Calibration of Hazard Rates from Credit Spreads and Bond Prices

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Outline

Today we will discuss credit spreads, bonds and the calibration procedures of survival probabilities

Main Learning Goals of the lecture:

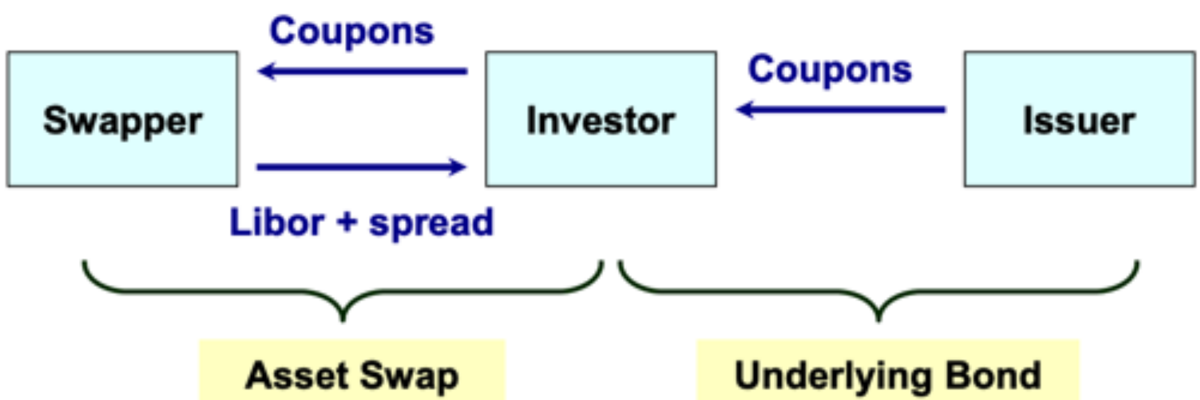
- Credit spreads
- Asset swaps as reference on credit spreads
- Calculation of default probabilities from credit spreads
- Risk neutral default probabilities based on bond prices (Hull, White paper)
- Risk neutral vs historical default probabilities
- Parametric forms of hazard rates

Credit Spreads

1. The Credit spread is the extra rate of interest per annum required by investors for bearing a particular credit risk. CDS spreads provide one measure of the credit spread.
2. Another one is the bond yield spread, difference between yield of a corporate bond and yield of a similar risk free bond.
3. A CDS can be used to hedge a position in a corporate bond. Example: An investor buys a five year corporate bond yielding 7% per year; at the same time she enters a five year CDS to buy a protection. Assume CDS spread is 200 b.p per annum.
4. The effect of CDS is to convert the corporate bond into risk free bond (approximately):
   - If there is no default, the investor earns $\overset{(7-2)\%}{5\%}$ per annum
   - If there is a default, then the investor earns 5% up to the time of default. Under CDS terms, the investor is able to exchange the bond for its face value, which can be invested in the risk free rate for the remainder of the 5 years period.
5. This argument shows that n-year CDS spread should be approximately equal to the excess of the par-yield on a n-year corporate bond over the par yield on a n-year risk free bond.

Asset Swaps 不重要

- Asset swaps provide a convenient reference point for traders in credit markets because they give direct estimate of the excess of bond yields over Libor/swap rates.
- Asset swaps combine an interest-rate swap with a bond and are seen as both cash market instruments and also as credit derivatives.
- The asset swap market is an important segment of the credit derivatives market since it explicitly sets out the price of credit as a spread over Libor.



Approximate Calculation of Default Probabilities

1. Suppose for a five year spread (CDS spread, bond yield spread or asset swap spread) for a company is 240 b.p. and the expected recovery rate in the case of default 40%.
2. The credit spread can be considered to be an average loss rate. Then the average probability of default per year over the five-year period, conditional on no earlier default, is

average hazard rate

$\frac{1}{T}\int_0^T \lambda(s)\,ds = \frac{1}{5}\sum \lambda_i \cdot 1$

$$\bar{\lambda} = \frac{s(T)}{1 - RR}$$

$(1-RR)\bar{\lambda} = S(T) = \text{CDS}$

approximation, not precise.

