MF 731 Corporate Risk Management
Practice Midterm Exam. Fall, 2021

This is the practice midterm exam. There are 5 questions for a total 100 points. Each question may contain multiple parts. You have 2 hours to complete the exam

To get the most out of the exam please take this exam as if it were the real exam! I.e. do not use notes, the class textbook, the internet, and especially do not look at the solutions ahead of time! Give yourself two hours to take the exam, and abide by this rule!

1. Comparing VaR$_\alpha$ across Loss Operators (20 Points) Consider a one stock portfolio where we hold $\lambda_t > 0$ shares of the stock constant over the period $[t, t+\Delta]$. Write $S_t > 0$ as the time t price and $X_{t+\Delta}$ as the log return over $[t, t+\Delta]$, and assume we have a model for $X_{t+\Delta}$ given our information at time t.

(a) **(5 Points)** Write the full, linear and quadratic loss operators for the portfolio over $[t, t+\Delta]$.

(b) **(10 Points)** Can you order $\mathrm{VaR}_\alpha(L)$, $\mathrm{VaR}_\alpha\left(L^{lin}\right)$ and $\mathrm{VaR}_\alpha\left(L^{quad}\right)$? If so, provide the ordering. If not, explain why not.

(c) **(5 Points)** Discuss value at risk using quadratic losses. Can you think of any reasons for why we (at least in this situation) might not want to use quadratic losses to estimate the value at risk?

2. Back-Testing VaR$_\alpha$ for a Discrete Loss Distribution (20 Points) In class we developed the back-testing methodology assuming the conditional loss distribution of $L_{t+\Delta}$ given our time t information $\mathcal{F}_t$ was continuous. In this problem we will see that a similar methodology can be developed for discrete distributions.

For each t assume that conditional upon our information $\mathcal{F}_t$, the losses over $[t, t+\Delta]$ take the form

$$L_{t+\Delta} = \begin{cases} -1 & \text{Probability } = \frac{1-p}{2} \\ 1 & \text{Probability } = \frac{1-p}{2} \\ \mathcal{L}_{t+\Delta} & \text{Probability } = p \end{cases}.$$

Here, $\mathcal{L}_{t+\Delta} >> 1$ is not random (i.e. NOT an $\mathcal{F}_t$ measurable random variable), and $0 < p < 1/2$ is the same for all t.

(a) **(6 Points)** For $(1-p)/2 < \alpha < 1$, identify VaR_α^t and ES_α^t.

(b) **(14 Points)** Assume we have historical loss data $\left\{L_{t-(m-1)\Delta}\right\}_{m=1}^M$ over the last M days. Describe the back-testing methodology for how we would check if our methodology for estimating VaR$_\alpha$ is sound. Here, assume $(1-p)/2 < \alpha < 1-p$.

3. True/False (5 points each). Identify if each of the statements below is true or false. If it is true provide a short explanation or proof for why it is true. If it is false, provide a short explanation, proof, or counter-example for why it is false. Answers with no explanation will not be given any credit.

(a) For loss random variables L_1 and L_2, if $L_1 \geq L_2$ with probability one then $\mathrm{ES}_\alpha(L_1) \geq \mathrm{VaR}_\alpha(L_2)$.

(b) Assume that given $\mathcal{F}_t$ we model $X_{t+\Delta} = \sigma_{t+\Delta} Z$ where $\sigma_{t+\Delta}$ is known given $\mathcal{F}_t$, and Z is independent of $\mathcal{F}_t$ with mean 0 and variance 1. Then, EWMA produces a higher estimate of $\mathbb{E}\left[\sigma_{t+2\Delta}^2 \middle| \mathcal{F}_t\right]$ than GARCH.

(c) BASEL introduced VaR$_\alpha$ as an appropriate tool for measuring risk in the amendment to the first BASEL accord.

(d) All coherent risk measures are convex.

4. (20 Points) Coherent measures and portfolio variance. Consider d stocks $S^{(1)}, ..., S^{(d)}$ and an investment window $[t, t+\Delta]$. Write $X = (X^{(1)}, ..., X^{(d)})$ as the log return over $[t, t+\Delta]$, and assume that given t, $X \sim N(\mu, \Sigma)$. Assume we have two portfolios P^1, P^2 with respective dollar positions (at t) of θ and ψ. We use linearized losses $L^{1,'}, L^{2,'}$ and further assume given t

(1) $L^{1,'}$ and $L^{2,'}$ have the same conditional mean.
(2) $L^{1,'}$ has a strictly lower conditional variance than $L^{2,'}$.

Let ϱ be a risk measure. Is it possible for ϱ to be coherent if $\varrho(L^{2,'}) < \varrho(L^{1,'})$?

5. Time Aggregation for a Forward Contract using GARCH (20 Points). Consider a stock model where the log return and variance follow (under the physical measure) a GARCH process but with drift $r > 0$, the constant interest rate. The physical and risk neutral measures are the same, and the GARCH model is

$$\begin{aligned} X_{t+\Delta} &= \left(r\Delta - \frac{1}{2}\sigma^2_{t+\Delta}\right) + \sigma_{t+\Delta} Z_{t+\Delta}, \\ \sigma^2_{t+\Delta} &= \alpha_0 + \alpha_1 \left(X_t - \left(r\Delta - \frac{1}{2}\sigma_t^2\right)\right)^2 + \beta_1 \sigma_t^2. \end{aligned} \tag{0.1}$$

Here, $\alpha_0, \alpha_1, \beta_1 > 0$, $\alpha_1 + \beta_1 < 1$ and the $\{Z_{t+k\Delta}\}_{k=-\infty}^{\infty}$ are iid $N(0,1)$ random variables. The information at t is generated by $\{Z_{t+k\Delta}\}_{k=-\infty}^{0}$. You may assume this model has a solution.

Fix a strike $K > 0$ and maturity $T = k_T\Delta$. A forward contract with maturity T and strike K is an option with payoff $S_T - K$ at T. To ease notation write $t_k = k\Delta$ for any integer k.

(a) **(6 Points)** As the physical and risk neutral measures are the same, for integers $k \leq k_T$ the forward price at t_k is

$$V_{t_k} = \mathbb{E}\left[e^{-r(T-t_k)}(S_T - K)\middle|\mathcal{F}_{t_k}\right].$$

Show that $\mathbb{E}\left[e^{-r(T-t_k)}S_T\middle|\mathcal{F}_{t_k}\right] = S_{t_k}$ and hence if $K = S_{t_0}e^{r(T-t_0)}$ the forward contract has 0 value at $t_0 = 0$. **Note:** you must prove your result. Just saying the discounted stock price is a martingale under the risk neutral measure is not sufficient!

(b) **(14 Points)** Fix K from part (a). Assume our portfolio consists of N forward contracts, and our horizon is $[t_m, t_M]$, where $0 < m < M < k_T$. Write the pseudo code for how you estimate/obtain the value at risk for the m day full losses $L := -(V_{t_M} - V_{t_m})$, $\mathrm{VaR}_\alpha(L)$ given our information at t_m. Here, you may assume that $\alpha_0, \alpha_1, \beta_1$ are given and $S_{t_m}, X_{t_m}, \sigma_{t_m+\Delta}$ are known at t_m.