Bond Pricing Analytics and Introduction to CDS

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Outline

Today, we will generalize bonds pricing analytics to the case of non-zero recovery and intermediate coupons. We will introduce credit derivatives and discuss the pricing of basic single reference Credit Default Swap (CDS)

Main Learning Goals of the lecture:

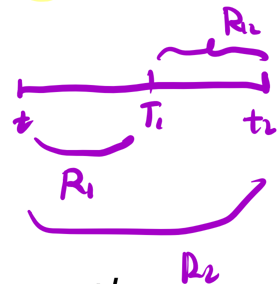
- Forward rates of defaultable and non-defautable bonds, similarities between bonds and survival probabilities
- Inclusion of recovery and coupons
- Introduce Credit Derivatives
- Purpose and mechanics a CDS
- Simple Pricing Example
- Reference: Chapter 7 "Credit Derivatives" in the main textbook; the example is taken from John Hull book.

Forward rates

Definition 1

- *Recall, that when using a simple rate R , the investment of $\$1$ at time t grows to the value $1 + R(T - t)$ at time T.*
- *The non-defaultable simply compounded forward rate over the period $[T_1, T_2]$ as seen from date t, $0 \le t \le T_1$ is:*

$$R(t, T_1, T_2) := \frac{B(t, T_1)/B(t, T_2) - 1}{T_2 - T_1}.$$



- *The defaultable simply compounded forward rate over the same period is:*

$$\overline{R}(t, T_1, T_2) := \frac{\overbrace{\overline{B}(t, T_1)/\overline{B}(t, T_2) - 1}^{\text{return}}}{T_2 - T_1}, \quad T_2 > T_1 \ge t.$$

Continuously compounded forward rates

- For continuously compounded rate r , the investment of \$1 at time t grows to the value $e^{r(T-t)}$ at time T.
- The forward rate over the period $[T_1, T_2]$ as seen from date t, can be calculated as

$$r(t, T_1, T_2) = \frac{(\ln B(t,T_1) - \ln B(t,T_2))}{T_2 - T_1}$$

$= -\ln(B$

$= \frac{\ln[B(t,t+\Delta t) - B(t,T)]}{\Delta t}$

- Now, let $T_1 = T$, $T_2 = T_1 + \Delta t$. Then when $\Delta t \downarrow 0$, we get the instantaneous continuously compounded forward rates for risk free and defaultable bonds:

$= -\frac{\partial}{\partial T} \ln B(t,T)$

$B(t,T_1) = e^{r_1(t_1-t)}$

$e^{r_2(T_2-t)}$

$e^{-r_1(T_1-t)}$ $e^{-r_{12}(t_2-t_1)}$

$$r(t,T) := -\frac{\partial}{\partial T} \ln B(t,T).$$

instantaneous,

Short rate

$$\overline{r}(t,T) := -\frac{\partial}{\partial T} \ln \overline{B}(t,T).$$

Connection to hazard rates

$B(t,T) = e^{-\int_t^T r(s)ds}$

$\overline{B}(t,T) = e^{-\int_t^T (r_s+h_s)ds}$

- Recalling the previous lecture:

Survival

$$\overline{B}(t,T) = B(t,T)P(t,T) = B(t,T)e^{-\int_t^T h(t,s)ds}$$

- Therefore, the implied hazard rate of default at time $T > t$ as seen at time t is given

$$h(t,T) = \underbrace{\overline{r}(t,T)}_{\overline{B}} - \underbrace{r(t,T)}_{B}$$

- There is a close structural connection between the way in which a hazard rate can be used to model uncertain arrival of default, and the continuous-time discounting in classical interest rate models.
- Survival probability $P(t,T)$ looks like zero coupon bond : non-negative, decreasing , starting at $P(t,t) = 1$.

Recovery

D → positive payoff

no D

- An asset with positive recovery can be viewed as an asset with zero recovery and an additional positive payoff at default.

到期

Let T_M be the maturity of the credit derivative we want to price. A partition of the interval $(0, T_M]$ is $\mathbb{T} = \{T_0, T_1, \cdots, T_M\}$, with $T_0 = 0$.

Assumption 1

a) *The expected recovery rate is assumed to be time and default independent percentage, π, of the reference entity's par value.*

b) *If a default occurs in $(T_{n-1}, T_n]$, we assume that the payment is made at T_n*

这D 这pay

Remark 1

The above partition can be equal to the one determined by the cash flows of the financial asset or a finer one.

