NYU Tandon Bridge **Spring 2024**

Homework 1

Student: Shichen Zhang (sz4968)

Question 1

A.

1.

$$\begin{aligned} 10011011_2 &= 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 128 + 0 + 0 + 16 + 8 + 0 + 2 + 1 \\ &= 155 \end{aligned}$$

2.

$$\begin{aligned} 456_7 &= 4 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 \\ &= 196 + 35 + 6 \\ &= 237 \end{aligned}$$

3.

$$\begin{aligned} 38A_{16} &= 3 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 \\ &= 768 + 128 + 10 \\ &= 906 \end{aligned}$$

4.

$$\begin{aligned} 2214_5 &= 2 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 4 \times 5^0 \\ &= 250 + 50 + 5 + 4 \\ &= 309 \end{aligned}$$

B.

1.

$$\begin{aligned}
69 \div 2 &= 34 \quad \text{mod } 1 \\
34 \div 2 &= 17 \quad \text{mod } 0 \\
17 \div 2 &= 8 \quad \text{mod } 1 \\
8 \div 2 &= 4 \quad \text{mod } 0 \\
4 \div 2 &= 2 \quad \text{mod } 0 \\
2 \div 2 &= 1 \quad \text{mod } 0 \\
1 \div 2 &= 0 \quad \text{mod } 1
\end{aligned}$$

$$69_{10} = 1000101_2$$

2.

$$\begin{aligned}
485 \div 2 &= 242 \quad \text{mod } 1 \\
242 \div 2 &= 121 \quad \text{mod } 0 \\
121 \div 2 &= 60 \quad \text{mod } 1 \\
60 \div 2 &= 30 \quad \text{mod } 0 \\
30 \div 2 &= 15 \quad \text{mod } 0 \\
15 \div 2 &= 7 \quad \text{mod } 1 \\
7 \div 2 &= 3 \quad \text{mod } 1 \\
3 \div 2 &= 1 \quad \text{mod } 1 \\
1 \div 2 &= 0 \quad \text{mod } 1
\end{aligned}$$

$$485_{10} = 111100101_2$$

3.

Each hexadecimal digit can be translated into its 4-bit binary equivalent:

$$\begin{aligned}
&6_{16} = 0110_2 \\
&D_{16} = 13_{10} = 1101_2 \\
&1_{16} = 0001_2 \\
&A_{16} = 10_{10} = 1010_2
\end{aligned}$$

Therefore,

$$6D1A_{16} = 0110110100011010_2$$

C.

1.

$$1101011_2 = 01101011_2$$

Each 4-bit binary can be translated into its hexadecimal digit equivalent:

$$0110_2 = 6_{16}$$
$$1011_2 = B_{16}$$

Therefore,

$$1101011_2 = 6B_{16}$$

2.

$$\begin{aligned} 895 \div 16 &= 55 \qquad && \text{mod } 15 \\ 55 \div 16 &= 3 && \text{mod } 7 \\ 3 \div 16 &= 0 && \text{mod } 3 \end{aligned}$$

$$\begin{aligned} 15_{10} &= F_{16} \\ 895_{10} &= 37F_{16} \end{aligned}$$

Question 2

1.

$$\begin{array}{r}
{\scriptstyle 1\,1\,1\,1} \\
7\,5\,6\,6_8 \\
+\,4\,5\,1\,5_8 \\
\hline
1\,4\,3\,0\,3_8
\end{array}$$

$$7566_8 + 4515_8 = 14303_8$$

2.

$$\begin{array}{r}
{\scriptstyle 1\,1\,1\,1\,1} \\
1\,0\,1\,1\,0\,0\,1\,1_2 \\
+\,0\,0\,0\,0\,1\,1\,0\,1_2 \\
\hline
1\,1\,0\,0\,0\,0\,0\,0_2
\end{array}$$

$$10110011_2 + 1101_2 = 11000000_2$$

3.

