Due date: Sep 23 , 2021 8am

Problem 1
A five year credit default swap entered on Mar 20 2018, requires quarterly payments at the rate of 200 basis point per year. 1 The principal is \$ 100 Million. A default occurs after 3 years and two months. The auction process finds the price of the cheapest deliverable bond to be 40% of its face value. Describe the cashflows and their timing for the seller of the credit default swap.

Solution: The seller receives:

$$100000000 * 0.02 * 0.25 = \$500000$$

at time 0.25, 0.5, 0.75,..., 3. (June 20 2018, Sep 20 2018, and Dec 20 2018, Mar 20 2019, ...)
The seller also receives a final accrual payment of about

$$100000000 * 0.02 * \frac{2}{12} = \$333333$$

The seller pays $100000000 * (1 - 40\%) = \$60$ million after 3 years and 2 months.

Problem 2
We consider a five year digital CDS, which pays notional \$1 in case of default. We need to find its fair value spread s, giving the following information:

- The continuous hazard rate per year is h = 0.03.
- The risk free rate r, continuously compounded is 4%.
- The premium is paid each 6 months.
- Default can happens each 3 months (a quarter), and the payment in case of default is made at the end of the quarter.

Solution:

Inputs		
HR	RF	RR
0.03	0.04	0.4

Table 1	Unconditional Default Prob and Suvival Prob		
	Time	Prob S	Prob D
	0.5	0.985112	0.014888
	1	0.970446	0.014666
	1.5	0.955997	0.014448
	2	0.941765	0.014233
	2.5	0.927743	0.014021
	3	0.913931	0.013812
	3.5	0.900325	0.013607
	4	0.88692	0.013404
	4.5	0.873716	0.013205
	5	0.860708	0.013008
Sum			

Table 2	Calculation of PV of Expected Payments(scaled by s)			
	Prob S	Expected Payment	DF	PV of EP
	0.985112	0.492556	0.980199	0.482803
	0.970446	0.485223	0.960789	0.466197
	0.955997	0.477999	0.941765	0.450162
	0.941765	0.470882	0.923116	0.434679
	0.927743	0.463872	0.904837	0.419729
	0.913931	0.456966	0.88692	0.405292
	0.900325	0.450162	0.869358	0.391352
	0.88692	0.44346	0.852144	0.377892
	0.873716	0.436858	0.83527	0.364894
	0.860708	0.430354	0.818731	0.352344
				4.145344

Table 3	Calculation of the Present Value of Expected Payoff (noitional principal = \$1)					
	Time	Prob S	Prob D	Expected Payoff	df	PV of expected payoff
	0.5	0.985112	0.014888	0.014888	0.980199	0.014593
	1	0.970446	0.014666	0.014666	0.960789	0.014091
	1.5	0.955997	0.014448	0.014448	0.941765	0.013607
	2	0.941765	0.014233	0.014233	0.923116	0.013139
	2.5	0.927743	0.014021	0.014021	0.904837	0.012687
	3	0.913931	0.013812	0.013812	0.88692	0.01225
	3.5	0.900325	0.013607	0.013607	0.869358	0.011829
	4	0.88692	0.013404	0.013404	0.852144	0.011422
	4.5	0.873716	0.013205	0.013205	0.83527	0.011029
	5	0.860708	0.013008	0.013008	0.818731	0.01065
						0.125298

spread	0.029776

Table 4	Calculation of the Present Value of Accrual Payment (scaled by s)			
	Prob D	Expected Accrual Payment	df	PV of Expected Accrual Payment
	0.014888	0.007444	0.980199	0.007297
	0.014666	0.007333	0.960789	0.007046
	0.014448	0.007224	0.941765	0.006803
	0.014233	0.007116	0.923116	0.006569
	0.014021	0.007011	0.904837	0.006343
	0.013812	0.006906	0.88692	0.006125
	0.013607	0.006803	0.869358	0.005915
	0.013404	0.006702	0.852144	0.005711
	0.013205	0.006602	0.83527	0.005515
	0.013008	0.006504	0.818731	0.005325
				0.062649

Figure 1: Prob 2 Answer

The annualized spread is approximately **0.029776**.

Problem 3
Go back to the defaultable bond we analyzed in the class (spreadsheet "DefaultableBond Pricing" in Questrom, except we don't fix the coupon rate). Consider two cases:

- The hazard rate h is constant, h = 0.04 (we did it in the class), so default times follow the exponential distribution.
- The default times follow the Weibull distribution with parameters $\lambda = 0.04$ and p = $\frac{3}{4}$. For each

of these cases, find the semi-annual coupon rate c, so that the bonds are valued at par, thus the value of the bond = 1. Which coupon rate is higher and why?

Solution:

Defaultable Bond Pricing

Hazard Rate (per year)	Risk free rate (continuous) per	Recovery Rate	Coupon Rate (semi-annual)	Time in years	Survival Prob	Probability of Default	scale by coupon rate Expected value coupons	Discount	PV of exp coupons	PV of expected notional	Recovery Value
0.04	0.05	0.4		0	1	0		1			
				0.5	0.980198673	0.019801327	0.03719854	0.97531	0.036280104		0.00772
			0.0759	1	0.960789439	0.019409234	0.036461959	0.95123	0.034683688		0.00739
				1.5	0.941764534	0.019024906	0.035739964	0.92774	0.033157519		0.00706
				2	0.923116346	0.018648187	0.035032265	0.90484	0.031698505	0.835270211	0.00675
			Summation						0.135819816	0.835270211	0.02892
							Value	1.0000			

Defaultable Bond Pricing

	Risk free rate (continuous) per	Recovery Rate	Coupon Rate (semi-annual)	Time in years	Survival Prob	Probability of Default	scale by coupon rate Expected value coupons	Discount	PV of exp coupons	PV of expected notional	Recovery Value
	0.05	0.4		0	1	0		1			
0.053182959				0.5	0.948206514	0.051793486	0.047099314	0.97531	0.045936428		0.02021
0.089442719			0.099344	1	0.914440644	0.03376587	0.045422096	0.95123	0.043206834		0.01285
0.12123093				1.5	0.885829371	0.028611272	0.044000917	0.92774	0.040821564		0.01062
0.150424124				2	0.860343007	0.025486364	0.042734958	0.90484	0.038668189	0.778470545	0.00922
			Summation						0.168633014	0.778470545	0.052
							Value	1.0000			

The coupon rate for **(a)** is about 0.0759 and **(b)** is 0.099344. We can see that the coupon rate with default time with Weibull distribution is higher, since once the default time follows Weibull distribution, it will be scaled by a parameter p (p = 0.75) and therefore the probability of survival will be lower than that in (a). If survival probability is lower, the coupon rate should be higher. This effect as can be seen mathematically from the formula in slides.

$$\bar{C}(0) = \sum_{n=1}^{M} \bar{c}_n \bar{B}\left(0, T_n\right) + \bar{B}\left(0, T_M\right) + \pi \sum_{n=1}^{M} e\left(0, T_{n-1}, T_n\right)$$

$$\bar{B}(t, T) = B(t, T) P(t, T), \quad T > t$$

Where $P(t, T)$ is the survival probability and Cn-bar is coupon rate.