NYU Tandon Bridge **Spring 2024**

Homework 3

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Question 7

(a) Exercise 3.1.1

section a

True.
Since 27 is an integer multiple of 3, $27 = 3 \times 9$, this statement is true.

section b

False.
27 is not a perfect square because it can not be express as the square of two integers.

section c

True.
Since $100 = 10 \times 10$, 100 is a perfect square and is in set B.

section d

False.
Set E is not a subset of set C because it is not true that every object in set E is also in set C. 3 is in set E but not in set C. Set C is not a subset of set E neither because it is not true that every object in set C is also in set E. 4 is in set C but not in set E.

section e

True
Since every object in set E is in set A, the statement is true.

section f

False
It is not true that every object in set A is in set E, 12 is in A but not E. Therefore the statement is false.

section g

False
E is a set, A does not have any set in it, so the statement is false.

(b) exercise 3.1.2

section a

False.
15 is an object not a set, so it is not a proper subset of A.

section b

True.
If x is in $\{15\}$, then x is in A. Therefore, $\{15\}$ is a proper subset of A.

section c

True.
Empty set is a proper subset of C because it has no object.

section d

True.
If an object is in D, it is also in D. Therefore, D is a subset of itself.

section e

False.
There is no set in B, so the statement is false.

(c) Exercise 3.1.5

section b

$$A = \{x \in \mathbb{Z}^+ : x \text{ is an integer multiple of } 3\}$$

The set is infinite.

section d

$$B = \{x \in \mathbb{N} : 0 \leq x \leq 1000 \text{ and } x \text{ is an integer multiple of } 10\}$$

The cardinality of the set $|B|$ is 101.

(d) Exercise 3.2.1

section a

True.
2 is an object in X.

section b

True.
$\{2\}$ is a subset of X because every objects in $\{2\}$ is also in X.

section c

False.
$\{2\}$ is not in X.

section d

False.
3 is not in X.

section e

True.
$\{1,2\}$ is in X.

section f

True.
$\{1,2\}$ is a subset of X because every objects in $\{1,2\}$ is also in X.

section g

True.
$\{2,4\}$ is a subset of X because every objects in $\{2,4\}$ is also in X.

section h

False.
$\{2,4\}$ is not in X.

section i

False.
$\{2,3\}$ is not a subset of X because 3 is in $\{2,3\}$ but not in X.

section j

False.
$\{2,3\}$ is not in X.

section k

False.
$|X| = 6$

Question 8

Exercise 3.2.4

section b

If $X \in P(A)$, then X is a subset of A, X can be:

$$\begin{aligned}
&\emptyset \\
&\{1\}, \{2\}, \{3\} \\
&\{1,2\}, \{1,3\}, \{2,3\} \\
&\{1,2,3\}
\end{aligned}$$

If $2 \in X$, then X can be $\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}$.

Question 9

(a) Exercise 3.3.1

section c

$$\begin{aligned} A \cap C &= \{-3, 0, 1, 4, 17\} \cap \{x \in \mathbb{Z} : x \text{ is odd}\} \\ &= \{-3, 1, 17\} \end{aligned}$$

section d

$$\begin{aligned} A \cup (B \cap C) &= \{-3, 0, 1, 4, 17\} \cup (\{-12, -5, 1, 4, 6\} \cap \{x \in \mathbb{Z} : x \text{ is odd}\}) \\ &= \{-3, 0, 1, 4, 17\} \cup \{-5, 1\} \\ &= \{-5, -3, 0, 1, 4, 17\} \end{aligned}$$

section e

$$\begin{aligned} A \cap B \cap C &= \{-3, 0, 1, 4, 17\} \cap \{-12, -5, 1, 4, 6\} \cap \{x \in \mathbb{Z} : x \text{ is odd}\} \\ &= \{1, 4\} \cap \{x \in \mathbb{Z} : x \text{ is odd}\} \\ &= \{1\} \end{aligned}$$

