HW772 Credit Risk Homework 3

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Due Date: 2021/9/30

- 1. Suppose that the CDS spreads for 1-, 3-, 5-, 7- year instruments are 50, 75, 80, 90 basis points, respectively and the expected recovery rate is 50%. Calculate:
 - Unconditional default probability in the interval between 2 and 4 years
 - Survival probability up in the first 6 years.

Conditional Survival Prob
1.0000
0.9900
0.9729
0.9560
0.9394
0.9231
0.9021

Unconditional Default Probability between 2 and 4 years	0.033

See more details in Excel.

The Unconditional Default Probability between 2 and 4 years is around **0.033**.

The survival probability up in the first 6 years is **0.9021**.

2. A company issued three and five year bonds:

- Each bond has a coupon of 5% per year, paid annually, at the end of the year.
- The yields on the bonds (continuous compounding) are 6% and 6.5%, respectively.
- Risk-free interest rates are 4% with continuous compounding
- The recovery rate is 40%
- Default can take place halfway through each year, so at times 0.5, 1.5, 2.5, 3.5, 4.5
- The unconditional risk-neutral default probabilities per year are Q_1 for years 1, 2, 3 and Q_2 for years 4, 5.

Estimate Q_1 and Q_2 , by the procedure we discussed in the class:

- Calculate prices of a corporate bonds and coupons, using the given yields.
- Calculate prices of risk-free bonds with the same coupons, same maturity, using risk-free yields.
- The differences between the corporate and the corresponding risk free bonds will
 give the total loss on three and five years periods.
- Calculate Q₁ using the same procedure as in the class, placing default times at 0.5, 1.5, 2.5 years.
- Use Q_1 from the previous step to calculate the loss in the first three years for the 5 year bond, and then get Q_2 , placing defaults at 3.5, 4.5 years. ¹

0.5 Q1 1.0462 0.6462 0.9802 0.6334 1.5 Q1 1.0379 0.6379 0.9418 0.6007 2.5 Q1 1.0292 0.6292 0.9048 0.5693 Q1 0.031602984 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118		Coupon Rate	Yield of 3Y Bond	Yield of 5Y Bond	Risk-free Rate	Recovery Rate
1 0.0471 0.0469 0.0480 0.0480 2 0.0443 0.0439 0.0462 0.0462 3 0.0418 0.0411 0.0443 0.0443 4 0.0386 0.0409 0.0409 5 0.0361 0.0409 0.0409 Total Coupon 0.1332 0.2066 0.1385 0.2221 PV of notional 0.8353 0.7225 0.8869 0.8187 P 0.9685 0.9291 1.0255 1.0408 Expected loss 0.0570 0.1117 Discount Factor PV of Expected 0.5 Q1 1.0462 0.6462 0.9802 0.6334 1.5 Q1 1.0379 0.6379 0.9418 0.6007 2.5 Q1 1.0292 0.6292 0.9048 0.5693 Q1 0.31602984 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 <t< td=""><td>Inputs</td><td>5%</td><td>6%</td><td>6.5%</td><td>4%</td><td>40%</td></t<>	Inputs	5%	6%	6.5%	4%	40%
1 0.0471 0.0469 0.0480 0.0480 2 0.0443 0.0439 0.0462 0.0462 3 0.0418 0.0411 0.0443 0.0443 4 0.0386 0.0409 0.0409 5 0.0361 0.0409 0.0409 Potal Coupon 0.1332 0.2066 0.1385 0.2221 PV of notional 0.8353 0.7225 0.8869 0.8187 P 0.9685 0.9291 1.0255 1.0408 Expected loss 0.0570 0.1117 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q1 1.0379 0.6379 0.9418 0.6007 2.5 Q1 1.0292 0.6292 0.9048 0.5693 1.5 Q1 1.0542 0.65642 0.990198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.6160706 2.5 Q2 1.0542 0.6542 0.941764534 0.6160706 2.5 Q2						
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P 0.9685 0.9291 1.0255 1.0408 Expected loss 0.0570 0.1117 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q1 1.0462 0.6462 0.9802 0.6334 1.5 Q1 1.0379 0.6379 0.9418 0.6007 2.5 Q1 1.0292 0.6292 0.9048 0.5693 Q1 0.31602984 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.525555920	Total Coupon	0.1332	0.2066	0.1385	0.2221	
Expected loss 0.0570	PV of notional	0.8353	0.7225	0.8869	0.8187	
Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q1 1.0462 0.6462 0.9802 0.6334 1.5 Q1 1.0379 0.6379 0.9418 0.6007 2.5 Q1 1.0292 0.6292 0.9048 0.5693 Q1 0.31602984 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.525555920	Р	0.9685	0.9291	1.0255	1.0408	
0.5 Q1 1.0462 0.6462 0.9802 0.6334 1.5 Q1 1.0379 0.6379 0.9418 0.6007 2.5 Q1 1.0292 0.6292 0.9048 0.5693 Q1 0.031602984 Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.525555920	Expected loss	0.0570			0.1117	
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Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.52555920	1.5	Q1	1.0379	0.6379	0.9418	0.6007
Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.52555920	2.5	Q1	1.0292	0.6292	0.9048	0.5693
Year Def.Prob Default Free Value Loss Discount Factor PV of Expected 0.5 Q2 1.0618 0.6618 0.980198673 0.64873632 1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.52555920						
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1.5 Q2 1.0542 0.6542 0.941764534 0.61607050 2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.52555920	Year	Def.Prob	Default Free Value	Loss	Discount Factor	PV of Expected Los
2.5 Q2 1.0462 0.6462 0.904837418 0.58468553 3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.52555920	0.5	Q2	1.0618	0.6618	0.980198673	0.648736322
3.5 Q2 1.0379 0.6379 0.869358235 0.55453118 4.5 Q2 1.0292 0.6292 0.835270211 0.52555920	1.5	Q2	1.0542	0.6542	0.941764534	0.616070506
4.5 Q2 1.0292 0.6292 0.835270211 0.52555920	2.5	Q2	1.0462	0.6462	0.904837418	0.584685535
	3.5	Q2	1.0379	0.6379	0.869358235	0.554531186
Q2 0.049310813	4.5	Q2	1.0292	0.6292	0.835270211	0.525559206
Q2 0.049310813						
				Q2	0.049310813	

See more details in excel.

Q1 is **0.0316** and Q2 is **0.0493**.