Stefanie Shidoosh

CS 161: HW #5

1. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither. Justify your answer using truth tables (worlds):

(a) (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

Smoke	Fire	(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
F	F	T
T	F	T
F	T	F
T	T	T

Neither.

(b) (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

Smoke	Fire	Heat	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
F	F	F	T
T	F	F	T
F	F	T	F
T	F	T	T
F	T	F	T
T	T	F	T
F	T	T	T
T	T	T	T

Neither.

(c) ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Smoke	Fire	Heat	((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
F	F	F	T
T	F	F	T
F	F	T	T
T	F	T	T
F	T	F	T
T	T	F	T
F	T	T	T
T	T	T	T

Valid.

Stefanie Shidoosh

2. Consider the following:

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

	Knowledge base
If the unicorn is mythical, then it is immortal	P1: Mythical → ¬Mortal
If it is not mythical, then it is a mortal mammal	P2: ¬Mythical → Mortal ∧ Mammal
If the unicorn is either immortal or a mammal, then it is horned	P3: ¬Mortal ∨ Mammal → Horned
The unicorn is magical if it is horned.	P4: Horned → Magical

(b) Convert the knowledge base into CNF

	CNF
If the unicorn is mythical, then it is immortal	¬Mythical ∨ ¬Mortal
If it is not mythical, then it is a mortal mammal	(Mythical ∨ Mortal) ∧ (Mythical ∨ Mammal)
If the unicorn is either immortal or a mammal, then it is horned	(Mortal ∨ Horned) ∧ (¬Mammal ∨ Horned)
The unicorn is magical if it is horned.	¬Horned ∨ Magical

(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

1. Mortal → ~Mythical	Contrapositive of P1
2. Mortal → ~Immortal /\ Mammal	Hypothetical syllogism w.r.t. #1 and P2
3. ¬Mortal \/ (~Immortal /\ Mammal)	Definition of implication applied to #2
4. (¬Mortal \/ Mortal) /\ (Mortal l \/ Mammal)	#3 converted into CNF
5. ¬Mortal \/ Mammal	Simplify tautologically of #4

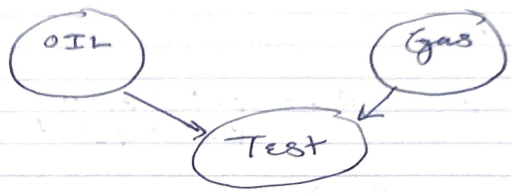
Stefanie Shidoosh

6. Horned	Modus pones w.r.t. #5 and P3
7. Mythical	Modus pones w.r.t. #6 and P4

The knowledge base cannot be used to prove that the unicorn is mythical. It can, however, be used to prove that the unicorn is horned and magical.

3. An oil well may be drilled on Mr. Y's farm in Texas. Based on what has happened to similar farms, we judge the probability of only oil being present to be .5, the probability of only natural gas being present to be .2, and the probability of neither being present to be .3. Oil and gas never occur together. If oil is present, a geological test will give a positive result with probability .9; if natural gas is present, it will give a positive result with probability .3; and if neither are present, the test will be positive with probability .1.

(a) Model this problem as a Bayesian network over three variables: Oil, Gas, and Test.



Oil	P(Oil)
T	0.5
F	0.5

Gas	P(Gas)
T	0.2
F	0.8

Oil	Gas	Test	P(Test | Oil, Gas)
F	F	T	0.1
F	T	T	0.3
T	F	T	0.9
T	T	T	0

(b) Suppose the test comes back positive. What's the probability that oil is present?

$$P(Oil \mid Test) = \frac{P(Test \mid Oil) * P(Oil)}{P(Test)} = \frac{0.9 * 0.5}{P(Test)},$$

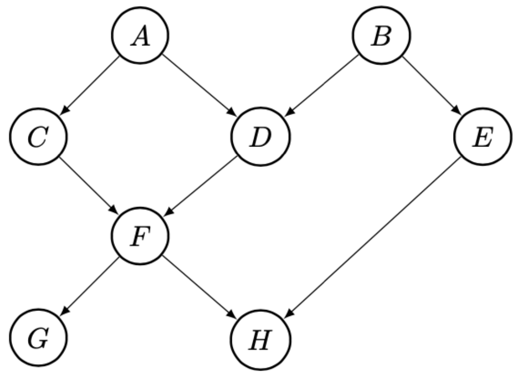
$$P(Test) = \Sigma_i P(Test, \{Oil, \neg Oil\})$$

$$\begin{aligned} P(Test) &= P(Test, Oil) + P(Test, \neg Oil) \\ &= P(Test \mid Oil) * P(Oil) + P(Test \mid \neg Oil) * P(\neg Oil) \\ &= 0.9 \times 0.5 + 0.4 \times 0.5 \\ &= 0.65 \end{aligned}$$

$$P(Oil \mid Test) = \frac{P(Test \mid Oil) \times P(Oil)}{P(Test)} = \frac{0.9 \times 0.5}{0.65} = 0.69$$

Stefanie Shidoosh

4. Consider the Bayesian network in Figure 1:



A	$\Pr(A)$
1	.2
0	.8

B	$\Pr(B)$
1	.7
0	.3

B	E	$\Pr(E\|B)$
1	1	.1
1	0	.9
0	1	.9
0	0	.1

A	B	D	$\Pr(D\|AB)$
1	1	1	.5
1	1	0	.5
1	0	1	.6
1	0	0	.4
0	1	1	.1
0	1	0	.9
0	0	1	.8
0	0	0	.2

Figure 1: A Bayesian network with some of its CPTs.