where $s(T)$ is the credit spread for a maturity T, RR is the recovery rate, and $\bar{\lambda}$ is the average hazard rate between time 0 and time T:

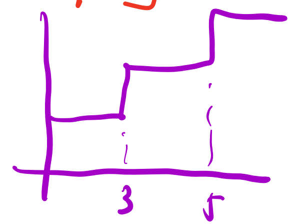
$$\bar{\lambda} = \frac{1}{T}\int_0^T \lambda(s)\,ds = \frac{1}{T}\sum \lambda_i \cdot 1$$

3. In our case $\bar{\lambda} = \frac{0.024}{1-0.4} = 4\%$.

Case of different maturities

- If credit spreads are known for a number of different maturities, the term structure of the hazard rate can be "bootstrapped" (at least approximately)
- Example: Suppose, the CDS spreads for a company are given as follows:
  1. 3 -year CDS is 50 b.p $S_3 = 0.5\%$
  2. 5 year CDS is 60 b.p. $S_5 = 0.6\%$
  3. 10 year CDS is 100 b.p $S_{10} = 1\%$
  4. The expected recovery rate is 60% $rr = 0.6$

upward-sloping



- First we can calculate the average hazard rates over 3, 5 and 10 years.
- From this, we can estimate the average hazard rates between 3 and 5 years, and 5 and 10 years by the formula, similar to the calculation of forward rates:

$$\bar{\lambda}_{i,i+1} = \frac{\bar{\lambda}_{i+1} T_{i+1} - \bar{\lambda}_i T_i}{T_{i+1} - T_i}$$

Risk neutral probabilities of default from bond prices

- Here, we consider a more exact calculation when the underlying bod's price is not close to par.
- We consider an example in Excel of a corporate 5 year bond and similar risk free bond:
  1. Coupon with 6% per year, paid semi-annually 
  2. The yield on the bond is 7% (continuous compounding)
  3. The yield on a similar risk free bond is 5%
  4. We assume defaults on the corporate bond can happen only at times $0.5, 1.5, 2.5, 3.5$ and 4.5 years (immediately before the coupon payments)
  5. Risk free rates are assumed to be $r = 5\%$ per year for all maturities $\lambda_T = 0.0875$ calculate $\bar{B}$ and B
  6. The recovery rate is 40%. 0.97 1.01

  We can calculate the default probabilities, making the following simplifying assumption:

  The unconditional probability of default is the same per each year and equal Q

Steps in calculating default probability Q

1. First, we calculate the prices of the corporate bond $\bar{B}(0)_T$ and the risk free bond $B(0)_T$: same maturity T, the same coupons; quoted by different yields. The expected loss is

$\tau = \frac{1}{2}$

$\frac{1}{2}$

$$L_T = B(0)_T - \bar{B}(0)_T$$

Default prob is constant.

$B(\tau)$

0.5	3%
1	3%

2. We assumed a default can happen at $0.5, 1.5, 2.5, 3.5, 4.5$, right before the coupon payment date. In each case we stand at time $\tau = 0.5, \ldots$ and will loose a portion of all our future coupons, including the immediate coupon, as well as the principal. We calculate the price of risk free bond $B(\tau)_T$ at time τ

only for notional

3. We assume that we recover $RR = 40\%$ of the value, thus loose $1 - RR = 60\%$. Thus our loss at time τ is $B(\tau)_T - RR$. Finally, we discount it to time 0

$$L_\tau = e^{-r\tau}(B(\tau)_T - RR)$$

4. These losses are scaled by the probability of default Q, and the total sum should be equal L_T; a simple equation for Q.

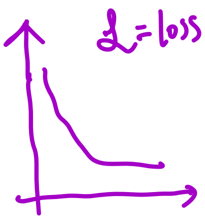
Results of the calculations

Following the outlined steps, we should get the following expected losses in the each year and the estimate of Q:

