Due on Thursday Morning.

Problem 1
Three dimensional Copula.

Solution: **(a)** $H(x, y, 0) = 0$, $H(x, 0, z) = 0$, $H(-1, y, z) = 0$
Thus, H is grounded.
(b) $H_1(x) = H(x, \infty, \pi/2) = \lim_{y\to\infty} \frac{(x+1)(e^y-1)\sin z}{x+2e^y-1} = \frac{x+1}{2}$
$H_2(y) = H(1, y, \pi/2) = \frac{2(e^y-1)}{2e^y} = 1 - e^{-y}$
$H_3(z) = H(1, \infty, z) = \frac{2(e^y-1)\sin z}{2e^y} = \sin z$
(c) $H_{1,2}(x, y) = H(x, y, \pi/2) = \frac{(x+1)(e^y-1)}{x+2e^y-1}$
$H_{2,3}(y, z) = H(1, y, z) = (1 - e^{-y})\sin z$
$H_{1,3}(x, z) = H(x, \infty, z) = \frac{(x+1)\sin z}{2}$
(d) $V_H(B) = H(x_2, y_2, z_2) - H(x_2, y_2, z_1) - H(x_2, y_1, z_2) - H(x_1, y_2, z_2) +$
$H(x_2, y_1, z_1) + H(x_1, y_2, z_1) + H(x_1, y_1, z_2) - H(x_1, y_1, z_1) = H(\frac{1}{2}, 2, \frac{\pi}{2}) - H(\frac{1}{2}, 2, \frac{\pi}{4}) - H(\frac{1}{2}, 1, \frac{\pi}{2}) - H(0, 2, \frac{\pi}{2}) +$
$H(\frac{1}{2}, 1, \frac{\pi}{4}) + H(0, 2, \frac{\pi}{4}) + H(0, 1, \frac{\pi}{2}) - H(0, 1, \frac{\pi}{4}) = 0.02129011357656302$

Problem 2
t-copula

Solution: For $t1, t2 <= 3$ cases, the $t-$copula actually bumped the probability up more or less. Oppositely, in terms of the case of $t1 \geq 5, t2 \leq 2$, the probability is lower than the Gaussian case a little bit. I assume that this is reasonable, since t-copula more fits to the extremes. Thus, when $t1, t2 \leq 3$, the probability will increase. Also, when the ρ goes up, whatever the copula we use, the effects were almost neutral.

```
P(t1, t2 <= 3) 0 =  0.02452
P(t1>=5, t2 <=2) 0 =  0.10431
P(t1, t2 <= 3) 0.2 =  0.03369
P(t1>=5, t2 <=2) 0.2 =  0.09342
P(t1, t2 <= 3) 0.85 =  0.07577
P(t1>=5, t2 <=2) 0.85 =  0.0389
```

