HW9
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1.

1.a) $u_1 = 1$ $u_2 = 1$ $v_1 = v$ $v_2 = 1$ by properties.

$u_1 \leq u_2$ $v_1 \leq v_2$

$C(1,1) - C(u,1) - C(1,v) + C(u,v) \geq 0$

$1 - u - v + C(u,v) \geq 0$

$C(u,v) \geq 0$

$C(u,v) \geq u+v-1$ $C(u,v) \geq \max\{u+v-1, 0\}$

$u_1 = u$ $u_2 = 1$ $v_1 = 0$ $v_2 = v$

$C(1,v) - C(u,v) - C(1,0) + C(u,0) \geq 0$

$C(u,v) \leq C(1,v) - C(1,0) + C(u,0)$

$C(u,v) \leq v$

$u_1 = 0$ $u_2 = u$ $v_1 = v$ $v_2 = 1$

$C(u,1) - C(u,v) - C(0,1) + C(0,v) \geq 0$

$C(u,v) \leq u$

$C(u,v) \leq \min\{u,v\}$

b) $C(u,v) = \lambda C_1(u,v) + (1-\lambda) C_2(u,v)$

$C(u,0) = \lambda C_1(u,0) + (1-\lambda) C_2(u,0)$

$= 0$

$C(u,1) = \lambda u + (1-\lambda) u = u$

$C(v,0) = 0$

$C(v,1) = \lambda v + (1-\lambda) v = v$

$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1)$

$= \lambda(C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1))$

$+ (1-\lambda)(C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1))$

≥ 0

c) $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$

let $u_2 = 1$ $v_2 - v_1 \geq C(u_1, v_2) - C(u_1, v_1)$

let $v_2 = 1$ $u_2 - u_1 \geq C(u_2, v_1) - C(u_1, v_1)$

$C(u_1, v_2) + C(u_2, v_1) - 2C(u_1, v_1) \leq (u_2 - u_1) + (v_2 - v_1)$

$2C(u_2, v_2) - C(u_1, v_2) - C(u_1, v_1) \leq (u_2 - u_1) + (v_2 - v_1)$

$C(u_2, v_2) - C(u_1, v_1) \leq (u_2 - u_1) + (v_2 - v_1)$

$\therefore$ for $\forall u_1, u_2, v_1, v_2$ $|C(u_2, v_2) - C(u_1, v_1)| \leq (u_2 - u_1) + |v_2 - v_1|$

2.

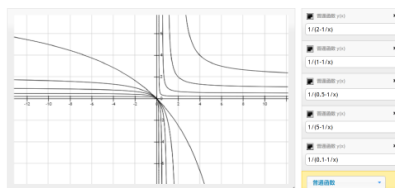(a) $F(x) = H(x, \infty) = \dfrac{1}{1 + e^{-x}}$

$G(y) = H(\infty, y) = \dfrac{1}{1 + e^{-y}}$

(b) $H(x, y) = C(F(x), G(y))$

$= \left(\dfrac{1}{F(x)} + \dfrac{1}{G(x)} - 1 \right)^{-1}$

$C(u, v) = \left(\dfrac{1}{u} + \dfrac{1}{v} - 1 \right)^{-1}$
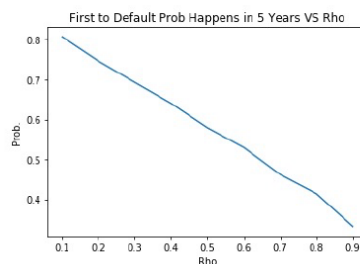
(c) $\left(\dfrac{1}{u} + \dfrac{1}{v} - 1 \right)^{-1} = C$

$\therefore \dfrac{1}{u} + \dfrac{1}{v} = C$

3.

[0.8021, 0.7532, 0.6921, 0.6394, 0.5786, 0.5294, 0.4761, 0.4027, 0.3345]

First to Default Prob Happens in 5 Years VS Rho

(a) $\lambda = 4$ $\rho = 0.1 : 0.1 : 0.9$