Homework 7: Portfolio Credit Risk

Credit Risk (MF772) Fall 2021 Instructor: Roza Galeeva

Due date: 8 am, Thursday November 11. Please, note that `late assignments will not be accepted.`

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1. [Expected shorfall for Normal Distribution]

We assume that the distribution of losses L of a portfolio is normal $N(\mu, \sigma_L^2)$. For a given confidence level α, find analytical expression for expected shortfall

$$ES_\alpha(L) = \mathbf{E}[L \mid L \geq VaR_\alpha(L)].$$

2. [Expected shorfall for Binomial Distribution]

Consider a homogenous portfolio of $m = 10,000$ obligators with $EAD = 100$, $LGD = 40\%$. Let probability of default for each obligator is $p = 0.04$.

Find the expected shortfall at 99% confidence level for the following diversity scores:

$$D = 1000, 500, 200$$

Use the expression from question 1 and CLT (central limit theorem) to approximate the distribution of number of defaults by a normal distribution.

3. [Randomization of default probability]

Set up

Let Z be a random variable with density function $f_Z(z)$ and, let $p(Z) \in [0,1]$ be a random variable with distribution function F and mean $\overline{p}$. That is,

$$F(x) := \mathcal{P}(p(Z) \leq x),$$

$$\overline{p} := E[p(Z)] = \int_{-\infty}^{\infty} p(z) f_Z(z) dz.$$

The variable Z is called **mixing variable**. The economic intuition behind this randomization of the default probability $p(Z)$ is that Z should be a common background variable affecting all obligors in the portfolio.

a) conditional on Z, each obligator i has default probability $p(Z)$, that is,

$$\mathcal{P}(X_i = 1 \mid Z) = p(Z) \quad \text{and} \quad \mathcal{P}(X_i = 0 \mid Z) = 1 - p(Z);$$

b) conditional on Z, the indicator random variables $X_1, X_2, \cdots, X_m$ are i.i.d.

Now, we make a specific choice:

- Let a random variable Z is a standard normal from $N(0,1)$ and let probability of default $p(Z)$ be given by

$$p(Z) = e^{-aZ^2}, \qquad \text{for some } a > 0.$$

- There are two obligators $m = 2$. X_1 and X_2 are the indicators of default.

$$\mathbb{P}(X_i = 1) = p(Z), i = 1, 2$$

Questions

a) Find the value of a so that such that the correlation between random variables X_1 and X_2 is equal to 0.4.

b) Let now we have $m = 100$ obligators and X_i is the indicator of default for obligator i. For each obligator, conditioned on Z, the probability of default is $p(Z) = e^{-aZ^2}$ Let a be as in q1. Find the standard deviation of the number of defaults, compare it with standard deviation of portfolio with $m = 100$ independent defaults, with probability of default $p = \bar{p} = \mathbb{E}(p(Z))$.