

Homework 1

Student: Shichen Zhang (sz4968)

Question 1**A.****1.**

$$\begin{aligned}10011011_2 &= 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\&= 128 + 0 + 0 + 16 + 8 + 0 + 2 + 1 \\&= 155\end{aligned}$$

2.

$$\begin{aligned}456_7 &= 4 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 \\&= 196 + 35 + 6 \\&= 237\end{aligned}$$

3.

$$\begin{aligned}38A_{16} &= 3 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 \\&= 768 + 128 + 10 \\&= 906\end{aligned}$$

4.

$$\begin{aligned}2214_5 &= 2 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 4 \times 5^0 \\&= 250 + 50 + 5 + 4 \\&= 309\end{aligned}$$

B.

1.

$$\begin{aligned}69 \div 2 &= 34 && \text{mod } 1 \\34 \div 2 &= 17 && \text{mod } 0 \\17 \div 2 &= 8 && \text{mod } 1 \\8 \div 2 &= 4 && \text{mod } 0 \\4 \div 2 &= 2 && \text{mod } 0 \\2 \div 2 &= 1 && \text{mod } 0 \\1 \div 2 &= 0 && \text{mod } 1\end{aligned}$$

$$69_{10} = 1000101_2$$

2.

$$\begin{aligned}485 \div 2 &= 242 && \text{mod } 1 \\242 \div 2 &= 121 && \text{mod } 0 \\121 \div 2 &= 60 && \text{mod } 1 \\60 \div 2 &= 30 && \text{mod } 0 \\30 \div 2 &= 15 && \text{mod } 0 \\15 \div 2 &= 7 && \text{mod } 1 \\7 \div 2 &= 3 && \text{mod } 1 \\3 \div 2 &= 1 && \text{mod } 1 \\1 \div 2 &= 0 && \text{mod } 1\end{aligned}$$

$$485_{10} = 111100101_2$$

3.

Each hexadecimal digit can be translated into its 4-bit binary equivalent:

$$\begin{aligned}6_{16} &= 0110_2 \\D_{16} &= 13_{10} = 1101_2 \\1_{16} &= 0001_2 \\A_{16} &= 10_{10} = 1010_2\end{aligned}$$

Therefore,

$$6D1A_{16} = 0110110100011010_2$$

C.

1.

$$1101011_2 = 01101011_2$$

Each 4-bit binary can be translated into its hexadecimal digit equivalent:

$$0110_2 = 6_{16}$$

$$1011_2 = B_{16}$$

Therefore,

$$1101011_2 = 6B_{16}$$

2.

$$895 \div 16 = 55 \quad \text{mod } 15$$

$$55 \div 16 = 3 \quad \text{mod } 7$$

$$3 \div 16 = 0 \quad \text{mod } 3$$

$$15_{10} = F_{16}$$

$$895_{10} = 37F_{16}$$

Question 2

1.

$$\begin{array}{r} 1111 \\ 7566_8 \\ + 4515_8 \\ \hline 14303_8 \end{array}$$

$$7566_8 + 4515_8 = 14303_8$$

2.

$$\begin{array}{r} 11111 \\ 1011011_2 \\ + 00001101_2 \\ \hline 1100000_2 \end{array}$$

$$10110011_2 + 1101_2 = 11000000_2$$

3.

$$\begin{array}{r} 11 \\ 7A66_{16} \\ + 45C5_{16} \\ \hline C02B_{16} \end{array}$$

$$7A66_{16} + 45C5_{16} = C02B_{16}$$

4.

$$\begin{array}{r} 241 \\ 3022_5 \\ - 2433_5 \\ \hline 34_5 \end{array}$$

$$3022_5 - 2433_5 = 34_5$$

Question 3

A.

1.

$$\begin{array}{rcl} 124 \div 2 = 62 & \text{mod } 0 \\ 62 \div 2 = 31 & \text{mod } 0 \\ 31 \div 2 = 15 & \text{mod } 1 \\ 15 \div 2 = 7 & \text{mod } 1 \\ 7 \div 2 = 3 & \text{mod } 1 \\ 3 \div 2 = 1 & \text{mod } 1 \\ 1 \div 2 = 0 & \text{mod } 1 \end{array}$$

$$124_{10} = 01111100_8 \text{ bit 2's comp}$$

2.

