Homework : Copulas

Credit Risk (MF772) Fall 2020 Instructor: Roza Galeeva

Due date: Sunday Nov 29 5 pm . Please, note that `late assignments will not be accepted.`

---

1. **[Properties of copulas]**
   Prove the following statements:

   a) Upper and lower bounds: For any copula $C(u,v)$

   $$\max(u+v-1,0) \leq C(u,v) \leq \min(u,v)$$

   b) Linear combinations of copulas: If $C_1(u,v)$ and $C_2(u,v)$ are copulas then

   $$C(u,v) = \lambda C_1(u,v) + (1-\lambda)C_2(u,v)$$

   where $0 \leq \lambda \leq 1$, is also a copula. Prove it, so check all the properties.

   c) Let $C(u,v)$ be a copula with domain $DomC$. Show that $C(u,v)$ is uniformly continuous: For any u_1,u_2,v_1,v_2 in $DomC$

   $$|C(u_2,v_2) - C(u_1,v_1)\ \leq |u_2-u_1| + |v_2-v_1|$$

2. **[Copula from a distribution function]**
   Gumbel's bivariate logistic distribution: Let X and Y be r.v. with a joint distribution function given by

   $$H(x,y) = (1+e^{-x}+e^{-y})^{-1}$$

   (a) Find the marginal d.f. of X and Y (as $H(x,\infty)$, $H(\infty,y)$)

   (b) Write the corresponding copula $C(u,v)$

   (c) Draw the contour diagrams

   $$C(u,v) = const$$

3. **[Gaussian Copula]**
   Consider 10 obligators, whose default times are given by exponential distribution with the same hazard rates $\lambda = 0.04$ (per year). Assume that their joint distribution is given by a Gaussian copula, with equicorrelation matrix, so for any $i,j = 1,..10$ $\rho_{i,j} = \rho$. Let

   $$\rho = 0.1 : 0.1 : 0.9$$

   Work on the following problems:

   (a) For each correlation value calculate and plot the probability that first to default happens in 5 years (so in less that 5 years at least one obligator defaults).