DT-MRI Interpolation: at What Level?

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Abstract—**This paper studies two levels of interpolation techniques for improving spatial resolution of diffusion tensor magnetic resonance imaging (DTI): a) at the level of scalar gray-level images using anisotropic interpolation and b) at the level of diffusion tensor fields using Log-Euclidean interpolation and quaternion interpolation. The performance of these different level interpolations is evaluated both qualitatively and quantitatively using criteria such as fractional anisotropy (FA), mean diffusivity (MD), fiber length, etc. We conclude that tensor field level interpolations avoids undesirable swelling effect in DTI, which is not the case with scalar gray-level interpolation, and that scalar gray-level image interpolation and Log-Euclidean tensor field interpolation suffer from decreases in FA and MD, which is however avoided by the quaternion tensor field interpolation.**

Keywords-Diffusion tensor imaging; magnetic resonance imaging; interpolation; diffusion weighted image; tensor field interpolation; fiber tractography.

I. INTRODUCTION

Diffusion tensor magnetic resonance imaging (DTI) has gained considerable attention and importance in recent years in the field of medical imaging and clinic researches. However, due to technical limitations of MRI machines, DTI is sensitive to the difficult compromise between spatial resolution and noise or artifacts, and the data is often acquired with a low resolution. To improve DTI data resolution, interpolation provides an interesting software solution.

Although there exists only little works on interpolation of diffusion weighted (DW) gray-level images, such interpolation still remains basically an issue of scalar image interpolation. In this sense, this is a classical image processing problem for which a variety of methods exist [1]. Based on these methods, anisotropic interpolation was proposed for diffusion tensor images [2]. In contrast to scalar image interpolation, the interpolation of diffusion tensor fields is a relatively new problem in the field of image processing; the available techniques are anisotropic interpolation [3], PDE interpolation [4], tensor spline interpolation [5], Cholesky factorization-based interpolation [6], geodesic and rotational interpolations [7, 8], Log-Euclidean interpolation [9], geodesic-loxodromes interpolation [10], and feature-based interpolation [11]. Since DTI yields both scalar DW gray-level images and tensor fields, it is nature to question whether interpolation should be performed at the level of DW images or at that of tensor fields. Up to now, there is no such comparative study of the different levels of interpolation in DTI.

Our goal is to provide a qualitative and quantitative comparisons of different levels of interpolation on the same datasets and to evaluate their feasibility and performance to improve the spatial resolution of DTI. For the interpolation of scalar DW gray-level images, we focus on the anisotropic interpolation method because it is space-variant and can reduce noise without compromising the sharpness of boundaries. For the interpolation of diffusion tensor fields, we use the Log-Euclidean and quaternion interpolation methods because they preserve the positive-definiteness of the tensors, they do not introduce a swelling effect, and they are computationally efficient.

II. STRATEGIES OF INTERPOLATION FOR DTI DATA

The raw data provided by DTI consists in DW gray-level images from which the tensors are computed. Therefore, a straightforward way of interpolating DTI data would be to interpolate the DW images and then to compute the tensors from the interpolated DW images. Alternatively, we can also directly interpolate the diffusion tensor fields.

A. DW image interpolation

Anisotropic interpolation is implemented with the sigmoid kernel whose sharpness is adapted to the local image intensity gradient in each spatial direction using $a_i = a_{\min} + (a_{\max} - a_{\min}) \times \left|\nabla I_i / \nabla I_{\max}\right|$, where $\left|\nabla I_{\max}\right|$ is the maximum intensity gradient of the image, and where $a_{\max}$ and $a_{\min}$ delimit the linear mapping between a and $\left|\nabla I\right|$ so that a is in a proper range. In the present study, η is set to 0.5. The sigmoid kernel function in the 1-D case is defined by

$$h(x) = \begin{cases} \dfrac{1}{1+\exp(a(x-\eta))}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}. \tag{1}$$

