

By Donsker's invariance principle,

$$I(\bar{m}_{i\Delta}^n : i=0, \dots, n) \Rightarrow W$$

where  $I(\cdot)$  means linear interpolation.

Note that

$$\{I(m_{i\Delta}^n)(t)\} = \left\{ \sigma I(\bar{m}_{i\Delta}^n)(t) + \mu t + O\left(\frac{1}{\sqrt{n}}\right) \right\}$$

$$\stackrel{\text{D.}}{\Rightarrow} \{\mu t + \sigma W_t : t \in T\}$$

$$I(S_{i\Delta}^n) = I(\exp(m_{i\Delta}^n)) \Rightarrow \{\exp(\mu t + \sigma W_t) : t \in T\}$$

$$= S$$