1. Based on the given coupons and yields, the corporate bond price is $\bar{B}(0)_5 = 0.9534$ and the risk free (the same coupon and maturity) is $B(0)_5 = 1.0409$. Thus the total expected loss $L_5 = 0.0875$

$$Q = \frac{\text{Expected Loss}}{\sum \text{PV of expected loss}}$$

2. Calculation of expected losses in each year:



Time (yrs)		Coupon	6%			Risk Free	5%
		Def. Prob	Recovery amount	Default Free Value	Loss	Discount	PV of expected loss
0.5		Q	0.4	1.0673	0.6673	0.97531	0.65081
1.5		Q	0.4	1.0597	0.6597	0.92774	0.61204
2.5		Q	0.4	1.0517	0.6517	0.8825	0.57516
3.5		Q	0.4	1.0434	0.6434	0.83946	0.54008
4.5		Q	0.4	1.0346	0.6346	0.79852	0.50671
						Total	2.88481

Then the default probability $Q = \frac{0.0875}{2.8845} = 0.0303$ or 303 b.p.

$\lambda \mathcal{L}_{0.5} + \lambda(1-\lambda)\mathcal{L}_{1.5} + \lambda(1-\lambda)^2\mathcal{L}_{2.5}$
$= \mathcal{L}\lambda$

$(1-\lambda)^2 - (1-\lambda)^3 = (1-\lambda)^2\lambda$

Extensions of the procedure

- We can extend the calculations to assume that defaults can take place more frequently,
- Instead of assuming a constant unconditional probability of default, we can assume a constant hazard rate, or even a particular pattern of variation of the default probability with time.
- With several bonds, we can estimate several parameters describing the term structure of default probabilities:
  1. Suppose we have bonds maturing in $3, 5, 7$ and 10 years.
  2. We assume a piece-wise constant function for default probability
  3. We use the first bond to estimate the default probability Q_1 per year for the first three years;
  4. Use the second bond to estimate default for years 4 and 5 (applying the calculated Q_1 is the first three years); etc.

Risk neutral vs historical (physical) default probabilities

use market information

- Under the risk neutral evaluation arguments we can value cash flows on the assumption that all investors are risk neutral
- The risk neutral default probabilities (sometimes called implied default probabilities) can be obtained from credit spreads or bonds prices
- On the other hand, the default probabilities estimated from historical data are real-world probabilities (sometime called physical default probabilities)
- They are not the same, typically the implied hazard rate from credit spread is substantially higher than the historical hazard rates (many reasons: illiquidity, premium for systemic and non-systemic risk)
- In applications, when valuing credit derivatives or estimating the impact of default risk on the pricing of the instruments, we should use the risk neutral default probabilities
- In scenario analysis to calculate the potential future losses from defaults, (for example for regulatory capital), we should use real-world default probabilities

The term structure of CDS spreads

The term structure of CDS spreads is normally $\boxed{\text{upward-sloping}}$.

- The investors may perceive the firm's credit quality to be declining over time, resulting in rising costs of default protection and CDS spreads. Sometimes known as the expectations hypothesis, this explanation essentially adopts the view that current CDS spreads are good forecasts of future default probabilities and recovery rates.
- For relatively riskier, lower-quality firms – especially those experiencing financial distress – the opposite is true, and the term structure of CDS spreads is generally downwardsloping

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Parametric forms of Hazard Rates

Calibration of a parameterized version of the spread curve has several advantages, especially in the situations of sparse data. There are several choices:

1. Constant

$$h(0,T) = \beta_0$$

2. Offset to a given reference curve for the obligator rating class

$$h(0,T) = \beta_0 + f(T)$$

3. Linear with two parameters:

$$h(0,T) = \beta_0 + \beta_1 T$$

   The slope for highly rated credits is typically positive, while the slope for speculative credits is negative.

4. Nelson and Siegel (four parameters)

$$h(\beta,\gamma;T) = \beta_0 + (\beta_1+\beta_2)\frac{1-e^{-T/\gamma}}{T/\gamma} - \beta_2 e^{-T/\gamma}$$

Calibration procedure

We use some market based quotes, for example CDS spreads information to find the optimal parameters.
We assume that the number of parameters does not exceed the number of calibration instruments. The standard procedure of calibration is to find the minimum of the target function:

$$min_\beta \sum_i \omega_i \left(C^i_{market} - C^i_{model}(\beta, (B_i, \pi)) \right)^2$$

where β are the parameters of the model, ω_i are weights to fit more liquid markets more accurately, (B_i, π) contains information about risk free ZCB and the recovery rate.