Homework : Application of Copulas to Credit Modelling

Credit Risk (MF772) Fall 2021 Instructor: Roza Galeeva

Due date: 8 am, Thursday Dec 9 . Please, note that `late assignments will not be accepted.`

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1. Three dimensional Copula

Let function $H(x, y, z)$ be a function with domain $[-1, 1] \times [0, \infty] \times [0, \pi/2]$:

$$H(x, y, z) = \frac{(x+1)(e^y - 1)\sin z}{x + 2e^y - 1}$$

a) Evaluate $H(x, y, 0)$, $H(x, 0, z)$ and $H(-1, y, z)$ (check "groundness").

b) Calculate one-dimensional margins

$$H_1(x) = H(x, \infty, \pi/2), H_2(y) = H(1, y, \pi/2), H_3(z) = H(1, \infty, z)$$

c) Calculate two-dimensional margins (three)

$$H_{1,2}(x, y) = H(x, y, \pi/2), ...$$

d) Calculate the volume $V_H(B)$, $B = [0, 1/2] \times [1, 2] \times [\pi/4, \pi/2]$ [1]

2. t-copula

Use the same assumptions as in the quiz you did in class [2]

Now instead of two-dimensional Gaussian copula, use t-copula with the same correlation.

a) Calculate the same probabilities as in the quiz and compare the answers

b) For each correlation value, compare scatter plots of default times for two obligators. Make observations.

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[1] revisit definition of volume defined by a function in n-dimensional space, take care of signs propperly

[2] look at Questrom for your particular session if you missed it

3. n**-dimensional Gaussian copula**

Consider a homogenous portfolio of m obligators with the same hazard rate $\lambda = 0.05$ (per year), and with a Gaussian copula of default times, given by an equicorrelation matrix (all pairwise correlations are same $= \rho$.)

Evaluate a simplified version of a *first to default swap*:

- The default event is the first default of any m obligators. The contract terminates after the first default (provided it happens during lifetime of the swap).
- Protection buyer pays a regular fee s, as percentage of the total exposure; the payments occur at the end of each year.
- In case of default, the protection seller covers the loss on one obligator.

We consider the following specifications:

a) The number of obligators $m = 5$.

b) The correlation $\rho = 0.4$ and the recovery rate for each obligator is $RR = 0.5$

c) The swap covers 5 years.

d) For simplicity we assume zero interest rate (no discount)

e) The exposure per obligator is the same, in this case 20% of the total exposure.

Evaluate the fair price of the protection s and its sensitivity to correlation $\frac{\partial s}{\partial \rho}$, ("Rhoza").