Practice Exam 1

Tables Provided on Exam 1

Table 1.5.1: Laws of propositional logic.

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee F \equiv p$	$p \wedge T \equiv p$
Domination laws:	$p \wedge F \equiv F$	$p \vee T \equiv T$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv F$ $\neg T \equiv F$	$p \vee \neg p \equiv T$ $\neg F \equiv T$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Table 1.12.1: Rules of inference known to be valid arguments.

Rule of inference	Name
p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification

Rule of inference	Name
p q $\therefore p \wedge q$	Conjunction
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Table 1.13.1: Rules of inference for quantified stateme

Rule of Inference	Name
c is an element (arbitrary or particular) ∀x P(x) ∴ P(c)	Universal instantiation
c is an arbitrary element P(c) ∴ ∀x P(x)	Universal generalization
∃x P(x) ∴ (c is a particular element) ∧ P(c)	Existential instantiation*
c is an element (arbitrary or particular) P(c) ∴ ∃x P(x)	Existential generalization

Table 3.6.1: Set identities.

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double Complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

Set Theory

Given:

$A = \{ 1, \{2\}, \{\{3, 4\}\} \}$

For each of the following statements, state whether they are **true** or **false.**

true a. $1 \in A$

false b. $1 \subseteq A$

true c. $\{2\} \in A$

false d. $\{2\} \subseteq A$

false e. $\{3, 4\} \in A$

false f. $\{3, 4\} \subseteq A$

true g. $\{\{3,4\}\} \in A$

false h. $\{\{3,4\}\} \subseteq A$

false i. $\emptyset \in A$

true j. $\emptyset \subseteq A$

1.2 Let $A = \{1, 2, 3, 4\}$. Select the statement that is **false**.

a. $\emptyset \in P(A)$

b. $\emptyset \subseteq P(A)$

false c. $\{2, 3\} \in A$

d. $\{2, 3\} \subseteq A$

Functions

Choose the property for which the function satisfies if well defined.

a. Neither one-to-one, nor onto

b. One-to-one, but not onto

c. Onto, but not one-to-one

d. one-to-one and onto

e. not well defined

Given a function whose domain is the **set of all integers and whose target is the set of all positive integers:**

a)

$$f(x) = 2x + 1$$

e

b)

$$f(x) = |x| + 1$$

c

c)

$$f(x) = x\hat{}2 + 1$$

a

d)

$$f(x) = \{(x > 0 : 2x + 1) \land (x <= 0 : -2x)\}$$

e

e)

$$f(x) = \{(x >= 0 : 2x + 1) \land (x < 0 : -2x + 2)\}$$

b

Proofs

4.1 Direct Proof

Prove that the product of two odd integers is an odd integer.

Define two odd integers as 2k+1 and 2j+1, where k and j are both integers.
we have:
(2k+1)*(2j+1)=4kj+2k+2j+1=2(2kj+k+j)+1, since k and j are both integers, 2kj+k+j is also an integer.
Let m = 2kj+k+j, (2k+1)*(2j+1)=2m+1.

4.2 Proof by Contrapositive

Prove that if n^2 is even, then n is even.

Consider the contraposition of the proposition which is ~q→~p. Show that if n is odd, then n^2 is odd.

Suppose n is odd, we shall prove n^2 is odd.
Since n is odd, n=2k+1 for some integer k.
n^2 = (2k+1)^2 = 4k^2+4k+1=2(2k^2+2k)+1
Since k is an integer, 2k^2+2k is also an integer, define m = 2k^2+2k;
n^2 = 2m + 1, so n^2 is odd;

4.2 Proof by Contradiction

Prove by contradiction that if 3n+5 is odd, then n is even.

suppose 3n+5 is odd and n is also odd, we have n = 2k+1 for some k,
3n+5 = 3(2k+1)+5=6k+6=2(3k+3).
Since k is an integer, 3k+3 is an integer. 3n+5 = 2(3k+3) is an even integer.
This contradicts to the hypothesis that 3n+5 is odd.
So if 3n+5 is odd, then n is even.

Number Systems Conversion

5.1 Decimal to 8-bit Two's Complement

$(-43)_{10}$ = (11010101)2

5.2 Binary to Hexadecimal

$(110011100)_2$ = 19C

(1100)2 = 12 = C

(1001)2 = 9

(0001)2 = 1

Coding

```
    a
   b  b
  c    c
 d      d
e        e
 d      d
  c    c
   b  b
    a
```

Make a hollowed-out diamond made up of in-order alphabet letters. An example is below when `n=5`.

Challenge Question

For this question, you have to create the following Barn Door shape with `n=10`:

```
##########
#$      $#
# $    $ #
#  $  $  #
#   $$   #
#   $$   #
#  $  $  #
# $    $ #
#$      $#
##########
```