Convert 124 to its 8 bits binary expression,

$$\begin{array}{rcl} 124 \div 2 = 62 & \text{mod } 0 \\ 62 \div 2 = 31 & \text{mod } 0 \\ 31 \div 2 = 15 & \text{mod } 1 \\ 15 \div 2 = 7 & \text{mod } 1 \\ 7 \div 2 = 3 & \text{mod } 1 \\ 3 \div 2 = 1 & \text{mod } 1 \\ 1 \div 2 = 0 & \text{mod } 1 \end{array}$$

$$124_{10} = 01111100_2$$

Invert the bits,

$$10000011$$

Add 1,

$$10000011 + 00000001 = 10000100$$

Therefore,

$$-124_{10} = 10000100_8 \text{ bit 2's comp}$$

3.

$$\begin{aligned}
109 \div 2 &= 54 \quad \text{mod } 1 \\
54 \div 2 &= 27 \quad \text{mod } 0 \\
27 \div 2 &= 13 \quad \text{mod } 1 \\
13 \div 2 &= 6 \quad \text{mod } 1 \\
6 \div 2 &= 3 \quad \text{mod } 0 \\
3 \div 2 &= 1 \quad \text{mod } 1 \\
1 \div 2 &= 0 \quad \text{mod } 1
\end{aligned}$$

$$109_{10} = 01101101_8 \text{ bit 2's comp}$$

4.

Convert 79 to its 8 bits binary expression,

$$\begin{aligned}
79 \div 2 &= 39 \quad \text{mod } 1 \\
39 \div 2 &= 19 \quad \text{mod } 1 \\
19 \div 2 &= 9 \quad \text{mod } 1 \\
9 \div 2 &= 4 \quad \text{mod } 1 \\
4 \div 2 &= 2 \quad \text{mod } 0 \\
2 \div 2 &= 1 \quad \text{mod } 0 \\
1 \div 2 &= 0 \quad \text{mod } 1
\end{aligned}$$

$$79_{10} = 01001111_2$$

Invert the bits,

$$10110000$$

Add 1,

$$10110000 + 00000001 = 10110001$$

Therefore,

$$-79_{10} = 10110001_8 \text{ bit 2's comp}$$

B

1.

$$\begin{aligned}
00011110_8 \text{ bit 2's comp} &= -0 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
&= -0 + 0 + 0 + 16 + 8 + 4 + 2 + 0 \\
&= 30
\end{aligned}$$

2.

$$\begin{aligned}
11100110_8 \text{ bit 2's comp} &= -1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
&= -128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 \\
&= -26
\end{aligned}$$

3.

$$\begin{aligned}00101101_8 \text{ bit } 2's \text{ comp} &= -0 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\&= -0 + 0 + 32 + 0 + 8 + 4 + 0 + 1 \\&= 45\end{aligned}$$

4.

$$\begin{aligned}10011110_8 \text{ bit } 2's \text{ comp} &= -1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\&= -128 + 0 + 0 + 16 + 8 + 4 + 2 + 0 \\&= -98\end{aligned}$$

Question 4

1. Exercise 1.2.4

section b

| p | q | $p \vee q$ | $\neg(p \vee q)$ |
|-----|-----|------------|------------------|
| T | T | T | F |
| T | F | T | F |
| F | T | T | F |
| F | F | F | T |

section c

| p | q | r | $p \wedge \neg q$ | $r \vee (p \wedge \neg q)$ |
|-----|-----|-----|-------------------|----------------------------|
| T | T | T | F | T |
| T | T | F | F | F |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

2. Exercise 1.3.4

section b

| p | q | $p \rightarrow q$ | $q \rightarrow p$ | $(p \rightarrow q) \rightarrow (q \rightarrow p)$ |
|-----|-----|-------------------|-------------------|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

section d

| p | q | $p \leftrightarrow q$ | $p \leftrightarrow \neg q$ | $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$ |
|-----|-----|-----------------------|----------------------------|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | T | F | T |

