Nov 15

§ SDE Models.

§1. GBM.

$$dX_t = X_t(\mu\, dt + \sigma\, dW_t)$$

§2. Gaussian short rate models.

§2.1. Vasicek/HW/HL

§2.2. Multifactor models.

§3. Square root diffusion

§3.1 CIR rate model.

Model, Given $r(0)>0$, $\sigma, \alpha, b > 0$

$$d\cancel{r_t} = \alpha(b - r(t))\,dt + \sigma\sqrt{r(t)}\, dW_t$$

Facts

1. No explicit soln.
2. $r(t) \geq 0$
3. If $2\alpha b \geq \sigma^2$, then $r(t) > 0$, $\cancel{\forall t}$

Think, Given r_t

$$r_{t+\delta} = r_t + \int_t^{t+\delta} \alpha(b - r_s)\,ds + \int_t^{t+\delta} \sigma\sqrt{r_s}\,dW_s$$

$$r_{t+\delta} = r_t + \alpha(b - r_t)\delta + \sigma\sqrt{r_t}\,\underbrace{W_{t,t+\delta}}_{\sim \sqrt{\delta} Z}$$

$W_{t+\delta} - W_t$

$$r_{t+\delta} = r_t + \alpha(b - r_t)\delta + \sigma\sqrt{r_t}\,\sqrt{\delta}\,Z$$

Algo

① Set r_0

② Repeat

$$r_\delta = r_0 + \alpha(b - r_0)\delta + \sigma\sqrt{r_0}\,\sqrt{\delta}\,Z_0$$

$$r_{2\delta} = r_\delta + \cdots r_\delta \cdots \boxed{\sqrt{r_\delta}} \cdots Z_1$$

$\vdots$

could be negative

Algo (modified Euler)

① Set r_0

② Repeat.

$$r_{i+1} = r_i + \alpha(b - r_i)\delta + \sigma\sqrt{(r_i)^+}\,\sqrt{\delta}\,Z_i$$

$(x)^+ = \max(x, 0)$

§3.2 Heston stoch volatility model

<u>Model</u> Stock S_t follows

$$\begin{cases} dS_t = S_t\left(\mu\, dt + \sqrt{V(t)}\, dW_1(t)\right) \\ dV_t = \alpha\,(b - V(t))\,dt + \sigma\sqrt{V_t}\,dW_2(t). \end{cases}$$

ex. ① Design approx. of put/call price under Heston

② Calibrate Heston with Market data.