$$\begin{array}{r}
{\scriptstyle 1\;\;1}\phantom{\;6\;6_{16}} \\
7\,A\;6\;6_{16} \\
+\;4\;5\,C\;5_{16} \\
\hline
C\;0\;2\,B_{16}
\end{array}$$

$$7A66_{16} + 45C5_{16} = C02B_{16}$$

4.

$$\begin{array}{r}
{\scriptstyle 2\;4\;1} \\
\cancel{3}\,\cancel{0}\,\cancel{2}\,2_5 \\
-\,2\,4\,3\,3_5 \\
\hline
3\,4_5
\end{array}$$

$$3022_5 - 2433_5 = 34_5$$

Question 3

A.

1.

$$\begin{aligned}
124 \div 2 &= 62 && \text{mod } 0 \\
62 \div 2 &= 31 && \text{mod } 0 \\
31 \div 2 &= 15 && \text{mod } 1 \\
15 \div 2 &= 7 && \text{mod } 1 \\
7 \div 2 &= 3 && \text{mod } 1 \\
3 \div 2 &= 1 && \text{mod } 1 \\
1 \div 2 &= 0 && \text{mod } 1
\end{aligned}$$

$$124_{10} = 01111100_{8\ bit\ 2's\ comp}$$

2.

Convert 124 to its 8 bits binary expression,

$$\begin{aligned}
124 \div 2 &= 62 && \text{mod } 0 \\
62 \div 2 &= 31 && \text{mod } 0 \\
31 \div 2 &= 15 && \text{mod } 1 \\
15 \div 2 &= 7 && \text{mod } 1 \\
7 \div 2 &= 3 && \text{mod } 1 \\
3 \div 2 &= 1 && \text{mod } 1 \\
1 \div 2 &= 0 && \text{mod } 1
\end{aligned}$$

$$124_{10} = 01111100_2$$

Invert the bits,

$$10000011$$

Add 1,

$$10000011 + 00000001 = 10000100$$

Therefore,

$$-124_{10} = 10000100_{8\ bit\ 2's\ comp}$$

3.

$$\begin{aligned}
109 \div 2 &= 54 \quad &\text{mod } 1\\
54 \div 2 &= 27 \quad &\text{mod } 0\\
27 \div 2 &= 13 \quad &\text{mod } 1\\
13 \div 2 &= 6 \quad &\text{mod } 1\\
6 \div 2 &= 3 \quad &\text{mod } 0\\
3 \div 2 &= 1 \quad &\text{mod } 1\\
1 \div 2 &= 0 \quad &\text{mod } 1
\end{aligned}$$

$$109_{10} = 01101101_{8\ bit\ 2's\ comp}$$

4.

Convert 79 to its 8 bits binary expression,

$$\begin{aligned}
79 \div 2 &= 39 \quad &\text{mod } 1\\
39 \div 2 &= 19 \quad &\text{mod } 1\\
19 \div 2 &= 9 \quad &\text{mod } 1\\
9 \div 2 &= 4 \quad &\text{mod } 1\\
4 \div 2 &= 2 \quad &\text{mod } 0\\
2 \div 2 &= 1 \quad &\text{mod } 0\\
1 \div 2 &= 0 \quad &\text{mod } 1
\end{aligned}$$

$$79_{10} = 01001111_2$$

Invert the bits,

$$10110000$$

Add 1,

$$10110000 + 00000001 = 10110001$$

Therefore,

$$-79_{10} = 10110001_{8\ bit\ 2's\ comp}$$

B

1.

$$\begin{aligned}
00011110_{8\ bit\ 2's\ comp} &= 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 + 0 \times 2^5 + 0 \times 2^6 - 0 \times 2^7\\
&= 0 + 2 + 4 + 8 + 16 + 0 + 0 - 0\\
&= 30
\end{aligned}$$

2.