Question 5

1. Exercise 1.2.7

section b

$$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$$

section c

$$B \vee (D \wedge M)$$

2. Exercise 1.3.7

section b

$$(s \vee y) \rightarrow p$$

section c

$$p \rightarrow y$$

section d

$$p \leftrightarrow (s \wedge y)$$

section e

$$p \rightarrow (s \vee y)$$

3. Exercise 1.3.9

section c

$$c \rightarrow p$$

section d

$$c \rightarrow p$$

Question 6

1. Exercise 1.3.6

section b

If Joe is eligible for the honors program, then he has maintained a B average.

section c

If Rajiv can go on the roller coaster, then he is at least four feet tall.

section d

If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10

section c

| p | q | r | $p \vee r$ | $q \wedge r$ | $(p \vee r) \leftrightarrow (q \wedge r)$ |
|-----|-----|-----|------------|--------------|---|
| T | F | T | T | F | F |
| T | F | F | T | F | F |

According to the truth table, the truth value of the expression $(p \vee r) \leftrightarrow (q \wedge r)$ is false.

section d

| p | q | r | $p \wedge r$ | $q \wedge r$ | $(p \wedge r) \leftrightarrow (q \wedge r)$ |
|-----|-----|-----|--------------|--------------|---|
| T | F | T | T | F | F |
| T | F | F | F | F | T |

According to the truth table, the truth value of the expression $(p \wedge r) \leftrightarrow (q \wedge r)$ is unknown.

section e

| p | q | r | $r \vee q$ | $p \rightarrow (r \vee q)$ |
|-----|-----|-----|------------|----------------------------|
| T | F | T | T | T |
| T | F | F | F | F |

According to the truth table, the truth value of the expression $p \rightarrow (r \vee q)$ is unknown.

section f

| p | q | r | $p \wedge q$ | $(p \wedge q) \rightarrow r$ |
|-----|-----|-----|--------------|------------------------------|
| T | F | T | F | T |
| T | F | F | F | T |

According to the truth table, the truth value of the expression $(p \wedge q) \rightarrow r$ is true.

Question 7

Exercise 1.4.5

section b

1st sentence can be expressed in symbolic logic as,

$$\neg j \rightarrow (l \vee \neg r)$$

2nd sentence can be expressed in symbolic logic as,

$$(r \wedge \neg l) \rightarrow j$$

Proof:

| j | l | r | $\neg j \rightarrow (l \vee \neg r)$ | $(r \wedge \neg l) \rightarrow j$ |
|-----|-----|-----|--------------------------------------|-----------------------------------|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | T | T |
| F | T | T | T | T |
| F | T | F | T | T |
| F | F | T | F | F |
| F | F | F | T | T |

According to the truth table, the two expressions have the same truth value in all situations, therefore they are logically equivalent.

section c

1st sentence can be expressed in symbolic logic as,

$$j \rightarrow \neg l$$

2nd sentence can be expressed in symbolic logic as,

$$\neg j \rightarrow l$$

Proof:

| j | l | $j \rightarrow \neg l$ | $\neg j \rightarrow l$ |
|-----|-----|------------------------|------------------------|
| T | T | F | T |
| T | F | T | T |
| F | T | T | T |
| F | F | T | F |

According to the truth table, the two expressions have different truth values, therefore they are not logically equivalent.

section d

1st sentence can be expressed in logic as,

$$(r \vee \neg l) \rightarrow j$$

2nd sentence can be expressed in logic as,

$$j \rightarrow (r \wedge \neg l)$$

Proof:

| j | l | r | $(r \vee \neg l) \rightarrow j$ | $j \rightarrow (r \wedge \neg l)$ |
|-----|-----|-----|---------------------------------|-----------------------------------|
| T | T | T | T | F |
| T | T | F | T | F |
| T | F | T | T | T |
| T | F | F | T | F |
| F | T | T | F | T |
| F | T | F | T | T |
| F | F | T | F | T |
| F | F | F | F | T |

According to the truth table, the two expressions have different truth values, therefore they are not logically equivalent.

