Get the horizontal ellipse from an ellipse in arbitrary direction

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- Link: https://github.com/shidafu/ViewConeCalibration.git
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- Algorithom:

Get [*HEllipse*] [*Rotate*] and [*Shift*] From [*Ellipse*]:

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \cdot [Ellipse] \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} x & y & 1 \end{bmatrix} \cdot [Rotate]^{\mathrm{T}} \cdot [Shift]^{\mathrm{T}} \cdot [HEllipse] \cdot [Shift] \cdot [Rotate] \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Where:

$$[Ellipse] = \begin{bmatrix} k/a^2 & 0 & 0 \\ 0 & k/b^2 & 0 \\ 0 & 0 & -k \end{bmatrix}$$

$$[Rotate] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[Shift] = \begin{bmatrix} 1 & 0 & -x_s \\ 0 & 1 & -y_s \\ 0 & 0 & 1 \end{bmatrix}$$

- Inputs:

```
Ellipse----3 by 3 real symmetric matrix;
```

- Outputs:

```
HEllipse----3 by 3 diagonal matrix;
            [k/(a^2),0,0;
             0,k/(b^2),0;
```

```
                    0,0,k];
            Rotate----3 by 3 rotation matrix;
                        [cos(phi),sin(phi),0;
                         -sin(phi),cos(phi),0;
                         0,0,1];
            Shift----3 by 3 shift matrix;
                        [0,0,-xs;
                         0,0,-xs;
                         0,0,1];
            a,b,c,e,phi----double value of ellipse para;
            Center----2 by 1 array,Center point of the source ellipse;
            Focus----2 by 1 array,Focus point of the source ellipse;
            Peaks1----2 by 1 array,Peak points in long axis of the source ellipse;
            Peaks2----2 by 1 array,Peak points in short axis of the source ellipse;

function
 [HEllipse,Rotate,Shift,a,b,c,e,phi,Center,Focus,Peaks1,Peaks2] =
 GetHorizontalEllipse(Ellipse)

% Initial
[hE, wE]=size(Ellipse);
if ~(hE==3 && wE==3)
    error('Input matrix size error!');
end
if ~det(Ellipse)>0 || Ellipse~=Ellipse
    error('Input matrix is not real symmetric matrix!');
end
% Algorithm
% Find a rotate matrix [Rotate]:
%                 [cos(phi) sin(phi) 0
%      [Rotate] = -sin(phi) cos(phi) 0
%                    0        0     1]
% Fit the equation:
%       [x]'*[Ellipse]*[x]=[x]'*[Rotate]'*[D]*[Rotate]*[x]=0
% Thus: [Ellipse]=[Rotate]'*[D]*[Rotate]
%       [D]=[Rotate]*[Ellipse]*[Rotate]'
% Find a rotate matrix [Rotate] to make[D]:
%       [D](1,2) and [D](2,1)=0
% By solving orthogonal eigenvectors of[Ellipse](1:2,1:2):
%       [Ellipse(1:2,1:2)]*[Rotate]' = [Rotate]'*[D]
[V,D] = eig(Ellipse(1:2,1:2));
Rotate=orth(V)';
if Rotate(1,2)*Rotate(2,1)>0 || Rotate(1,1)*Rotate(2,2)<0
    Rotate=Rotate(:,2:-1:1);
    D=[D(2,2) 0;0 D(1,1)];
end
Rotate=[Rotate [0;0];0 0 1];
% Fix rotation 90
R90=[1 0 0;...
     0 1 0 ;...
     0 0 1];
if(a<b)
    R90=[cos(pi/2)  sin(pi/2) 0;...
        -sin(pi/2)  cos(pi/2) 0 ;...
```

```
          0          0       1];
else
end
Rotate=Rotate*R90;
D=Rotate*Ellipse*Rotate';
% Find a shift matrix [Shift] and [HEllipse]
% Since:
%     clc
%     syms x0 y0 real;
%     S=[1 0 -x0;0 1 -y0;0 0 1];
%     syms d11 d13 d22 d23 d33 real;
%     D=[d11 0 d13;...
%        0 d22 d23;...
%        d13 d23 d33];
%     syms l1 l2 l3 real;
%     HEllipse=[l1 0 0;...
%               0 l2 0;...
%               0 0 l3];
%     D=S'*L*S=
%     [     l1,      0,                  -l1*x0]
%     [      0,     l2,                  -l2*y0]
%     [ -l1*x0, -l2*y0, l1*x0^2 + l2*y0^2 + l3]
% Thus:
%     HEllipse(1,1)=D(1,1)
%     HEllipse(2,2)=D(2,2)
%     HEllipse(3,3)=D(3,3)-D(1,3)*D(1,3)/D(1,1)-D(2,3)*D(2,3)/D(2,2);
HEllipse=D;
HEllipse(3,3)=HEllipse(3,3)-HEllipse(1,3)*HEllipse(1,3)/
HEllipse(1,1)-...
             HEllipse(2,3)*HEllipse(2,3)/HEllipse(2,2);
HEllipse(1,3)=0;
HEllipse(2,3)=0;
HEllipse(3,1)=0;
HEllipse(3,2)=0;
HEllipse=HEllipse./-HEllipse(3,3);
a=sqrt(1/HEllipse(1,1));
b=sqrt(1/HEllipse(2,2));

xs=-D(1,3)/D(1,1);
ys=-D(2,3)/D(2,2);
x0y0=Rotate'*[xs;ys;1];
Center=x0y0(1:2,1);
Shift=[1 0 -xs;0 1 -ys;0 0 1];

phi=acossin([Rotate(1,1) Rotate(1,2)]);
c=sqrt((a^2-b^2));
e=c/(a);

% Get Focus
Focus=[c*cos(theta),-c*cos(theta);c*sin(theta),-c*sin(theta)];
Focus(:,1)=Focus(:,1)+Center;
Focus(:,2)=Focus(:,2)+Center;
% Get Peaks1,Peaks2
Peaks1=[a*cos(theta),-a*cos(theta);a*sin(theta),-a*sin(theta)];
```

```
Peaks1(:,1)=Peaks1(:,1)+Center;
Peaks1(:,2)=Peaks1(:,2)+Center;
Peaks2=[b*cos(theta+pi/2),-b*cos(theta+pi/2);b*sin(theta+pi/2),-
b*sin(theta+pi/2)];
Peaks2(:,1)=Peaks2(:,1)+Center;
Peaks2(:,2)=Peaks2(:,2)+Center;
```

Error using GetHorizontalEllipse (line 70)
Not enough input arguments.

Get the angle from cos and sin

- Inputs:

```
R---[cos(theta) sin(theta)]
```

- Outputs:

```
phi---Between [-pi pi)
```

```
function [phi] = acossin(R)
if R(1,1)>0 %cos(theta)
    if R(1,2)>0 %sin(theta) 1st quad
        if(R(1,1)>R(1,2)) %<45'
            phi=asin(R(1,2));
        else%>45'
            phi=acos(R(1,1));
        end
    else %4th quad
        if(R(1,1)>-R(1,2)) %>-45'
            phi=asin(R(1,2));
        else%<-45'
            phi=-acos(R(1,1));
        end
    end
else
    if R(1,2)>0 %sin(theta) 2nd quad
        if(-R(1,1)>R(1,2)) %>135'
            phi=pi-asin(R(1,2));
        else%<135'
            phi=acos(R(1,1));
        end
    else %3rd quad
        if(-R(1,1)>-R(1,2)) %<-135'
            phi=-pi-asin(R(1,2));
        else%<135'
            phi=-acos(R(1,1));
        end
    end
end
```

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