Pricing Default

Let $e(t, T_{n-1}, T_n)$ be the price (value) of $\$1$ at time $t \geq 0$, that is paid if default happens in $(T_{n-1}, T_n]$. That is,

违约后给的钱

payment $$e(t, T_{n-1}, T_n) := \mathbb{E}\left[B(t, T_n)\mathbf{1}_{\{T_{n-1} < \tau \leq T_n\}} | \mathcal{F}_t \wedge \{\tau > t\}\right].$$

$\mathbb{P}(T_{n-1} < \tau \leq T_n)$

Default happened after t.

Proposition 1

a) *The price $e(t, T_{n-1}, T_n)$ is given by*

$$e(t, T_{n-1}, T_n) = B(t, T_n)\left(P(t, T_{n-1}) - P(t, T_n)\right)$$

two R.V: default rate
interest rate

b) $e(0, 0, T_M)$ *is the price (value) of $\$1$ that is paid if default happens in $(0, T_M]$.*

$$e(0, 0, T_M) = \sum_{n=1}^{M} e(0, T_{n-1}, T_n).$$

Fundamental quantities of the model

- Term structure of the default-free forward interest rates $R(0, T_{n-1}, T_n)$ and hazard rates $h(0, T_{n-1}, T_n)$.
- The term structure of (implied) survival probabilities $P(0, T)$
- Expected recovery rate π (rate of recovery as percentage of par).

Pricing Building Blocks

Continuously compounded rates

$r = 5\%$ $h = 4\%$

$B(T) = e^{-0.05T}$

1. The price of a default-free zero coupon bond $\overline{B}(T) = e^{-0.09T}$

$$B(0,T) = \exp\left(-\int_0^T r(0,s)ds\right)$$

2. The price of the defaultable zero-coupon bond with zero recovery

$$\overline{B}(0,T) = \exp\left(-\int_0^T (r(0,s) + h(0,s))ds\right)$$

3. Default Value

$$e(t, T_{n-1}, T_n) = B(t, T_n)\left(\overbrace{P(t, T_{n-1}) - P(t, T_n)}^{\text{prob of default}}\right)$$

Defaultable fixed-coupon bond

semi-annual

Proposition 2

The price of a defaultable fixed-coupon bond is:

$$\overline{C}(0) = \overbrace{\sum_{n=1}^{M} \overline{c}_n \overline{B}(0, T_n)}^{\text{coupon part}} + \overbrace{\overline{B}(0, T_M)}^{\text{pricipal}} + \overbrace{\pi \sum_{n=1}^{M} e(0, T_{n-1}, T_n)}^{\text{scale by recovery}}$$

where $\overline{c}_n$ is the fixed-coupon payment at time T_n.

$p(T) = e^{-hT}$ ①

$p^{\text{default (unconditional)}}(T_{i-1}, T_i)$

$= p(T_{i-1}) - p(T_i)$ ②

③ Coupon x survival

④ e^{-rT}

⑧ Coupon x ④

⑥ notional x ⑦

τx

⑦ prob D x discount to t_i

Credit Derivatives

Definition 1

Credit derivatives *are financial instruments that are designed to transfer the credit exposure of one or more underlying assets between two parties.*

→ They are designed to reduce or eliminate credit risk exposure by providing **insurance** against losses suffered due to **credit events**, which trigger the payout.

→ The terms under which a credit derivative is executed include a specification of what constitutes a **credit event** (**not** necessarily a default). A credit event can be a:

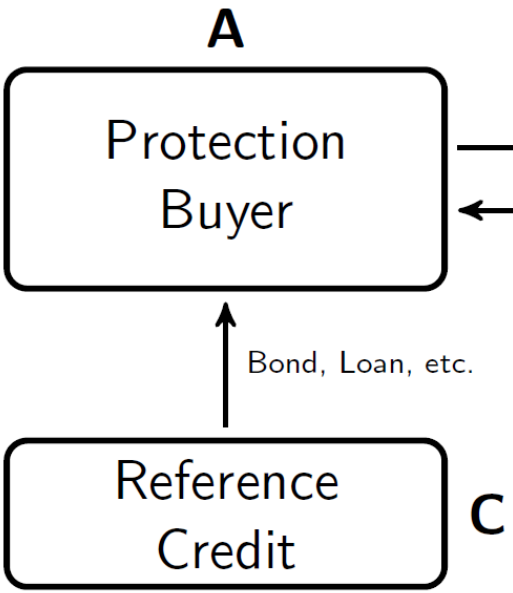
- bankruptcy or insolvency of the reference asset obligor;
- default on payments obligations (bond coupons, etc.)
- debt restructuring,
- downgrade in credit rating below a specified minimum level; changes in credit spread

The International Swaps and Derivatives Association (ISDA) has by now standardized the definition of a credit event.

