Due date: Sep 23, 2021 8am

Problem 1

A five year credit default swap entered on Mar 20 2018, requires quarterly payments at the rate of 200 basis point per year. 1 The principal is \$ 100 Million. A default occurs after 3 years and two months. The auction process finds the price of the cheapest deliverable bond to be 40% of its face value. Describe the cashflows and their timing for the seller of the credit default swap.

Solution: The seller receives:

$$100000000 * 0.02 * 0.25 = $500000$$

at time 0.25, 0.5, 0.75,..., 3. (June 20 2018, Sep 20 2018, and Dec 20 2018, Mar 20 2019, ...) The seller also receives a final accrual payment of about

$$100000000 * 0.02 * \frac{2}{12} = \$333333$$

The seller pays 100000000 * (1 - 40%) = 60 million after 3 years and 2 months.

Problem 2

We consider a five year digital CDS, which pays notional \$1 in case of default. We need to find its fair value spread s, giving the following information:

- The continuous hazard rate per year is h = 0.03.
- The risk free rate r, continuously compounded is 4%.
- The premium is paid each 6 months.
- Default can happens each 3 months (a quarter), and the payment in case of default is made at the end of the quarter.

Solution:

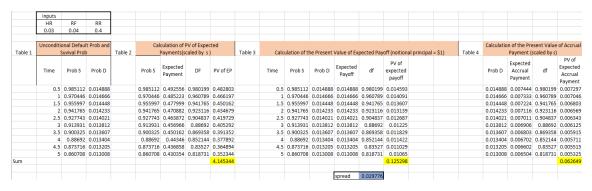


Figure 1: Prob 2 Answer

The annualized spread is approximately **0.029776**.

Problem 3

Go back to the defaultable bond we analyzed in the class (spreadsheet "DefaultableBond Pricing" in Questrom, except we don't fix the coupon rate). Consider two cases:

- \bullet The hazard rate h is constant, h = 0.04 (we did it in the class), so default times follow the exponential distribution.
- The default times follow the Weibull distribution with parameters $\lambda = 0.04$ and $p = \frac{3}{4}$. For each

of these cases, find the semi-annual coupon rate c, so that the bonds are valued at par, thus the value of the bond = 1. Which coupon rate is higher and why?

Solution:

Defaultabl	le Bond Pricing											
	Hazard Rate (per	Risk free rate		Coupon Rate (semi-			Probability of	scale by coupon rate Expected value			PV of expected	Recovery
	year)	(continuous) per	Recovery Rate	annual)	Time in years	Survival Prob	Default	coupons	Discount	PV of exp coupons	notional	Value
	0.04	0.05	0.4		0	1	0		1			
					0.5	0.980198673	0.019801327	0.03719854	0.97531	0.036280104		0.00772
				0.0759	1	0.960789439	0.019409234	0.036461959	0.95123	0.034683688		0.00739
					1.5	0.941764534	0.019024906	0.035739964	0.92774	0.033157519		0.00706
					2	0.923116346	0.018648187	0.035032265	0.90484	0.031698505	0.835270211	0.00675
				Summation						0.135819816	0.835270211	0.02892
								Value 1.0000				

Defaultable	Bond Pricing											
								scale by coupon rate				+
		Risk free rate	Recovery Rate	Coupon Rate (semi- annual)	Time in	Survival Prob	Probability of Default	Expected value	Discount	PV of exp coupons	PV of expected notional	Recovery Value
		(continuous) per	necovery nate	annual)	Time in years	SULVIVAL PLOD	Delault	coupons	Discount	PV of exp coupons	notional	Value
		0.05	0.4		0	1	0		1			
	0.053182959				0.5	0.948206514	0.051793486	0.047099314	0.97531	0.045936428		0.02021
	0.089442719			0.099344	1	0.914440644	0.03376587	0.045422096	0.95123	0.043206834		0.01285
	0.12123093				1.5	0.885829371	0.028611272	0.044000917	0.92774	0.040821564		0.01062
	0.150424124				2	0.860343007	0.025486364	0.042734958	0.90484	0.038668189	0.778470545	0.00922
				Summation						0.168633014	0.778470545	0.0529
								Value 1.0000				

The coupon rate for (a) is about 0.0759 and (b) is 0.099344. We can see that the coupon rate with default time with Weibull distribution is higher, since once the default time follows Weibull distribution, it will be scaled by a parameter p (p = 0.75) and therefore the probability of survival will be lower than that in (a). If survival probability is lower, the coupon rate should be higher. This effect as can be seen mathematically from the formula in slides.

$$\bar{C}(0) = \sum_{n=1}^{M} \bar{c}_n \bar{B}(0, T_n) + \bar{B}(0, T_M) + \pi \sum_{n=1}^{M} e(0, T_{n-1}, T_n)$$

$$\bar{B}(t,T) = B(t,T)P(t,T), \quad T > t$$

Where P(t,T) is the survival probability and Cn-bar is coupon rate.