Question 8

1. Exercise 1.5.2

section c

$$\begin{aligned}(p \rightarrow q) \wedge (p \rightarrow r) &\equiv (\neg p \vee q) \wedge (p \rightarrow r) && \text{(Conditional identity)} \\ &\equiv (\neg p \vee q) \wedge (\neg p \vee r) && \text{(Conditional identity)} \\ &\equiv \neg p \wedge (q \vee r) && \text{(Distributive law)} \\ &\equiv p \rightarrow (q \vee r) && \text{(Conditional identity)}\end{aligned}$$

section f

$$\begin{aligned}\neg(p \vee (\neg p \wedge q)) &\equiv \neg((p \vee \neg p) \wedge (p \vee q)) && \text{(Distributive law)} \\ &\equiv \neg(T \wedge (p \vee q)) && \text{(Complement law)} \\ &\equiv \neg(p \vee q) && \text{(Identity law)} \\ &\equiv \neg p \wedge \neg q && \text{(De Morgan's law)}\end{aligned}$$

section i

$$\begin{aligned}(p \wedge q) \rightarrow r &\equiv \neg(p \wedge q) \vee r && \text{(Conditional identity)} \\ &\equiv (\neg p \vee \neg q) \vee r && \text{(De Morgan's law)} \\ &\equiv r \vee (\neg p \vee \neg q) && \text{(Commutative law)} \\ &\equiv (r \vee \neg p) \vee \neg q && \text{(Associative law)} \\ &\equiv (\neg p \vee r) \vee \neg q && \text{(Commutative law)} \\ &\equiv (\neg p \vee \neg \neg r) \vee \neg q && \text{(Double negation law)} \\ &\equiv \neg(p \wedge \neg r) \vee \neg q && \text{(De Morgan's law)} \\ &\equiv (p \wedge \neg r) \rightarrow \neg q && \text{(Conditional identity)}\end{aligned}$$

2. Exercise 1.5.3

section c

$$\begin{aligned}\neg r \vee (\neg r \rightarrow p) &\equiv \neg r \vee (\neg \neg r \vee p) && \text{(Conditional identity)} \\ &\equiv \neg r \vee (r \vee p) && \text{(Double negation law)} \\ &\equiv (\neg r \vee r) \vee p && \text{(Associative law)} \\ &\equiv T \vee p && \text{(Complement law)} \\ &\equiv T && \text{(Domination law)}\end{aligned}$$

section d

| | | |
|--|--|------------------------|
| $\neg(p \rightarrow q) \rightarrow \neg q$ | $\equiv \neg\neg(p \rightarrow q) \vee \neg q$ | (Conditional identity) |
| | $\equiv (p \rightarrow q) \vee \neg q$ | (Double negation law) |
| | $\equiv (\neg p \vee q) \vee \neg q$ | (Conditional identity) |
| | $\equiv \neg p \vee (q \vee \neg q)$ | (Associative law) |
| | $\equiv \neg p \vee T$ | (Complement law) |
| | $\equiv T$ | (Domination law) |

Question 9

1. Exercise 1.6.3

section c

$$\exists x (x = x^2)$$

section d

$$\forall x (x \leq x^2 + 1)$$

2. Exercise 1.7.4

section b

$$\forall x (\neg S(x) \wedge W(x))$$

section c

$$\forall x (S(x) \rightarrow \neg W(x))$$

section d

$$\exists x (S(x) \wedge W(x))$$

Question 10

1. Exercise 1.7.9

section c

True.

This expression means that there exists an x such that either $x = c$ is false or $P(x)$ is true. According to the truth table, if $x = a$, $x = c$ is false, so the expression is true.

section d

True.

According to the truth table, if $x = e$, then $Q(x)$ is true and $R(x)$ is true. It follows $Q(x) \wedge R(x)$ is true. Therefore, there exists an x such that $Q(x) \wedge R(x)$ is true.

section e

True.

According to the truth table, $Q(a)$ is true and $P(d)$ is true. Therefore, $Q(a) \wedge P(d)$ is true.

section f

True.

