CS 161: HW 6

1. a) $P(A, B, B)$, $P(A, B, z)$: $\{x/A, y/B, z/B\}$

b) $Q(y, G(A, B))$, $Q(G(x, x), y)$: $\{y/G(x, x)\}$,
$Q(G(x, x), G(A, B))$, $Q(G(x, x), G(x, x))$: $\{y/G(x, x)\}$
$Q(G(x, x), G(A, B))$, $Q(G(x, x), G(x, x))$: $\{y/G(x, x), x/A\}$ $Q(G(A, A), G(A, B))$,
$Q(G(A, A), G(A, A))$: $\{y/G(x, x), x/A\}$
A cannot be unified with B, MGU does not exist.

c) Older(Father(x), x), Older(Father(x), John): {x/John, y/John}

d) Knows(Father(y), y), Knows(x, x): {x/Father(y)}
Knows(Father(y), y), Knows(Father(y), Father(y)): {x/Father(y)}
Father(y) cannot be unified with y, MGU does not exist.

2. a) First-order logic:

i. $\forall x$ (Food(x) $\rightarrow$ Likes(John, x))
ii. Food(Apples)
iii. Food(Chicken)
iv. $\forall x \forall y$ (Eats(x, y) $\wedge$ ~MadeSick(x, y) $\rightarrow$ Food(y))
v. $\forall x \forall y$(MadeSick(x, y) $\rightarrow$ ~Well(x))
vi. Eats(Bill, Peanuts) $\wedge$Well(Bill)
vii. $\forall x$ (Eats(Bill, x) $\rightarrow$ Eats(Sue, x))

First-order logic converted to CNF:

I. ~Food(x) $\vee$ Likes(John, x)
II. Food(Apples)
III. Food(Chicken)
IV. ~Eats(x, y) $\vee$ MadeSick(x, y) $\vee$ Food(y)
V. ~MadeSick(x, y) $\vee$ ~Well(x)
VI. {a. (Well(Bill) b. (Eats(Bill, Peanuts))}
VII. ~Eats(Bill, x) $\vee$ Eats(Sue, x)

b. Resolution to show that John likes Apples and Chicken :

~Likes(John, Apples)	Contradiction to hypothesis
~Likes(John, Chicken)	Contradiction to hypothesis
~Food(Apples)	Resolve with I, {x/Apples}
N/A	Resolve with II
~Food(Chicken)	Resolve with I, {x/Chicken}
N/A	Resolve with III

Since both John does not like Apples is false and John does not like Chicken is false, John likes Apples and Chicken must be true.

c. Resolution to show that Sue eats Peanuts

~Eats(Sue, Peanuts)	Contradiction to hypothesis
~Eats(Bill, Peanuts)	Resolve with VII, {x/Peanuts}
N/A	Resolve with VI, part b

Since Sue does not eat peanuts is false, Sue eats peanuts must be true.

3. Knowledge base:

Mother(Mary, Tom)
Alive(Mary)
$\forall x \forall y$ Mother(x, y) $\rightarrow$ Parent(x, y)
$\forall x \forall y$(Parent(x, y) $\wedge$ Alive(x)) $\rightarrow$ Older(x, y)

I. Mother(Mary, Tom)
II. Alive(Mary)
III. ~Mother(x, y) $\vee$ Parent(x, y)
IV. (~Parent(x, y) $\vee$ ~Alive(x)) $\vee$ Older(x, y)

Resolution to show that Mary is older than Tom

~Older(Mary, Tom)	Contradiction to hypothesis
~Parent(Mary, Tom) $\vee$ ~Alive(Mary)	Resolve with IV, {x/Mary, y/Tom}
~Parent(Mary, Tom)	Resolve with II
~Mother(Mary, Tom)	Resolve with III
N/A	Resolve with I

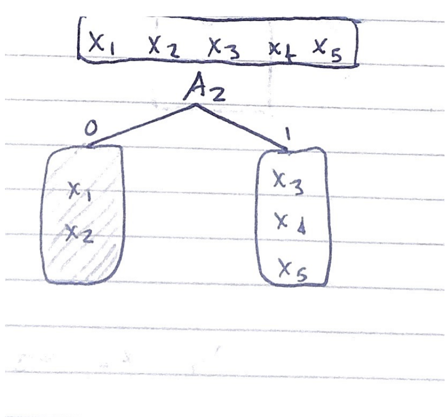
Since Mary is not older than Tom is false, Mary is older than Tom must be true.

4.

Consider the following data set comprised of three binary input attributes (A_1, A_2, and A_3) and one binary output:

Example	A_1	A_2	A_3	Output y
x1	1	0	0	0
x2	1	0	1	0
x3	0	1	0	0
x4	1	1	1	1
x5	1	1	0	1

Construct a decision tree with one split by choosing an attribute to split on that maximizes information gain.



Splitting on Attribute 2 brings us the closest to the desired output y classification. It misclassifies x_3, but this is adequate considering the other choices. Splitting on Attribute 1 gives 4 possible options for the output y, two of which are 1. Splitting on Attribute 3 classifies only 2 of the examples, and only one of them is 1. Therefore, splitting on Attribute 2 is the best option relative to the other attributes.