(a) Express Pr(A,B,C,D,E,F,G,H) as a multiplication of conditional and marginal probabilities, according to the factorization encoded in the network structure:

Pr(A) x Pr(B) x Pr(C | A) x Pr(D | A, B) x Pr(E | B) x Pr(F | C, D) x Pr(G | F) x Pr(H | F, E)

(b) Express Pr(E, F, G, H) in terms of factors instead of (conditional) probabilities.

Pr(E | B) x Pr(F | C, D) x Pr(G, F) x Pr(H | F, E)

(c) Express Pr(a, ¬b, c, d, ¬e, f, ¬g, h) in terms of the parameters in the CPTs (a denotes A = 1 and ¬a denotes A = 0). Use placeholder symbols for the parameters that are not shown in the CPTs.

= 0.2 ∗ 0.3 ∗ Pr(c | a) ∗ 0.6 ∗ 0.1 ∗ Pr(f | c, d) ∗ Pr(¬g | f) ∗ Pr(h | d, ¬e)

= 0.0036 ∗ Pr(c | a) ∗ Pr(f | c, d) ∗ Pr(¬g | f) ∗ Pr(h | f, ¬e)

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(d) Compute Pr(¬a, b) and Pr(¬e | a). Justify your answers. Hint: leaf nodes that are not part of the probability query can be removed from the network without affecting the computed probability.

Because of independence:

Pr(¬a, b) = Pr(a) * Pr(b)

= 0.2 * 0.3

= 0.06

Pr(¬e | a):

$Pr(\neg\varepsilon | a)$

ε is independent of a:

$$Pr(\neg\varepsilon | a) = \frac{Pr(\neg\varepsilon, a)}{Pr(a)} = \frac{Pr(\neg\varepsilon) \times Pr(a)}{Pr(a)} = Pr(\neg\varepsilon)$$

ε is dependent on b:

$$Pr(\varepsilon) = Pr(\neg\varepsilon, b) + Pr(\neg\varepsilon, \neg b)$$

By conditioning:

$$Pr(\neg\varepsilon) = (\neg\varepsilon, b) \times Pr(b) + Pr(\neg b)$$

$$= 0.4 \times 0.7 + 0.1 \times 0.3$$

$$= Pr(\neg\varepsilon | a)$$

$$= 0.66$$

(e) List the Markovian assumptions (also known as topological semantics) encoded in the Bayesian network structure.

I(A, Ø, B, E)

I(B, Ø, AC)

I(C, A, DBE)

I(D, AB, CE)

I(E, B, ACDFG)

I(F, CD, ABE)

I(G, F, ABCDEH)

I(H, EF, ABCDG)

(f) Provide the Markov blanket for variable D.

{A, B, C, F}

Stefanie Shidoosh

(g) Multiply the factors (tables) corresponding to Pr(D|AB) and Pr(E|B).

A	B	D	Pr(D|AB)
F	F	F	0.5
F	F	T	0.5
F	T	F	0.6
F	T	T	0.4
T	F	F	0.1
T	F	T	0.9
T	T	F	0.8
T	T	T	0.2

B	E	Pr(E|B)
F	F	0.1
F	T	0.9
T	F	0.9
T	T	0.1

A	B	D	E	Pr(D|AB) * Pr(E|B)
F	F	F	F	0.05
F	F	F	T	0.45
F	F	T	F	0.05
F	F	T	T	0.45
F	T	F	F	0.54
F	T	F	T	0.06
F	T	T	F	0.36
F	T	T	T	0.04
T	F	F	F	0.01
T	F	F	T	0.09
T	F	T	F	0.09
T	F	T	T	0.81
T	T	F	F	0.72
T	T	F	T	0.08
T	T	T	F	0.18
T	T	T	T	0.02

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(h) Sum out D from the factor (table) computed above.
$x(A, B, D, E) = Pr(D \mid A, B) * Pr(E \mid B)$
$x(A, B, E) = x(A, B, D, E) + x(A, B, \sim D, E)$

A	B	E	x(A, B, E)
T	T	T	0.1
T	T	F	0.9
T	F	T	0.9
T	F	F	0.1
F	T	T	0.1
F	T	F	0.9
F	F	T	0.9
F	F	F	0.1