Problem 1

Finish the problem we started in the class, using the set up as in the spreadsheet "TwoBonds0.xlsx" at Questrom tools (slightly modified). Now we are given only yields on risk free bonds and constant hazard rates h(T) = 0.04 (different).

- Calculate yields for the defaultable bonds ("Issuer") by calculating survival probabilities based on given hazard rates, using formulas for the discrete case
- Calculate yields for the defaultable bonds ("Issuer") by calculating survival probabilities based on given hazard rates, using formulas for the continuous case

Solution: See Excel for detailed calculations.

MATURITY (years)	YIELD(%)												
	Gov. (risk free)	Issuer	Conditional Survival	Conditional Default	Hazard rate	Given Hazard rates	Prob. Survival, contionuos case	Prob. Survival, discrete case	Price of Gov	Price Issuer Continuos	Price Issuer discrete	Yield contunous, %	Yield Discrete, %
0.0							1	1					
0.5	2.00000	5.00000	0.98000	0.02000	0.03000	0.0400	0.98019867	0.98	0.99004983	0.97044553	0.97024884	6	6.04054146
1.0	2.50000	5.40000	0.98000	0.02000	0.03000	0.0400	0.96078944	0.9604	0.97530991	0.93706746	0.93668764	6.5	6.54054146
3.0	3.00000	5.60000	0.92000	0.08000	0.03000	0.0400	0.88692044	0.883568	0.91393119	0.81058425	0.80752035	7	7.12623412
5.0	3.25000	5.75000	0.92000	0.08000	0.03000	0.0400	0.81873075	0.81288256	0.85001609	0.69593431	0.69096326	7.25	7.39337265
7.0	3.45000	6.05000	0.92000	0.08000	0.03000	0.0400	0.75578374	0.74785196	0.7854488	0.59362944	0.58739942	7.45	7.60071774
10.0	4.00000	6.85000	0.88000	0.12000	0.03000	0.0400	0.67032005	0.65810972	0.67032005	0.44932896	0.44114414	8	8.18383613

Figure 1: Excel Solution

Problem 2

Consider the following Weibull distribution with parameters:

- $\lambda = 0.04$, p = 1 (exponential)
- $\lambda = 0.04$, p = 1/2
- $\lambda = 0.04$, p = 1/2

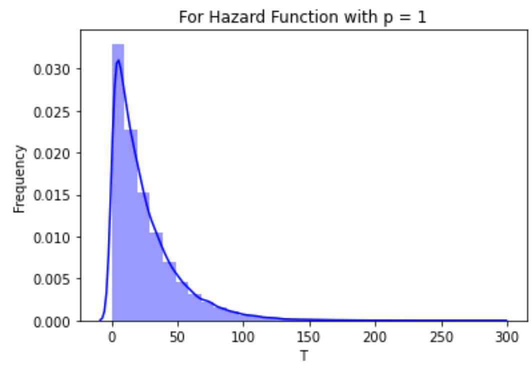
Perform the following:

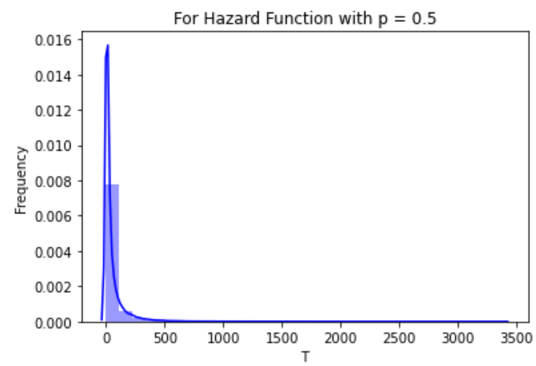
a) Simulate random default times (at least 10, 000) from the these three distributions
b) Make histograms, & observations how they compare to each other.
c) Calculate their means, standard deviations.
d) Compare those with the theoretical values, and check the relations between them

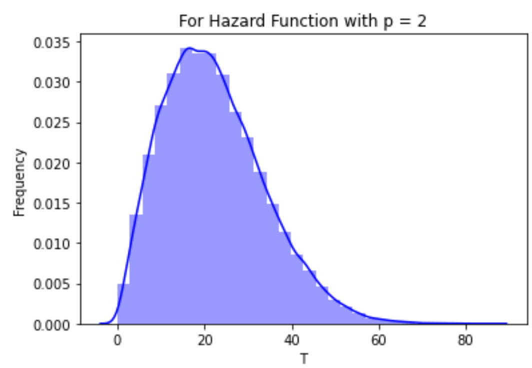
Solution: **(a)(b)** For these two questions, I used n = 50000 which means the number of simulation is 50000, in order for more accurate result. From the figures below, we can clear see that when p = 1, the distribution is almost exponential, it fits our theoretical result when p = 1, the Weibull distribution returns to exponential distribution.

The second graph is quite interesting since when p = 0.5, the distribution is not exponential anymore and represents that moist of the simulation is centered at 0 and the distribution has a long tail which means there are a lot of extreme value occurred even if the probability is low and the hazard rate is decreasing as P < 1.

The third picture compared to the first two is more plump and the mean looks not centered around 0 anymore, we can guess that this case happens since ,for p = 0, the Weibull distribution expresses some memory and the hazard rate is increasing as P > 1 ,which is opposite of exponential case.







(c)(d) The simulation mean and standard deviation is very closed to theoretical value as the number of simulation is increasing. I used 50000 which is a relatively large number, so the values are closed to each other. **I also create a graph for when number of simulation increases, how the absolute error changes for mean value in P = 1 case. It is apparent that as n goes larger and larger, the absolute error goes lower and lower.**

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The mean value for p = 1 case is: 24.93958227540938
The mean value for p = 0.5 case is: 49.59566649963096
The mean value for p = 2 case is: 22.131565860941325
The std for p = 1 case is: 24.857773400263756
The std for p = 0.5 case is: 109.38728289378234
The std for p = 2 case is: 11.562151591639457
The theoretical mean for p = 1 is :  25.0
The theoretical mean for p = 0.5 is :  50.0
The theoretical mean for p = 2 is :  22.15567313631895
The theoretical std for p = 1 is :  25.0
The theoretical std for p = 0.5 is :  111.80339887498948
The theoretical std for p = 2 is :  11.58128437940261
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