NYU Tandon Bridge **Spring 2024**

Homework 6

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Question 5

a.

In order to show $5n^3 + 2n^2 + 3n = \Theta(n^3)$, we need to find c_1, c_2 and n_0 such that for all $n \geq n_0$,

$$c_1 \cdot n^3 \leq 5n^3 + 2n^2 + 3n \leq c_2 \cdot n^3$$

If we let $c_1 = 5$, $c_2 = 10$ and $n_0 = 1$, we have,

$$5n^3 \leq 5n^3 + 2n^2 + 3n \leq 10n^3 \quad \text{for all } n \geq 1$$

Therefore, $5n^3 + 2n^2 + 3n = \Theta(n^3)$.

b.

In order to show $\sqrt{7n^2 + 2n - 8} = \Theta(n)$, we need to find c_1, c_2 and n_0 such that for all $n \geq n_0$,

$$c_1 \cdot n \leq \sqrt{7n^2 + 2n - 8} \leq c_2 \cdot n$$

If we let $n_0 = 4$, then $n \geq 4$, we can square the inequalities to get:

$$c_1^2 \cdot n^2 \leq 7n^2 + 2n - 8 \leq c_2^2 \cdot n^2 \quad \text{for all } n \geq 4$$

Since $n \geq 4$,

$$\begin{aligned} 2n - 8 &\geq 0 \\ 7n^2 + 2n - 8 &\geq 7n^2 \end{aligned}$$

$$\begin{aligned} 2n &\leq 2n^2 \\ 7n^2 + 2n - 8 &\leq 7n^2 + 2n^2 \\ 7n^2 + 2n - 8 &\leq 9n^2 \end{aligned}$$

By combining these two inequalities, we can get,

$$7n^2 \leq 7n^2 + 2n - 8 \leq 9n^2 \quad \text{for all } n \geq 4$$

By taking the square root, we can get,

$$\sqrt{7}n \leq \sqrt{7n^2 + 2n - 8} \leq 3n \quad \text{for all } n \geq 4$$

Therefore, we show that when $c_1 = \sqrt{7}$, $c_2 = 3$ and $n_0 = 4$, we have $c_1 \cdot n \leq \sqrt{7n^2 + 2n - 8} \leq c_2 \cdot n$ for all $n \geq n_0$, , so $\sqrt{7n^2 + 2n - 8} = \Theta(n)$.