$$\begin{aligned}
11100110_{8\ bit\ 2's\ comp} &= 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 0 \times 2^4 + 1 \times 2^5 + 1 \times 2^6 - 1 \times 2^7\\
&= 0 + 2 + 4 + 0 + 0 + 32 + 64 - 128\\
&= -26
\end{aligned}$$

3.

$$\begin{aligned}00101101_{8\ bit\ 2's\ comp} &= 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 0 \times 2^5 + 1 \times 2^6 - 0 \times 2^7 \\ &= 1 + 0 + 4 + 8 + 0 + 0 + 32 + 0 - 0 \\ &= 45\end{aligned}$$

4.

$$\begin{aligned}10011110_{8\ bit\ 2's\ comp} &= 0 \times 2^0 + 1 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 + 0 \times 2^5 + 0 \times 2^6 - 1 \times 2^7 \\ &= 0 + 2 + 4 + 8 + 16 + 0 + 0 - 128 \\ &= -98\end{aligned}$$

Question 4

1. Exercise 1.2.4

section b

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

section c

p	q	r	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

2. Exercise 1.3.4

section b

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

section d

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5

1. Exercise 1.2.7

section b

$$(B \land D) \lor (B \land M) \lor (D \land M)$$

section c

$$B \lor (D \land M)$$

2. Exercise 1.3.7

section b

$$(s \lor y) \to p$$

section c

$$p \to y$$

section d

$$p \leftrightarrow (s \land y)$$

section e

$$p \to (s \lor y)$$

3. Exercise 1.3.9

section c

$$c \to p$$

section d

$$c \to p$$

Question 6

1. Exercise 1.3.6

section b

If Joe is eligible for the honors program, then he has maintained a B average.

section c

If Rajiv can go on the roller coaster, then he is at least four feet tall.

section d

If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10

section c

p	q	r	$p \vee r$	$q \wedge r$	$(p \vee r) \leftrightarrow (q \wedge r)$
T	F	T	T	F	F
T	F	F	T	F	F

According to the truth table, the truth value of the expression $(p \vee r) \leftrightarrow (q \wedge r)$ is false.

section d

p	q	r	$p \wedge r$	$q \wedge r$	$(p \wedge r) \leftrightarrow (q \wedge r)$
T	F	T	T	F	F
T	F	F	F	F	T

According to the truth table, the truth value of the expression $(p \wedge r) \leftrightarrow (q \wedge r)$ is unknown.

section e

p	q	r	$r \vee q$	$p \to (r \vee q)$
T	F	T	T	T
T	F	F	F	F

According to the truth table, the truth value of the expression $p \to (r \vee q)$ is unknown.

section f

p	q	r	$p \wedge q$	$(p \wedge q) \to r$
T	F	T	F	T
T	F	F	F	T

According to the truth table, the truth value of the expression $(p \wedge q) \to r$ is true.

Question 7

Exercise 1.4.5

section b

1st sentence can be expressed in symbolic logic as,

$$\neg j \to (l \vee \neg r)$$

2nd sentence can be expressed in symbolic logic as,

$$(r \wedge \neg l) \to j$$

Proof:

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

According to the truth table, the two expressions have the same truth value in all situations, therefore they are logically equivalent.

section c

1st sentence can be expressed in symbolic logic as,

$$j \to \neg l$$

2nd sentence can be expressed in symbolic logic as,

$$\neg j \to l$$

Proof:

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

According to the truth table, the two expressions have different truth values, therefore they are not logically equivalent.

section d

1st sentence can be expressed in logic as,

$$(r \vee \neg l) \to j$$

2^{nd} sentence can be expressed in logic as,

$$j \to (r \wedge \neg l)$$

Proof:

j	l	r	$(r \vee \neg l) \to j$	$j \to (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

According to the truth table, the two expressions have different truth values, therefore they are not logically equivalent.