According to the truth table, for all x , if $x \neq b$, $Q(x)$ is always true.

section g

False.

According to the truth table, if $x = c$, $P(x)$ is false and $R(x)$ is false. It follows $P(x) \vee R(x)$ is false. Therefore, it is false that for all x , $P(x) \vee R(x)$ is true.

section h

True.

According to the truth table, for all x , either $P(x)$ is true or $R(x)$ is false. If $P(x)$ is true, then $R(x) \rightarrow P(x)$ is true. If $R(x)$ is false, then $R(x) \rightarrow P(x)$ is also true. Therefore, for all x , $R(x) \rightarrow P(x)$ is always true.

section i

True.

According to the truth table, when $x = a$, $Q(x)$ is true and $R(x)$ is false. It follows $Q(x) \vee R(x)$ is true. Therefore, there exists an x such that $Q(x) \vee R(x)$ is true.

2. Exercise 1.9.2

section b

True.

According to the truth table, when $x = 2$, $Q(x, y)$ is true for all y .

section c

True.

According to the truth table, when $y = 1$, $P(x, y)$ is true for all x .

section d

False.

According to the truth table, $S(x, y)$ is always false. Therefore, there does not exist an x and a y such that $S(x, y)$ is true.

section e

False.

According to the truth table, when $x = 1$, there does not exist a y such that $Q(x, y)$ is true. Therefore, it false that, for all x , there exist a y such that $Q(x, y)$ is true.

section f

True.

According to the truth table, for all x , there exists a y such that $P(x, y)$ is true. When $x = 1$, if $y = 1$, then $P(x, y)$ is true. When $x = 2$, if $y = 1$, then $P(x, y)$ is true. When $x = 3$, if $y = 1$, then $P(x, y)$ is true.

section g

False.

According to the truth table, when $x = 1$ and $y = 2$, $P(x, y)$ is false. So, it is false that, for all x and y , $P(x, y)$ is true.

section h

True.

According to the truth table, when $x = 2$ and $y = 1$, $Q(x, y)$ is true. So, there exists an x and a y such that $Q(x, y)$ is true.

section i

True.

According to the truth table, for all x and y , $S(x, y)$ is always false. Therefore, $\neg Q(x, y)$ is always true.

Question 11

1. Exercise 1.10.4

section c

$$\exists x \exists y (x + y = xy)$$

section d

$$\forall x \forall y (((x > 0) \wedge (y > 0)) \rightarrow \left(\frac{x}{y} > 0\right))$$

section e

$$\forall x (((x > 0) \wedge (x < 1)) \rightarrow \left(\frac{1}{x} > 1\right))$$

section f

$$\forall x \exists y (y < x)$$

section g

$$\forall x \exists y ((x \neq 0) \rightarrow \left(y = \frac{1}{x}\right))$$

2. Exercise 1.10.7

section c

$$\exists x (N(x) \wedge D(x))$$

section d

$$\forall y (D(y) \rightarrow P(\text{Sam}, y))$$

section e

$$\exists x \forall y (N(x) \wedge P(x, y))$$

section f

$$\exists x \forall y ((N(x) \wedge D(x)) \wedge ((N(y) \wedge (y \neq x)) \rightarrow \neg D(y)))$$

3. Exercise 1.10.10

section c

$$\forall x \exists y (T(x, y) \wedge (y \neq \text{Math 101}))$$

section d

$$\exists x \forall y ((y \neq \text{Math 101}) \rightarrow T(x, y))$$

section e

$$\forall x \exists y_1 \exists y_2 ((x \neq \text{Sam}) \rightarrow (T(x, y_1) \wedge T(x, y_2) \wedge (y_1 \neq y_2)))$$

section f

$$\exists y_1 \exists y_2 \forall y_3 (T(\text{Sam}, y_1) \wedge T(\text{Sam}, y_2) \wedge (y_1 \neq y_2) \wedge (y_3 \neq y_1) \wedge (y_3 \neq y_2)) \rightarrow \neg T(\text{Sam}, y_3))$$

