

# Homework : Portfolio Credit Risk

Credit Risk (MF772) Fall 2020

Instructor: Roza Galeeva

Due date: Sunday Nov 22 5 pm . Please, note that late assignments will not be accepted.

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## 1. [Expected shortfall]

Consider a homogenous portfolio of  $m = 10,000$  obligors with  $EAD = 100$ ,  $LGD = 40\%$ . Let probability of default for each obligator is  $p = 0.04$ .

*Question* Find the expected shortfall at 99% confidence level for the following diversity scores:

$$D = 1000, 500, 200$$

You can use the CLT to approximate the distribution of number of defaults by a normal distribution.

## 2. [On mixed binomial model]

- (a) Let a random variable  $Z$  is a standard normal from  $N(0, 1)$  and let probability of default  $p(Z)$  be given by

$$p(Z) = e^{-aZ^2}, \quad \text{for some } a > 0.$$

- Consider the mixed binomial Merton model with  $m = 2$ , and the indicator r.v.  $X_1$  and  $X_2$  with probabilities  $\mathbb{P}(X_i = 1) = p(Z)$ ,  $i = 1, 2$

*Question*

- a) Find the value of  $a$  so that such that the correlation of indicator random variables  $X_1$  and  $X_2$  is equal to 0.25.
- b) Let  $m = 100$  and  $X_i$  is the indicator of default for obligator  $i$ . The probability of default for obligator is  $p(Z) = e^{-aZ^2}$ . Let  $a$  be as in q1. Find the standard deviation of the number of defaults, compare it with standard deviation of portfolio with  $m = 100$  independent defaults, with probability of default  $p = \bar{p} = \mathbb{E}(p(Z))$ .

## 3. [Mapping Default Correlation to Diversity Score ]

- (a) Let  $Z$  be a r.v. (its distribution does not matter in this problem). Consider a portfolio of  $m = 1000$  similar obligors whose default indicator variables  $X_i$  are conditionally independent given  $Z$  such that  $X_i | Z = z \sim \text{Bernoulli}(z)$ . Assume that for every obligor the exposure is  $EAD_i = 1$  and  $LGD_i = 100\%$ .
- (b) Suppose that the default probability for each obligor is  $p_i = 1\%$  and the default correlation  $\rho_{i,j} = 0.00901$  for  $i \neq j$ .

*Questions:*

- a) Find standard deviation of the loss of the portfolio.
- b) Find an equivalent portfolio of  $m = 1000$  with diversity score  $D$ , the same total exposure,  $LGD_i$  and probability of default  $p = 1\%$  so that its loss variance matches the loss variance with the given default correlation.