Question 8

1. Exercise 1.5.2

section c

$$\begin{aligned}
(p \to q) \wedge (p \to r) &\equiv (\neg p \vee q) \wedge (p \to r) && \text{(Conditional identity)} \\
&\equiv (\neg p \vee q) \wedge (\neg p \vee r) && \text{(Conditional identity)} \\
&\equiv \neg p \wedge (q \vee r) && \text{(Distributive law)} \\
&\equiv p \to (q \vee r) && \text{(Conditional identity)}
\end{aligned}$$

section f

$$\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\equiv \neg((p \vee \neg p) \wedge (p \vee q)) && \text{(Distributive law)} \\
&\equiv \neg(T \wedge (p \vee q)) && \text{(Complement law)} \\
&\equiv \neg(p \vee q) && \text{(Identity law)} \\
&\equiv \neg p \wedge \neg q && \text{(De Morgan's law)}
\end{aligned}$$

section i

$$\begin{aligned}
(p \wedge q) \to r &\equiv \neg(p \wedge q) \vee r && \text{(Conditional identity)} \\
&\equiv (\neg p \vee \neg q) \vee r && \text{(De Morgan's law)} \\
&\equiv r \vee (\neg p \vee \neg q) && \text{(Commutative law)} \\
&\equiv (r \vee \neg p) \vee \neg q && \text{(Associative law)} \\
&\equiv (\neg p \vee r) \vee \neg q && \text{(Commutative law)} \\
&\equiv (\neg p \vee \neg\neg r) \vee \neg q && \text{(Double negation law)} \\
&\equiv \neg(p \wedge \neg r) \vee \neg q && \text{(De Morgan's law)} \\
&\equiv (p \wedge \neg r) \to \neg q && \text{(Conditional identity)}
\end{aligned}$$

2. Exercise 1.5.3

section c

$$\begin{aligned}
\neg r \vee (\neg r \to p) &\equiv \neg r \vee (\neg\neg r \vee p) && \text{(Conditional identity)} \\
&\equiv \neg r \vee (r \vee p) && \text{(Double negation law)} \\
&\equiv (\neg r \vee r) \vee p && \text{(Associative law)} \\
&\equiv T \vee p && \text{(Complement law)} \\
&\equiv T && \text{(Domination law)}
\end{aligned}$$

section d

$$
\begin{aligned}
\neg(p \to q) \to \neg q &\equiv \neg\neg(p \to q) \lor \neg q && \text{(Conditional identity)}\\
&\equiv (p \to q) \lor \neg q && \text{(Double negation law)}\\
&\equiv (\neg p \lor q) \lor \neg q && \text{(Conditional identity)}\\
&\equiv \neg p \lor (q \lor \neg q) && \text{(Associative law)}\\
&\equiv \neg p \lor T && \text{(Complement law)}\\
&\equiv T && \text{(Domination law)}
\end{aligned}
$$

Question 9

1. Exercise 1.6.3

section c

$$\exists x \ (x = x^2)$$

section d

$$\forall x \ (x \leq x^2 + 1)$$

2. Exercise 1.7.4

section b

$$\forall x \ (\neg S(x) \wedge W(x))$$

section c

$$\forall x \ (S(x) \rightarrow \neg W(x))$$

section d

$$\exists x \ (S(x) \wedge W(x))$$

Question 10

1. Exercise 1.7.9

section c

True.

This expression means that there exists an x such that either $x = c$ is false or $P(x)$ is true. According to the truth table, if $x = a$, $x = c$ is false, so the expression is true.

section d

True.

According to the truth table, if $x = e$, then $Q(x)$ is true and $R(x)$ is true. It follows $Q(x) \land R(x)$ is true. Therefore, there exists an x such that $Q(x) \land R(x)$ is true.

section e

True.

According to the truth table, $Q(a)$ is true and $P(d)$ is true. Therefore, $Q(a) \land P(d)$ is true.

section f

True.

According to the truth table, for all x, if $x \neq b$, $Q(x)$ is always true.

section g

False.