B. Diffusion tensor field interpolation

The Log-Euclidean interpolation of diffusion tensors consists of first computing the logarithm of the original tensors, then interpolating the logarithmical matrices, and finally reconstructing the interpolated tensors using matrix exponentiation [9]. The procedure of Log-Euclidean interpolation can be represented by:

$$\begin{cases} \mathbf{L}_0 = \log(\mathbf{D}_0),\ \mathbf{L}_1 = \log(\mathbf{D}_1) \\ \mathbf{L}(t) = (1\text{-}t)\log(\mathbf{L}_0) + t\log(\mathbf{L}_1)\ , \\ \mathbf{D}(t) = \exp(\mathbf{L}(t)) \end{cases} \tag{2}$$

The matrix logarithm of a tensor is calculated as follows:

a) Diagonalize the diffusion tensor $\mathbf{D}$, which provides a rotational matrix $\mathbf{R}$ and a diagonal matrix $\mathbf{\Lambda}$ with the eigenvalues of $\mathbf{D}$ on its diagonal, with the equality $\mathbf{D} = \mathbf{R\Lambda R}^T$;
b) Transform each diagonal element of $\mathbf{\Lambda}$ (which is necessarily positive, since it is an eigenvalue of $\mathbf{D}$) into its natural logarithm in order to obtain a new diagonal matrix $\mathbf{\Lambda'}$;
c) Recompose $\mathbf{\Lambda'}$ and $\mathbf{R}$ to obtain the logarithm of $\mathbf{D}$ with the formula $\mathbf{L} = \log(\mathbf{D}) = \mathbf{R\Lambda' R}^T$. Conversely, the matrix exponential can be obtained by replacing the natural logarithm with the scalar exponential.

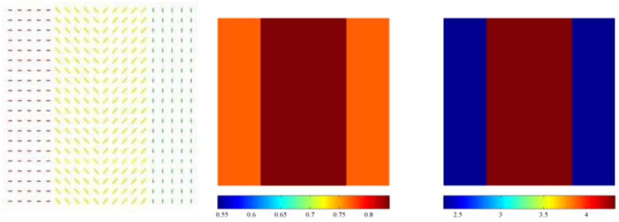
The tensor field interpolation using quaternion was accomplished using the following five steps:

a) Diagonalize the diffusion tensor $\mathbf{D}$, which gives $\mathbf{D} = \mathbf{E\Lambda E}^T$, then arrange the eigenvalues of the diagonal matrix $\mathbf{\Lambda}$ in a decreasing order, and arrange the eigenvector matrix $\mathbf{E}$ accordingly.
b) Compute the unit quaternion $\mathbf{q}$ from the ordered eigenvector matrix to represent the diffusion tensor orientation using Eqs. (19)-(22) in [11].
c) Logarithmically transform the three eigenvalues.
d) Interpolate the eigenvalues in logarithm and the unit quaternion using spherical linear interpolation (SLERP).
e) Construct the interpolated diffusion tensor using the interpolated eigenvalues and quaternions.

III. Experiments and Results

To evaluate the performance of the different interpolation strategies for the improving the spatial resolution of DTI and its impact on the analysis of clinical parameters, both synthetic tensor data and real DTI data were used, and the interpolation results were both qualitatively and quantitatively assessed.

A. *Synthetic data*



(a) (b) (c)

Fig. 1. Synthetic tensor field and the corresponding parameter maps. (a) original tensor field. (b) FA map. (c) MD map.

We generated a 20×20 synthetic tensor field containing 4 different homogeneous regions separated by discontinuities of different shapes or orientations of tensors (Fig. 1a). We used the Stejskal-Tanner diffusion equations to compute the DW images from this synthetic tensor field. The image without diffusion gradient was chosen to be constant and we considered the truncated octahedron encoding scheme (12 directions) to simulate the gradient directions. Fig. 1b and Fig. 1c indicate respectively the FA and MD maps calculated from the original tensor field. It should be stressed that the original parameter maps present three homogeneous regions.