Case of rho = 0,

```
P(t1, t2 <= 3) =  0.01658
P(t1>=5, t2 <=2) =  0.11237
```

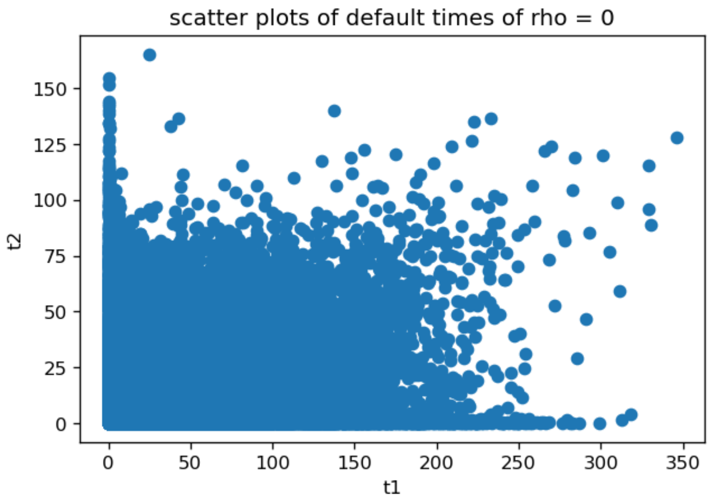
Case of rho = 0.2,

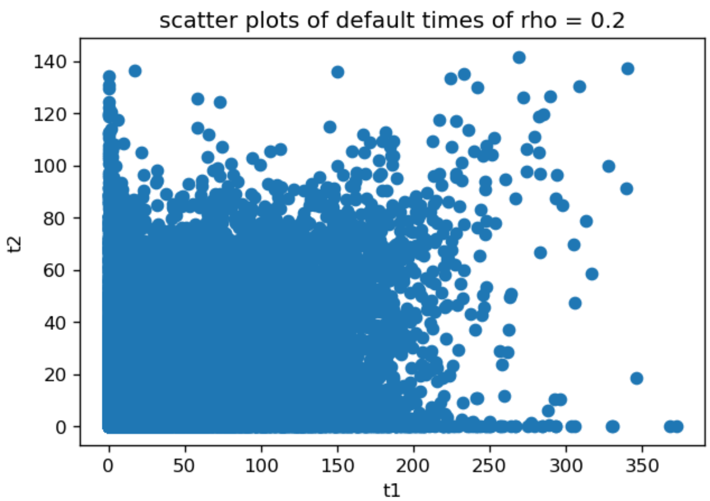
```
P(t1, t2 <= 3) =  0.01655
P(t1>=5, t2 <=2) =  0.11299
```

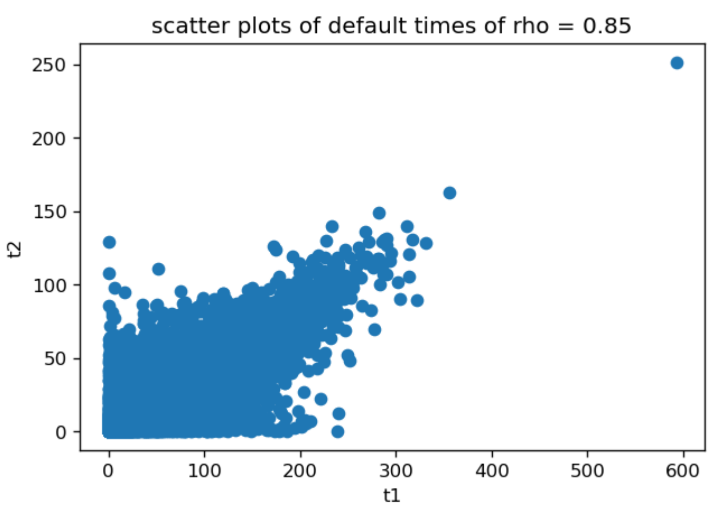
Case of rho = 0.85,

```
P(t1, t2 <= 3) =  0.07349
P(t1>=5, t2 <=2) =  0.043
```

From the following pics, as ρ increases, $t1$ and t_2 are gradually close to each other and it becomes concentrated and symmetric.


scatter plots of default times of rho = 0
t2
t1
0
25
50
75
100
125
150
0
50
100
150
200
250
300
350


scatter plots of default times of rho = 0.2
t2
t1
0
20
40
60
80
100
120
140
0
50
100
150
200
250
300
350


scatter plots of default times of rho = 0.85
t2
t1
0
50
100
150
200
250
0
100
200
300
400
500
600

Problem 3
n-dimensional Gaussian copula

Solution: I used $N(\#of simu) = 50000$ to get the result, thus the value could variate a little. Please, see more details in python file. Example results and codes is as below:

```
Spread:  0.01685290968368299
Sentivity dS/dR: -0.013686916334690637
```

```python
def s(T,rho,lam,RR,N,r):
    mu = np.zeros(5)
    sig = np.zeros((5,5)) + rho +  np.eye(5) * (1-rho)

    x1,x2,x3,x4,x5 = np.random.multivariate_normal(mu,sig,N).T
    u1 = norm.cdf(x1)
    u2 = norm.cdf(x2)
    u3 = norm.cdf(x3)
    u4 = norm.cdf(x4)
    u5 = norm.cdf(x5)

    t1 = -np.log(1-u1)/lam
    t2 = -np.log(1-u2)/lam
    t3 = -np.log(1-u3)/lam
    t4 = -np.log(1-u4)/lam
    t5 = -np.log(1-u5)/lam

    premium = np.zeros(N)
    deft = np.zeros(N)

    ts = np.linspace(0,T,M)
    for i in range(N):
        tau = min(t1[i],t2[i],t3[i],t4[i],t5[i])
        for j in range(M-1):
            if tau >= ts[j+1]:
                premium[i] += np.exp(-r*ts[j+1])
        for k in range(M-1):
            if (tau<ts[k+1]) & (tau>ts[k]):
                premium[i] += (tau-ts[k])*np.exp(-r*tau)
        if tau <= ts[-1]:
            deft[i] = np.exp(-r*tau)

    vp = premium.mean()
    vd = deft.mean()*LGD*(1-RR)

    s = vd/vp
    return s
spread = s(T,rho,lam,RR,N,r)
print("Spread: ",spread)

s1 = s(T,rho + rho * delta,lam,RR,N,r)
s2 = s(T,rho - rho * delta,lam,RR,N,r)
Rhoza_galeeva = (s1-s2) / delta
print("Sentivity dS/dR:",Rhoza_galeeva)
```