According to the truth table, if $x = c$, $P(x)$ is false and $R(x)$ is false. It follows $P(x) \lor R(x)$ is false. Therefore, it is false that for all x, $P(x) \lor R(x)$ is true.

section h

True.

According to the truth table, for all x, either $P(x)$ is true or $R(x)$ is false. If $P(x)$ is true, then $R(x) \to P(x)$ is true. If $R(x)$ is false, then $R(x) \to P(x)$ is also true. Therefore, for all x, $R(x) \to P(x)$ is always true.

section i

True.

According to the truth table, when $x = a$, $Q(x)$ is true and $R(x)$ is false. It follows $Q(x) \lor R(x)$ is true. Therefore, there exists an x such that $Q(x) \lor R(x)$ is true.

2. Exercise 1.9.2

section b

True.

According to the truth table, when $x = 2$, $Q(x, y)$ is true for all y.

section c

True.

According to the truth table, when $y = 1$, $P(x, y)$ is true for all x.

section d

False.

According to the truth table, $S(x, y)$ is always false. Therefore, there does not exist an x and a y such that $S(x, y)$ is true.

section e

False.

According to the truth table, when $x = 1$, there does not exist a y such that $Q(x, y)$ is true. Therefore, it false that, for all x, there exist a y such that $Q(x, y)$ is true.

section f

True.

According to the truth table, for all x, there exists a y such that $P(x, y)$ is true. When $x = 1$, if $y = 1$, then $P(x, y)$ is true. When $x = 2$, if $y = 1$, then $P(x, y)$ is true. When $x = 3$, if $y = 1$, then $P(x, y)$ is true.

section g

False.

According to the truth table, when $x = 1$ and $y = 2$, $P(x, y)$ is false. So, it is false that, for all x and y, $P(x, y)$ is true.

section h

True.

According to the truth table, when $x = 2$ and $y = 1$, $Q(x, y)$ is true. So, there exists an x and a y such that $Q(x, y)$ is true.

section i

True.

According to the truth table, for all x and y, $S(x, y)$ is always false. Therefore, $\neg Q(x, y)$ is always true.

Question 11

1. Exercise 1.10.4

section c

$$\exists x \exists y\ (x + y = xy)$$

section d

$$\forall x \forall y\ (((x > 0) \land (y > 0)) \to \left(\frac{x}{y} > 0\right))$$

section e

$$\forall x\ (((x > 0) \land (x < 1)) \to \left(\frac{1}{x} > 1\right))$$

section f

$$\forall x \exists y\ (y < x)$$

section g

$$\forall x \exists y\ ((x \neq 0) \to \left(y = \frac{1}{x}\right))$$

2. Exercise 1.10.7

section c

$$\exists x\ (N(x) \land D(x))$$

section d

$$\forall y\ (D(y) \to P(\text{Sam}, y))$$

section e

$$\exists x \forall y\ (N(x) \land P(x, y))$$

section f

$$\exists x \forall y\ ((N(x) \land D(x)) \land ((N(y) \land (y \neq x)) \to \neg D(y)))$$

3. Exercise 1.10.10

section c

$$\forall x \exists y\ (T(x, y) \land (y \neq \text{Math } 101))$$

section d

$$\exists x \forall y\ ((y \neq \text{Math } 101) \to T(x, y))$$

section e

$$\forall x \exists y_1 \exists y_2\ ((x \neq \text{Sam}) \to (T(x, y_1) \land T(x, y_2) \land (y_1 \neq y_2)))$$

section f

$$\exists y_1 \exists y_2 \forall y_3\ (T(\mathrm{Sam}, y_1) \wedge T(\mathrm{Sam}, y_2) \wedge (y_1 \neq y_2) \wedge (y_3 \neq y_1) \wedge (y_3 \neq y_2)) \rightarrow \neg T(\mathrm{Sam}, y_3))$$