B. *Real DTI data*

The *ex vivo* DTI data sets were acquired on a clinical 1.5 T MR scanner (Magnetom Avanto, Siemens). Cardiac DW images (12 or 30 gradient directions, 4 averages, b=1000) and T2 images were obtained from 16 *ex vivo* human heart. Each excised heart was placed in a plastic container and filled by a non-destructive hydrophilic gel to maintain a diastolic shape.

C. *Performance evaluation*

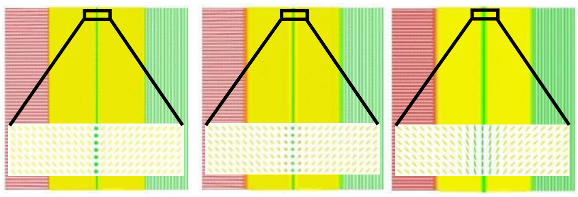
The comparison of the different interpolation methods was performed in a qualitative and a quantitative manner, by means of tensor fields, tensor parameter maps such as FA and MD and fiber tractography.

D. *Results*

1) *Synthetic data*

The anisotropic interpolation was applied to synthetic DW images and Log-Euclidean and quaternion interpolations to the synthetic tensor fields (Fig. 1a). The original size of 20×20 was interpolated to the size of 153×153. To compare the different levels of interpolation, we consider the tensor fields and their associated parameter.

Figure 2 shows the interpolated tensor fields obtained using anisotropic interpolation at the level of DW images and Log-Euclidean and quaternion interpolations at the level of tensor fields. The colors of the ellipsoids are determined by the primary eigenvector directions of the tensors. The tensor fields being too dense, we zoomed the black boxed regions. Compared to the original tensor field, it is observed that tensors with different orientations or shapes (vertical green line and/or its neighbors) have been introduced by both DW interpolation and Log-Euclidean tensor field interpolation although the original tensors all have the same shape, whereas with the Log-Euclidean tensor field interpolation introduces sphere-like tensors not only on the vertical green line but also on both sides of it. Note that on the green vertical line, the volume of tensors produced by anisotropic interpolation at the level of DW images is bigger than that interpolated by Log-Euclidean interpolation at the level of tensor fields. Besides, for the same DW interpolation, the interpolated tensors on and around the vertical green line have bigger volume than the other tensors. Also note that the tensor shapes have been preserved by the quaternion tensor level interpolation and thus no sphere-like tensors were introduced.



(a) (b) (c)

Fig. 2. Interpolated tensor fields obtained after performing interpolation (a) at the level of DW image, (b) at the level of tensor field with Log-Euclidean method, and (c) at the level of tensor field with quaternions. The color codes the direction of the principal eigenvector (red: left/right, blue: anterior/posterior, green: bottom/top).

Figure 3 shows the FA and MD maps obtained using anisotropic interpolations at the level of DW images and Log-Euclidean and quaternion interpolations at the level of tensor fields. As observed, both DW image interpolation and Log-Euclidean tensor field interpolation produced a significant decrease of FA between two original homogeneous regions, as indicated by the vertical lines with sharp different colors. The DW image interpolation produced much sharper edges than the Log-Euclidean tensor field interpolation. By contrast, the quaternion method at the tensor field level preserved the FA values during interpolation and therefore preserved the homogeneous regions in the original FA map. A similar phenomenon is observed in the MD maps and profiles, in which the decrease of MD values with DW image interpolation and Log-Euclidean tensor field interpolation occurred between the two central regions where the tensors have the same shape but different orientations. Note that the decreasing effect introduced by Log-Euclidean interpolation at the level of tensor fields is much greater than that at the level of DW images.

