MF731

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part 1.

1. (a) $x \in \{1, 2, \cdots, k\}$, k is fixed integer $p(k) = \mathbb{P}[X=k] > 0$

cdf $F(x) = \sum_{k=1}^{\lfloor x \rfloor} p(k)$

suppose their twist $c_n > 0$ d_n that $F(c_n x + d_n)^n \sim H(x)$ with $H(x)$ degenerated

$F(c_n x_n + d_n) = \left(\sum_{k=1}^{\lfloor c_n x + d_n} p(x)\right)^n$.

when $n \to \infty$ $\sum_{k=1}^{c_n x + d_n} p(k) \begin{cases} = 1 & \text{if } c_n x + d_n \geq K \\ < 1 & \text{if } c_n x + d_n < K \end{cases}$

So $\lim_{n\to\infty} F(c_n x + d_n)^n = 1_{c_n x + d_n \geq k}$

This shows that $H(x)$ is degenerated, so there are no $c_n > 0$, d_n that make $H(x)$ a non-degenerated cdf.

(b) pmf $p(k) = \int_k^{k+1} \frac{1}{y^2} dy = \frac{1}{k} - \frac{1}{k+1} = \frac{1}{(k+1)k}$, $k = 1, 2 \ldots$

$F(c_n x + d_n)^n = \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{c_n x + d_n} - \frac{1}{c_n x + d_{n+1}}\right)^n$

$= \left(1 - \frac{1}{c_n x + d_n + 1}\right)^n$

let $c_n = n$, $d_n = n-1$

$F(c_n x + d_n)^n = \left(1 - \frac{1}{n(x+1)}\right)^n \to \exp\left(-\frac{1}{x+1}\right)$ which is a GEV

and we have $\exp\left(-\frac{1}{x+1}\right) = \exp\left(-(1+x)^{-1}\right)$

$\Rightarrow \xi = 1$, $\mu = 0$, $\sigma = 1$

2.

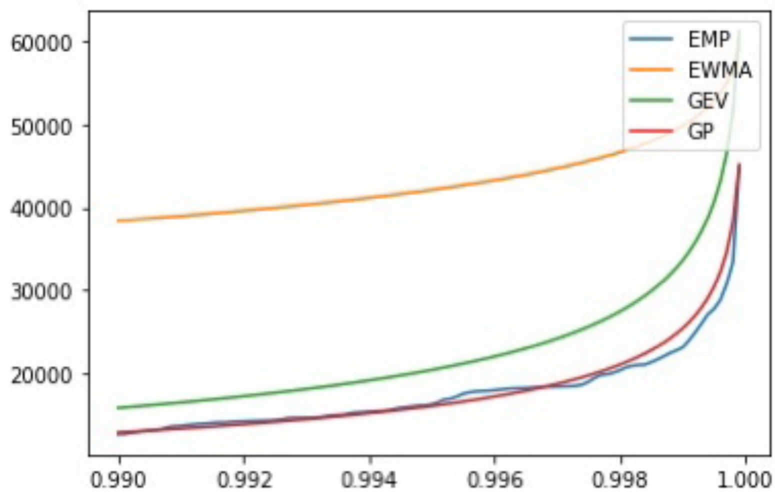
For $u \in [0,1]$, $x \in [0, 1-u]$

$F_u(x) = P(x \leq x + \mu \mid x > \mu) = \frac{P[u < x \leq x + \mu]}{P[x > u]} = \frac{x}{1-u}$

$G_{-1, \beta(u)}(x) = 1 - \left(1 + \frac{(-1)x}{\beta}\right)^{-1/(-1)} = \frac{x}{\beta}$

Thus, if we let $\beta(\mu) = 1 - \mu$,

$\lim_{u \to 1} \sup \left[F_{(u)} \infty - G_{-1, \beta(u)}(x)\right| -$



the highest value:
EMP: 44788.057
EWMA: 59361.880
GEV: 61290.322
GP: 61290.321

part 2.

1.

