MF 772 Assignment 9 Yuhe Xiao

Problem 1

(a)

$C(u,0) = C(0,v) = 0, \qquad C(u,1) = u, \qquad C(1,v) = v$

$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geq 0$

Let $u_2 = v_2 = 1, u_1 \leq 1, v_1 \leq 1$

We have $C(u_1,v_1) \geq u_1 + v_1 - 1$, and by definition, C>=0, so

$C(u,v) \geq \max\{u+v-1,0\}$

Let $u_2 = 1, v_1 = 0, u_1 \leq 1, v_2 \geq 0$

We have $C(u_1,v_2) \leq v_2$, by the same argument we can have $C(u_1,v_2) \leq u_1$, so

$C(u,v) \leq \min\{u,v\}$

$\Rightarrow \max\{u+v-1,0\} \leq C(u,v) \leq \min\{u,v\}$

(b)

$C(u,0) = C(0,v) = 0 + 0 = 0$

$C(u,1) = \lambda u + (1-\lambda)u = u$

$C(1,v) = \lambda v + (1-\lambda)v = v$

$$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1)$$
$$= \lambda[C_1(u_2,v_2) - C_1(u_2,v_1) - C_1(u_1,v_2) + C_1(u_1,v_1)]$$
$$+ (1-\lambda)[C_2(u_2,v_2) - C_2(u_2,v_1) - C_2(u_1,v_2) + C_2(u_1,v_1)] \geq 0$$

So C is copula

(c)

Let $u_2 = 1, u_1 \leq 1, 1 \geq v_2 \geq v_1 \geq 0$

$v_2 - v_1 \geq C(u_1,v_2) - C(u_1,v_1) \Rightarrow |C(u,v_2) - C(u,v_1)| \leq |v_2 - v_1|$

Let $v_2 = 1, v_1 \leq 1, 1 \geq u_2 \geq u_1 \geq 0$

$u_2 - u_1 \geq C(u_2,v_1) - C(u_1,v_1) \Rightarrow |C(u_2,v) - C(u_1,v)| \leq |u_2 - u_1|$

$$|C(u_2,v_2) - C(u_1,v_1)| \leq |C(u_2,v_2) - C(u_2,v_1)| + |C(u_2,v_1) - C(u_1,v_1)|$$
$$\leq |u_2 - u_1| + |v_2 - v_1|$$

Problem 2

(a)

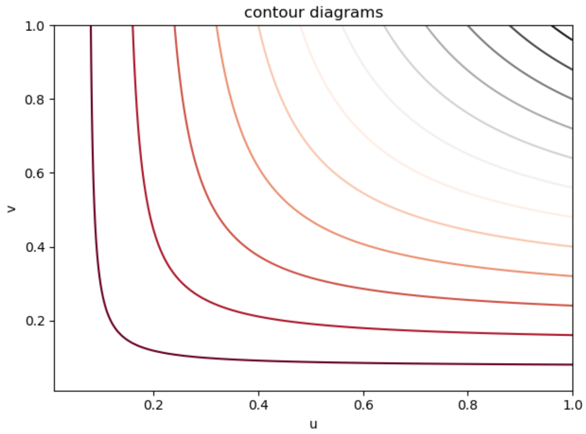
$$H(x,+\infty) = \frac{1}{1+e^{-x}}, \qquad H(+\infty,y) = \frac{1}{1+e^{-y}}$$

(b)

$$u = H(x,+\infty), e^{-x} = \frac{1}{u} - 1, \qquad v = H(+\infty,y), e^{-y} = \frac{1}{v} - 1$$

$$C(u,v) = \frac{1}{1+e^{-x}+e^{-y}} = \frac{1}{\frac{1}{u} - 1 + \frac{1}{v}} = \frac{uv}{u+v-uv}$$

(c)



Problem 3

Steps from Copula to default times:

1. Correlated variables $x_i, i = 1, \dots, 10$ by Gaussian copula
2. $u_i = \Phi(x_i)$
3. $\tau_i = -\frac{\log(1-u_i)}{\lambda}$

rho_list: [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]

probability that first default in 5 years:

[0.80335, 0.74591, 0.69194, 0.63917, 0.58026, 0.52703, 0.46871, 0.4075, 0.32993]