(b) Exercise 3.3.3

section a

$$\begin{aligned} \bigcap_{i=2}^{5} A_i &= A_2 \cap A_3 \cap A_4 \cap A_5 \\ &= \{2^0, 2^1, 2^2\} \cap \{3^0, 3^1, 3^2\} \cap \{4^0, 4^1, 4^2\} \cap \{5^0, 5^1, 5^2\} \\ &= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\} \cap \{1, 5, 25\} \\ &= \{1\} \end{aligned}$$

section b

$$\begin{aligned} \bigcup_{i=2}^{5} A_i &= A_2 \cap A_3 \cap A_4 \cap A_5 \\ &= \{2^0, 2^1, 2^2\} \cup \{3^0, 3^1, 3^2\} \cup \{4^0, 4^1, 4^2\} \cup \{5^0, 5^1, 5^2\} \\ &= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\} \\ &= \{1, 2, 3, 4, 5, 9, 16, 25\} \end{aligned}$$

section e

$$\begin{aligned}\bigcap_{i=1}^{100} C_i &= \{x \in \mathbb{R}\} \cap \left\{-\frac{1}{1} \leq x \leq \frac{1}{1}\right\} \cap \left\{-\frac{1}{2} \leq x \leq \frac{1}{2}\right\} \cap \left\{-\frac{1}{3} \leq x \leq \frac{1}{3}\right\} ... \cap \left\{-\frac{1}{100} \leq x \leq \frac{1}{100}\right\} \\ &= \left\{x \in \mathbb{R} : -\frac{1}{100} \leq x \leq \frac{1}{100}\right\}\end{aligned}$$

section f

$$\begin{aligned}\bigcup_{i=1}^{100} C_i &= \{x \in \mathbb{R}\} \cap \left\{-\frac{1}{1} \leq x \leq \frac{1}{1}\right\} \cup \left\{-\frac{1}{2} \leq x \leq \frac{1}{2}\right\} \cup \left\{-\frac{1}{3} \leq x \leq \frac{1}{3}\right\} ... \cup \left\{-\frac{1}{100} \leq x \leq \frac{1}{100}\right\} \\ &= \left\{x \in \mathbb{R} : -\frac{1}{1} \leq x \leq \frac{1}{1}\right\}\end{aligned}$$

(c) Exercise 3.3.4

section b

$$\begin{aligned}A \cup B &= \{a, b\} \cup \{b, c\} \\ &= \{a, b, c\} \\ P(A \cup B) &= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}\end{aligned}$$

section d

$$\begin{aligned}P(A) &= \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \\ P(B) &= \{\emptyset, \{b\}, \{c\}, \{b, c\}\} \\ P(A) \cup P(B) &= \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \cup \{\emptyset, \{b\}, \{c\}, \{b, c\}\} \\ &= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}\end{aligned}$$

Question 10

(a) Exercise 3.5.1

section b

$$\text{(foam, tall, non-fat)}$$

section c

$$\{\text{(foam, non-fat),(foam, whole), (no-foam, non-fat), (no-foam, whole)}\}$$

(b) Exercise 3.5.3

section b

True.

$$\begin{aligned}\mathbb{Z}^2 &= \mathbb{Z} \times \mathbb{Z} = \{(a, b) : a, b \text{ are integers}\} \\ \mathbb{R}^2 &= \mathbb{R} \times \mathbb{R} = \{(a, b) : a, b \text{ are real numbers}\}\end{aligned}$$

If (a, b) is in $\mathbb{Z}^2$, they will be in $\mathbb{R}^2$, so $\mathbb{Z}^2 \subseteq \mathbb{R}^2$.

section c

True.

$$\begin{aligned}\mathbb{Z}^2 &= \mathbb{Z} \times \mathbb{Z} = \{(a, b) : a, b \text{ are integers}\} \\ \mathbb{Z}^3 &= \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z} = \{(a, b, c) : a, b, c \text{ are integers}\}\end{aligned}$$

$$\begin{aligned}\mathbb{Z}^2 \cap \mathbb{Z}^3 &= \{(a, b)\} \cap \{(a, b, c)\} \\ &= \emptyset\end{aligned}$$

section e

True.

$$\begin{aligned}A \times C &= \{(a, c) : a \in A \text{ and } c \in C\} \\ B \times C &= \{(b, c) : b \in B \text{ and } c \in C\}\end{aligned}$$

If $A \subseteq B$, then every object a in A will be an object in B. Therefore, every ordered pair (a, c) in $A \times C$ will be an ordered pair in $B \times C$, which means $A \times C \subseteq B \times C$.