(a). mean. $E(S_N) = E(X)\,E(N)$

variance $Var(S_N) = E^2(X)\,Var(N) + E(N)\cdot Var(X)$

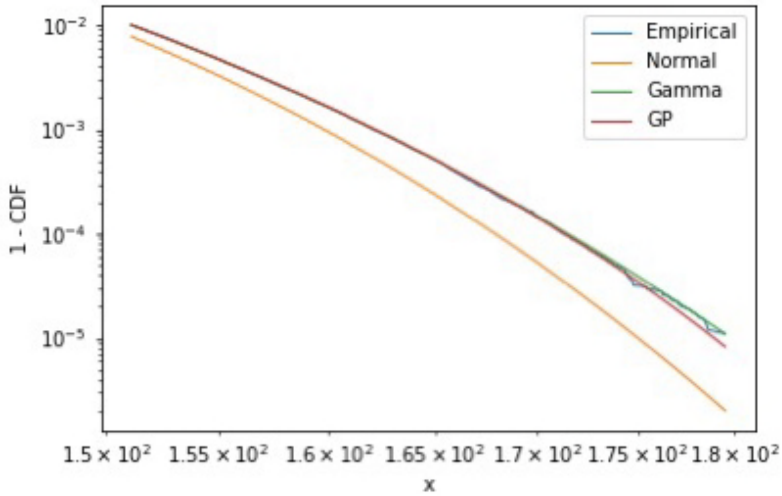
(b) Gamma, distribution.

mean: $\frac{\alpha}{\beta} + k = E[S_N] = E[X]E[N]$

Variance: $\frac{\alpha}{\beta^2} = Var[S_N] = E[X]^2\,Var(N) + E(N)\,Var(X)$.

Skewness: $\frac{2}{\sqrt{\alpha}} = \frac{E[X^3]}{\sqrt{\alpha E(X^2)^3}}$.

$\alpha = \left[\frac{2}{Skew[S_N]}\right]^2$. $\beta = \sqrt{\frac{\alpha}{Var(S_N)}}$ $k = E[S_N] - \frac{\alpha}{\beta}$



by the pic above we have:

GPD > Gamma > Normal.

$\therefore$ Gamma and GPD are much better than Normal

2.

(a). Note that $S_1^b = S_1(1 - \frac{1}{2}s_1)$

$\therefore L_1 = -\Delta S_1(1-\frac{1}{2}s_1) + \Delta S_0 = -\Delta S_0 e^{X_1}(1-\frac{1}{2}s_1) + \Delta S_0 = -\Delta S_0(e^{X_1}(1-\frac{1}{2}s_1) - 1)$

(b) To compare VaR_α we have loss:

$\bar{L}_1 = -\Delta S_0(e^{X_1} - 1)$

Using $X_1 \sim \mu_1 + \sigma_1 Z$ with $Z \sim N(0,1)$

$\tilde{P}[L_1^u \le T] = P[Z \ge \frac{1}{\sigma_1}(\log(1 - \frac{T}{\Delta S_0}) - \mu_1)]$

$= 1 - N(\frac{1}{\sigma_1}(\log(1 - \frac{T}{\Delta S_0}) - \mu_1))$

with $T = VaR_\alpha$.

$VaR_\alpha = \Delta S_0 \cdot (1 - e^{\mu_1 + \sigma_1 N^{-1}(1-\alpha)})$

$\therefore LC = \frac{1}{2}\Delta S_0(\nu_1 + k\xi_1)$ $LVaR_\alpha^{ind} = VaR_\alpha + LC$.

x

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(1) The confidence alpah:  0.99
(2) estimate of LVaR via simulation: 341.15
(3) estimate of VaR via simulation: 335.62
(4) The estimated liquidity cost LC: 5.53
(5) The estimated pct increase in the RM: 1.65%
(6) The industry approximate LVaR: 348.89
(7) The industry liquidity cost LC: 12.98
(8) The industry pct increase in the RM: 3.86%
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