HW772 Credit Risk Homework 3

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Due Date: 2021/9/30

1. Suppose that the CDS spreads for 1-, 3-, 5-, 7- year instruments are 50, 75, 80, 90 basis points, respectively and the expected recovery rate is 50%. Calculate:

- Unconditional default probability in the interval between 2 and 4 years
- Survival probability up in the first 6 years.

Conditional Survival Prob
1.0000
0.9900
0.9729
0.9560
0.9394
0.9231
0.9021

Unconditional Default Probability between 2 and 4 years	0.033

See more details in Excel.

The Unconditional Default Probability between 2 and 4 years is around **0.033**.

The survival probability up in the first 6 years is **0.9021**.

2. A company issued three and five year bonds:

- Each bond has a coupon of 5% per year, paid annually, at the end of the year.
- The yields on the bonds (continuous compounding) are 6% and 6.5%, respectively.
- Risk-free interest rates are 4% with continuous compounding
- The recovery rate is 40%
- Default can take place halfway through each year, so at times $0.5, 1.5, 2.5, 3.5, 4.5$
- The unconditional risk-neutral default probabilities per year are Q_1 for years $1, 2, 3$ and Q_2 for years $4, 5$.

Estimate Q_1 and Q_2, by the procedure we discussed in the class:

- Calculate prices of a corporate bonds and coupons, using the given yields.
- Calculate prices of risk-free bonds with the same coupons, same maturity, using risk-free yields.
- The differences between the corporate and the corresponding risk free bonds will give the total loss on three and five years periods.
- Calculate Q_1 using the same procedure as in the class, placing default times at $0.5, 1.5, 2.5$ years.
- Use Q_1 from the previous step to calculate the loss in the first three years for the 5 year bond, and then get Q_2, placing defaults at $3.5, 4.5$ years. [1]

	Coupon Rate	Yield of 3Y Bond	Yield of 5Y Bond	Risk-free Rate	Recovery Rate
Inputs	5%	6%	6.5%	4%	40%
Year	Price of 3Y Corporate Bond	Price of 5Y Corporate Bond	Price of 3Y Risk-free Bond	Price of 5Y Risk-free Bond	
1	0.0471	0.0469	0.0480	0.0480	
2	0.0443	0.0439	0.0462	0.0462	
3	0.0418	0.0411	0.0443	0.0443	
4		0.0386		0.0426	
5		0.0361		0.0409	
Total Coupon	0.1332	0.2066	0.1385	0.2221	
PV of notional	0.8353	0.7225	0.8869	0.8187	
P	0.9685	0.9291	1.0255	1.0408	
Expected loss	**0.0570**			**0.1117**	
Year	Def.Prob	Default Free Value	Loss	Discount Factor	PV of Expected Loss
0.5	Q1	1.0462	0.6462	0.9802	0.6334
1.5	Q1	1.0379	0.6379	0.9418	0.6007
2.5	Q1	1.0292	0.6292	0.9048	0.5693
			Q1	0.031602984	
Year	Def.Prob	Default Free Value	Loss	Discount Factor	PV of Expected Loss
0.5	Q2	1.0618	0.6618	0.980198673	0.648736322
1.5	Q2	1.0542	0.6542	0.941764534	0.616070506
2.5	Q2	1.0462	0.6462	0.904837418	0.584685535
3.5	Q2	1.0379	0.6379	0.869358235	0.554531186
4.5	Q2	1.0292	0.6292	0.835270211	0.525559206
			Q2	0.049310813	

See more details in excel.

Q1 is **0.0316** and Q2 is **0.0493**.