
$$

$= \frac{\delta^S \gamma^S}{\gamma}\int_0^T \tilde{V}(u)^+ e^{-ru} du$

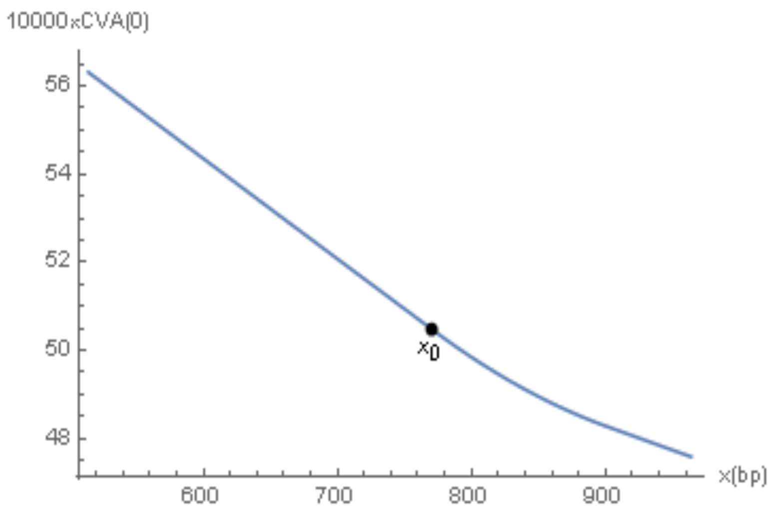
Similarly, we have

$$\mathrm{DVA}(0) = \delta^B\gamma^B \int_0^T \tilde{V}(t)^- e^{-(r+\gamma)t}dt.$$

CDS: Numerical Application

$r = 5\%$, $\gamma^R = 20\%$, $\gamma^S = 10\%$, $\gamma^B = 3\%$.
$\delta^R = 40\%$, $\delta^S = 50\%$, $\delta^B = 60\%$. $T = 5$, $\Delta = 4$.

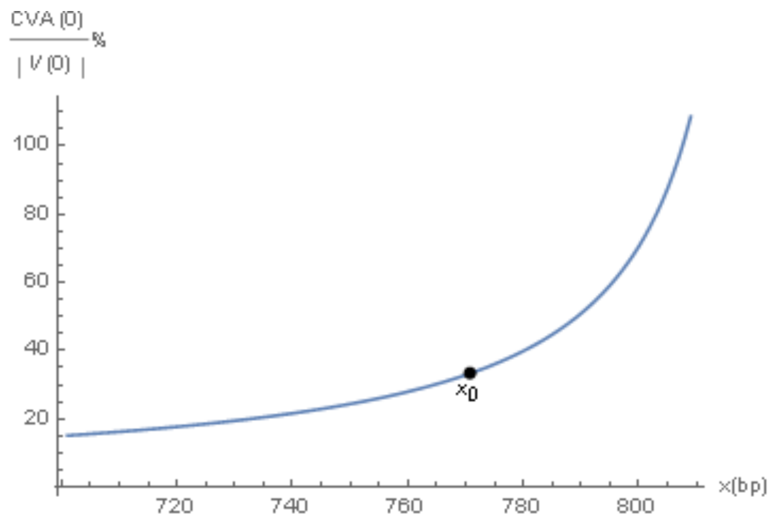
$\mathrm{CVA}(0)$ (bp) versus CDS spread x (bp).



x_0: fair time 0 spread (including accrued interest).

CDS: Numerical Application

$\mathrm{CVA}(0)$ as % of $|V(0)|$ versus CDS spread x:



Though the $\mathrm{CVA}(0)$ is small in absolute terms, it is a relatively large percentage of the swap value.