Question 12

1. Exercise 1.8.2

section b

Statement:

$$\forall x\ (D(x) \lor P(x))$$

Negation:

$$\neg\forall x\ (D(x) \lor P(x))$$

Applying De Morgan's law:

$$\exists x\ \neg(D(x) \lor P(x))$$

Applying De Morgan's law:

$$\exists x\ (\neg D(x) \land \neg P(x))$$

English: there is a patient who was neither given the medication nor the placebo.

section c

Statement:

$$\exists x\ (D(x) \land M(x))$$

Negation:

$$\neg\exists x\ (D(x) \land M(x))$$

Applying De Morgan's law:

$$\forall x\ \neg(D(x) \land M(x))$$

Applying De Morgan's law:

$$\forall x\ (\neg D(x) \lor \neg M(x))$$

English: every patient was not given the medication or did not have migraines or both.

section d

Statement:

$$\forall x(P(x) \to M(x))$$

Negation:

$$\neg\forall x(P(x) \to M(x))$$

Applying De Morgan's law:

$$\exists x\neg(P(x) \to M(x))$$

Applying conditional identity:

$$\exists x\neg(\neg P(x) \lor M(x))$$

Applying De Morgan's law:

$$\exists x(\neg\neg P(x) \land \neg M(x))$$

Applying double negation law:

$$\exists x(P(x) \land \neg M(x))$$

English: there is a patient who was given the placebo and did not have migraines.

section e

Statement:

$$\exists x(M(x) \land P(x))$$

Negation:

$$\neg\exists x\ (M(x) \land P(x))$$

Applying De Morgan's law:

$$\forall x\ \neg(M(x) \land P(x))$$

Applying De Morgan's law:

$$\forall x\ (\neg M(x) \lor \neg P(x))$$

English: every patient did not have migraines or was not given not have placebo.

2. Exercise 1.9.4

section c

$$\begin{aligned}
\neg\exists x\forall y(P(x,y) \to Q(x,y)) &\equiv \forall x\neg\forall y(P(x,y) \to Q(x,y)) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\neg(P(x,y) \to Q(x,y)) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\neg(\neg P(x,y) \lor Q(x,y)) && \text{(Conditional identity)}\\
&\equiv \forall x\exists y(\neg\neg P(x,y) \land \neg Q(x,y)) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y(P(x,y) \land \neg Q(x,y)) && \text{(Double negation law)}
\end{aligned}$$

section d

$$\begin{aligned}
\neg\exists x\forall y(P(x,y) \leftrightarrow P(y,x)) &\equiv \forall x\neg\forall y(P(x,y) \leftrightarrow P(y,x)) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\neg(P(x,y) \leftrightarrow P(y,x)) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\neg\left((P(x,y) \to P(y,x)) \land (P(y,x) \to P(x,y))\right) && \text{(Conditional identity)}\\
&\equiv \forall x\exists y\left(\neg(P(x,y) \to P(y,x)) \lor \neg(P(y,x) \to P(x,y))\right) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\left(\neg(\neg P(x,y) \lor P(y,x)) \lor \neg(\neg P(y,x) \lor P(x,y))\right) && \text{(Conditional identity)}\\
&\equiv \forall x\exists y\left((\neg\neg P(x,y) \land \neg P(y,x)) \lor (\neg\neg P(y,x) \land \neg P(x,y))\right) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\left((P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y))\right) && \text{(Double negation law)}
\end{aligned}$$

section e

$$\begin{aligned}
\neg(\exists x\exists y\, P(x,y) \land \forall x\forall y\, Q(x,y)) &\equiv \neg\exists x\exists y\, P(x,y) \lor \neg\forall x\forall y\, Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\neg\exists y\, P(x,y) \lor \exists x\neg\forall y\, Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\forall y\neg P(x,y) \lor \exists x\exists y\neg Q(x,y) && \text{(De Morgan's law)}
\end{aligned}$$