(c) Exercise 3.5.6

section d

$$\begin{aligned}\{0\} \cup \{0\}^2 &= \{0\} \cup \{00\} \\ &= \{0, 00\}\end{aligned}$$

Therefore, $x \in \{0, 00\}$

$$\begin{aligned}\{1\} \cup \{1\}^2 &= \{1\} \cup \{11\} \\ &= \{1, 11\}\end{aligned}$$

Therefore, $y \in \{1, 11\}$

$$xy = \{01, 011, 001, 0011\}$$

section e

$$\begin{aligned}\{a\} \cup \{a\}^2 &= \{a\} \cup \{aa\} \\ &= \{a, aa\}\end{aligned}$$

Therefore, $y \in \{a, aa\}$.

Since $x \in \{aa, ab\}$,

$$xy = \{aaa, aaaa, aba, abaa\}$$

(d) Exercise 3.5.7

section c

$$\begin{aligned}(A \times B) \cup (A \times C) &= (\{a\} \times \{b, c\}) \cup (\{a\} \times \{a, b, d\}) \\ &= \{ab, ac\} \cup \{aa, ab, ad\} \\ &= \{ab, ac, aa, ad\}\end{aligned}$$

section f

$$\begin{aligned}P(A \times B) &= P(\{a\} \times \{b, c\}) \\ &= P(\{ab, ac\}) \\ &= \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}\end{aligned}$$

section g

$$
\begin{aligned}
P(A) \times P(B) &= P(\{a\}) \times P(\{b, c\}) \\
&= \{\emptyset, \{a\}\} \times \{\emptyset, \{b\}, \{c\}, \{b, c\}\} \\
&= \{(\emptyset, \emptyset), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{b, c\}), (\{a\}, \emptyset), (\{a\}, \{b\}), (\{a\}, \{c\}), (\{a\}, \{b, c\})\}
\end{aligned}
$$

Question 11

(a) Exercise 3.6.2

section b

$$(B \cup A) \cap (\overline{B} \cup A) = A$$

Proof:

$$\begin{aligned}
(B \cup A) \cap (\overline{B} \cup A) &= (A \cup B) \cap (A \cup \overline{B}) && \text{Commutative law} \\
&= A \cup (B \cap \overline{B}) && \text{Distributed law} \\
&= A \cup \emptyset && \text{Complement law} \\
&= A && \text{Identity law}
\end{aligned}$$

section c

$$\overline{A \cap \overline{B}} = \overline{A} \cup B$$

Proof:

$$\begin{aligned}
\overline{A \cap \overline{B}} &= \overline{A} \cup \overline{\overline{B}} && \text{De Morgan's law} \\
&= \overline{A} \cup B && \text{Double complement law}
\end{aligned}$$

(b) Exercise 3.6.3

section b

If $A = \{1\}$ and $B = \{1, 2\}$, then $B \cap A = \{1\}$ and $A - (B \cap A) = \emptyset$, which means $A - (B \cap A) \neq A$.

section d

If $A = \{1\}$ and $B = \{1, 2\}$, then $B - A = \{2\}$ and $(B - A) \cup A) = \{1, 2\}$, which means $(B - A) \cup A \neq A$.

(c) Exercise 3.6.4

section b

$$A \cap (B - A) = \emptyset$$

Proof:

$$
\begin{aligned}
A \cap (B - A) &= A \cap (B \cap \overline{A}) && \text{Set subtraction law} \\
&= (B \cap \overline{A}) \cap A && \text{Commutative law} \\
&= B \cap (\overline{A} \cap A) && \text{Asscociative law} \\
&= B \cap \emptyset && \text{Complement law} \\
&= \emptyset && \text{Domination law}
\end{aligned}
$$

section c

$$A \cup (B - A) = A \cup B$$

Proof:

$$
\begin{aligned}
A \cup (B - A) &= A \cup (B \cap \overline{A}) && \text{Set subtraction law} \\
&= (A \cup B) \cap (A \cup \overline{A}) && \text{Distributed law} \\
&= (A \cup B) \cap U && \text{Complement law} \\
&= A \cup B && \text{Identity law}
\end{aligned}
$$