Credit derivatives market

1. As a market segment of derivative securities, the market for credit derivatives was created in the early 1990s in London and New York.
2. There is a large variety of credit derivatives; they are customized to fit the investor's needs. They now become a major investment tool as well.
3. The reference security of a credit derivative can be any financial instrument that is subject to credit risk. The vast majority of credit derivatives take the form of the *credit default swap*.
4. A *credit defaults swap* is a contractual agreement to transfer the default risk of the reference credit **C** from counterparty **A** (protection buyer) to counterparty **B** (protection seller) during the term of the CDS.
5. Credit default swap are the most common type of credit derivatives. Their quotes are considered as an indicator of investors' perception of credit risk.

Diagram of a single-name CDS



Credit Default Swaps (CDS)

A CDS is a bilateral agreement between a *protection buyer* and a *protection seller* , in which the buyer agrees to make fixed periodic payment to the seller in exchange for protection against a credit event on underlying asset or portfolio of assets. The underlying might be

- a Single reference entity (single name CDS)
- portfolio of reference entities (CDS index)
- particular amount of losses in a basket of reference entities

For now, we will be dealing with Single-Name CDS contracts.

1. *Reference entity* is the issuer of the debt instrument. It could be a corporation, a government or a bank loan.
2. Collectively, the payments made by the protection buyer are called the premium leg;
3. The contingent payment that might have to be made by the protection seller is called the protection leg.

Settlement of CDS

- The buyer of the CDS makes periodic payments to the seller until the end of the life of the life of the CDS or until a credit event occurs. The payments are typically made quarterly, every half year or every year.
- The settlement in the event of a default involves either physical delivery of the bonds or cash payment.
- **Example** :
  1. Two parties enter a 5year CDS on March 1 2017. The notional principal is 100 MM USD.
  2. The buyer agrees to pay 100 b.p. annually for protection against default by the reference entity.
  3. If the reference entity does not default, the buyer pays 1MM at the end of 2017, 2018, 2019, 2020, 2021; no payoff.
  4. Suppose on Jun 1 2020. the buyer notifies the seller of a credit event.
     - In case of a physical delivery, the buyer has the right to sell bonds issued by a reference entity for 100M
     - In case of a cash settlement, an independent calculation agent will conduct a poll of dealers to determine the cheapest deliverable bond, say 35MM, the buyer will receive 65 MM.

5年 3月1日 2017 notional 100

若 no Default, 100bp × 5 100bp = 1%

buyer $\xrightarrow{\text{100bp 每年}}$ seller

bond etc.

reference party

e^{-rt} e^{-ht_i}

surp(P) discount

① E of premium (buyer)

(seller)

payments from A

0	pre
1	pre
2	⋮
3	⋮
4	⋮
5	⋮

Default 在0.5

0		0 ← $(1-P_1)(1-R)D(0.5)$
1	P_1-P_2	0.5
2	P_2-P_3	$(P_1-P_2)(1-R)D(1.5)$
3	⋮	
4	⋮	
5	⋮	

pre: $S \times P_1 \times D$

$S \times P_2 \times D$

⋮

Default: 还没给premium, 得给半年的

0 1

(buyer)

$\frac{S}{2} \times (1-P_1) D(0.5)$

fair value:

$\Pi_1 = \Pi_2$

accrued

$S[\ ] + S[\ ] = \Pi_2$

premium

0 0.5 1 pay 2 3 4 5

default

default IP

$S = ?$

$(1-R)$

$(P_1 - P_2)$

Simple Example in the spreadsheet

We consider a 5 year CDS. The assumptions:

1. Hazard rate (constant) is constant, and equals 2 percent per year.
2. We assume that the risk free rate is 5 percent per year with continuous compounding, and the recovery rate is 40 percent.
3. We assume that the payments on the credit default swap happen once a year and made at the end of the year
4. Default assumptions
    - The default happens a half way through 半路

We evaluate two legs of the swap:

1. Premium leg $\Pi_{premium}$, is a collection of the premium payments, which is proportional to the CDS spread s
2. The payment in the case of default is $\Pi_{default}$.
3. The CDS spread s can be calculated from the condition that at the inception

$$\boxed{\Pi_{premium} = \Pi_{default}}$$

Accrual Payments

1. The default payments make up for the loss, $(LGD = 1 - \pi\ )$ and accounting properly for probability of defaults between years. They should be discounted properly, depending on the presumed times of default.
2. The premium leg will be a sum of two parts:
   (A) The first part Π_1 is the premium payments, which is collection of defaultable bonds scaled by the CDS spread s
   (B) In case of default the buyer of protection will have to make accrual payments Π_2 to cover half year payment of the premium. This part would be scaled by half of the CDS spread: $s/2$.
3. Details are given in the spreadsheet.