Question 12

1. Exercise 1.8.2

section b

Statement:

$$\forall x (D(x) \vee P(x))$$

Negation:

$$\neg \forall x (D(x) \vee P(x))$$

Applying De Morgan's law:

$$\exists x \neg(D(x) \vee P(x))$$

Applying De Morgan's law:

$$\exists x (\neg D(x) \wedge \neg P(x))$$

English: there is a patient who was neither given the medication nor the placebo.

section c

Statement:

$$\exists x (D(x) \wedge M(x))$$

Negation:

$$\neg \exists x (D(x) \wedge M(x))$$

Applying De Morgan's law:

$$\forall x \neg(D(x) \wedge M(x))$$

Applying De Morgan's law:

$$\forall x (\neg D(x) \vee \neg M(x))$$

English: every patient was not given the medication or did not have migraines or both.

section d

Statement:

$$\forall x (P(x) \rightarrow M(x))$$

Negation:

$$\neg \forall x (P(x) \rightarrow M(x))$$

Applying De Morgan's law:

$$\exists x \neg(P(x) \rightarrow M(x))$$

Applying conditional identity:

$$\exists x \neg(\neg P(x) \vee M(x))$$

Applying De Morgan's law:

$$\exists x (\neg \neg P(x) \wedge \neg M(x))$$

Applying double negation law:

$$\exists x (P(x) \wedge \neg M(x))$$

English: there is a patient who was given the placebo and did not have migraines.

section e

Statement:

$$\exists x(M(x) \wedge P(x))$$

Negation:

$$\neg \exists x (M(x) \wedge P(x))$$

Applying De Morgan's law:

$$\forall x \neg(M(x) \wedge P(x))$$

Applying De Morgan's law:

$$\forall x (\neg M(x) \vee \neg P(x))$$

English: every patient did not have migraines or was not given not have placebo.

2. Exercise 1.9.4

section c

$$\begin{aligned}\neg \exists x \forall y (P(x, y) \rightarrow Q(x, y)) &\equiv \forall x \neg \forall y (P(x, y) \rightarrow Q(x, y)) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y \neg (P(x, y) \rightarrow Q(x, y)) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y \neg (\neg P(x, y) \vee Q(x, y)) && \text{(Conditional identity)} \\ &\equiv \forall x \exists y (\neg \neg P(x, y) \wedge \neg Q(x, y)) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y (P(x, y) \wedge \neg Q(x, y)) && \text{(Double negation law)}\end{aligned}$$

section d

$$\begin{aligned}\neg \exists x \forall y (P(x, y) \leftrightarrow P(y, x)) &\equiv \forall x \neg \forall y (P(x, y) \leftrightarrow P(y, x)) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y \neg (P(x, y) \leftrightarrow P(y, x)) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y \neg ((P(x, y) \rightarrow P(y, x)) \wedge (P(y, x) \rightarrow P(x, y))) && \text{(Conditional identity)} \\ &\equiv \forall x \exists y (\neg (P(x, y) \rightarrow P(y, x)) \vee \neg (P(y, x) \rightarrow P(x, y))) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y (\neg (\neg P(x, y) \vee P(y, x)) \vee \neg (\neg P(y, x) \vee P(x, y))) && \text{(Conditional identity)} \\ &\equiv \forall x \exists y ((\neg \neg P(x, y) \wedge \neg P(y, x)) \vee (\neg \neg P(y, x) \wedge \neg P(x, y))) && \text{(De Morgan's law)} \\ &\equiv \forall x \exists y ((P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge \neg P(x, y))) && \text{(Double negation law)}\end{aligned}$$

section e

$$\begin{aligned}\neg (\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)) &\equiv \neg \exists x \exists y P(x, y) \vee \neg \forall x \forall y Q(x, y) && \text{(De Morgan's law)} \\ &\equiv \forall x \neg \exists y P(x, y) \vee \exists x \neg \forall y Q(x, y) && \text{(De Morgan's law)} \\ &\equiv \forall x \forall y \neg P(x, y) \vee \exists x \exists y \neg Q(x, y) && \text{(De Morgan's law)}\end{aligned}$$