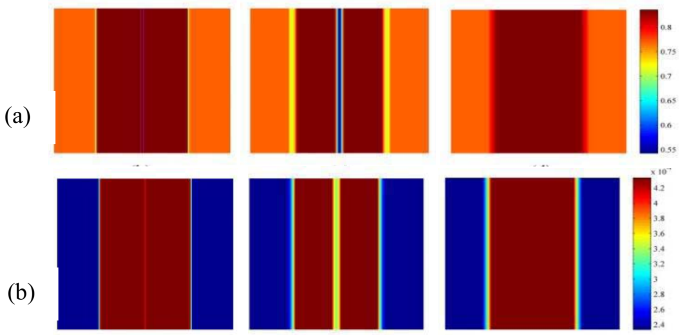


Fig. 3. FA and MD maps from interpolations at the DW image and tensor field levels. (a) FA maps with anisotropic interpolation at the level of DW images (left), Log-Euclidean (middle) and quaternion (right) interpolations at the level of tensor fields. (b) MD map with anisotropic interpolation at the level of DW images (left), Log-Euclidean (middle) and quaternion (right) interpolations at the level of tensor fields.

2) Real data

The interpolation methods at the two levels were applied to the datasets acquired in clinical conditions. We choose to illustrate the 10^{th} slice of a human cardiac DTI volume since it is near the heart base where both right and left ventricles are visible. The initial slices of size 128×128 were interpolated to a size of 512×512.

Figure 4 shows the FA map calculated from the original tensor field of the 10^{th} slice and the FA maps from the interpolated tensor fields obtained using anisotropic interpolation at the level of DW images and Log-Euclidean and quaternion interpolations at the level of tensor fields. The black pixels in the original FA map indicate where the eigenvalues are negative because of noise, and the subsequent FA values calculated are greater than 1. We observe that, with anisotropic interpolation at the level of DW images, noisy areas are enlarged, that is, outlier pixels (FA>1) are interpolated. By contrast, with Log-Euclidean and quaternion interpolations at the level of tensor fields, noisy pixels where FA values are greater than 1 have disappeared. Concerning the MD maps, the three interpolation methods produced similar results, except that DW image interpolation generates a border with higher values and that DW image interpolation and Log-Euclidean tensor interpolation led to a slight decrease in MD values in some voxels.

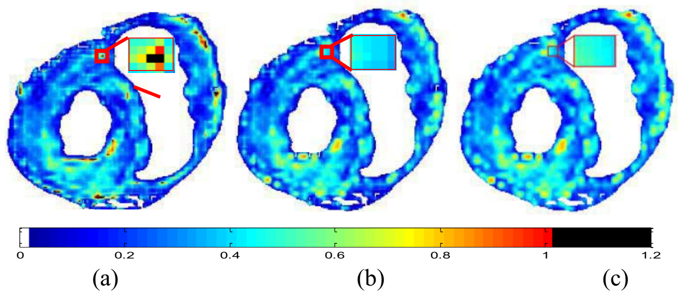


Fig. 4. FA maps calculated from the original tensor fields as well as the interpolated tensor fields using different interpolations, for real human cardiac DTI data. (a) FA map after anisotropic interpolation at the DW image level. (b) FA map from Log-Euclidean interpolation at the level of tensor fields. (c) FA maps from quaternion interpolation at the level of tensor fields.

Table 1. Parameters derived from fiber tractography of the whole human heart.

Method	N_f	Min (mm)	Max (mm)	Mean (mm)	Std
DW image interpolation	83800	20	148	37.64	16.86
Tensor field-Log-Euclidean	86937	20	149	37.97	17.49
Tensor field-Quaternion	126265	20	150	41.44	20.29

As regards fiber tractography, the three interpolation methods led to a similar spiral fiber architecture of the myocardium. In Table 1 gives the quantitative comparison between the two levels of interpolation in terms of the number of fibers, the mean length, and the minimum and maximum fiber lengths. We observe that, on average, Log-Euclidean and quaternion tensor field interpolations produced more and longer fibers than anisotropic DW interpolation. The mean length of fibers obtained with tensor field interpolations is at least 0.33 mm greater than that obtained with DW interpolation.

