HW Problems for Assignment 6 - Part 1
Due 6:30 PM Tuesday, December 7, 2021

1. (15 Points) Recover at Treasury. An alternative to the Recovery at Face (RF) convention discussed in class, is the Recovery at Treasury (RT) convention. Here, if the default time τ is prior to T, the bond holder receives $(1-\delta)$ per unit notional of a default free ZCB maturing at T. As such, the payment $(1-\delta)$ is at T, rather than at τ.

In the hazard rate model discussed in class, identify the time $t \leq T$ price of a defaultable ZCB using the RT methodology, given $\tau > t$. Assuming $r(t) \geq 0, t \leq T$, how do the RT and RF prices compare?

2. (15 Points) Credit Default Swap Spreads and Accrued Interest. Here, for the hazard rate model discussed in class, we will assume 0 interest rates (i.e. $r(t) = 0$ for all $t \geq 0$). As shown in class, if we ignore the accrued interest paid by the borrower to the seller up default, the time t credit default swap spread for entering into a N year swap as

$$x_t = \delta \frac{\int_t^{t+N} \gamma(s) e^{-\int_t^s \gamma(u)du} ds}{\frac{1}{\Delta}\sum_{n=1}^{N\Delta} e^{-\int_t^{t+n/\Delta}\gamma(u)du}}, \tag{0.1}$$

In this exercise we will not ignore accrued interest, and see how things change. Below, we set $t_n = t + n/\Delta$ for $n = 1, ..., N\Delta$. Recall that if $t_{n-1} < \tau \leq t_n$ then upon default, the buyer must pay the seller accrued interest in the amount of $x(\tau - t_{n-1})$ where x is the swap spread. As the interest rate is 0, the fair value of these payments is

$$E^{\mathbb{Q}}\left[\sum_{n=1}^{N\Delta} 1_{t_{n-1}<\tau\leq t_n} x(\tau - t_{n-1}) \;\middle|\; \mathcal{F}_t\right].$$

Simplify this formula to show that, including accrued interest, the time t swap rate is

$$x_t^{ai} = \delta \frac{\int_t^{t+N} \gamma(s) e^{-\int_t^s \gamma(u)du} ds}{\int_t^{t+N} e^{-\int_t^s \gamma(u)du} ds}, \tag{0.2}$$

which admits a simple interpretation as a weighted average of the intensity function. Looking at the formulas including and non including accrued interest, do you expect much of a difference?

3. (20 Points) Parameterized Logistic Copula. Let X_1 and X_2 be random variables with joint cdf

$$F_\theta(x_1, x_2) = \left(1 + e^{-x_1} + e^{-x_2} + (1-\theta)e^{-x_1-x_2}\right)^{-1}, \qquad x_1, x_2 \in \mathbb{R},$$

where $\theta \in [-1, 1]$.

(a) (**5 Points**) Identify the marginal distributions $F_{\theta,1}$, $F_{\theta,2}$ for X_1 and X_2 respectively.

(b) (**5 Points**) Show that when $\theta = 0$, X_1 and X_2 are independent.

(c) (**10 Points**) Show that the copula of X_1 and X_2, defined abstractly by $c_\theta(u_1, u_2) = F_\theta(F_{\theta,1}^{-1}(u_1), F_{\theta,2}^{-1}(u_2))$, takes the form

$$C_\theta(u_1, u_2) = \frac{u_1 u_2}{1 - \theta(1 - u_1)(1 - u_2)}.$$