INVERSE AND DETERMINANT OF A PARTITIONED MATRIX

Let A be an $N \times N$ nonsingular matrix which is partitioned as:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where A_{11} and A_{22} are, respectively, $N_1 \times N_1$ and $N_2 \times N_2$ nonsingular matrices where $N_1 + N_2 = N$. A_{21} and A_{12} are, respectively, $N_2 \times N_1$ and $N_1 \times N_2$ matrices. A^{-1} is partitioned in the same manner as:

$$A^{-1} = \begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix}$$

With these definitions,

- $|A| = |A_{22}||A_{11} - A_{12}A_{22}^{-1}A_{21}| = |A_{11}||A_{22} - A_{21}A_{11}^{-1}A_{12}|$, and
- the components of A^{-1} can be calculated as: $A^{11} = (A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1}$, $A^{22} = (A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}$, $A^{12} = -A_{11}^{-1}A_{12}A^{22}$ and $A^{21} = -A_{22}^{-1}A_{21}A^{11}$.

From Zellner, “Bayesian Inference” Appendix B1,
Gary Koop, “Bayesian Econometrics”, Theorem A.9