IV. DISCUSSION

Anisotropic interpolation at the level of DW images introduces less sphere-like tensors than Log-Euclidean interpolation at the level of tensor fields. This is due to the fact that the DW image interpolation technique is anisotropic, which preserves the sharpness of image edges. As a result, anisotropic interpolation produces tensor fields, FA and MD maps with sharp edges, which means that the edges are closer to original ones. However, DW image interpolation may introduce tensors with bigger volumes than the orginals, thus producing a swelling effect. By contrast, the quaternion interpolation at the level of tensor fields preserves tensor shape and consequently preserves the homogeneous regions in the original tensor field and in the FA and MD maps.

In the case of noisy DTI data such as *ex vivo* cardiac DTI data, anisotropic interpolation may amplify the noise and cannot guarantee the positive definiteness of the tensors. This explains the presence of enlarged black areas presenting unreasonable FA values (Fig. 4b). By contrast, Log-Euclidean and quaternion tensor interpolations does not produce tensors with outlier FA values (or unreasonable eigenvalues) since the positive definiteness of the tensors is ensured when taking their logarithms. This is cannot be guaranteed by DW image interpolation because it does not ensure the positivity of the interpolated eigenvalues due to the nonlinear process from the DW images to the tensor field. A similar observation can be made for FA and MD: the nonlinear transformation to obtain the diffusion tensors from the DW image can produce unrealistic FA and MD values. By contrast, even though Log-Euclidean interpolation at the level of tensor fields also introduces a decrease of the FA and MD values, it avoids the error propagation (not to say amplification) due to the nonlinear transformation. Based on these observations, it is possible to propose new methods to avoid the drawbacks of tensor field interpolation. For example, quaternion interpolation at the tensor field level allows to avoid the decreasing effect of FA and MD values.

The results of fiber tractography at the level of DW images and tensor fields are globally similar, but with some small differences. The two tensor field interpolation methods generate more and longer fibers than DW image interpolation does (the latter amplifies the noise and produces more unreasonable FA values). The difference in mean fiber length between tensor field and DW image interpolation is above 0.33 mm. This is perhaps due to the fact that the number of data points with outlier FA values is not large enough to influence the fiber tractography significantly. Indeed, in view of the fact that the myocytes in the myocardium are 80 to 100 μm in length [12], the difference between the two different levels of interpolation represents a difference of about three-four myocytes in length.

V. CONCLUSION

We have shown that interpolation is a feasible and effective way to improve the spatial resolution of DTI data. However, different levels of interpolation can produce rather different results. The interpolation at the level of DW gray-level images produces a swelling effect which can be avoided by the interpolating at the level of tensor fields. Moreover, the interpolation of tensor fields generates more and longer fibers than the interpolation of DW gray-level images. On the other hand, both the anisotropic DW image interpolation and Log-Euclidean tensor field interpolation introduce FA and MD decreases but to a different degree (FA and MD are better preserved by DW interpolation than by Log-Euclidean interpolation), whereas quaternion tensor field interpolation interpolates FA and MD monotonically. Although all the tensor field level interpolation techniques are not better than DW interpolation, they permit to predict attributes such as FA and MD directly from the interpolated tensors and thus to preserve these attributes, which is difficult with DW image interpolation because of the nonlinear transformation between DW image and diffusion tensors. Therefore, tensor field level interpolation would be more convenient than scalar image level interpolation for improving DTI. However, since there exist various possible interpolation methods and even many possible configurations for the same interpolation method (as for the quaternion method), a more systematic and exhaustive comparison based on the present study would be interesting. Finally, although we focused on *ex vivo* cardiac images, it would be interesting to extend this comparative study to brain DTI data.

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